Pion-pion Scattering with Physical Quark Masses (from Lattice QCD)

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July 25, 2018

Lattice 2018

Outline

Motivation: $K \to \pi\pi$

- Direct comparison of low energy QCD - experiment vs. lattice
- $K \to \pi\pi$ - an important decay to understand CP violation
- Lattice calculation difficult, Gparity calculation already done, but needs a check
- See talks tomorrow for more information (Weak decay session, T. Wang, C. Kelly)

G-parity vs. Periodic Boundary Conditions

We compute using spatially periodic boundary conditions (antiperiodic in time)

- G-parity - charge conjugation + 180 degree isospin rotation
- G-parity eliminates (kinematically) unphysical stationary $\pi\pi$ state
- G-parity and Periodic boundary conditions have different finite volume errors, useful check
- Periodic lattices ready for use, evolution cost amortized
- Good exercise for $K \to \pi\pi$ (same physics is present, strong phase common to both)

Outline

Lüscher Method

Lüscher gives a quantization condition which maps lattice spectra onto infinite volume scattering phase shifts.
Lüscher's formula[1]:

$$\delta(p) = -\phi(k) + \pi n, \;\; n \in \mathbb{Z}$$

$$\tan\;\phi(k) = \frac{\pi^{3/2}\kappa}{Z_{00}(1;\kappa^2)}$$

$$\kappa = \frac{pL}{2\pi}$$

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Extracting Spectra from Correlation Functions

We can define a generalized eigenvalue problem (GEVP) from an NxN matrix of correlators we compute on the lattice

$$C_{ij} \equiv \left\langle 0 | \hat{O}_i(t) \hat{O}(0)_j^\dagger | 0 \right\rangle = \sum_{n=1}^{\infty} \left(e^{-E_n t} + e^{-E_n(L_t - (t - t_0))} \right) \psi_{ni} \psi_{nj}^*$$

$$\psi_{ni}^* = \left\langle 0 | \hat{O}_i | n \right\rangle$$

$$\Rightarrow C(t) v_n(t, t_0) = \lambda_n(t, t_0) C(t_0) v_n(t, t_0)$$

$$\lambda_n(t, t_0) = e^{-E_n(t - t_0)} + e^{-E_n(L_t - (t - t_0))}$$

Important points:

- Systematic error in nth energy state: $\epsilon_n \sim e^{-(E_{N+1} - E_n)t}$ if $t_0 \geq t/2$[2]

Extracting Spectra (cont'd)

- Operator basis is composed of single particle operators $\overline{q}q$, $\overline{q}\gamma_\mu q$ and two particle operator $\overline{q}\gamma_5 q$, with various momentum combinations and non-zero $\vec{p}_{CM}$ up to $\pm(1,1,1)$.
- Operators are projected onto A_1 irrep and definite isospin $(0,1,2)$.
- t_0 is arbitrary, we fix it to be either $\left\lceil \frac{t}{2} \right\rceil$ or $t-1$ (if later times aren't very noisy)
- GEVP also exists for matrix elements [3]. We plan to use this for $K \to \pi\pi$.

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Lattice Details

- $V = 24^3$ ID lattice, 1.015 GeV lattice spacing
- $L_t = 64$
- $2+1$ flavor Mobius Domain Wall Fermions (generated by RBC/UKQCD)
- ~ 5 fm box
- Physical quark mass, unquenched (no chiral extrapolation needed)

Timing Summary

For our 24^3 run, we compute on 32 KNL nodes (64 cores, 192 GB memory) for 20 hours (current wall time). Timing breakdown (of expensive steps)

- Lanczos - low mode eigenvectors (amortized, not in an individual config run) - 6 hours
- 400 iterations of Zmobius Split CG ($ls = 12$) 1.3 hours (split-MADWF$=$ 6 hours)
- π meson field computation (all-to-all propagators[4]) 1 hour
- $\pi\pi \rightarrow \pi\pi$ contractions 10 hours
- σ meson field 1 hour
- ρ meson field 3 hours (3 polarizations, projected)
- σ (scalar) contractions 1 hour
- ρ (vector) contractions 2 hour

Our time is dominated not by inversions, but contractions. We are still improving this number.

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Momentum Combinations

Compute pions with $p_{lat,max} = \pm(1,1,1)$ (also permutations) in units of $\frac{2\pi}{L}$.

- 27 possible individual pion momenta.
- Cube this if you want to get a rough estimate of the combinations allowed
- Using momenta for irreps we care about (e.g. A_1) and crossing symmetry, we get 13890 separate $\pi\pi \to \pi\pi$ correlators.
- $\overline{q}q, \overline{q}\gamma^{\mu}q$ correlators are not included in this number.
- Initial run did not include enough of the correlators (recent bug found in crossing symmetry code) $\to$ spin contamination, but this is now fixed.

Auxiliary Diagrams

We have large number of momentum possibilities we could compute, but are all of them necessary? It turns out we can reduce this number by exactly a factor of 2 via a labeling symmetry between source and sink. If we swap these, we will get the exact same correlation function (with time valued quantities taken modulo the box size in time) up to an overall phase. Stated again, we have

Rule of Thumb:

Two lattice correlation functions are bit-by-bit identical (up to time reordering, phase) if they are related via a swap of source and sink labels.

We refer to the diagram derived (at the analysis stage) via this auxiliary symmetry as the auxiliary diagram[1]. In practice, our data set is not fully symmetric under auxiliary symmetry, so our (projected) gain is only 3/2.

[1]Hoying 2017, unpub.

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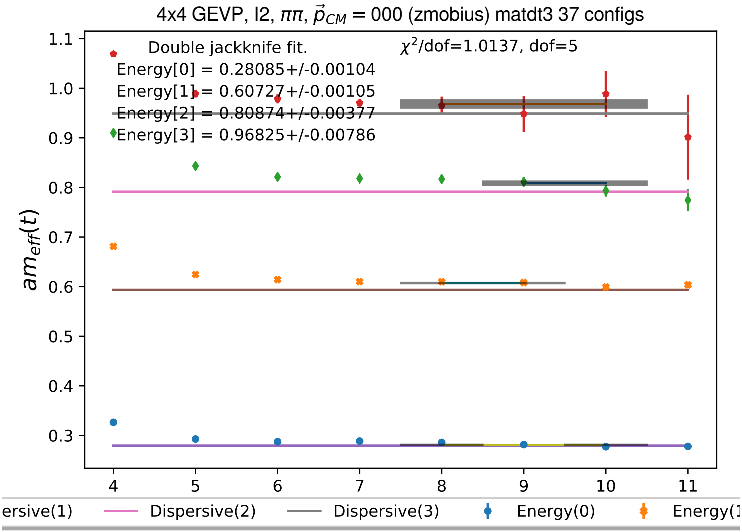
Caveat Emptor

- We must still calculate the correction from Zmobius→Mobius (these corrections are expected to be small).
- We go beyond 4π threshold, but experimental amplitude is small, so we are probably safe to neglect until higher energies (Lüscher formula does not exist for inelastic processes)
- Unmeasured systematic error due to having only one lattice spacing/size.

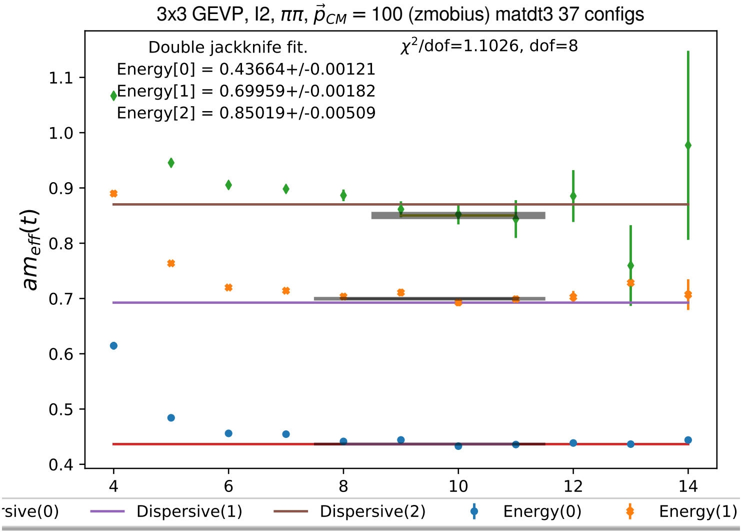
Now, results:

$I = 2,\ \vec{p}_{CM} = 0$

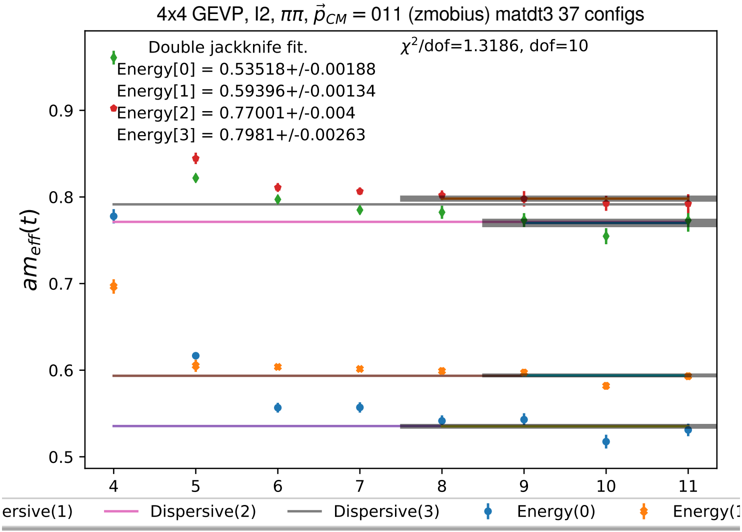


4x4 GEVP, I2, ππ, $\vec{p}_{CM}$ = 000 (zmobius) matdt3 37 configs
Double jackknife fit.
Energy[0] = 0.28085+/-0.00104
Energy[1] = 0.60727+/-0.00105
Energy[2] = 0.80874+/-0.00377
Energy[3] = 0.96825+/-0.00786
χ^2/dof=1.0137, dof=5
$am_{eff}(t)$
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
4
5
6
7
8
9
10
11
ersive(1)
Dispersive(2)
Dispersive(3)
Energy(0)
Energy(1

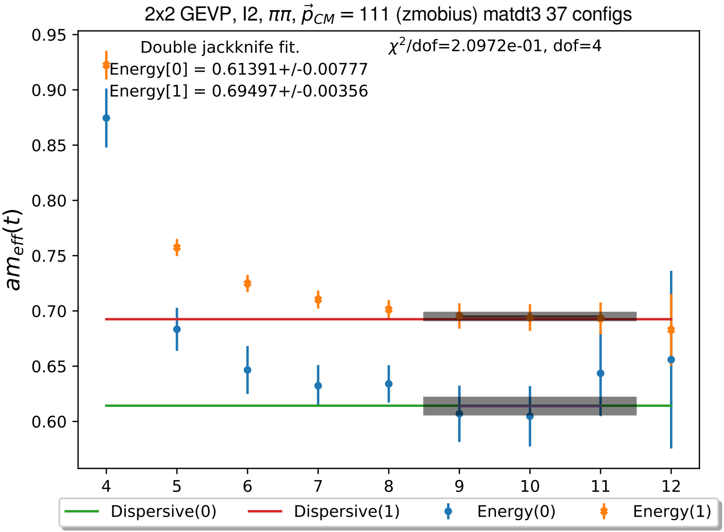
$I = 2, \ \vec{p}_{CM} = 001$



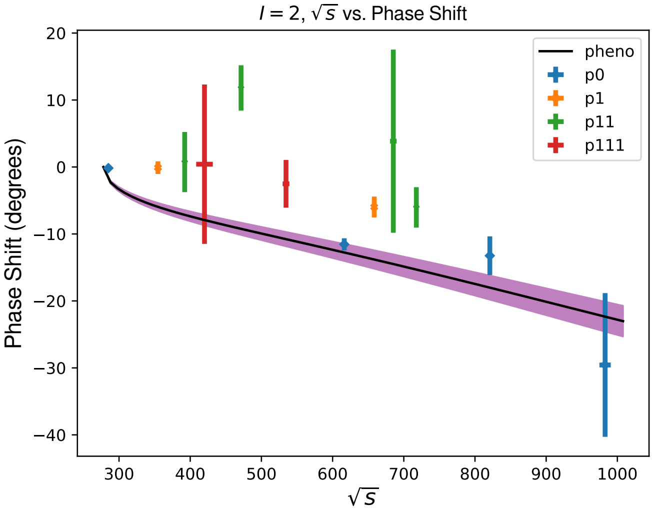
$I = 2, \ \vec{p}_{CM} = 011$



$I = 2, \ \vec{p}_{CM} = 111$

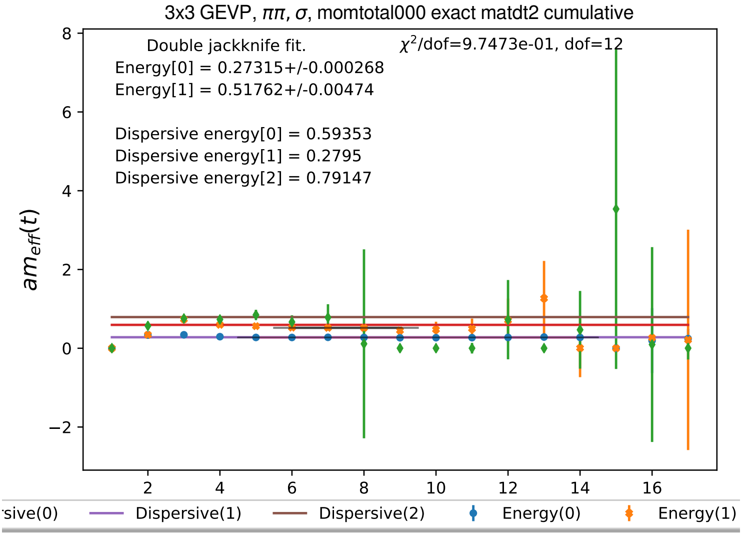


$I = 2$ phase shifts



$I = 0$, 3x3, Spin Contaminated

Ignore the tiny error bar green points past $t = 8$ and yellow past $t = 12$



Outline

Summary and Outlook

We have run on 170 gauge configurations and generated promising partial results. This is the first calculation of pipi scattering spectra from the lattice at physical pion mass.

- We have also run on 30+ configs of new, complete data and generated improved preliminary results, ($I = 1$ soon).
- $K \to \pi\pi$ periodic code ready to be tested statistically
- We will run next (fiscal) year on 32^3 lattice ($1, 1.4$ GeV lattice spacing) to test continuum limit
- Distillation study proposed as well
- Much of the analysis code and production code for these present and future runs is finished (but under-tested).
- $O(200)+$ configs likely needed to resolve $I = 0$, more a2a noise samples may be needed for σ.

Pipi data is available on request in standard binary format hdf5. My analysis code is also publicly available on github:
github.com/goracle/lattice-fitter

Thanks!

The RBC & UKQCD collaborations

BNL and BNL/RBRC

Mattia Bruno
Taku Izubuchi
Yong-Chull Jang
Chulwoo Jung
Christoph Lehner
Meifeng Lin
Aaron Meyer
Hiroshi Ohki
Shigemi Ohta (KEK)
Amarjit Soni

UC Boulder

Oliver Witzel

Columbia University

Ziyuan Bai
Norman Christ
Duo Guo
Christopher Kelly
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Masaaki Tomii
Jiqun Tu
Bigeng Wang
Tianle Wang
Evan Wickenden
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Tadeusz Janowski
Richard Kenway
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Brian Pendleton
Antonin Portelli
Tobias Tsang
Azusa Yamaguchi

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James Richings
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Renwick Hudspith

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All-to-All Propagators

$$D_{A2A}^{-1} \equiv \sum_{l=0}^{N_l-1} |\phi_l\rangle \frac{1}{\lambda_l} \langle\phi_l| + \sum_{h=0}^{N_h-1} \underbrace{\left(D^{-1} - \sum_{l=0}^{N_l-1} |\phi_l\rangle \frac{1}{\lambda_l} \langle\phi_l| \right)}_{D_{Defl}^{-1}} |\eta_h\rangle \langle\eta_h|$$

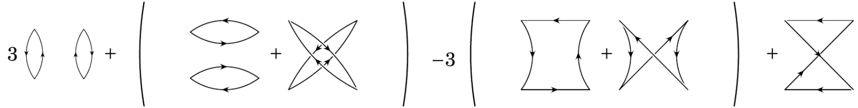
$$\mathbb{I} = \lim_{N_h\to\infty} |\eta_h\rangle \langle\eta_h| \,, \quad \Rightarrow \lim_{N_h\to\infty} D_{A2A}^{-1} = D^{-1}$$

- Deflate with 2000 low modes $|\phi_l\rangle$, Zmobius eigenvectors
- In practice, set $N_h = 1$ (more hits improve excited state noise. Gauge noise dominates lower energy states.)
- $12 * L_t * N_h$ high modes (spin, color, time diluted) $\rightarrow$ 768 high modes.
- We obtain the exact point-to-point propagator in this stochastic limit.[4]

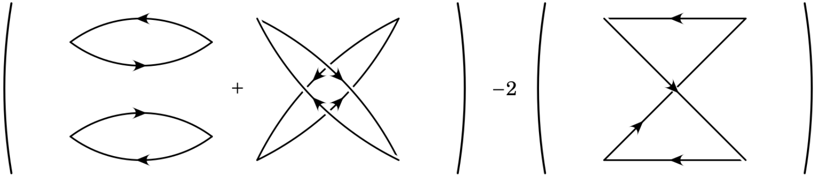
Operator Construction: Isospin Projection

On our lattices $(2+1)$, isospin is a good symmetry. We want to know the spectra of the different isospin channels $I = 0, 1, 2$ Examples below. Note the disconnected diagram in $I = 0$ which is very noisy, but important (needs large statistics).

$\langle I = 0 | I = 0 \rangle$:

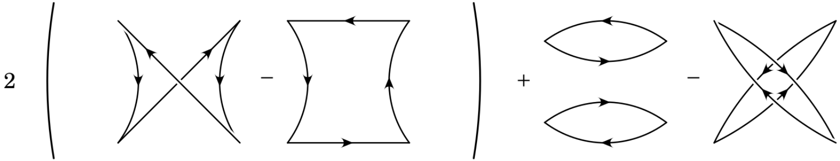


$\langle I = 2 | I = 2 \rangle$:



Isospin (cont'd)

$\langle I = 1 | I = 1 \rangle$:



Operator Construction:Spin Irrep Projection

- We would like to project our operator set onto the lowest spin in each isospin channel (K is spin 0).
- Continuum spin states have known correspondence to irreps of the group of allowed lattice rotations O (we can project continuum representations to lattice irreps)
- Each irrep in general corresponds to a tower of spin states
- We project onto irreps with the lowest spins $\rightarrow$ easier to resolve
- Bose symmetry $\Rightarrow$ $I = 1$ needs p-wave irrep: T_1
- $I = 0, 2$ needs s-wave irrep: A_1

Motivation: $K \to \pi\pi$

- Likely explanation for matter/antimatter asymmetry in Universe, baryogenesis, requires violation of CP.
- Amount of CPV in Standard Model appears too low to describe measured M/AM asymmetry: tantalizing hint of new physics.
- Direct CPV first observed in late 90s at CERN (NA31/NA48) and Fermilab (KTeV) in $K^0 \to \pi\pi$:

$$\eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} \quad \eta_{\pm} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)}$$

$$Re(\frac{\epsilon'}{\epsilon}) = \frac{1}{6}\left(1 - \left|\frac{\eta_{00}}{\eta_{\pm}}\right|^2\right) = 1.66(23)\ x\ 10^{-3}(\text{Experiment})$$

$K \to \pi\pi$ (cont'd.)

In terms of isospin states,

$\Delta I = 3/2$ decays to $I = 2$ final states, amplitude A_2
$\Delta I = 1/2$ decays to $I = 0$ final states, amplitude A_0

$$A(K^0 \to \pi^+\pi^-) = \sqrt{\frac{2}{3}} A_0 e^{i\delta_0} + \sqrt{\frac{1}{3}} A_2 e^{i\delta_2}$$

$$A(K^0 \to \pi^0\pi^0) = \sqrt{\frac{2}{3}} A_0 e^{i\delta_0} - 2\sqrt{\frac{1}{3}} A_2 e^{i\delta_2}$$

$$\Rightarrow \epsilon' = \frac{i\omega e^{i(\delta_2 - \delta_0)}}{\sqrt{2}} \left(\frac{Im\ A_2}{Re\ A_2} - \frac{Im\ A_0}{Re\ A_0} \right) \quad \boxed{\omega = \frac{ReA_2}{ReA_0}}$$

Small size of ϵ' makes it particularly sensitive to new direct-CPV introduced by most BSM models.

Infinite Volume Scattering Phase Shifts

We can then compare to experiment via phenomenological data on phase shift vs. energy.

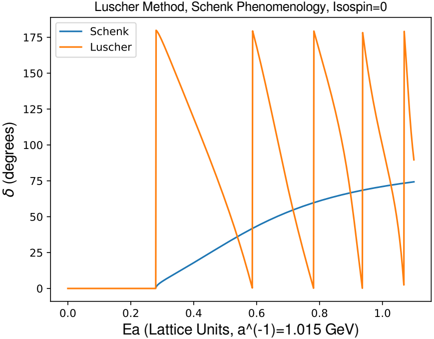


Figure: Phenomenology* v. Luscher, predictions for $I = 0$ on 24c, $a^{-1} = 1.015$ GeV. The ultimate goal is to obtain enough phase shift points to fit to a Breit-Wigner form and extract the mass and resonance width.

(*=phenomology data is outdated, useful for illustrative purposes only)