Comparison between models with and without dynamical charm quarks

Salvatore Calì[1,2], Francesco Knechtli[2], Tomasz Korzec[2]

[1]University of Cyprus, [2]University of Wuppertal

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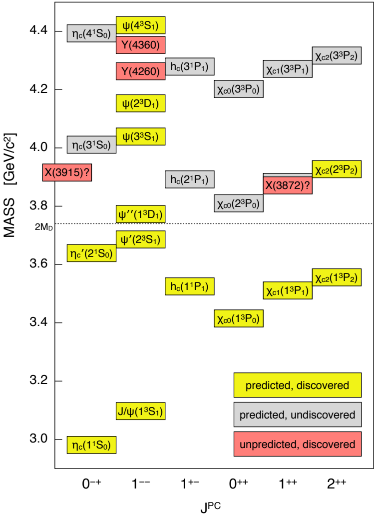
Charmonium

- Charmonium is a bound state $c\bar{c}$
- The high mass of a c quark allows a description of $c\bar{c}$ states in terms of non-relativistic potential models and relativistic corrections (spin-orbit and spin-spin forces)
- $\rightarrow$ Accurate measurements of mass give insight into the confining QCD potential.
- Experiments discovered a large number of unexpected charmonium-like states, many of which are poorly understood. This highlights the need for a more complete theoretical understanding from first principles.



From a presentation by R. Mitchell

Typical Lattice QCD simulations

$N_f = 2 + 1$ QCD (light quarks)

- pros
  - good approximation of QCD at energies much below the charm quark mass, $M_c \approx 1.3$ GeV (decoupling of heavy quarks), good agreement with experiments
  - it can also be used for charm physics, provided that charm loop effects are small (goal of our work)
- cons
  - unknown systematical errors

$N_f = 2 + 1 + 1$ QCD (light quarks + charm quark)

- pros
  - provide a better understanding of charm physics
- cons
  - multi-scale problem ($Lm_\pi \gg 1$ and $am_{J/\psi} \ll 1$), simulation costs
  - charm sea effects require high precision to be resolved (see Refs. [M. Bruno et al.: arXiv:1410.8374] and [F. Knechtli et al.: arXiv:1511.0491])

Goals and strategy

Main goals of this project

1. Evaluate the impact of a dynamical charm quark on various quantities, like the hyperfine splitting, quark masses and meson decay constants, in the continuum limit.
2. Study of lattice artifacts, exploring six lattice spacings in the range 0.02 fm $\lesssim a \lesssim$ 0.10 fm ($\lambda_{m_{J/\psi}} = \frac{1}{m_{J/\psi}} \approx 0.064$ fm)

Simplified setup

- As we aim at a precision that cannot be currently reached in Full QCD, we consider a model
  - $N_f = 2$ QCD (with two degenerate charm quarks)

  and we compare it to $N_f = 0$ QCD (quenched QCD)
- The absence of light quarks allows us to reach extremely fine lattice spacings which are crucial for reliable continuum extrapolations.

Matching

Decoupling

- $N_f = 2$ QCD at $M = M_c$ can be described by an effective Lagrangian for $E \ll M_c$ [Weinberg, Phys. Lett. B91 (1980)]

$$\mathcal{L}_{dec} = \mathcal{L}_{N_f=0} + \frac{1}{M_c^2}\mathcal{L}_6 + \cdots$$

- To match the two theories, we need to specify a value of the coupling at some scale or equivalently the Λ parameter.
- After matching, a low energy hadronic observable m^{had} satisfies

$$m^{had}(M_c)|_{N_f=2} = m^{had}|_{N_f=0} + \mathcal{O}\left(\frac{\Lambda^2}{M_c^2}\right)$$

- we use $m^{had} = 1/\sqrt{t_0}$ [M. Lüscher, 1006.4518] to match the two theories

Fixing the charm quark mass M_c

- to compare $N_f = 0$ and $N_f = 2$ QCD we fix M_c such that

$$\sqrt{t_0}m_{\eta_c}|_{N_f=2} = \sqrt{t_0}m_{\eta_c}|_{N_f=0} = 1.8075, \quad (\approx \text{ physical } m_{\eta_c})$$

Ensembles

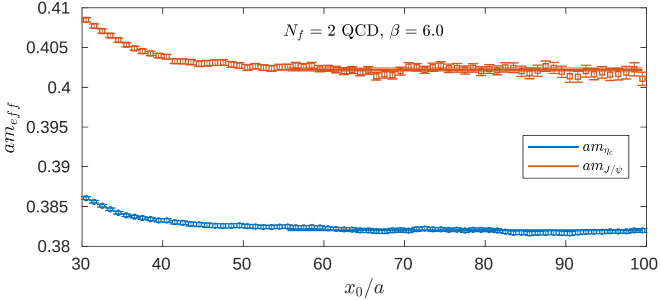
$\frac{T}{a} \times \left(\frac{L}{a}\right)^3$	β	a[fm]	κ	$a\mu$	M/Λ	t_0/a^2
96×24^3	5.300	0.097	0.135943	0.36151	4.87	1.23950(85)
120×32^3	5.500	0.051	0.136638	0.165997	4.87	4.4730(95)
192×48^3	5.600	0.042	0.136710	0.130949	4.87	6.609(15)
120×32^3	5.700	0.036	0.136698	0.113200	4.87	9.104(36)
192×48^3	5.880	0.028	0.136509	0.087626	4.87	15.622(62)
192×48^3	6.000	0.023	0.136335	0.072557	4.87	22.39(12)
120×32^3	6.100	0.052	–	–	∞	4.4329(32)
120×32^3	6.340	0.036	–	–	∞	9.034(29)
192×48^3	6.672	0.023	–	–	∞	21.924(81)
192×64^3	6.900	0.017	–	–	∞	39.41(15)

- S_G: Wilson's plaquette gauge action
- S_F: clover improved doublet of twisted mass fermions at maximal twist
- Open boundaries in time, periodic in space
- For further details, see [F. Knechtli et al., 1706.04982]

Finite volume check

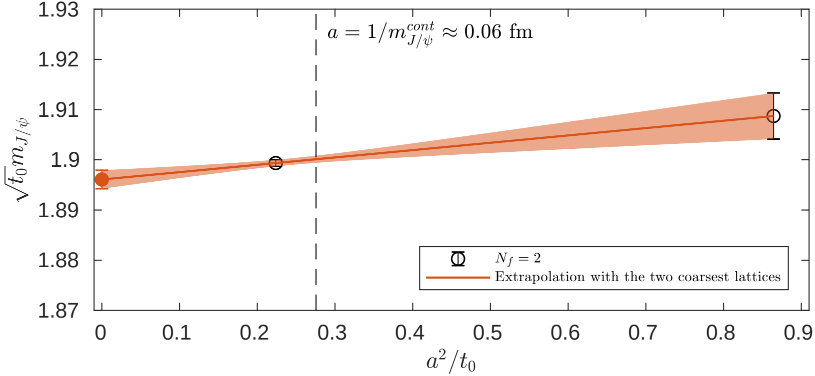
- We use $L/\sqrt{t_0} > 10$ and $Lm_{PS} \gg 4$: negligible finite volume effects.

Effective mass

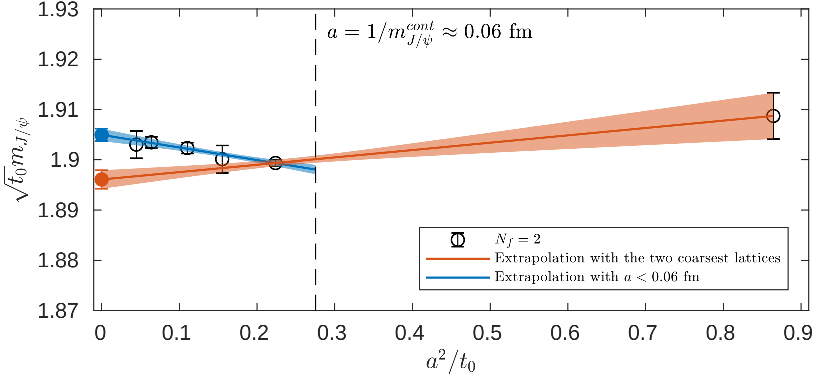


- we focus only on iso-triplet operators (no disconnected diagrams)
- $f(x_0, y_0) = \frac{a^6}{L^3} \sum_{\mathbf{x},\mathbf{y}} \langle O(x_0, \mathbf{x}) O^\dagger(y_0, \mathbf{y}) \rangle$
- $O \in \{\underbrace{\bar{c}'_1 \gamma_5 c'_2}_{m_{\eta_c}}, \underbrace{\bar{c}'_1 \gamma_i c'_2}_{m_{J/\psi}}, \cdots\}$ (physical basis)
- $am^{eff}\left(x_0 + \frac{a}{2}\right) = \log\left(\frac{f(x_0,y_0)}{f(x_0+a,y_0)}\right) \xrightarrow{(x_0-y_0)/a \gg 1} m_{eff}\left(x_0 + \frac{a}{2}\right) \approx m$

Study of lattice artifacts

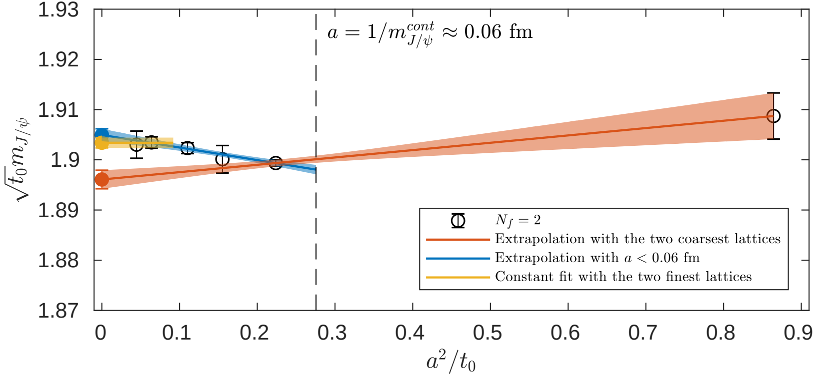


Study of lattice artifacts



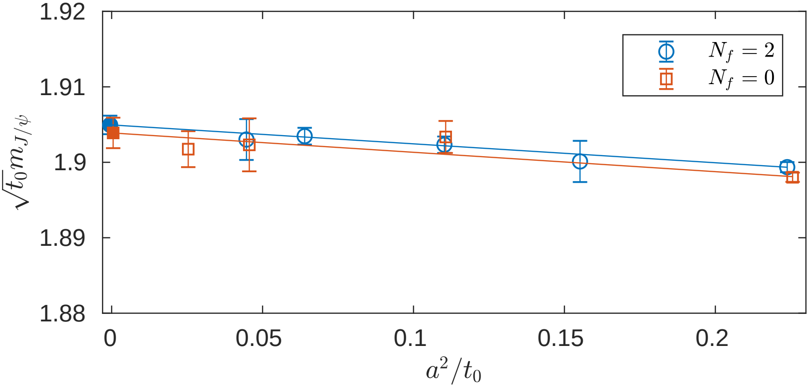
- If 1% precision (or better) is needed, continuum extrapolations linear in a^2 seem unsafe for lattice spacings 0.06 fm $\lesssim a \lesssim$ 0.10 fm.

Study of lattice artifacts



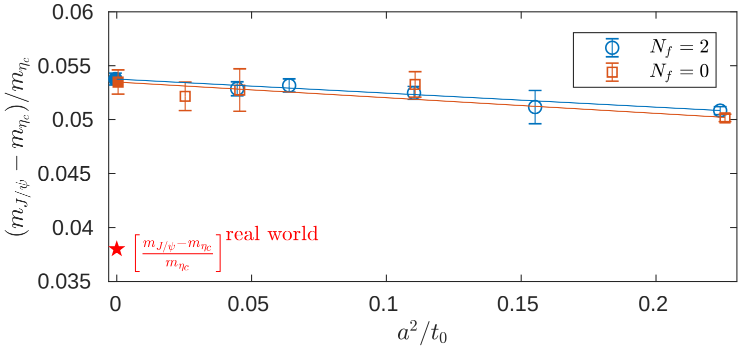
- If 1% precision (or better) is needed, continuum extrapolations linear in a^2 seem unsafe for lattice spacings 0.06 fm $\lesssim a \lesssim$ 0.10 fm.
- Constant fit ($0.02 \lesssim a/fm \lesssim 0.03$) $\approx$ linear fit in a^2 ($a \lesssim 0.05$ fm)

Continuum extrapolation of $m_{J/\psi}$



- Thanks to lattice spacings $a \lesssim 0.05$ fm continuum extrapolations linear in a^2 are under control
- No charm sea effects resolvable at a precision of 0.1%

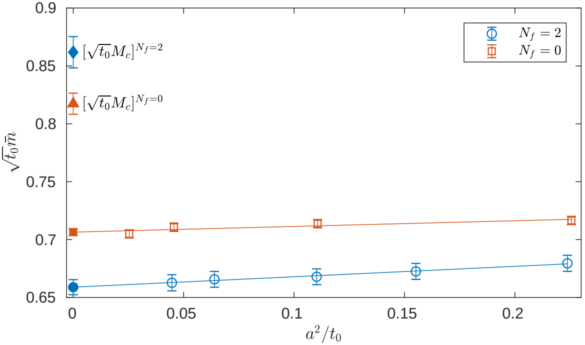
Hyperfine splitting: $(m_{J/\psi} - m_{\eta_c})/m_{\eta_c}$



- Quantity with large cutoff effects, see [Y-G. Cho et al.,1504.01630]
- Light sea quarks, disconnected contributions and electromagnetism are presumably responsible for the deviation to physical number
- No charm sea effects resolvable at a precision of 2%

RGI mass

- $\bar{m} = \frac{1}{Z_P}\sqrt{\mu^2 + Z_A^2 m_{PCAC}^2}$
- $N_f = 2$
  1. $Z_P, M/\bar{m}$
     [P. Fritzsch et al.: arXiv:1205.5380]
  2. Z_A
     [M. Della Morte et al.: arXiv:0505026]
- $N_f = 0$
  1. $Z_P, M/\bar{m}$
     [A. Jüttner: arXiv:0503040]
  2. Z_A
     [M. Lüscher et al.: arXiv:9611015]



- the running masses $\bar{m}$ are not renormalized at the same scale, but from $M/\bar{m}$ we can determine the RGI mass M_c, whose continuum values are comparable
- the relative size of charm sea effects is $\approx 5\%$

Meson decay constants in twisted mass QCD

f_{η_c}

- In twisted mass QCD we can define the pseudo-scalar decay constant f_{η_c} through [K. Jansen et al: arXiv:0312013]
  $f_{\eta_c} m_{\eta_c}^2 \equiv 2\mu \langle 0|\bar{c}_1 \gamma_5 c_2|\eta_c\rangle \equiv 2\mu \langle 0|P|\eta_c\rangle$
- The renormalization factors of the pseudo-scalar density Z_P and Z_μ obey $Z_P Z_\mu = 1 \rightarrow$ we can determine f_{η_c} without the need of any renormalization factor [R. Frezzotti et al: arXiv:0101001], [R. Frezzotti et al: arXiv:0104014]

$f_{J/\psi}$

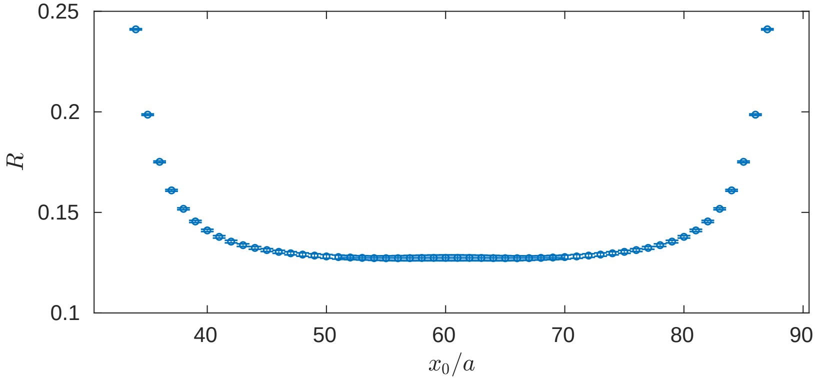
- Continuum definition: $\langle 0|\bar{c}'_1 \gamma_i c'_2|J/\psi\rangle = \langle 0|V'_i|J/\psi\rangle = \epsilon_i f_{J/\psi} m_{J/\psi}$
- $\langle 0|V'_i|J/\psi\rangle_{\text{phys}} = \langle 0|A_i|J/\psi\rangle_{\text{twisted}}$ (on a lattice we need Z_A)

  [K. Jansen et al: arXiv:0906.4720]

Dealing with open boundary conditions

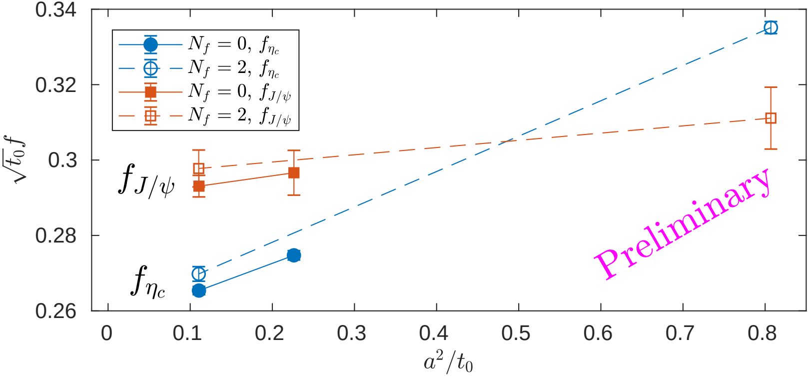
- With OB conditions, we need to take care of the boundary effects. We follow the strategy described in [M. Bruno et al: arXiv:1608.08900]

Computation of f_{η_c}: example for $N_f = 2$ QCD, $\beta = 5.7$



- $R(x_0, y_0) = \sqrt{\frac{|f_{PP}(x_0,y_0) f_{PP}(x_0,T-y_0)|}{f_{PP}(T-y_0,y_0)}}$, with $f_{PP} = \frac{a^6}{L^3} \sum_{\mathbf{x},\mathbf{y}} \langle P(x) P^\dagger(y) \rangle$
- $R(x_0, y_0) \propto f_{\eta_c}$ if $0 \ll x_0 \ll T$.

Preliminary results for f_{η_c} and $f_{J/\psi}$



- cutoff effects look milder in $f_{J/\psi}$
- charm sea effects seem small
- increase statistics and explore other lattice spacings

Summary

Conclusions

- Dynamical charm effects:
  1. not resolvable in charmonium masses ($\lesssim 0.1\%$ in $\sqrt{t_0} m_{J/\psi}$) and in the hyperfine splitting ($\lesssim 2\%$ in $(m_{J/\psi} - m_{\eta_c})/m_{\eta_c}$)
  2. considerable in the RGI mass ($\approx 5\%$)
  3. seem to be small in the mesons decay constants, but further investigations are needed
- Lattice artifacts for charmonium masses:
  1. $\mathcal{O}(a^2)$ below $a = 0.05$ fm
  2. linear extrapolations in a^2 for $0.06 \text{ fm} < a < 0.10$ fm seem unsafe

Future plans

- Increase the statistics and explore more lattice spacings for the decay constants of the mesons η_c and J/ψ $\rightarrow$ more accurate results and study of the lattice artifacts

Thank you for your attention

Dynamical Ensembles

Action

- S_G: Wilson's plaquette gauge action
- S_F: clover improved doublet of twisted mass fermions at maximal twist
- Open boundaries in time, periodic in space

$N_f = 2$

- $\beta \in \{5.3, 5.5, 5.6, 5.7, 5.88, 6.0\}$, 0.02 fm $\lesssim a \lesssim 0.10$ fm
- a is determined through the hadronic scale $L_1 \approx 0.4$ fm, which is defined at $m_{PCAC} = 0 \rightarrow$ Standard fermions and TM fermions are equivalent [M. Blossier et al.: arXiv:1203.6516], [P. Fritzsch et al.: arXiv:1205.5380]
- $a\mu = Z_p \times \frac{M}{\Lambda_2} \times \Lambda_2 L_1 \times \frac{\bar{m}}{M} \times \frac{a}{L_1}$, with $\frac{M}{\Lambda_2} = 4.87$
- κ_c interpolation of [P. Fritzsch et al.: arXiv:1205.5380], [P. Fritzsch et al.: arXiv:1508.0693]
- c_{sw} [K. Jansen and R. Sommer: arXiv:hep-lat/9709022]

Description of the matching procedure

Matching

- We compare the continuum limits of several quantities in $N_f = 0$ and in $N_f = 2$ QCD.
- The comparison is done at the mass point where $\sqrt{t_0} m_{\eta_c} = 1.8075$, which corresponds to the value obtained in $N_f = 2$ QCD with our finest lattice ($\beta = 6.0$).

$N_f = 0$ QCD

- 3 values of $\mu \rightarrow$ we determine $\mu^\star$ corresponding to the matching point through interpolation.

$N_f = 2$ QCD

- μ of the simulation $\rightarrow$ quantity R and its error δR
- Compute $dR/da\mu$
- Determine $\mu^\star$ such that: $\sqrt{t_0} m_{\eta_c} + (a\mu^\star - a\mu) \frac{d\sqrt{t_0} m_{\eta_c}}{da\mu} \equiv 1.8075$
- Find $R^\star$ at the tuning point using: $R^\star \equiv R + (a\mu^\star - a\mu) \frac{dR}{da\mu}$

Meson correlation functions and twisted mass derivatives

- We focus on
  1. masses and hyperfine splitting: $R = \sqrt{t_0} m_{\eta_c}, \sqrt{t_0} m_{J/\psi}, \frac{m_{J/\psi} - m_{\eta_c}}{m_{\eta_c}}$
  2. quark masses: $R = \sqrt{t_0}\bar{m}, \sqrt{t_0} M_c$
  3. decay constants: $R = \sqrt{t_0} f_{\eta_c}, \sqrt{t_0} f_{J/\psi}$
- R is extracted from the zero-momentum correlation function

$$f(x_0, y_0) = \frac{a^6}{L^3} \sum_{\mathbf{x},\mathbf{y}} \langle J(x_0, \mathbf{x}) J^\dagger(y_0, \mathbf{y}) \rangle, \quad J \in \overbrace{\{ \underbrace{\bar{c}'_1 \gamma_5 c'_2}_{m_{\eta_c}}, \underbrace{\bar{c}'_1 \gamma_i c'_2}_{m_{J/\psi}}, \cdots \}}^{\text{physical basis}}$$

- In $N_f = 2$ QCD, to find $R^\star$ at the tuning point $\mu^\star$ we need to compute $\frac{df(x_0,y_0)}{d\mu}$ and $\frac{dS}{d\mu}$
- We determine $f(x_0, y_0)$, $df(x_0, y_0)/d\mu$ and $dS/d\mu$ using stochastic sources
  $\langle \eta_{\alpha a}(u) \rangle^{\text{noise}} = 0, \quad \langle \eta^\star_{\alpha a}(u) \eta_{\beta b}(v) \rangle^{\text{noise}} = \delta_{u_0 x_0} \delta_{v_0 x_0} \delta_{\mathbf{uv}} \delta_{\alpha\beta} \delta_{ab}$

Dealing with open boundary conditions

- With OB, the two-point functions $f_{PP} = \frac{a^6}{L^3} \sum_{\mathbf{x},\mathbf{y}} \langle P(x) P^\dagger(y) \rangle$ and $f_{A_i A_i} = \frac{a^6}{L^3} \sum_{\mathbf{x},\mathbf{y}} \langle A_i(x) A_i^\dagger(y) \rangle$ have the following asymptotic behavior
  1. $f_{PP} = k_1(y_0) \langle 0 | P | \eta_c \rangle e^{-m_{\eta_c}(x_0 - y_0)}$,
  2. $f_{A_i A_i} = k_2(y_0) \langle 0 | A_i | J/\psi \rangle e^{-m_{J/\psi}(x_0 - y_0)}$,

  where $k_1(y_0)$ and $k_2(y_0)$ are two amplitudes that depend on the distance from the boundary.

- To extract the needed matrix elements and remove $k_1(y_0)$ and $k_1(y_0)$, we compute the ratios [M. Bruno et al: arXiv:1608.08900]
  1. $R_{\eta_c} = \sqrt{\frac{|f_{PP}(x_0, y_0) f_{PP}(x_0, T - y_0)|}{f_{PP}(T - y_0, y_0)}} = \frac{\langle 0 | P | \eta_c \rangle}{\sqrt{2 m_{\eta_c}}}$
  2. $R_{J/\psi} = \sqrt{\frac{|f_{A_i A_i}(x_0, y_0) f_{A_i A_i}(x_0, T - y_0)|}{f_{A_i A_i}(T - y_0, y_0)}} = \frac{\langle 0 | A_i | J/\psi \rangle}{\sqrt{2 m_{J/\psi}}}$