



# Nucleon form factors on a $(10.8\text{fm})^4$ lattice at the physical point in 2+1 flavor QCD

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## Plan of talk

- PACS Collaboration Members
- “PACS10” Configs with Physical Volume over  $(10 \text{ fm})^4$
- Improvements from Previous Work (arXiv:1807.03974)
- Results for Form Factors
  - Vector Current
  - Axial Vector Current
  - Generalized Goldberger-Treiman Relation
    - ⇒ Pseudoscalar Density
- Summary



## PACS Collaboration Members

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# “PACS10” Configs @ $\beta=1.82$ in 2+1 Flavor QCD

arXiv:1807.06237

- Wilson-clover quark action + Iwasaki gauge action
- Stout smearing with  $\alpha=0.1$  and  $N_{\text{smear}}=6$
- NP  $C_{\text{SW}}=1.11$  determined by SF
- $\beta=1.82 \Rightarrow a^{-1}=2.33 \text{ GeV}$
- **Lattice size= $128^4 \Rightarrow (10.8 \text{ fm})^3$  spatial volume**
- Hopping parameters:  $(\kappa_{\text{ud}}, \kappa_{\text{s}})=(0.126117, 0.124902)$   
 **$\Rightarrow m_{\pi} \approx 135 \text{ MeV}, m_{\pi}L \approx 7.5$**
- Simulation algorithm
  - (MP)<sup>2</sup>DDHMC w/ active link for ud quarks, RHMC for s quark
  - Block size= $16 \times 16 \times 8 \times 64$
  - MP parameters:  $(\rho_1, \rho_2)=(0.9997, 0.9940)$
  - Multi-time scale integrator:  $(N_0, N_1, N_2, N_3, N_4)=(8, 2, 2, 2, 22)$
  - trajectory length:  $\tau=1$
  - $N_{\text{RHMC}}=8$ ,  $[F_{\text{min}}, F_{\text{max}}]=[0.00025, 1.85]$
  - Chronological inverter guess for IR parts
  - Solver: mixed precision nested BiCGStab



# Measurement Details with Plateau Method (1)

## 2-pt correlator

$$\begin{aligned} C_{XS}(t_{\text{sink}} - t_{\text{src}}, \mathbf{p}) \\ = \frac{1}{4} \text{Tr} \{ \mathcal{P}_+ \langle N_X(t_{\text{sink}}, \mathbf{p}) \bar{N}_S(t_{\text{src}}, -\mathbf{p}) \rangle \} \end{aligned}$$

## 3-pt correlator

$$\begin{aligned} C_{O,\alpha}^{\mathcal{P}_k}(t, \mathbf{p}', \mathbf{p}) \\ = \frac{1}{4} \text{Tr} \{ \mathcal{P}_k \langle N(t_{\text{sink}}, \mathbf{p}') J_\alpha^O(t, \mathbf{q}) \bar{N}(t_{\text{src}}, -\mathbf{p}) \rangle \} \end{aligned}$$

## Ratio of 3-pt to 2-pt correlators

$$\mathcal{R}_{O,\alpha}^k(t, \mathbf{p}', \mathbf{p}) = \frac{C_{O,\alpha}^{\mathcal{P}_k}(t, \mathbf{p}', \mathbf{p})}{C_{SS}(t_{\text{sink}} - t_{\text{src}}, \mathbf{p}')} \sqrt{\frac{C_{LS}(t_{\text{sink}} - t, \mathbf{p}) C_{SS}(t - t_{\text{src}}, \mathbf{p}') C_{LS}(t_{\text{sink}} - t_{\text{src}}, \mathbf{p}')}{C_{LS}(t_{\text{sink}} - t, \mathbf{p}') C_{SS}(t - t_{\text{src}}, \mathbf{p}) C_{LS}(t_{\text{sink}} - t_{\text{src}}, \mathbf{p})}}$$



## Measurement Details with Plateau Method (2)

- AMA is used to gain high statistical precision
- $O(100)$  measurements/config  $\Rightarrow O(10^3 \sim 10^4)$  measurements so far
- 9 choices for spatial momenta:  
 $\vec{n}=(1,0,0),(1,1,0),(1,1,1),(2,0,0),(2,1,0),(2,1,1),(2,2,0),(3,0,0),(2,2,1)$   
minimum mom= $2\pi/L \sim 0.115$  GeV thanks to  $L=10.8$  fm
- Lattice size= $128^4 \Rightarrow (10.8 \text{ fm})^3$  spatial volume allows small  $q^2$  region
- Exp smeared src/sink operators for 2-pt and 3-pt functions
- Src-sink separation:  $t_{\text{sink}} - t_{\text{src}} = 10, 12, 14, 16$  ( $\sim 1.35$  fm)
- $Z_A=0.9650(68)(95)$ ,  $Z_V=0.95153(76)(1487)$  in SF scheme  
PoS(LATTICE2015)271



# How Large Spatial Size is Necessary?

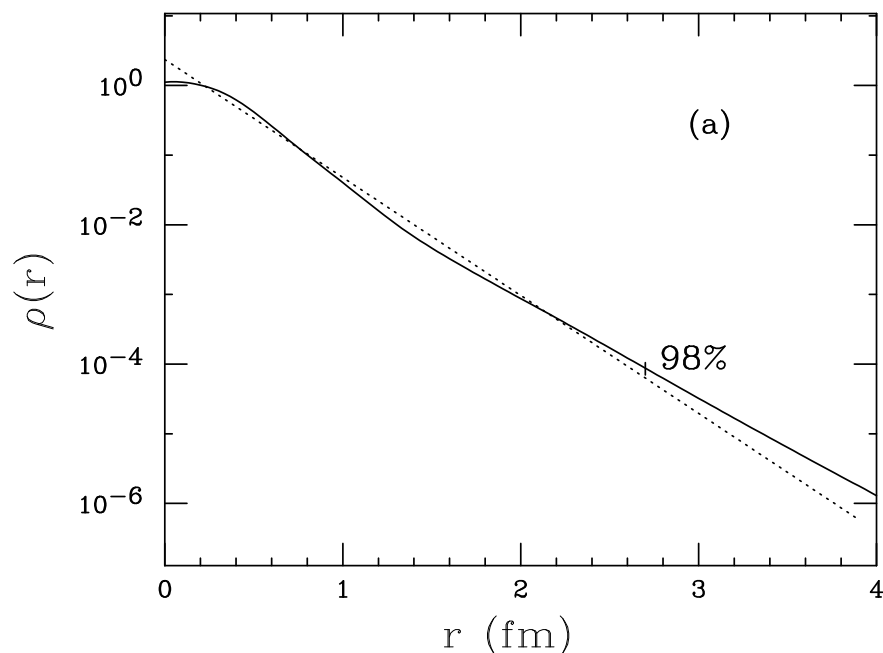
Sick, Atoms 6(2018)2

Charge RMS radius

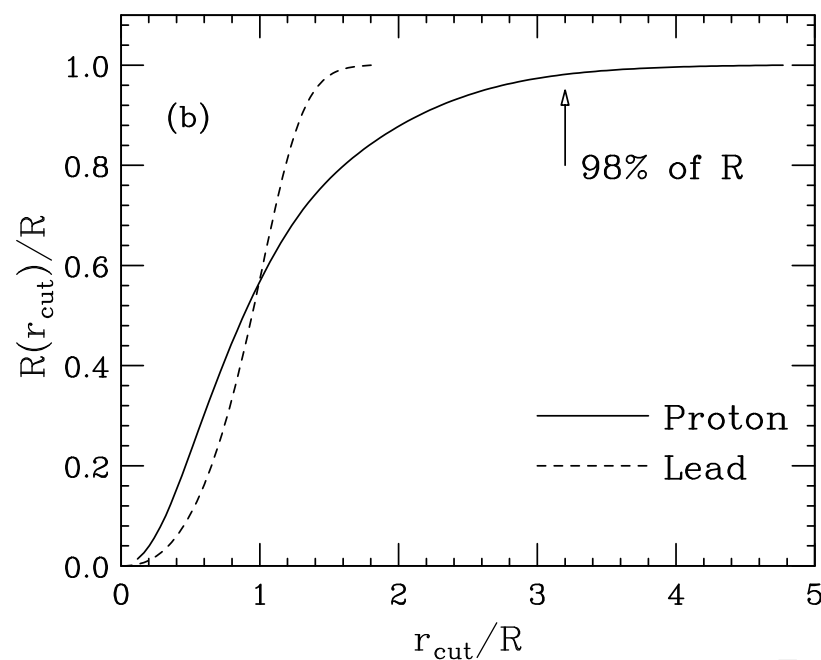
$$R^2 \equiv \int_0^\infty \rho(r) r^4 4\pi dr,$$

$$R(r_{\text{cut}}) = \left[ \int_0^{r_{\text{cut}}} \rho(r) r^4 dr \bigg/ \int_0^\infty \rho(r) r^4 dr \right]^{1/2}$$

Charge density



$R(r_{\text{cut}})$



Integration up to  $r_{\text{cut}}=2.7\text{fm} \Rightarrow$  Only 98% of charge RMS radius



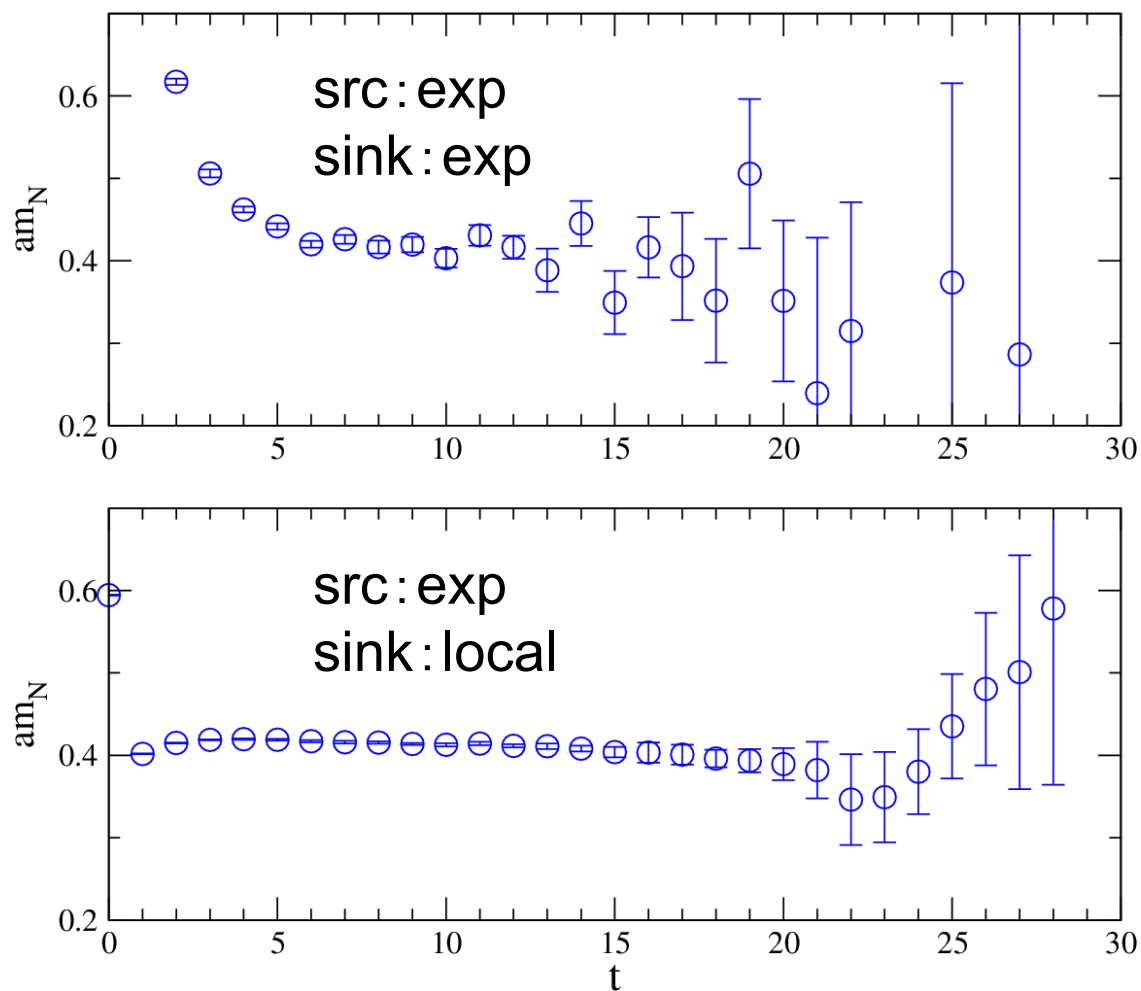
## Improvements from Our Previous Work

	Lattice 2018	arXiv:1807.03974
Volume	$128^4 (10.8 \text{ fm})^4$	$96^4 (8.1 \text{ fm})^4$
Minimum $q^2$	$0.013 \text{ GeV}^2$	$0.024 \text{ GeV}^2$
$m_\pi$	135 MeV (physical)	146 MeV
Measurement to increase statistics	w/ AMA	w/o AMA
$t_s =  t_{\text{sink}} - t_{\text{src}} $ dependence	$t_s = 10, 12, 14, 16$	$t_s = 15$





# Nucleon Effective Mass

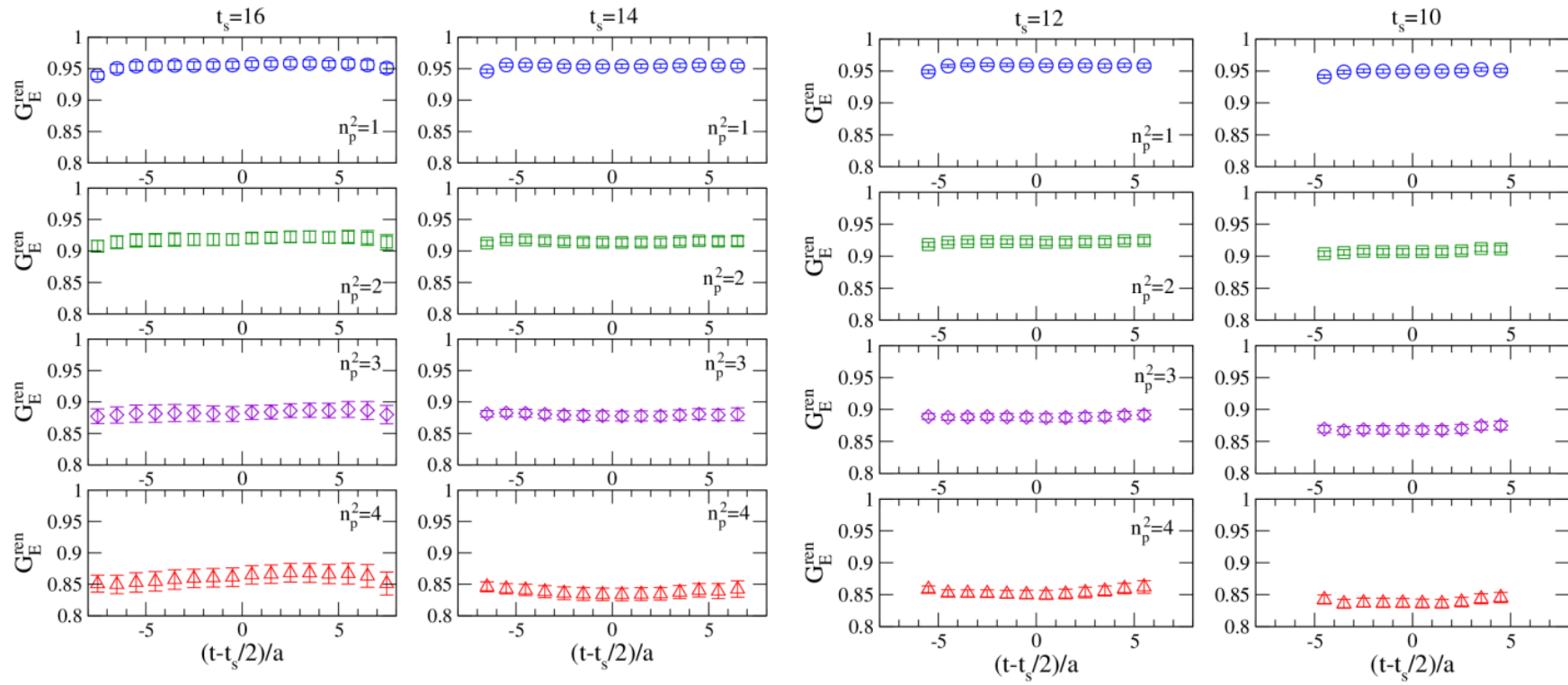


Good plateau is observed from small time slice



# Isovector Electric Form Factor (1)

Ratio of 3-pt to 2-pt correlators as a function of  $t$  (location of  $V$ )



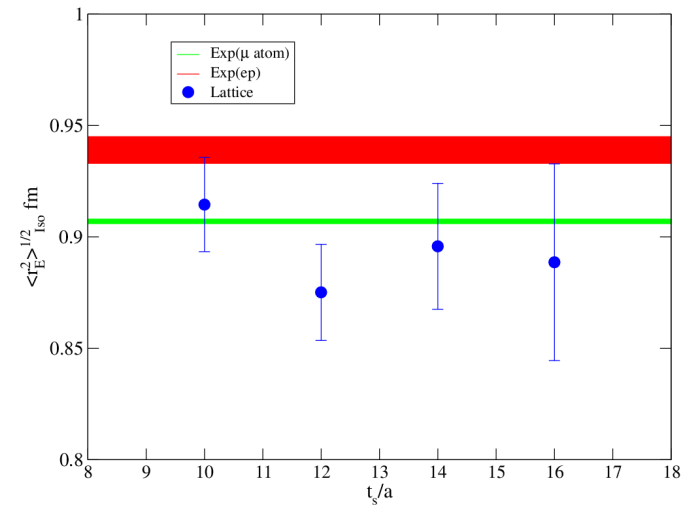
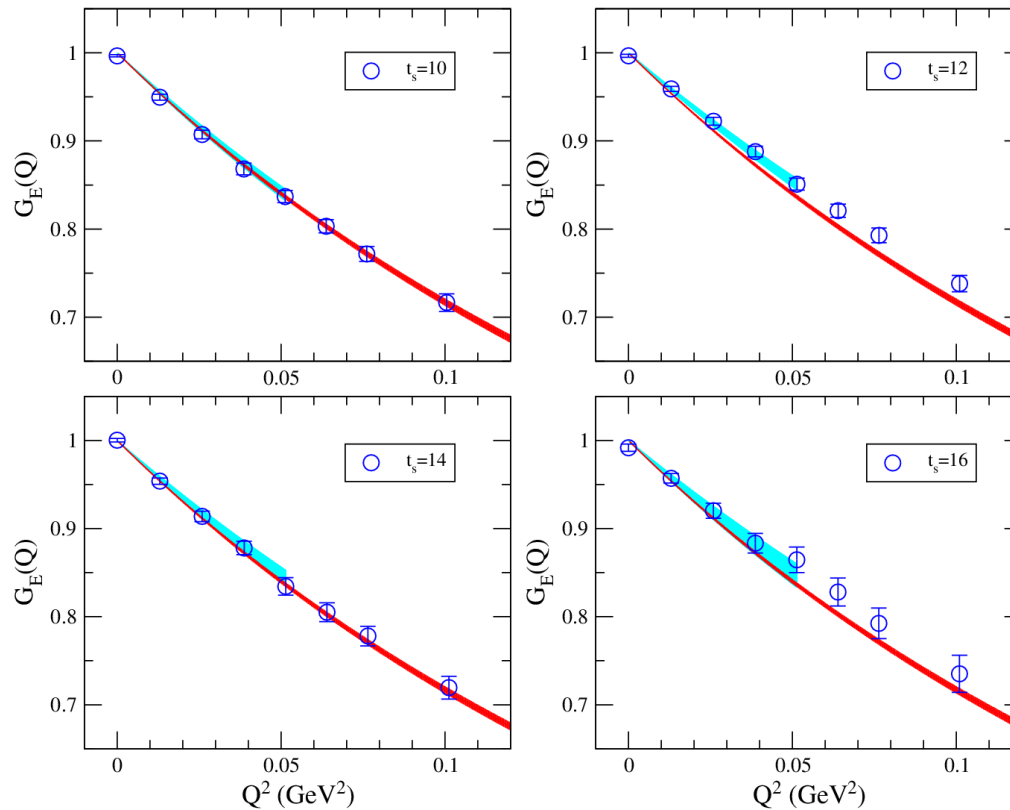
Good plateau for  $t_s=10, 12, 14, 16$



# Isovector Electric Form Factor (2)

$$G_E(q^2) = F_1(q^2) - \frac{q^2}{4m_N^2} F_2(q^2)$$

$$\langle r_E^2 \rangle = -6 \left. \frac{dG_E}{dQ^2} \right|_{Q^2=0}$$



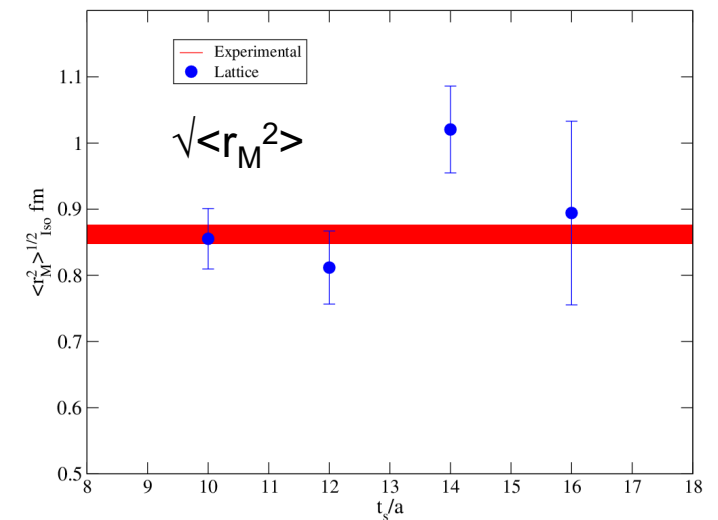
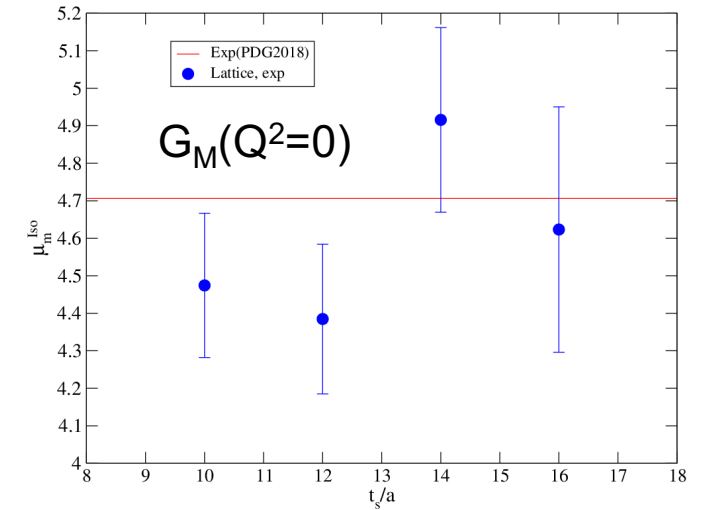
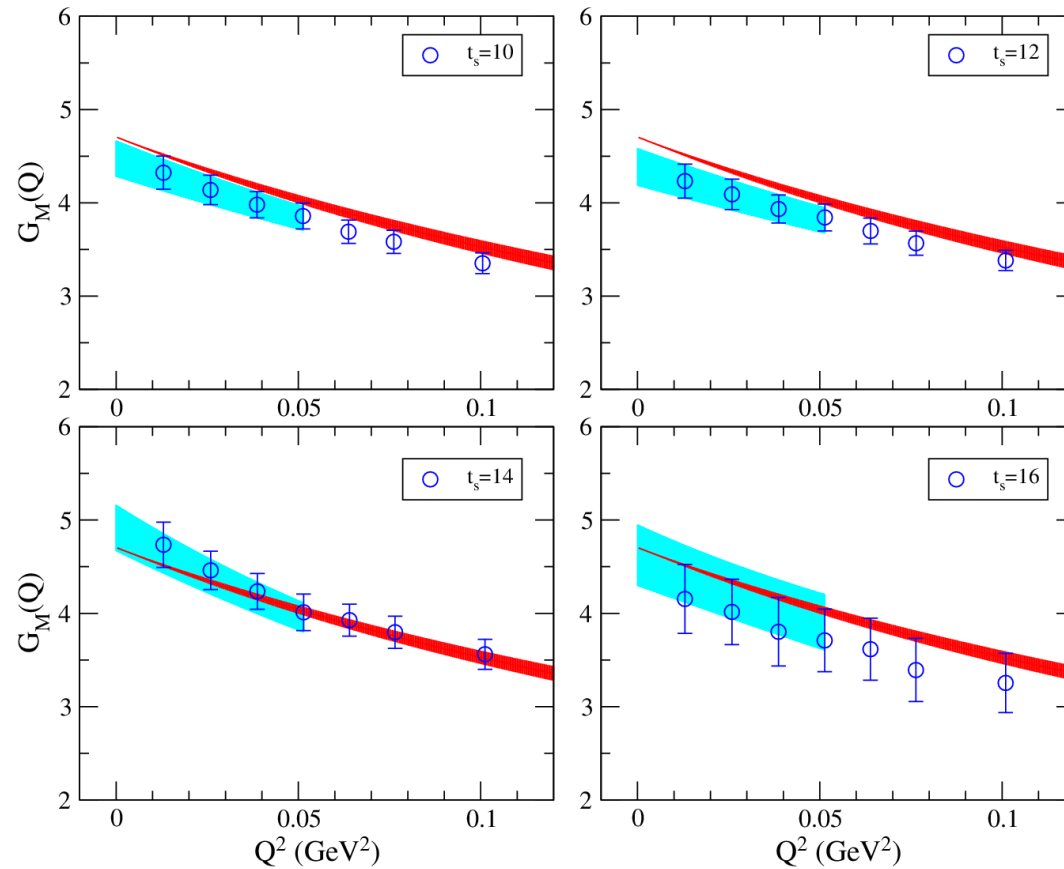
Seems to prefer  $\mu$ H experiment

⇒ Possibility to distinguish two experimental values



# Isovector Magnetic Form Factor

$$G_M(q^2) = F_1(q^2) + F_2(\hat{q}^2)$$

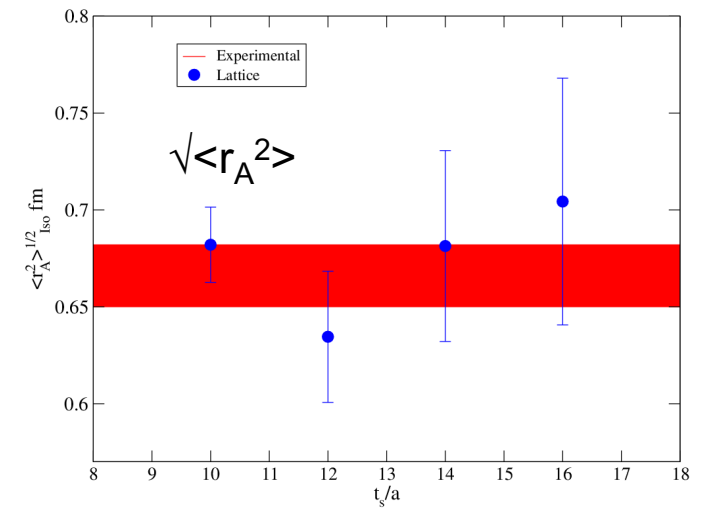
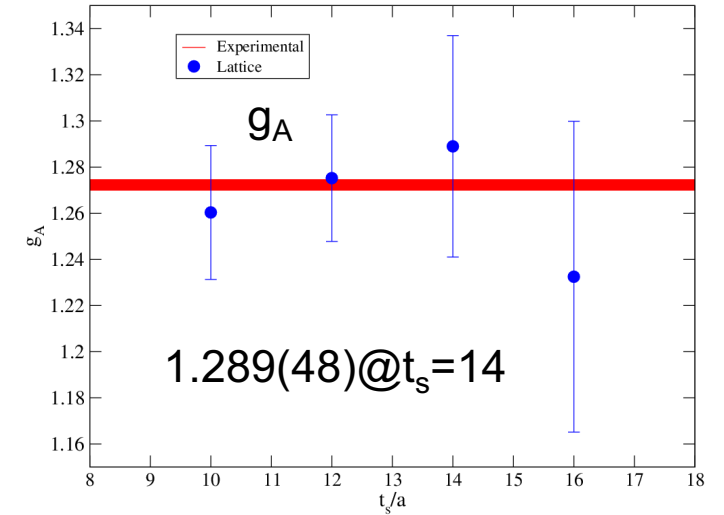
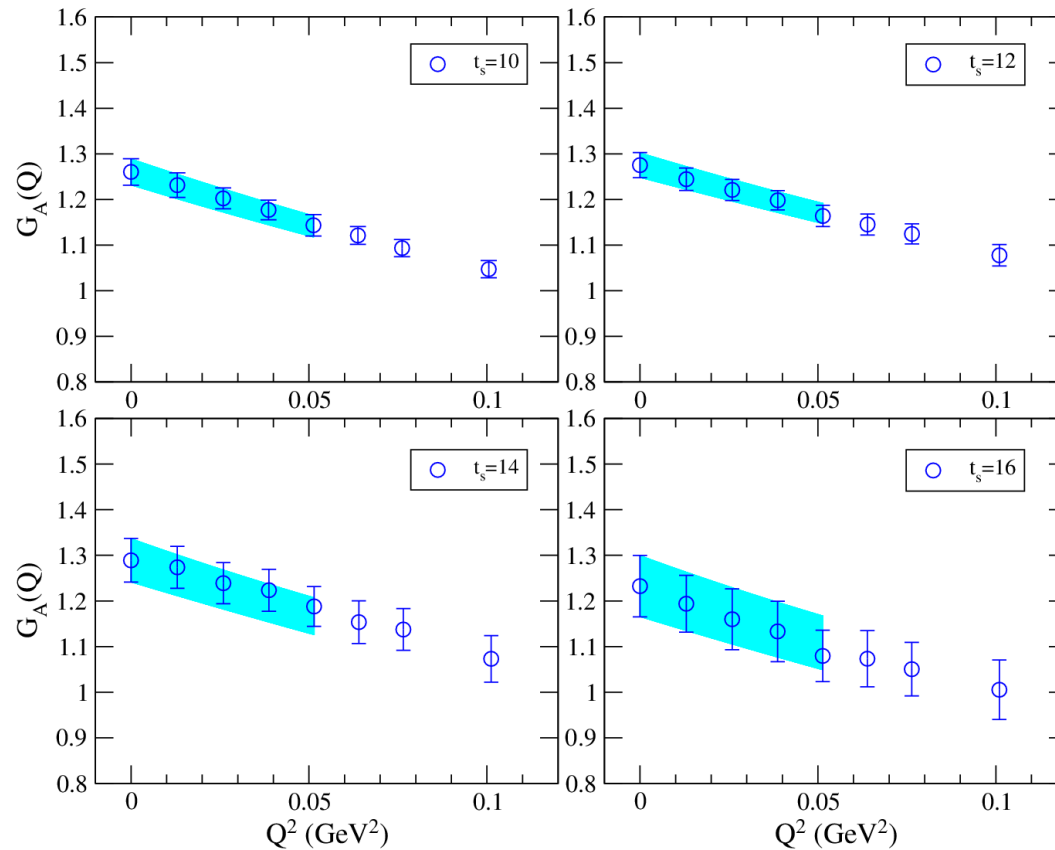


Consistent with  $\mu_M$  and  $\sqrt{\langle r_M^2 \rangle}$  within  $2\sigma$  error



# Axial Form Factor

Two form factors  $F_A$  and  $F_P$   
for axial vector current

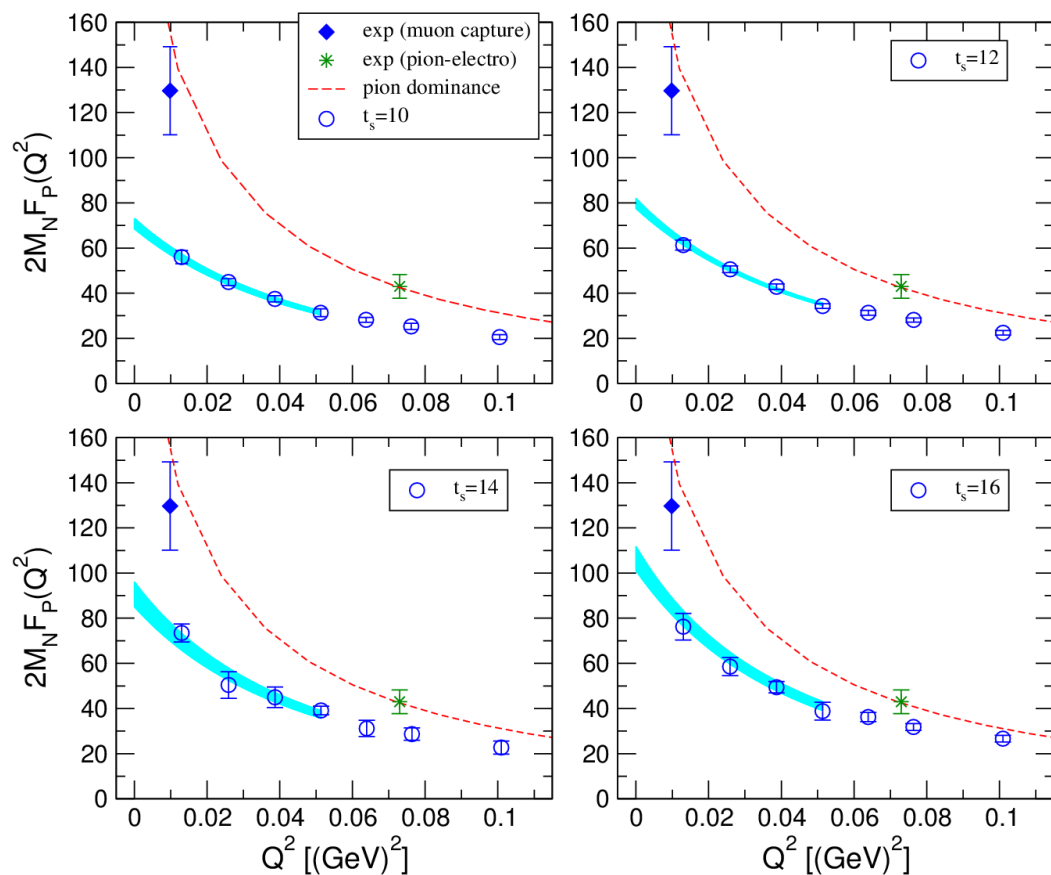


$g_A$  is consistent with experiment being independent of  $t_s$   
 $\sqrt{\langle r_A^2 \rangle}$  is also consistent with experiment

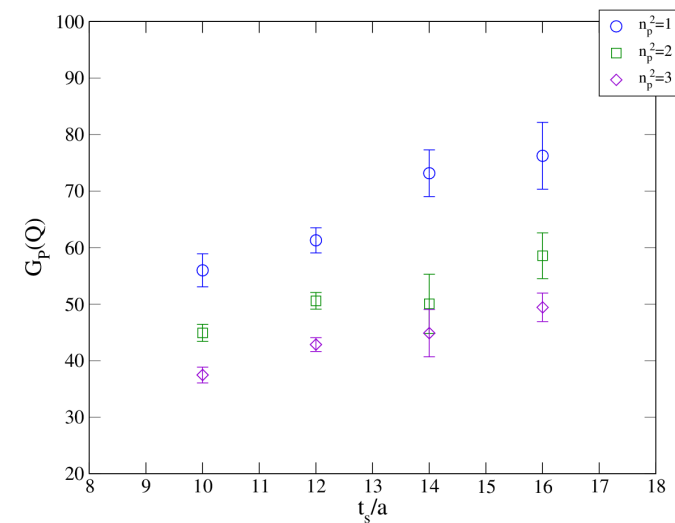


# Induced Pseudoscalar Form Factor $F_P$

$Q^2$  dependence of  $2M_N F_P$



$t_s$  dependence of  $F_P$



Clear  $t_s$  dependence for  $F_P \Rightarrow$  Excited state contributions

✂ ChPT analysis by Bär, Wed 14:00[HIS]



## Generalized Goldberger-Treiman (GT) Relation

$F_A$  and  $F_P$  are not independent

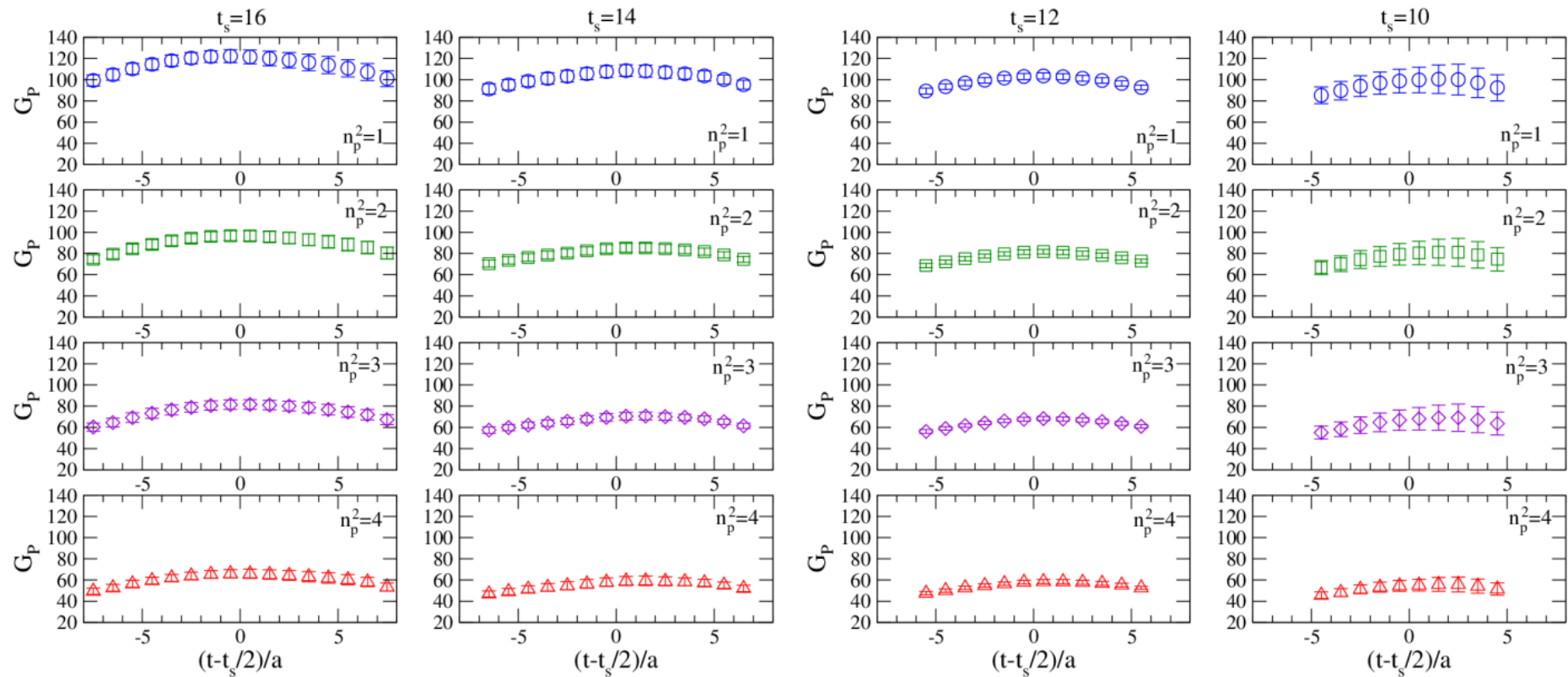
$$2M_N F_A(q^2) - q^2 F_P(q^2) = 2\hat{m} G_P(q^2)$$

$\Rightarrow$  Check Generalized GT relation with  $G_P$



# Pseudoscalar Form Factor $G_P(1)$

Ratio of 3-pt to 2-pt correlators as a function of  $t$  (location of  $P$ )



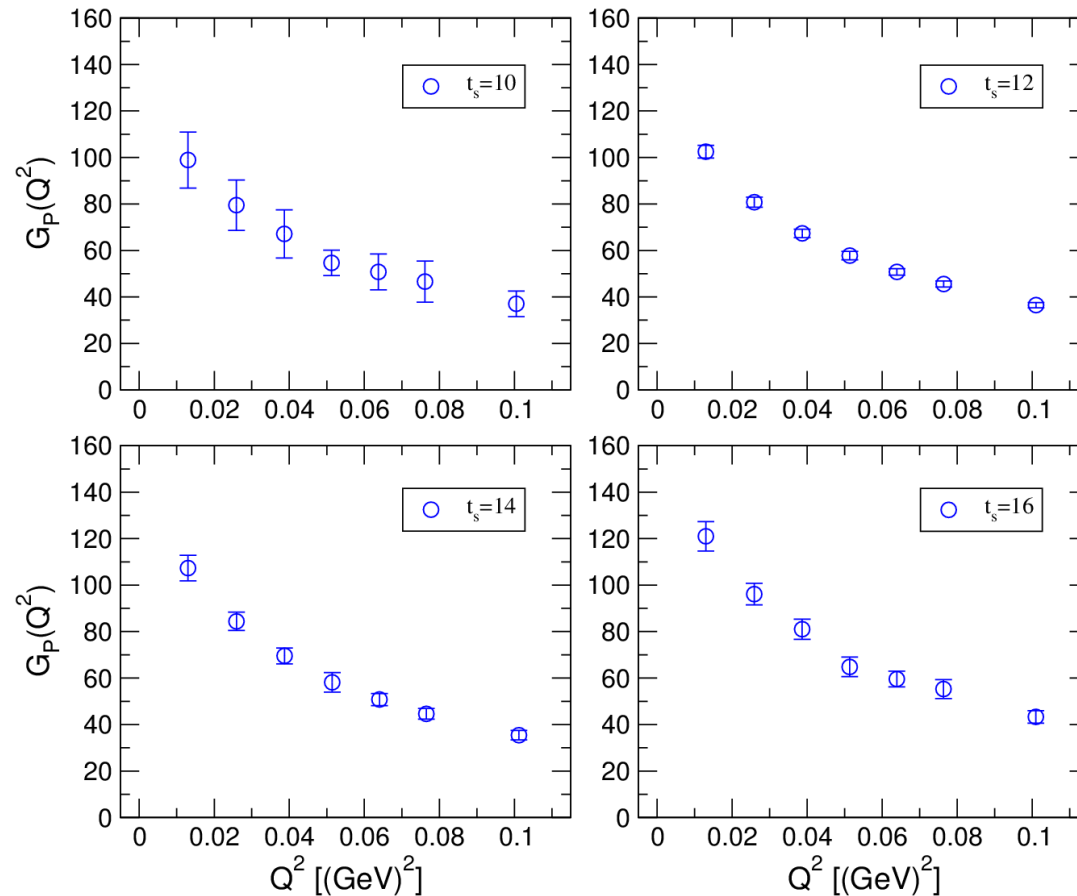
Mound like shape  $\Rightarrow$  Signal of excited state contributions





# Pseudoscalar Form Factor $G_P$ (2)

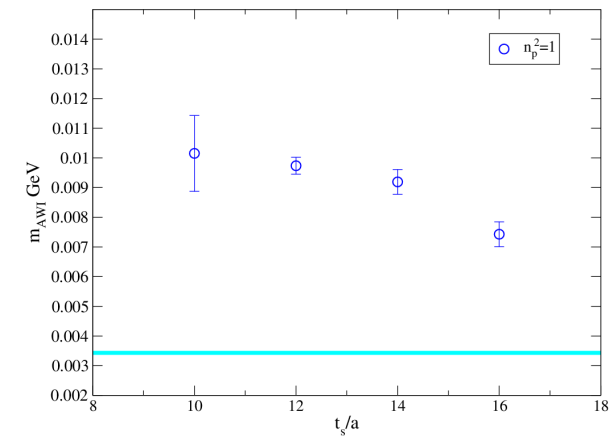
$Q^2$  dependence of  $G_P$



Generalized GT relation

$$m_{\text{AWTI}} = \frac{2M_N F_A^{\text{ren}}(q^2) - q^2 F_P^{\text{ren}}(q^2)}{2G_P(q^2)}$$

$t_s$  dependence of  $m_{\text{AWI}}(\text{GT})$



$m_{\text{AWI}}(\text{GT})$  becomes closer to  $m_{\text{AWI}}(\text{PCAC})$  for larger  $t_s$



## Summary

- 2+1 flavor QCD simulation at the physical point on  $(10.8 \text{ fm})^4$  lattice
  - Large spatial volume allows investigation at small  $Q^2$  region
- $t_s$  dependence is systematically investigated
  - $G_E$ ,  $G_M$  and  $F_A$  show no  $t_s$  dependence
  - Clear  $t_s$  dependence is observed for  $F_P$  and  $G_P$
- Results for  $G_E$ ,  $G_M$  and  $F_A$  are consistent with experiment including  $g_A$
- Violation of Generalized GT relation diminishes as  $t_s$  increases



# BACKUP