Nucleon electromagnetic form factors at high-Q^2 from Wilson-clover fermions

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Outline

- Introduction - Motivation
- Lattice setup and methodology
- Form factor results
- Summary and outlook

Motivation

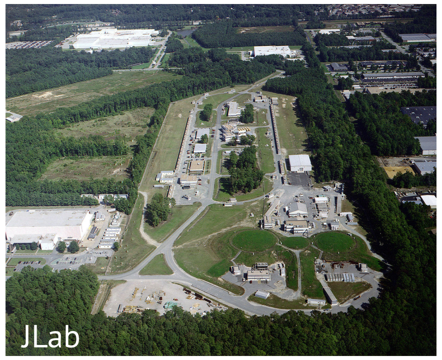
Nucleon electric and magnetic form factors are important probes of its internal structure

High-momentum transfer calculation from first principles:

- test validity of pQCD predictions, quark models and phenomenology
- required for DVCS measurements (EIC@BNL), probing GPDs
- nucleon FFs: good framework to test high-momentum region on the lattice

Rich experimental activity

- Super-BigBite Spectrometer at JLab Hall A S.B.S Program, updated 12-GeV CEBAF accelerator @ JLab
  - elastic *ep* scattering experiments up to $Q^2 \sim 18\ \mathrm{GeV}^2$
    - G_E/G_M dependence
    - scaling of F_1/F_2 at $Q^2 \to \infty$
    - individual contributions from up- and down-quarks
  - finalized/published results in ~ 5yr



Simulation details

- two Nf=2+1 Wilson-clover ensembles, produced by JLab lattice group
- different lattice volumes, similar lattice spacing

D5-ensemble: $\beta = 6.3$, $a = 0.094$ fm, $a^{-1} = 2.10$ GeV		
$32^3 \times 64$, $L = 3.01$ fm	$a\mu_l$	-0.2390
	$a\mu_s$	-0.2050
	κ	0.132943
	C_{sw}	1.205366
	m_π (MeV)	280
	$m_\pi L$	4.26
	Statistics	86144
D6-ensemble: $\beta = 6.3$, $a = 0.091$ fm, $a^{-1} = 2.17$ GeV		
$48^3 \times 96$, $L = 4.37$ fm	$a\mu_l$	-0.2416
	$a\mu_s$	-0.2050
	κ	0.133035
	C_{sw}	1.205366
	m_π (MeV)	170
	$m_\pi L$	3.76
	Statistics	50176

- Computational resources: BNL Institutional Cluster, USQCD 2017 allocation
- Calculation: Qlua interface: QUDA-MG for propagators, contractions on GPU

A.V. Pochinksy S. Syritsyn, C.K.

Form factor decomposition

Matrix element of the vector current: $\mathcal{V}_\mu(x) = \bar{\psi}(x)\gamma_\mu\psi(x)$

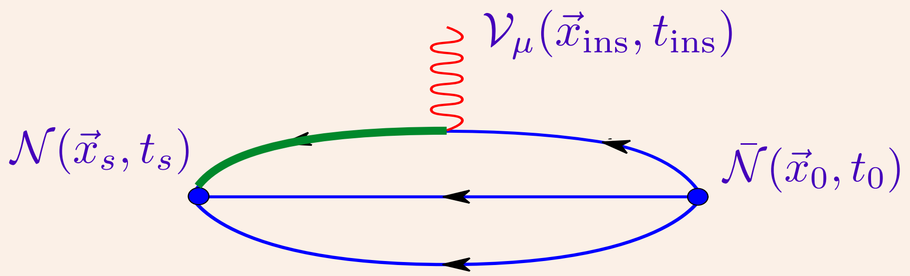
$$\langle N(p',s)|\mathcal{V}_\mu|N(p,s)\rangle = \sqrt{\frac{m_N^2}{E_N(\vec{p}')E_N(\vec{p})}}\,\bar{u}_N(p',s)\left[\gamma_\mu \underbrace{F_1(q^2)}_{\text{Dirac form factor}} + \frac{i\sigma_{\mu\nu}q^\nu}{2m_N}\underbrace{F_2(q^2)}_{\text{Pauli form factor}}\right]u_N(p,s)$$

Sachs Electric and Magnetic form factors:

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{(2m_N)^2}F_2(Q^2) \qquad G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

On the lattice:

Three-point correlation function

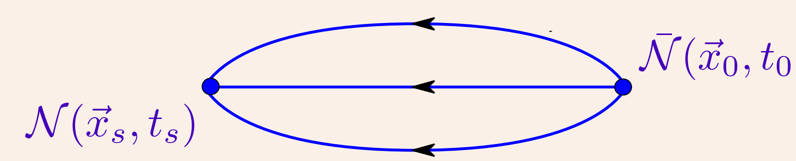


- seq. propagators: inversion through sink
- *(t_s-t_0) ~ 0.55* fm - *0.95* fm
- consider only connected contributions

$$G_\mu(\Gamma, \vec{p}', \vec{q}, t_s, t_{\rm ins}) = \sum_{\vec{x}_s, \vec{x}_{\rm ins}} e^{-i\vec{p}'\cdot(\vec{x}_s - \vec{x}_0)} e^{i\vec{q}\cdot(\vec{x}_{\rm ins} - \vec{x}_0)} \Gamma_{\beta\alpha}\langle \mathcal{N}_\alpha(\vec{x}_s, t_s)\mathcal{V}_\mu(\vec{x}_{\rm ins}, t_{\rm ins})\bar{\mathcal{N}}_\beta(\vec{x}_0, t_0)\rangle$$

Two-point correlation function

$$C(\vec{p}', t_s) = \sum_{\vec{x}_s} e^{-i\vec{p}'\cdot(\vec{x}_s - \vec{x}_0)}(\Gamma_4)_{\beta\alpha}\langle \mathcal{N}_\alpha(\vec{x}_s, t_s)\bar{\mathcal{N}}_\beta(\vec{x}_0, t_0)\rangle$$



Form factor decomposition

Ratio of 2pt and 3pt functions

$$R^{\mu}(\Gamma,\vec{q},\vec{p}\,';t_s,t_{\rm ins}) = \frac{G_\mu(\Gamma,\vec{p}\,',\vec{q},t_s,t_{\rm ins})}{C(\vec{p}\,',t_s-t_0)} \times \sqrt{\frac{C(\vec{p},t_s-t_{\rm ins})C(\vec{p}\,',t_{\rm ins}-t_0)C(\vec{p}\,',t_s-t_0)}{C(\vec{p}\,',t_s-t_{\rm ins})C(\vec{p},t_{\rm ins}-t_0)C(\vec{p},t_s-t_0)}}$$

1. Plateau method: $R^{\mu} \xrightarrow[t_s-t_{\rm ins}\gg 1]{t_{\rm ins}-t_0\gg 1} \Pi^{\mu}(\Gamma,\vec{q})$

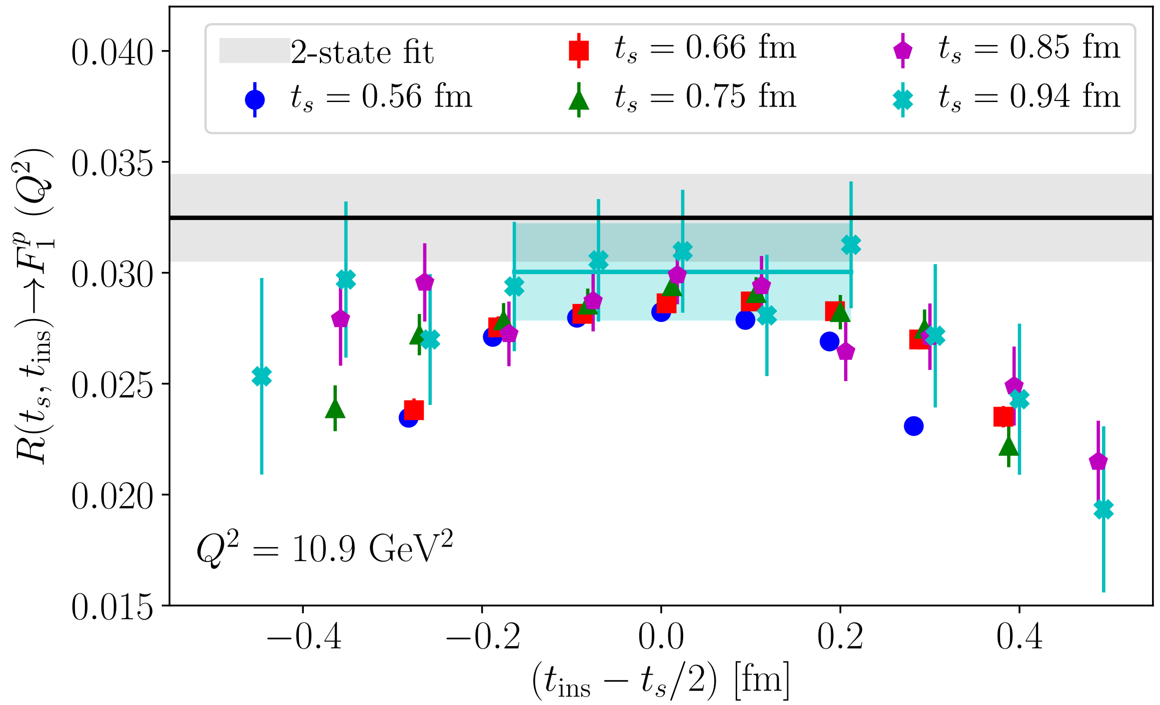
2. Two-state fit method:

$$C(\vec{p}\,',t_s) \simeq e^{-E(\vec{p}\,')t_s}\left[c_0(\vec{p}\,') + c_1(\vec{p}\,')e^{-\Delta E_1(\vec{p}\,')t_s}\right]$$

$$\begin{aligned} G_\mu(\Gamma,\vec{p}\,',\vec{p},t_s,t_{\rm ins}) &\simeq e^{-E_0(\vec{p}\,')(t_s-t_{\rm ins})}e^{-E_0(\vec{p})(t_{\rm ins}-t_0)} \times \\ \times\left[A_{00}(\vec{p},\vec{p}\,')\right. &+ A_{01}(\vec{p},\vec{p}\,')e^{-\Delta E_1(\vec{p})(t_{\rm ins}-t_0)} + \\ &+ A_{10}(\vec{p},\vec{p}\,')e^{-\Delta E_1(\vec{p}\,')(t_s-t_{\rm ins})} + \\ &+ \left. A_{11}(\vec{p},\vec{p}\,')e^{-\Delta E_1(\vec{p}\,')(t_s-t_{\rm ins})}e^{-\Delta E_1(\vec{p})(t_{\rm ins}-t_0)}\right] \end{aligned}$$

$$c_n(\vec{p}\,') = |\langle\mathcal{N}|n,\vec{p}\,'\rangle|^2/2E_n(\vec{p}\,')$$

$$A_{nm}(\vec{p},\vec{p}\,') = \langle\mathcal{N}|n,\vec{p}\,'\rangle\langle m,\vec{p}|\mathcal{N}\rangle\langle n,\vec{p}\,'|\mathcal{V}_\mu|m,\vec{p}\rangle/[2\sqrt{E_n(\vec{p})E_n(\vec{p}\,')}]$$



$$\langle 0,\vec{p}\,'|\mathcal{V}_\mu|0,\vec{p}\rangle = \frac{A_{00}(\vec{p},\vec{p}\,')}{\sqrt{c_0(\vec{p})c_0(\vec{p}\,')}}$$

$$\Pi^0(\Gamma_4,\vec{q}) = C\frac{E_N+m_N}{2m_N}G_E(Q^2) \qquad \Pi^i(\Gamma_4,\vec{q}) = C\frac{q_i}{2m_N}G_E(Q^2)$$

$$\Pi^i(\Gamma_k,\vec{q}) = C\frac{\epsilon_{ijk}q_j}{2m_N}G_M(Q^2)$$

$$\mathcal{S} = \sum_n^N \frac{\left(\sum_{m=E,M} A_{nm}G_m - \Pi^n\right)^2}{\sigma_n^2}$$

$$C = \sqrt{\frac{2m_N^2}{E_N(E_N+m_N)}}$$

$$Q^2 \equiv -q^2$$

Projectors:

unpolarized $\Gamma_4 = \frac{1+\gamma_4}{4}$

polarized $\Gamma_k = i\gamma_5\gamma_k\Gamma_4$

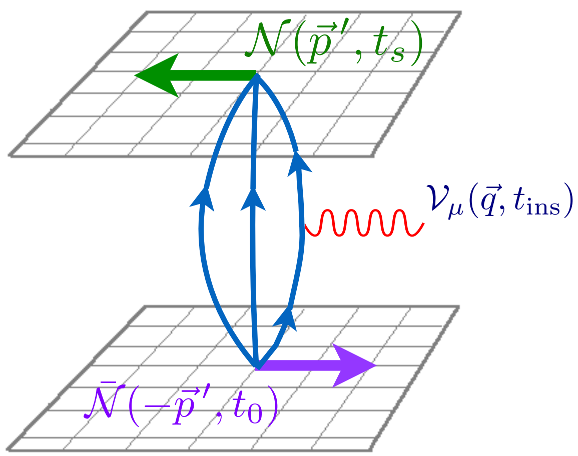
we incorporate **boosted** nucleon states
for increased signal in the high-Q^2 region

$$\mathcal{N}_\alpha(\vec{p}\,',t) = \sum_{\vec{x}} \epsilon^{abc}\left[u^a_\mu(x)(C\gamma_5)_{\mu\nu}d^b_\nu(x)\right]u^c_\alpha(x)e^{-i\vec{p}\,'\cdot\vec{x}}$$

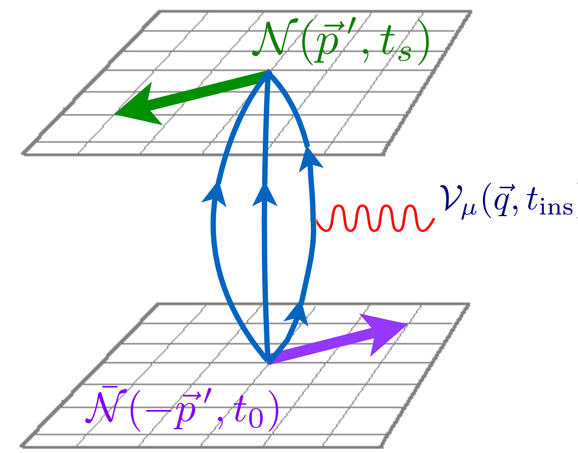
$$Q^2 = (\vec{p}-\vec{p}\,')^2 - (E-E')^2$$

Breit frame: $\vec{p} = -\vec{p}\,'\ ,\ E = E' \longrightarrow Q^2 = 4\vec{p}^{\,2}$

- boosting in single direction



- diagonal boosting in x-y plane



D5 $\longrightarrow$ $\vec{P}\,' = (-4,0,0) \to Q^2 \sim 10.9\ \mathrm{GeV}^2$

D6 $\longrightarrow$ $\vec{P}\,' = (-5,0,0) \to Q^2 \sim 8.1\ \mathrm{GeV}^2$

D5 $\longrightarrow$ $\vec{P}\,' = (-3,0,0) \to Q^2 \sim 6.1\ \mathrm{GeV}^2$

Still to be analyzed!

D5 $\longrightarrow$ $\vec{P}\,' = (-3,-3,0) \to Q^2 \sim 12.2\ \mathrm{GeV}^2$

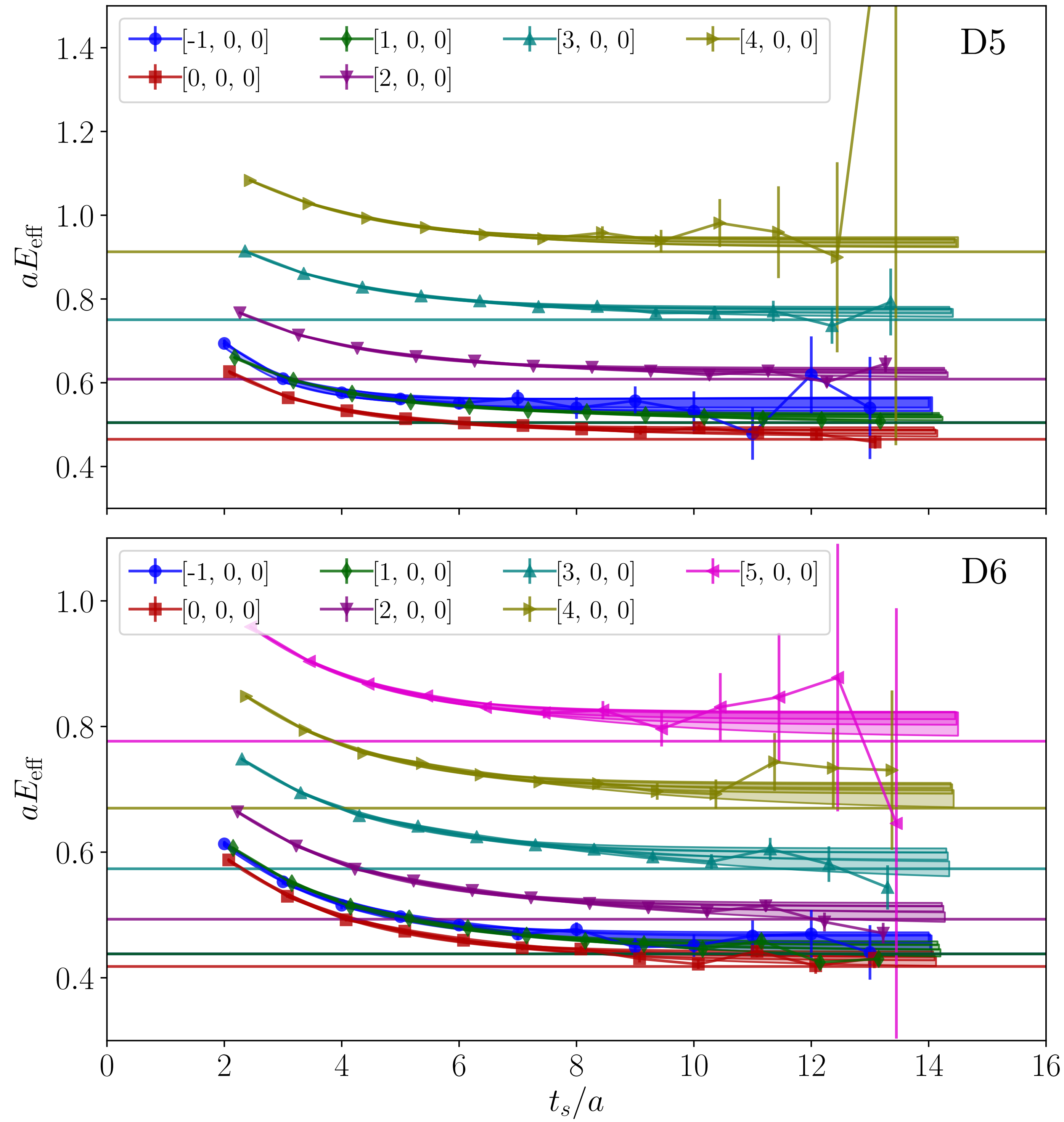
Gaussian "momentum" smearing:

$$\mathcal{S}_{\vec{k}_b}\psi(x) \equiv \frac{1}{1+6\alpha}\left[\psi(x) + \alpha\sum_{\mu=\pm1\ldots}^{3} U_\mu(x)e^{i\vec{k}_b\cdot\hat{\mu}}\psi(x+\hat{\mu})\right]$$

$\boxed{\vec{k}_b = 0.5\,\vec{p}\,'}$

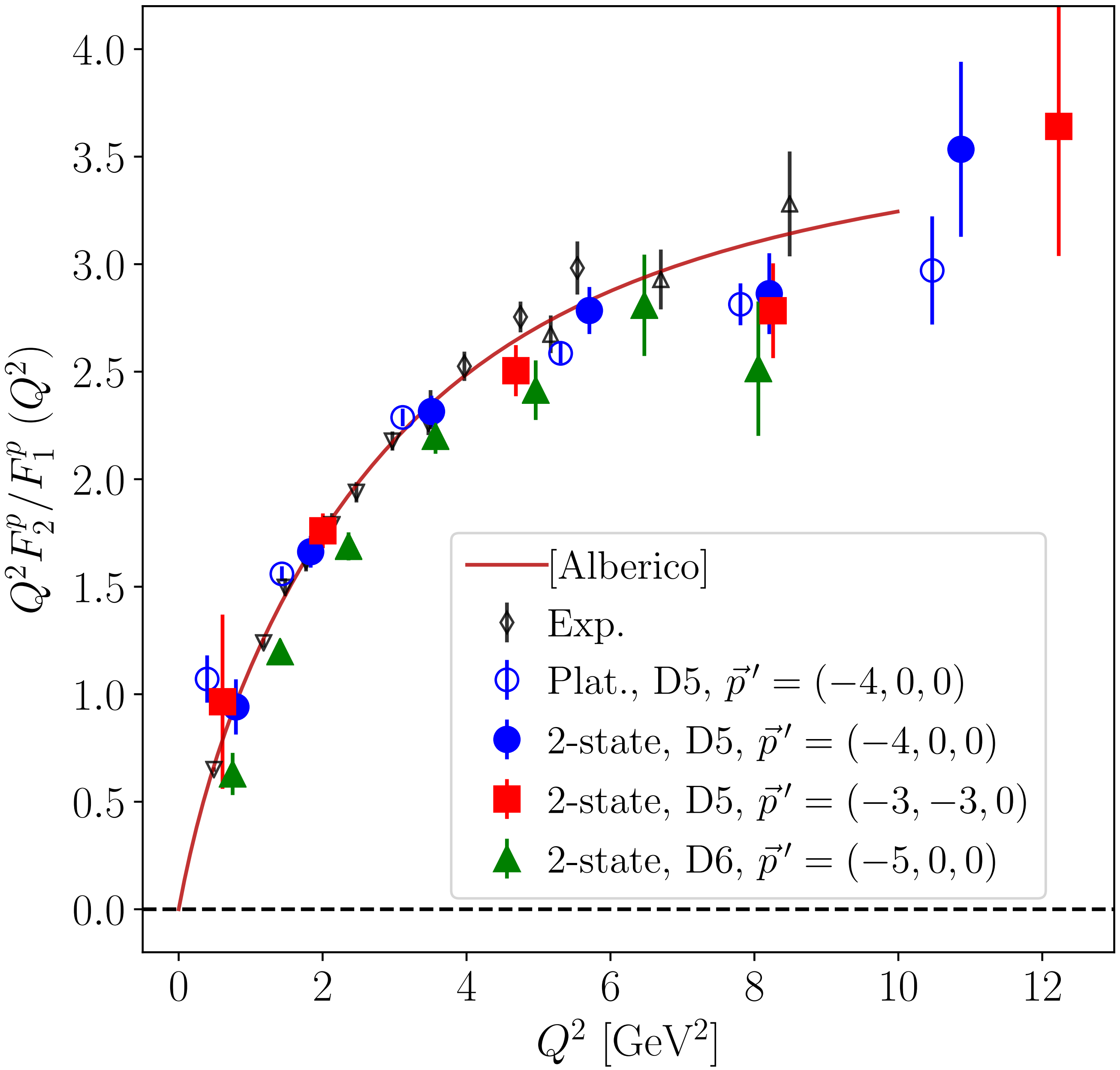
G. Bali et al. [arXiv: 1602.05525]

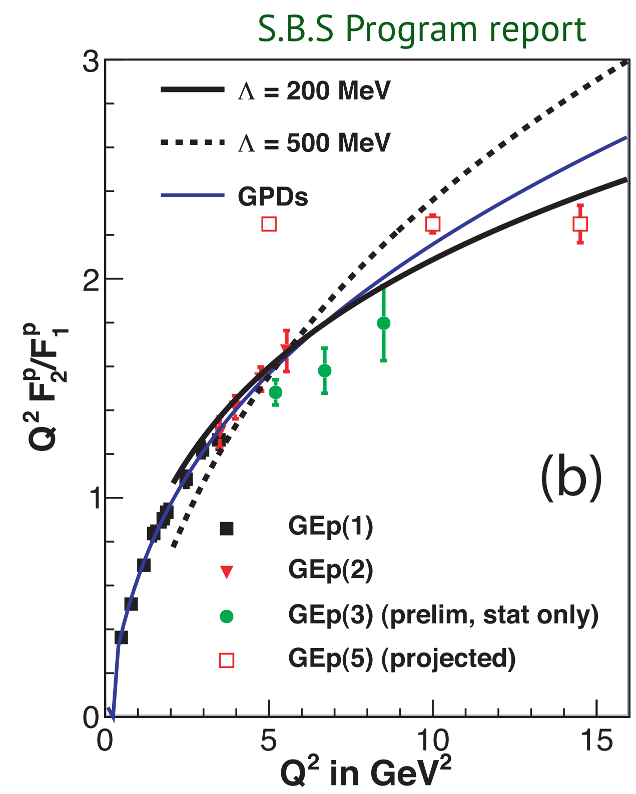
Effective Energy



- two-state fits to our lattice data are of good quality
- horizontal line: continuum dispersion relation using lattice value of m_N
- ground state energy slightly overestimates cont. dispersion relation
- excited states faint after $\sim t_s/a = 9$

Form Factor Results I: F_2/F_1 Ratio

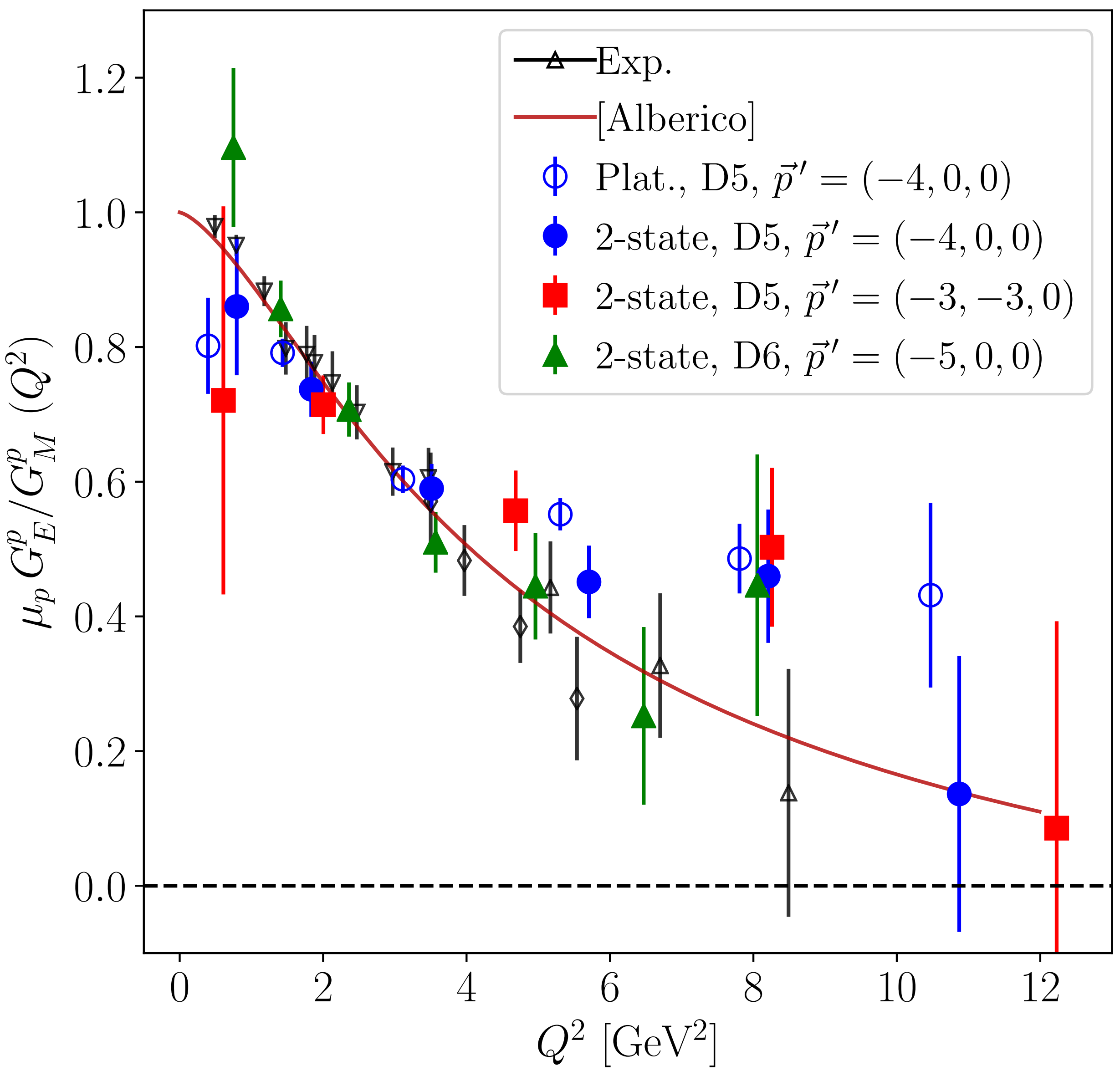


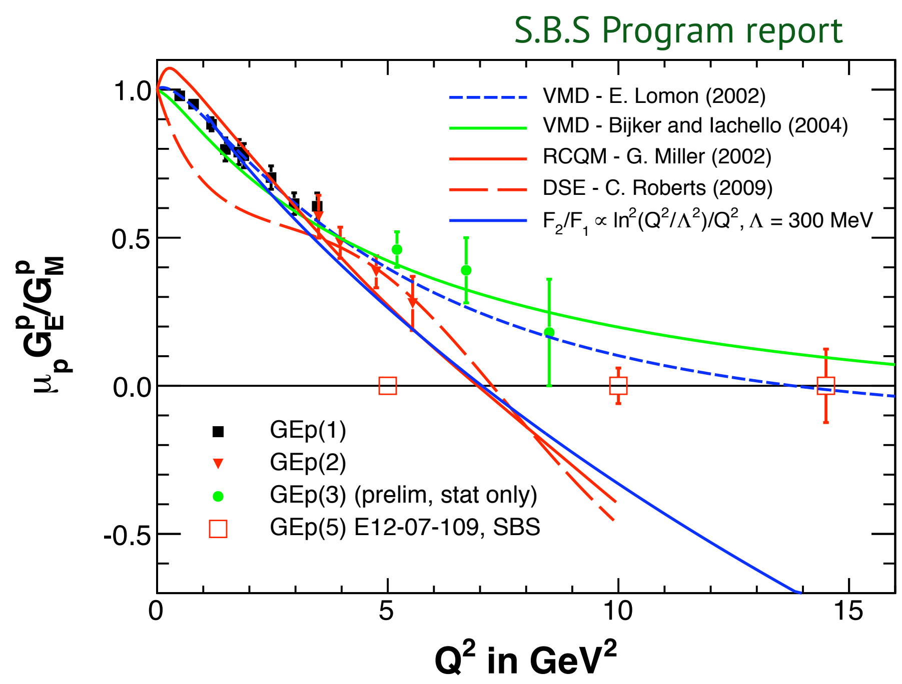


W. M. Alberico et al. [arXiv: 0812.3539]

- Q^2 - dependence compares well with exp. data and phenom. parametrization
- $Q^2 F_2^p / F_1^p (Q^2) \sim \log[Q^2/\Lambda]$ scaling reproduced A.V. Belitsky et al. [arXiv: hep-ph/0212351]
- consistency between on-axis / x-y diagonal boost momentum for D5

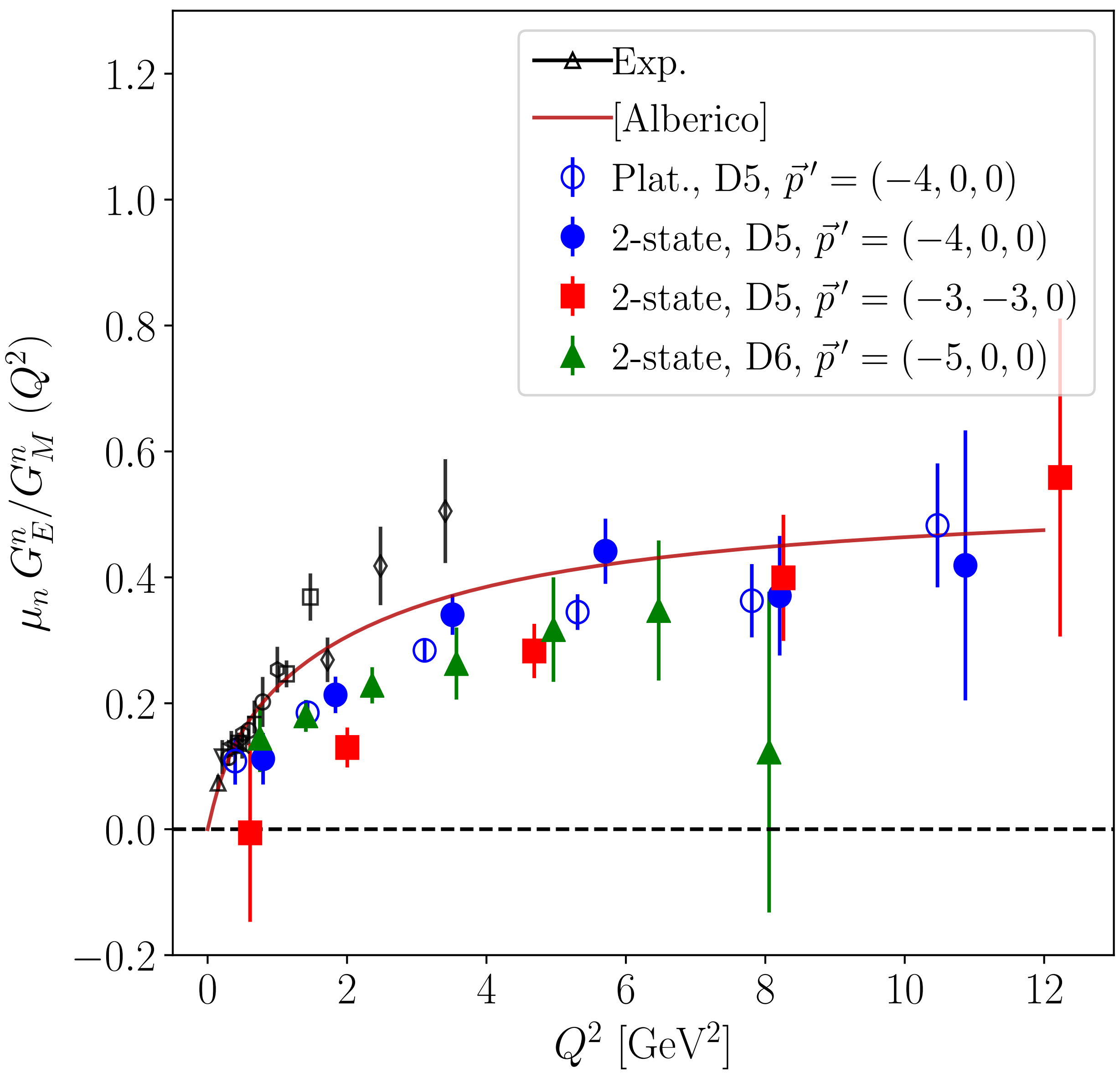
Form Factor Results II: G_E/G_M Ratio

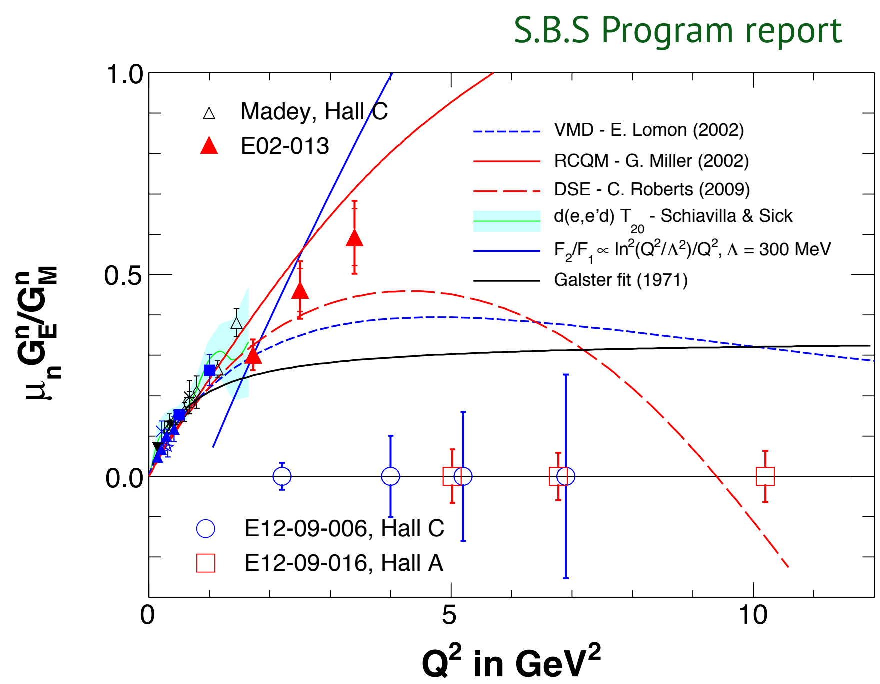




- consistency between our lattice data
- good agreement with experiment / phenomenology for proton up to $Q^2 \sim 6$ GeV^2
- lattice data support smoother approach towards zero

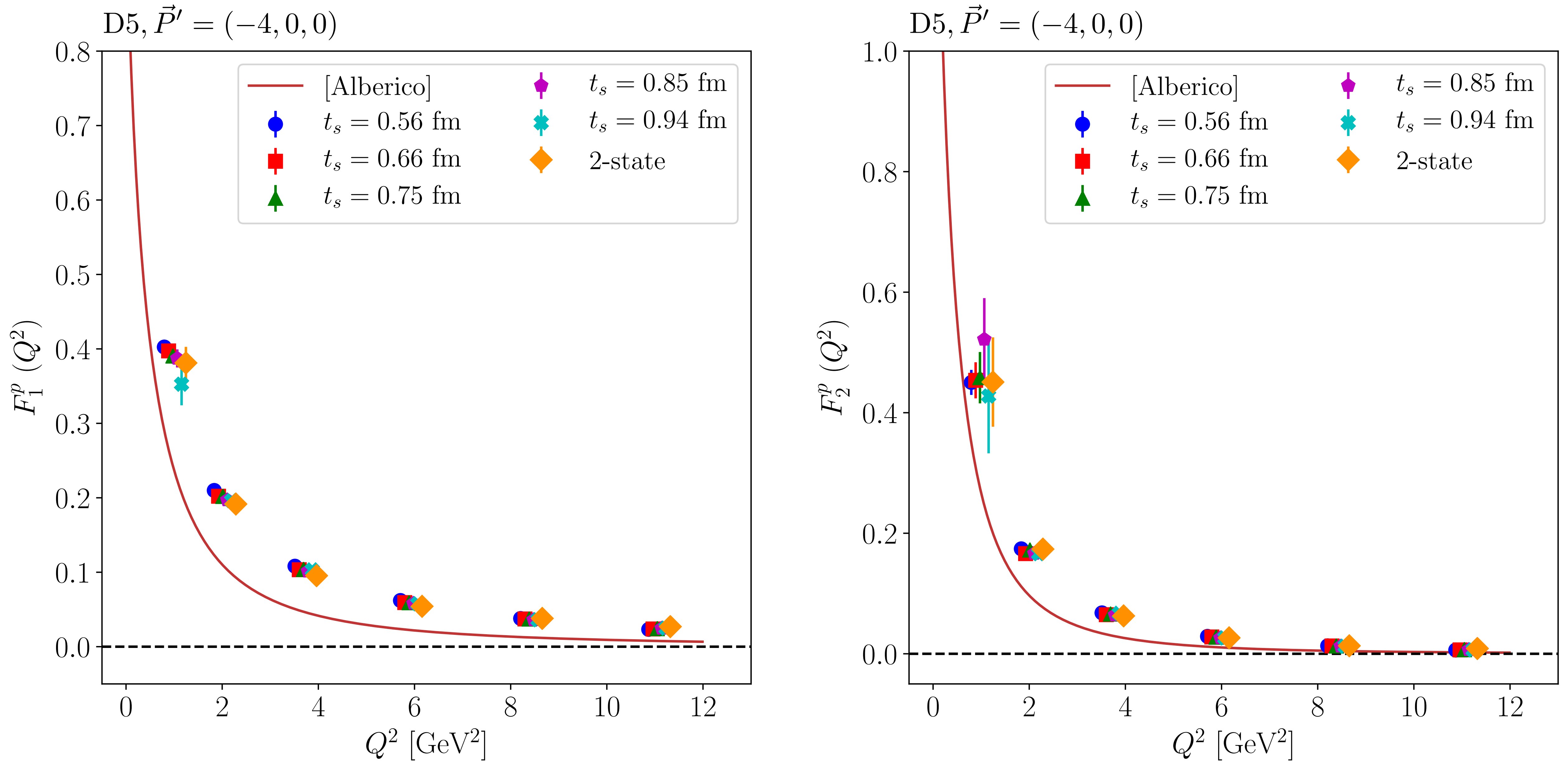
Form Factor Results II: G_E/G_M Ratio





- neutron: out lattice data underestimate experiment / phenomenology
- same qualitative behavior

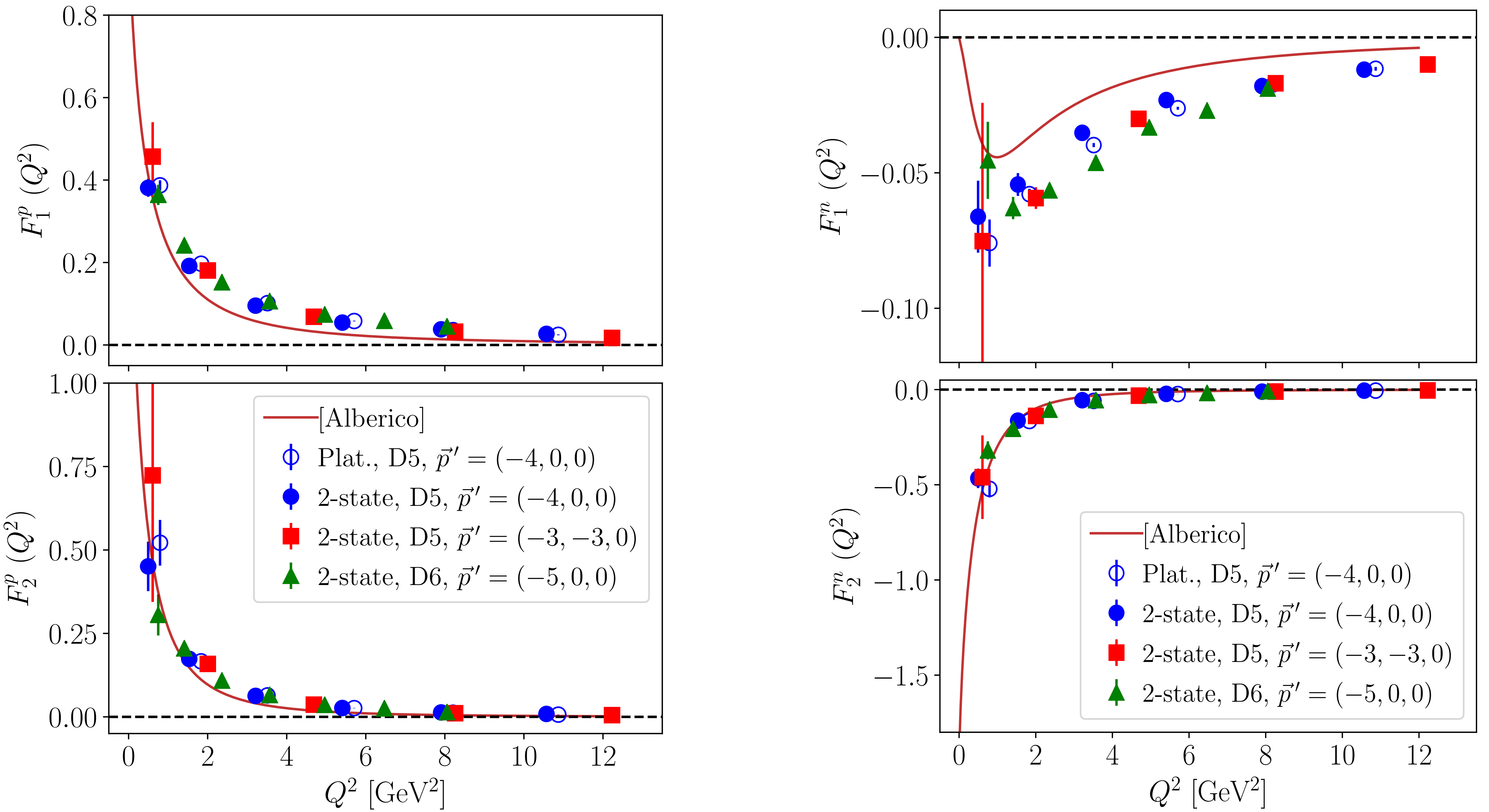
Form Factor Results III: F_1, F_2



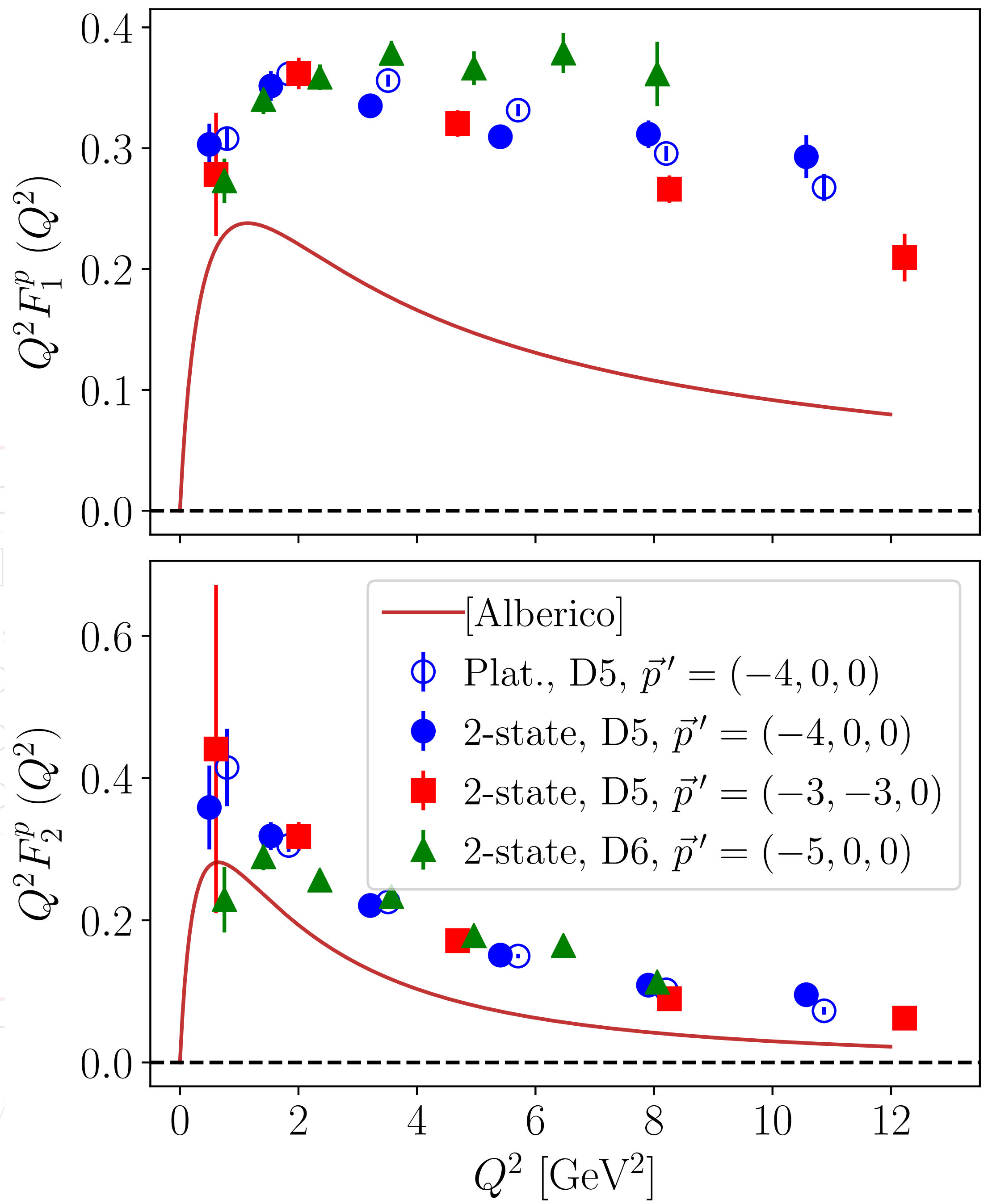
- shallow trend towards phenom. with increasing source-sink separation
- similar qualitative behavior, overestimation of phenom. prediction

W. M. Alberico et al. [arXiv: 0812.3539]

Form Factor Results III: F_1, F_2 : Two-state fits

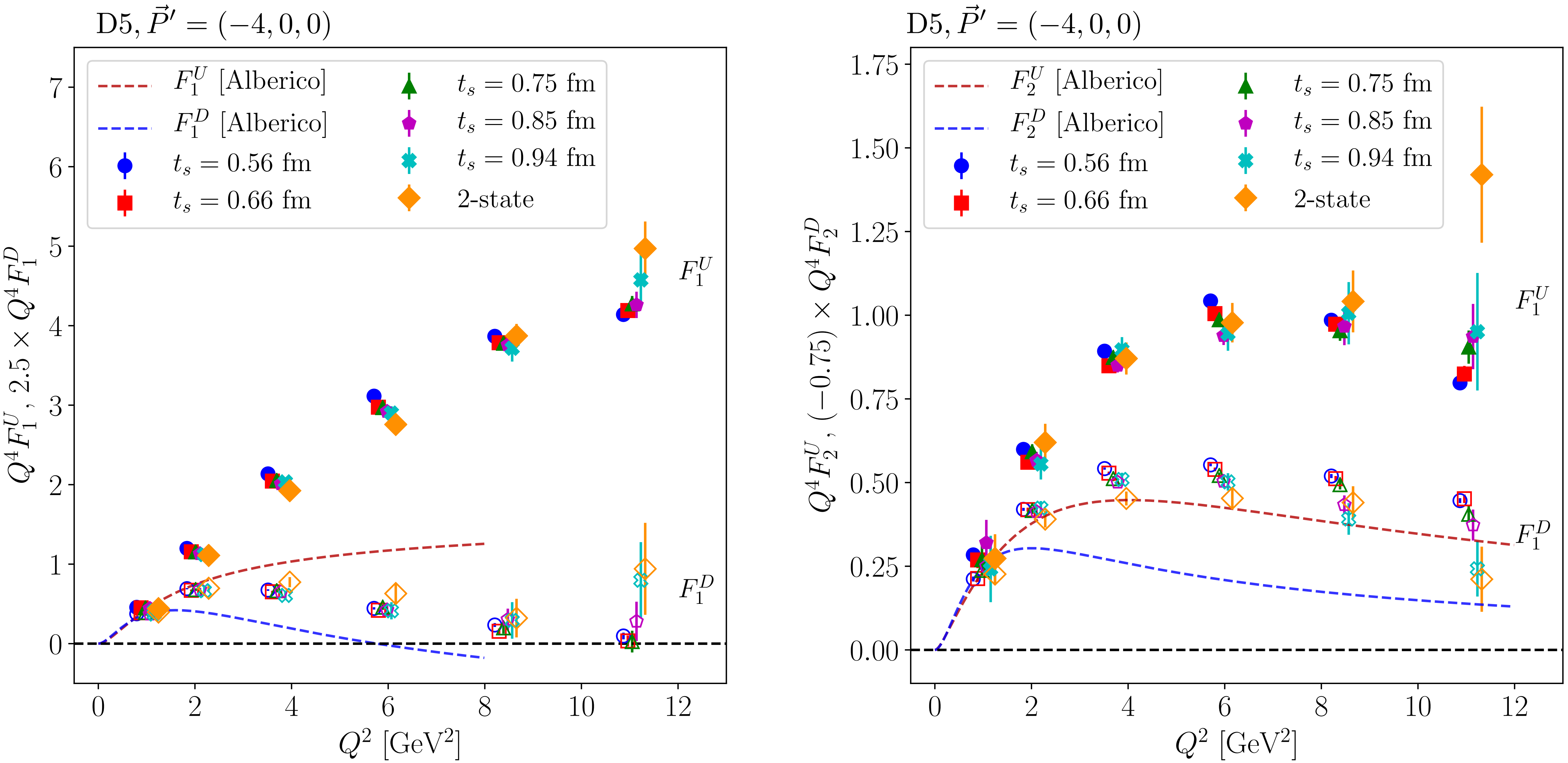


Form Factor Results III: F_1, F_2 : Two-state fits



- discrepancies for individual form factors
- **a thorough investigation is needed**

Form Factor Results III: F_1, F_2 : u,d quarks



- discrepancies observed for form factors of up- and down- quarks

Summary

- high-Q^2 on the lattice: feasible, but need to control systematics, noise-to-signal ratio

- our lattice results overestimate phenom. Q^2-dependence for F_1, F_2
- however: good agreement with experiment for F_2/F_1 and G_E/G_M ratios up to $Q^2 \sim 6$ GeV^2
- consistent results between $m_\pi = 170$ MeV (D5), $m_\pi = 280$ MeV (D6): small pion mass and volume effects

To-do:

- understand/resolve disagreement for individual form factors F_1, F_2
- complete investigation of excited state effects (perhaps larger t_s?)
- consider other systematic effects
  - $\mathcal{O}(a)$ **improvement**
  - continuum extrapolation
  - physical pion mass
  - disconnected diagrams

Thank you

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Bonus!

Bonus: Systematics I: Momentum discretization

Naive: $\vec{p} = \vec{\kappa}$, $\vec{\kappa} = \frac{2\pi}{L}\vec{n}$, $n_x, n_y, n_z = \frac{1}{a}\left[-\frac{L}{2}, \frac{L}{2}\right)$

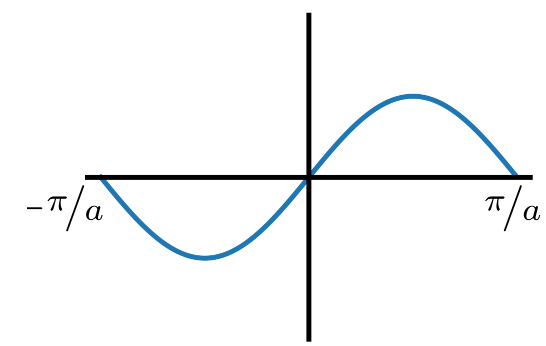
lattice momentum form:

- $\vec{p} \stackrel{?}{=} \vec{\kappa}$
- $\vec{p} \stackrel{?}{=} \vec{\kappa} - \frac{1}{6}\vec{\kappa}(a\vec{\kappa})^2$
- $\vec{p} \stackrel{?}{=} \frac{1}{a}\sin(a\vec{\kappa})$

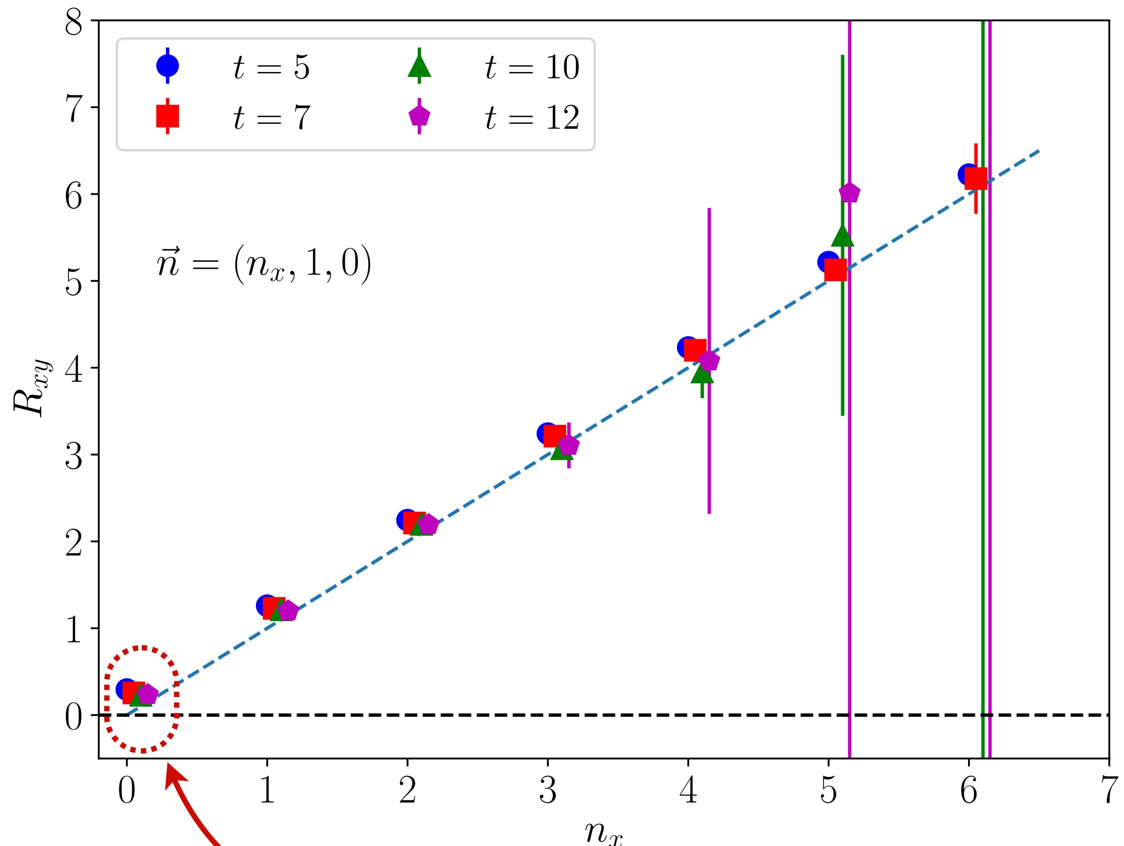
- take appropriate traces and ratios of two-point function to isolate momentum components

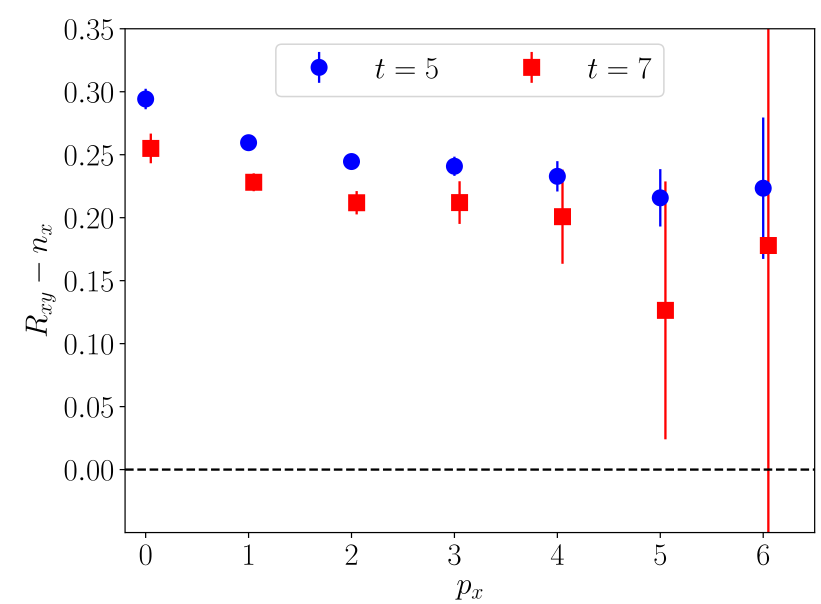
$$C(\vec{p},t) \stackrel{t \gtrsim 1}{=} |Z(\vec{p})|^2 \mathcal{S}(\vec{p}) e^{-E(\vec{p})t} \qquad \mathcal{S}(\vec{p}) = \frac{-i\not{p} + m}{2E(\vec{p})}$$

$$\mathrm{Im}\{\mathrm{Tr}[\gamma_k \mathcal{S}(\vec{p})]\} = -4p_k \to R_{xy}(\vec{p},t) \equiv \frac{\mathrm{Im}\{\mathrm{Tr}[\gamma_x C(\vec{p},t)]\}}{\mathrm{Im}\{\mathrm{Tr}[\gamma_y C(\vec{p},t)]\}} \xrightarrow{\text{cont.}} \frac{p_x}{p_y}$$



$n_x = 6 \to \kappa_x = {}^{3\pi}/_{8a}$





effect due to anisotropic quark (boosted) smearing??

- At non-zero momentum, correlators projected with $\Gamma^{\pm} \equiv \frac{1}{2}(\mathbb{1} + \gamma_4)$ include $\mathcal{O}((E-m)/2E)$ parity contaminations
- need to make sure that correlators from states at non-zero momentum correspond to the same zero-momentum states

F. M. Stokes et al. [arXiv: 1302.4152]

Parity-Expanded Variational Analysis (PEVA): Isolates parity of boosted hadron states

expand operator basis of correlation matrix $C_{ij}(\Gamma;\vec{p},t) = \mathrm{Tr}\left[\Gamma \sum_{\vec{x}} \langle \phi^i(x)\bar{\phi}^j(0)\rangle e^{-i\vec{p}\cdot\vec{x}}\right]$

$$\Gamma_p \equiv \frac{1}{4}(\mathbb{1} + \gamma_4)(\mathbb{1} - i\gamma_5\gamma_k\hat{p}_k)$$

$$\boxed{\begin{aligned} \phi_p^i &\equiv \Gamma_p \phi^i \\ \phi_p^{i'} &\equiv \Gamma_p \gamma_5 \phi^i \end{aligned}}$$

$$\boxed{\begin{aligned} \mathcal{G}_{ij}(\vec{p},t) &= C_{ij}(\Gamma_p;\vec{p},t) \\ \mathcal{G}_{ij'}(\vec{p},t) &= C_{ij}(-\gamma_5\Gamma_p;\vec{p},t) \\ \mathcal{G}_{i'j}(\vec{p},t) &= C_{ij}(\Gamma_p\gamma_5;\vec{p},t) \\ \mathcal{G}_{i'j'}(\vec{p},t) &= C_{ij}(-\gamma_5\Gamma_p\gamma_5;\vec{p},t) \end{aligned}}$$

$$\left(\begin{array}{cccc|cccc}
\mathbf{0\bar{0}} & \mathbf{0\bar{1}} & \mathbf{0\bar{2}} & \mathbf{0\bar{3}} & 0\bar{0}' & 0\bar{1}' & 0\bar{2}' & 0\bar{3}' \\
\mathbf{1\bar{0}} & \mathbf{1\bar{1}} & \mathbf{1\bar{2}} & \mathbf{1\bar{3}} & 1\bar{0}' & 1\bar{1}' & 1\bar{2}' & 1\bar{3}' \\
\mathbf{2\bar{0}} & \mathbf{2\bar{1}} & \mathbf{2\bar{2}} & \mathbf{2\bar{3}} & 2\bar{0}' & 2\bar{1}' & 2\bar{2}' & 2\bar{3}' \\
\mathbf{3\bar{0}} & \mathbf{3\bar{1}} & \mathbf{3\bar{2}} & \mathbf{3\bar{3}} & 3\bar{0}' & 3\bar{1}' & 3\bar{2}' & 3\bar{3}' \\
\hline
0'\bar{0} & 0'\bar{1} & 0'\bar{2} & 0'\bar{3} & 0'\bar{0}' & 0'\bar{1}' & 0'\bar{2}' & 0'\bar{3}' \\
1'\bar{0} & 1'\bar{1} & 1'\bar{2} & 1'\bar{3} & 1'\bar{0}' & 1'\bar{1}' & 1'\bar{2}' & 1'\bar{3}' \\
2'\bar{0} & 2'\bar{1} & 2'\bar{2} & 2'\bar{3} & 2'\bar{0}' & 2'\bar{1}' & 2'\bar{2}' & 2'\bar{3}' \\
3'\bar{0} & 3'\bar{1} & 3'\bar{2} & 3'\bar{3} & 3'\bar{0}' & 3'\bar{1}' & 3'\bar{2}' & 3'\bar{3}'
\end{array}\right)$$

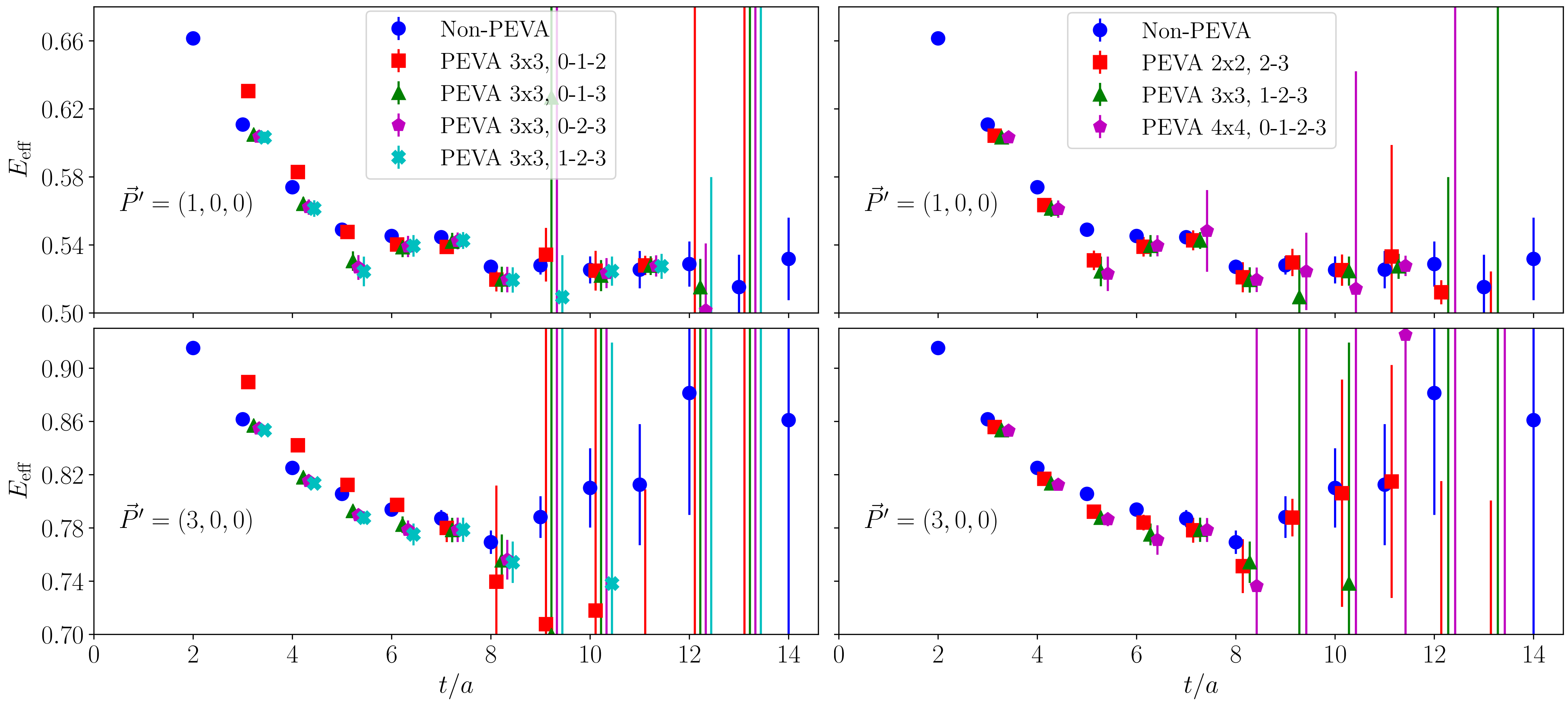
GEVP: $\mathcal{G}(\vec{p},t+\Delta t)\,\boldsymbol{u}^{\alpha}(\vec{p}) = e^{-E_{\alpha}(\vec{p})\Delta t}\mathcal{G}(\vec{p},t)\,\boldsymbol{u}^{\alpha}(\vec{p})$

Investigation:

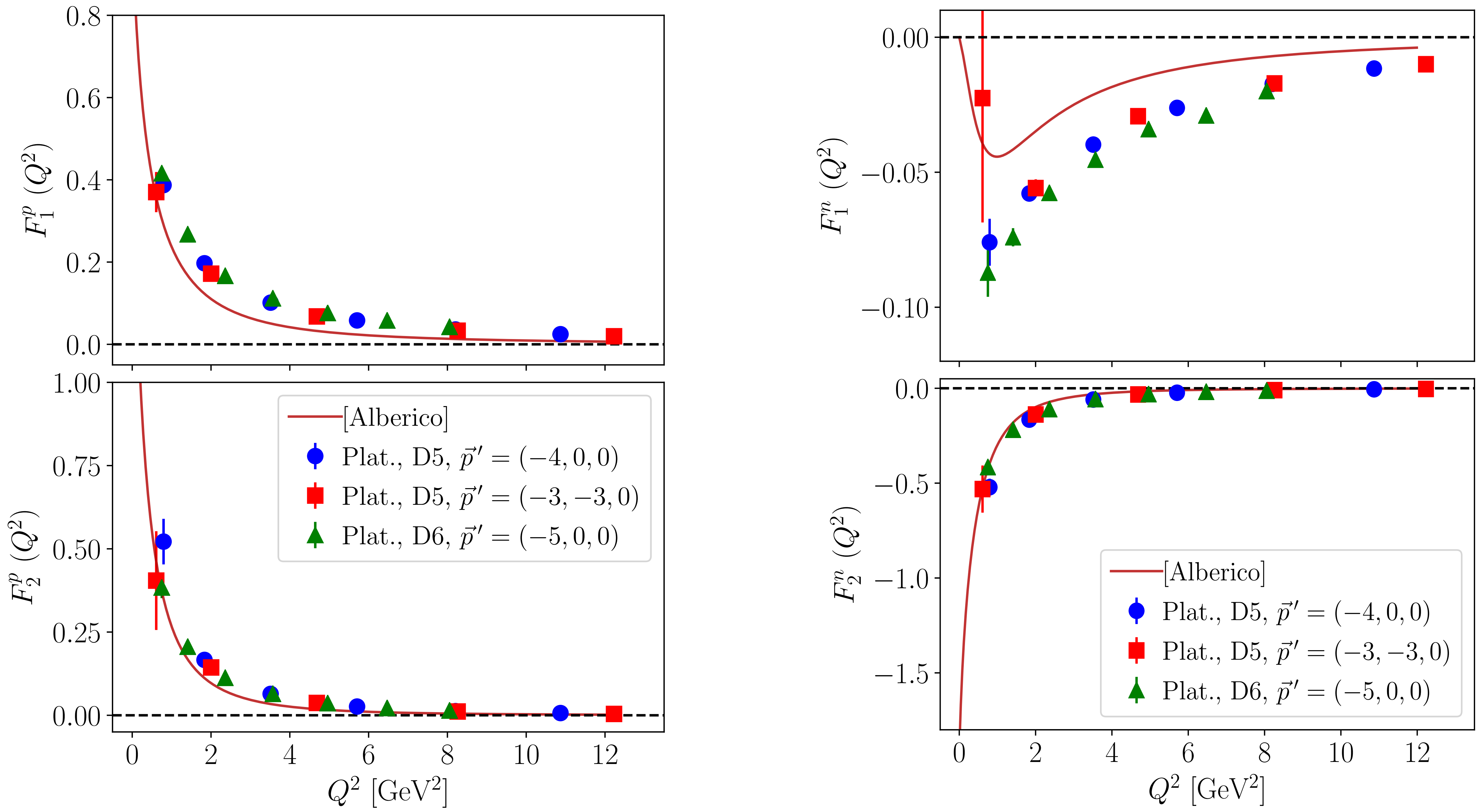
- two-point functions from nucleon interpolating operators at four different values of Gaussian smearing $\longrightarrow$ different overlap with nucleon ground state
- perform PEVA analysis for various sets of operators
- D5 ensemble, 240cfg x 32src statistics

$$\left(\begin{array}{cccc|cccc} 0\bar{0} & 0\bar{1} & 0\bar{2} & 0\bar{3} & 0\bar{0}' & 0\bar{1}' & 0\bar{2}' & 0\bar{3}' \\ 1\bar{0} & 1\bar{1} & 1\bar{2} & 1\bar{3} & 1\bar{0}' & 1\bar{1}' & 1\bar{2}' & 1\bar{3}' \\ 2\bar{0} & 2\bar{1} & 2\bar{2} & 2\bar{3} & 2\bar{0}' & 2\bar{1}' & 2\bar{2}' & 2\bar{3}' \\ 3\bar{0} & 3\bar{1} & 3\bar{2} & 3\bar{3} & 3\bar{0}' & 3\bar{1}' & 3\bar{2}' & 3\bar{3}' \\ \hline 0'\bar{0} & 0'\bar{1} & 0'\bar{2} & 0'\bar{3} & 0'\bar{0}' & 0'\bar{1}' & 0'\bar{2}' & 0'\bar{3}' \\ 1'\bar{0} & 1'\bar{1} & 1'\bar{2} & 1'\bar{3} & 1'\bar{0}' & 1'\bar{1}' & 1'\bar{2}' & 1'\bar{3}' \\ 2'\bar{0} & 2'\bar{1} & 2'\bar{2} & 2'\bar{3} & 2'\bar{0}' & 2'\bar{1}' & 2'\bar{2}' & 2'\bar{3}' \\ 3'\bar{0} & 3'\bar{1} & 3'\bar{2} & 3'\bar{3} & 3'\bar{0}' & 3'\bar{1}' & 3'\bar{2}' & 3'\bar{3}' \end{array}\right)$$

effect due to parity mixing is negligible within our statistics

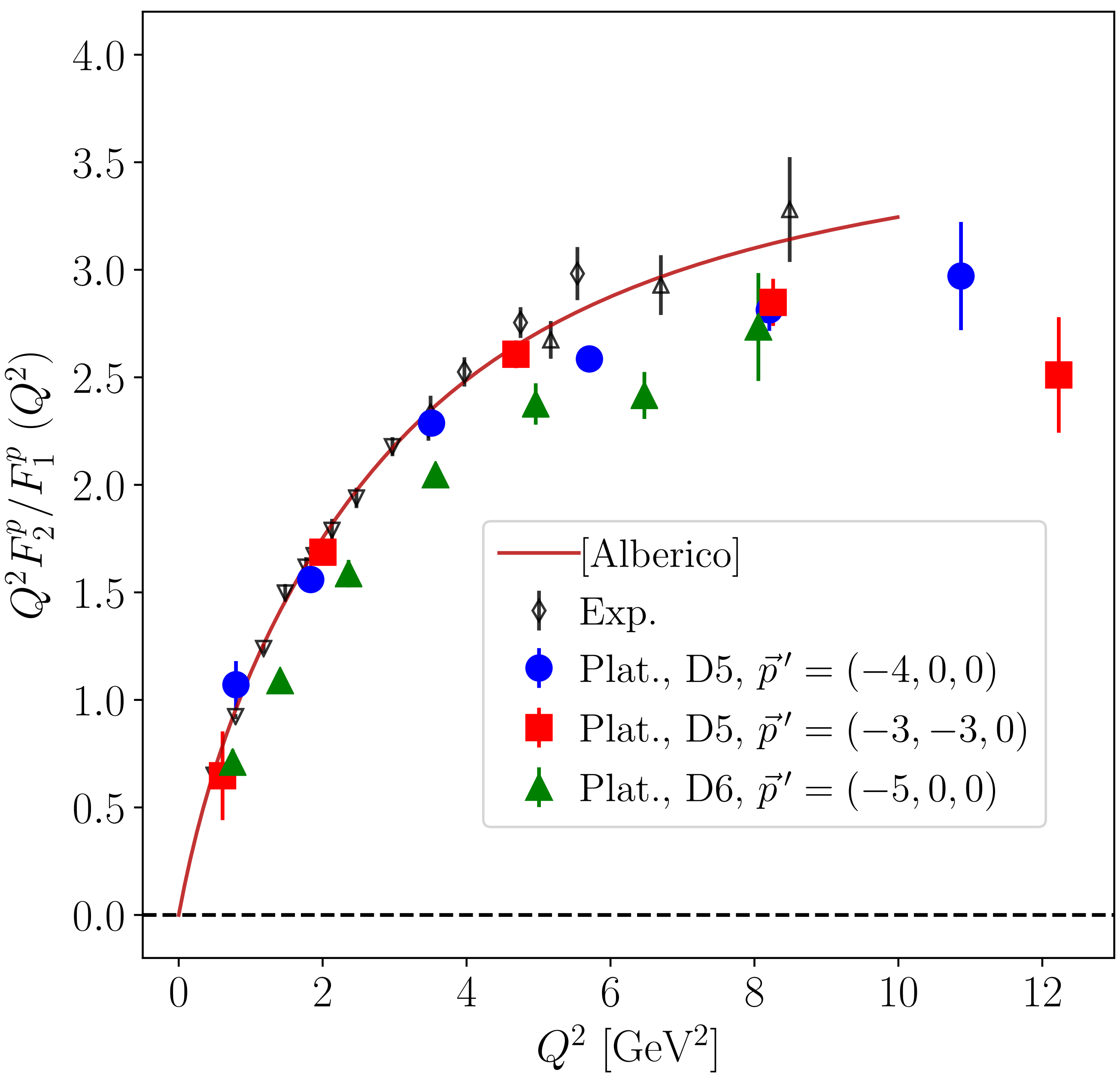


Bonus: Form Factors F_1, F_2, plateau values

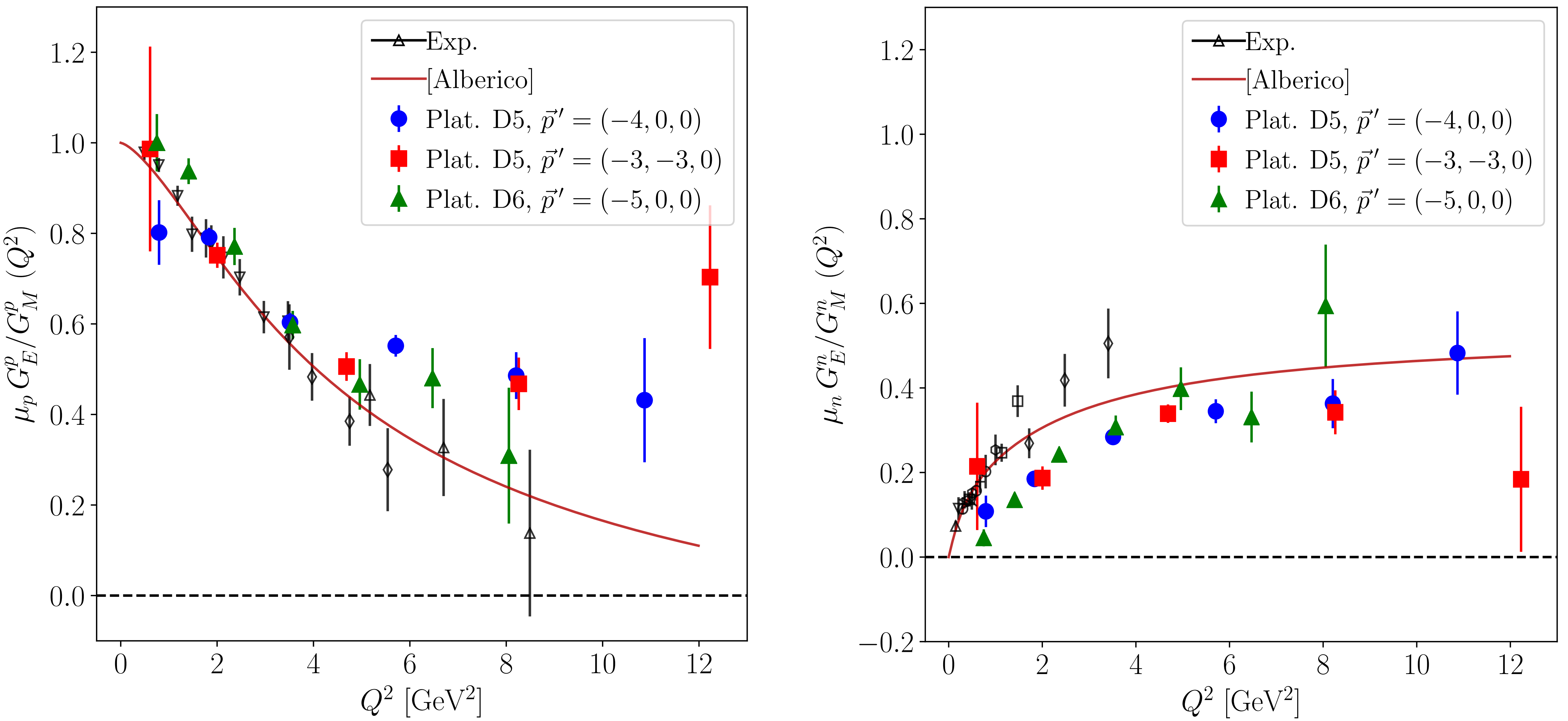


- consistent results across D5, D6 and sink boost momentum
- small effect from on-axis / x-y diagonal boost momentum
- source-sink separation $t_s \sim 0.9$ fm is shown

Bonus: Form factor ratios, plateau values



Bonus: Form factor ratios, plateau values



Bonus: Some more 3pt/2pt function ratios