Controlling Excited-State Contributions to Nucleon Isovector Charges using Distillation

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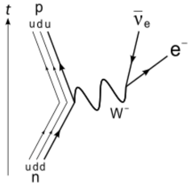
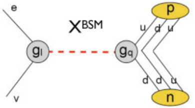
Why Calculate Nucleon Isovector Charges?

- $n \to pe^-\overline{\nu}_e$ coupling & proton-proton fusion
- $g_A^{u-d} = \langle 1 \rangle_{\Delta u - \Delta d}$
- $g_T^{u-d} = \langle 1 \rangle_{\delta u - \delta d}$
- g_S^{u-d} as related to DM direct detection
- novel BSM interactions at TeV scale





[LANL: T-2 Nuclear Physics]

Ab-initio QCD

- Despite actual QCD on Lattice ... inherent systematics
  1. finite lattice spacing a effects [J. Liang, et al., arXiv:hep-lat/1612.04388 Y.-B. Yang, et al., Phys. Rev. **D93**, 034503 (2016), arXiv:hep-lat/1509.
  2. volume dependence V [G. S. Bali, et al., Phys. Rev. **D91**, 054501 (2015).] [R. Horsley, et al., Phys. Lett. **B732** 41 (2014).]
  3. pion mass m_π [S. Ohta (LHP, RBC, UKQCD collabs.), arXiv:/hep-lat/1511.05126]
  4. number of quark flavors [H.-W. Lin, et al., arXiv:hep-lat/1806.10604].
  5. chiral symmetry [R. Gupta, et al., arXiv:hep-lat/1806.09006]. [R. Gupta, et al., arXiv:hep-lat/1801.03130]
  6. signal-to-noise [C. C. Chang, et al., Nature **558**, 91-94 (2018).]
  7. excited states

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Excited-State Contamination

$$C_{\mathsf{2pt}}\left(t\right) = \sum_{\vec{x}} \langle \mathcal{O}\left(\vec{x},t\right) \overline{\mathcal{O}}(\vec{0},0)\rangle = \sum_{n} \frac{1}{2M_n} \left|\langle 0|\,\mathcal{O}\,|n\rangle\right|^2 e^{-M_n t}$$

- Interpolating fields of point-like quark/gluon fields
  - hadronic states at all energy scales
- For ground-state quantities
  - large-Euclidean time
  - judicial choice of $\mathcal{O} \quad \rightarrow \quad \langle 0|\,\mathcal{O}\,|n\rangle\big|_{n>0} \ll \langle 0|\,\mathcal{O}\,|n=0\rangle$

Spatial Smearing

$$\tilde{\psi}(\vec{x},t) = \sum_{\vec{y}} S[U](\vec{x},\vec{y})\psi(\vec{y},t)$$

- Jacobi smearing

[C. Allton et al. (UKQCD Collaboration), Phys. Rev. D47, 5128 (1993).]

$$S[U]\left(\vec{x},\vec{y}\right) = J_{\sigma,n_\sigma}\left(t\right) \equiv \left(1 + \frac{\sigma\nabla^2\left(t\right)}{n_\sigma}\right)^{n_\sigma}$$
$$\longrightarrow \quad \text{Exp}\left(\sigma\nabla^2\left(t\right)\right)$$

- exp. suppression of high eigenmodes of Laplacian
  - lowest modes contribute appreciably

$$\tilde{\psi}\left(\vec{x},t\right) = J_{\sigma,n_\sigma}\left(t\right)\psi\left(\vec{x},t\right)$$

Distillation

- solutions to $-\nabla^2 \xi^{(k)} = \lambda^{(k)} \xi^{(k)}$ - ordered by $\lambda^{(k)}$

$$-\nabla^2_{ab}\left(\vec{x},\vec{y};t\right) = 6\delta_{xy}\delta_{ab} - \sum_{j=1}^{3}\left[\tilde{U}_j\left(\vec{x},t\right)_{ab}\delta_{x+\hat{j},y} + \tilde{U}_j^{\dagger}\left(\vec{x}-\hat{j},t\right)_{ab}\delta_{x-\hat{j},y}\right]$$

- define *Distillation* op. of rank $N \ll M = N_c \times N_X \times N_Y \times N_Z$

$$\square\left(\vec{x},\vec{y};t\right)_{ab} = \sum_{k=1}^{N} \xi_a^{(k)}\left(\vec{x},t\right)\xi_b^{(k)\dagger}\left(\vec{y},t\right)$$

[M. Peardon, et. al., (2009) arXiv: 0905.2160v1]

Distillation - Properties & Advantages

- interpolator construction separable from quark propagation
- can construct interpolators to probe angular structure of states w/o recalculating $M^{-1}(t, t')$
- momentum projection at source & sink

$$C\left(t', t\right) = \sum_{\vec{x},\vec{y}} e^{-i\vec{p}\cdot\vec{x}} e^{i\vec{p}\cdot\vec{y}} \langle \mathcal{O}\left(\vec{x}, t'\right) \overline{\mathcal{O}}\left(\vec{y}, t\right) \rangle$$

- number of eigenvectors scales with V_3
  - baryon contractions $\mathcal{O}\left(V_3^4\right)$

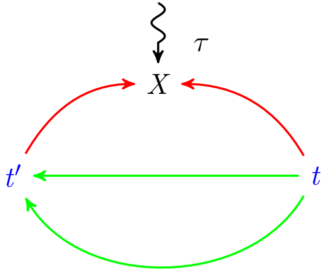
Components of a Calculation using Distillation

- **Solution Vectors**

$$S^k\left(\vec{x},t';t\right)=M_f^{-1}\left(t',t\right)\xi^{(k)}\left(t\right)$$

- **Perambulators**

$$\tau_{\alpha\beta}^{kl}\left(t',t\right)=\xi^{(k)\dagger}\left(t'\right)M_{\alpha\beta}^{-1}\left(t',t\right)\xi^{(l)}\left(t\right)$$



- **Elementals** - operator insertions in Distillation space

$$\Phi_{\alpha_1,\alpha_2,\alpha_3}^{(i,j,k)}\left(t\right)=\epsilon^{abc}\left(\mathcal{D}_1\xi^{(i)}\right)^a\left(\mathcal{D}_2\xi^{(j)}\right)^b\left(\mathcal{D}_3\xi^{(k)}\right)^c\left(t\right)S_{\alpha_1,\alpha_2,\alpha_3}$$

- Correlators comprised of perambulators and elementals

Variational Analysis

- $\mathcal{B} = \{\mathcal{O}_1, \ldots, \mathcal{O}_N\}$
- optimal linear combination of interpolators to project onto $|\mathrm{n}\rangle$, solve

$$C(t)\, u_{\mathrm{n}}(t, t_0) = \lambda_{\mathrm{n}}(t, t_0)\, C(t_0)\, u_{\mathrm{n}}(t, t_0)$$

- solved for fixed reference time t_0 and all later times t
- solutions organized by magnitude of eigenvalues $\lambda_{\mathrm{n}}(t, t_0)$ - principal correlators $\lambda_{\mathrm{n}}(t, t_0) \underset{t \gg t_0}{\sim} e^{-m_{\mathrm{n}}(t-t_0)}$
- components of eigenvectors $u_{\mathrm{n}}(t, t_0)$ yield weight of each $\mathcal{O}_i \in \mathcal{B}$ to interpolate $|\mathrm{n}\rangle$ from vacuum
- projected operator $\equiv$ $\mathcal{O}^{\mathrm{n}\dagger}_{\text{proj}} = \sum_i u^i_{\mathrm{n}} \mathcal{O}^\dagger_i$

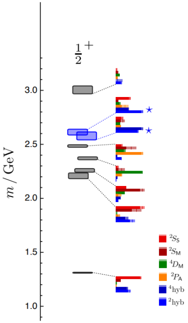
[B. J. Owen, et al., Phys. Lett. **B723**, 217 (2013).] $\sim 8\%$ improvement

Operator Construction

$$\mathcal{O}_i\left(t\right) \propto \epsilon^{abc}\mathcal{S}_i^{\alpha\beta\gamma}\left(\mathcal{D}_1\Box d\right)_a^\alpha\left(\mathcal{D}_2\Box u\right)_b^\beta\left(\mathcal{D}_3\Box d\right)_c^\gamma\left(t\right)$$

- $\left(N_M \otimes \left(\frac{1}{2}^+\right)^1_M \otimes D^{[0]}_{L=0,S}\right)^{J^P=\frac{1}{2}^+}$
- $\left(N_M \otimes \left(\frac{1}{2}^+\right)^1_M \otimes D^{[2]}_{L=0,M}\right)^{J^P=\frac{1}{2}^+}$
- $\left(N_M \otimes \left(\frac{1}{2}^+\right)^1_M \otimes D^{[2]}_{L=0,S}\right)^{J^P=\frac{1}{2}^+}$
- $\left(N_M \otimes \left(\frac{1}{2}^+\right)^1_M \otimes D^{[2]}_{L=1,A}\right)^{J^P=\frac{1}{2}^+}$
- $\left(N_M \otimes \left(\frac{1}{2}^+\right)^1_M \otimes D^{[2]}_{L=1,M}\right)^{J^P=\frac{1}{2}^+}$
- $\left(N_M \otimes \left(\frac{3}{2}^+\right)^1_S \otimes D^{[2]}_{L=1,M}\right)^{J^P=\frac{1}{2}^+}$
- $\left(N_M \otimes \left(\frac{3}{2}^+\right)^1_S \otimes D^{[2]}_{L=2,M}\right)^{J^P=\frac{1}{2}^+}$

$$N^{2S+1}L_{\mathcal{P}}J^P$$



N spectrum ordered by J^P for $m_\pi \sim 520$ MeV
[J. J. Dudek and R. G. Edwards (2012) arXiv:hep-ph/1201.2349]
[R. Edwards, et. al., (2011) arXiv: 1104.5152v2]

Lattice & Operator Specifics

- $32^3 \times 64$ isoclover lattices ($\beta = 6.3$) - 350cfgs
- $u/d + s$ flavor QCD $\quad m_s \rightsquigarrow \left(2M^2_{K+} - M^2_{\pi^0}\right)/M^2_{\Omega^-} = 0.1678$
- tree-level tadpole-improved Symanzik gauge action
- $m_\pi = 356$ MeV
- $a = 0.098$ fm

$$\left.\begin{matrix} \\ \\ \end{matrix}\right\} \quad m_\pi L \sim 5.7 \qquad m_\pi T \sim 11$$

- Distilled operators
  - 64 eigenvectors
- **Our Aim**: rather than precise determination of g_A, g_S, g_T, can Distillation demonstrate controlled excited-states?

First application of Distillation to structure calculations

2pt Decomposition

- Direct comparisons between 'local' ($N^2 S_S \frac{1}{2}^+$) and 'proj' ($\mathcal{O}^{\text{n}\dagger}_{\text{proj}} = \sum_i u^i_{\text{n}} \mathcal{O}^\dagger_i$)

$$C_{\text{2pt}}\left(\vec{p},t\right) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \sum_{\vec{y}} e^{i\vec{p}\cdot\vec{y}} \langle \mathcal{O}\left(\vec{x},t\right) \mathcal{O}^\dagger\left(\vec{y},0\right)\rangle$$

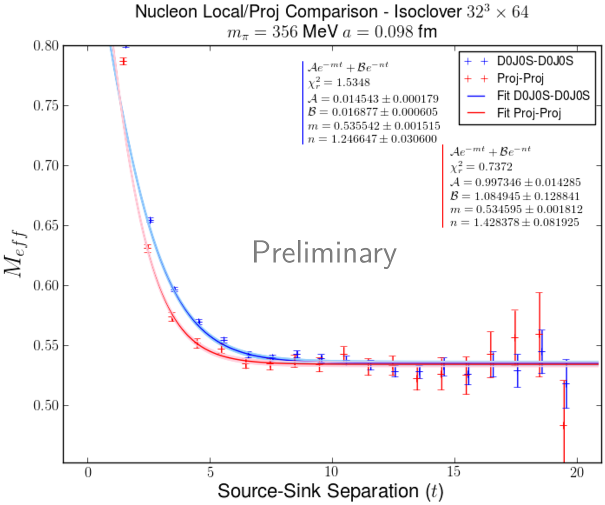
- For zero-momentum states, keeping ground and first-excited states:

$$C_{\text{2pt}}\left(t\right) = V_3 \left(\frac{\left|Z_0\right|^2}{2m_0} e^{-m_0 t} + \frac{\left|Z_1\right|^2}{2m_1} e^{-m_1 t} \right)$$

- Simple 2-state fit for masses/overlap factors

$$C_{\text{2pt}}\left(t\right) = \mathcal{A} e^{-m_0 t} + \mathcal{B} e^{-m_1 t}$$

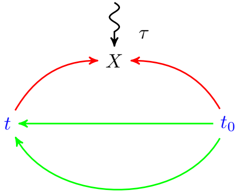
Effective Mass Comparison - Two State



3pt Decomposition

$$C_{\text{3pt}}\left(\vec{p}\,',t;\vec{q},\tau;t_0\right) = V_3\sum_{\vec{x}} e^{-i\vec{p}\,'\cdot\vec{x}}\sum_{\vec{z}} e^{i\vec{q}\cdot\vec{z}}\left\langle 0\right|\mathcal{O}\left(\vec{x},t\right)\mathcal{J}\left(\vec{z},\tau\right)\overline{\mathcal{O}}\left(t_0\right)\left|0\right\rangle$$

- For zero-momentum states at fixed t_{sep}



$$C_{\text{3pt}}\left(\tau\right) = \left(\frac{V_3\left|Z_0\right|^2}{4m_0^2}\mathcal{J}_{00}\,e^{-m_0 t_{\text{sep}}} + \frac{V_3\left|Z_1\right|^2}{4m_1^2}\mathcal{J}_{11}\,e^{-m_1 t_{\text{sep}}}\right)$$

$$+\frac{V_3 Z_0 Z_1^\dagger}{4m_0 m_1}\mathcal{J}_{01}\,e^{-m_0 t_{\text{sep}}}\,e^{-(m_1-m_0)\tau} + \frac{V_3 Z_1 Z_0^\dagger}{4m_0 m_1}\mathcal{J}_{10}\,e^{-m_1 t_{\text{sep}}}\,e^{(m_1-m_0)\tau}$$

- Fit form: $C_{\text{3pt}}\left(\tau\right) = \mathscr{C} + \mathscr{D}\cosh\left[\Delta m\left(\tau - t_{\text{sep}}/2\right)\right]$

Extraction of Ground-state Matrix Element

$$\mathscr{C} = \frac{V_3 \left|Z_0\right|^2}{4m_0^2} \mathcal{J}_{00}\, e^{-m_0 t_{\mathsf{sep}}} + \frac{V_3 \left|Z_1\right|^2}{4m_1^2} \mathcal{J}_{11}\, e^{-m_1 t_{\mathsf{sep}}}$$

- Fit t_{sep}-dependence of $\mathscr{C}$ to extract $\mathcal{J}_{00}$

$$\mathscr{C}\left(t_{\mathsf{sep}}\right) = Xe^{-m_0 t_{\mathsf{sep}}} + Ye^{-m_1 t_{\mathsf{sep}}}$$
$$\Rightarrow \mathcal{J}_{00} \propto X$$

- "effective charges"

$$g_\Gamma^{\mathsf{eff}}\left(t_{\mathsf{sep}}, \tau\right) = \frac{C_\Gamma^{\mathsf{3pt}}\left(t_{\mathsf{sep}}, \tau\right)}{C_{\mathsf{fit}}^{\mathsf{2pt}}\left(t_{\mathsf{sep}}\right)}$$

Extraction of Ground-state Matrix Element

$$\mathscr{C} = \frac{V_3 \left|Z_0\right|^2}{4m_0^2} \mathcal{J}_{00} e^{-m_0 t_{\text{sep}}} + \frac{V_3 \left|Z_1\right|^2}{4m_1^2} \mathcal{J}_{11} e^{-m_1 t_{\text{sep}}}$$

- Fit t_{sep}-dependence of $\mathscr{C}$ to extract $\mathcal{J}_{00}$

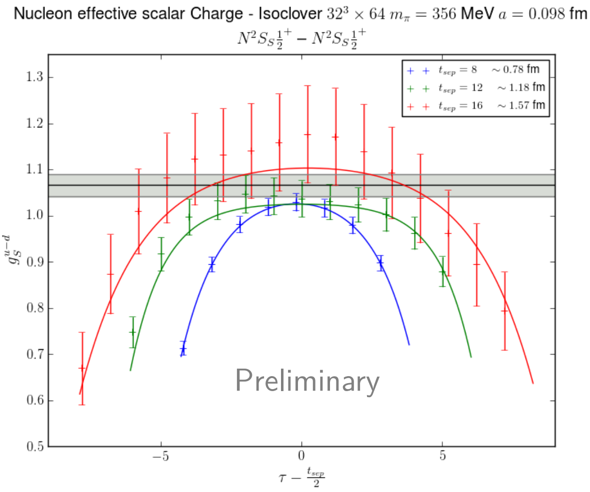
$$\mathscr{C}\left(t_{\text{sep}}\right) = Xe^{-m_0 t_{\text{sep}}} + \boxed{Ye^{-m_1 t_{\text{sep}}}}$$
$$\Rightarrow \mathcal{J}_{00} \propto X$$

little effect in extracting $\mathcal{J}_{00}$...

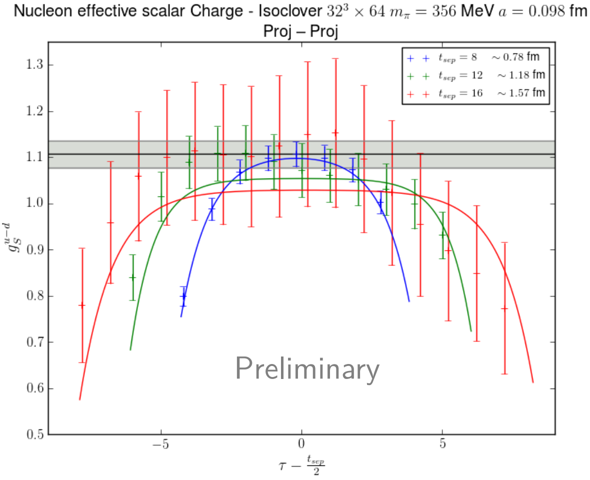
- "effective charges"

$$g_{\Gamma}^{\text{eff}}\left(t_{\text{sep}}, \tau\right) = \frac{C_{\Gamma}^{\text{3pt}}\left(t_{\text{sep}}, \tau\right)}{C_{\text{fit}}^{\text{2pt}}\left(t_{\text{sep}}\right)}$$

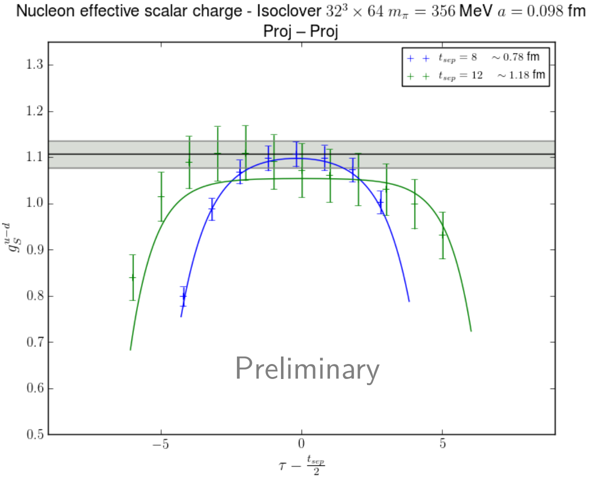
Determination of g_S - Local Operator



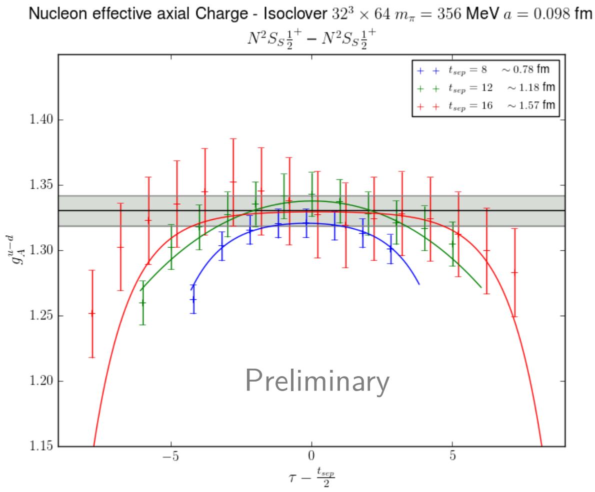
Determination of g_S - Projected Operator



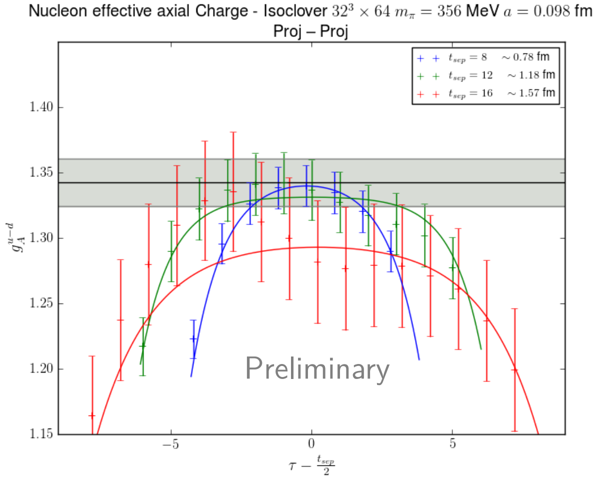
Determination of g_S - Projected Operator (w/o $t_{\text{sep}} = 16$)



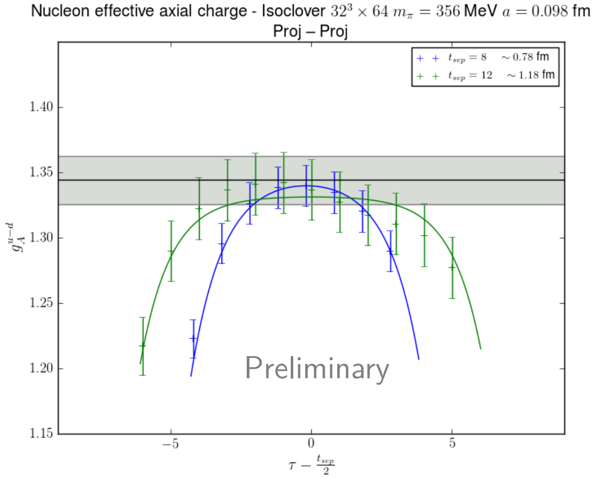
Determination of g_A - Local Operator



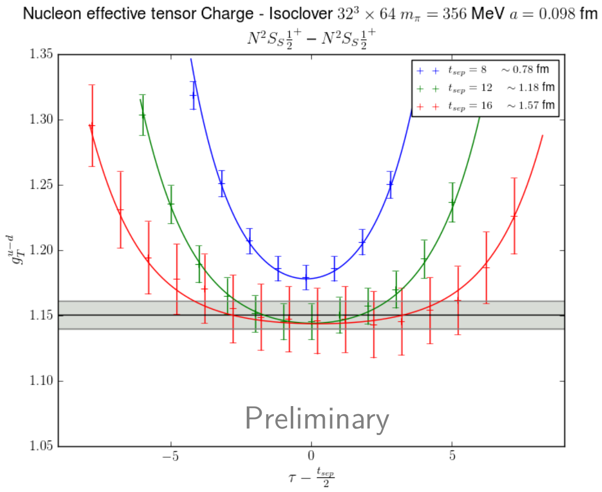
Determination of g_A - Projected Operator



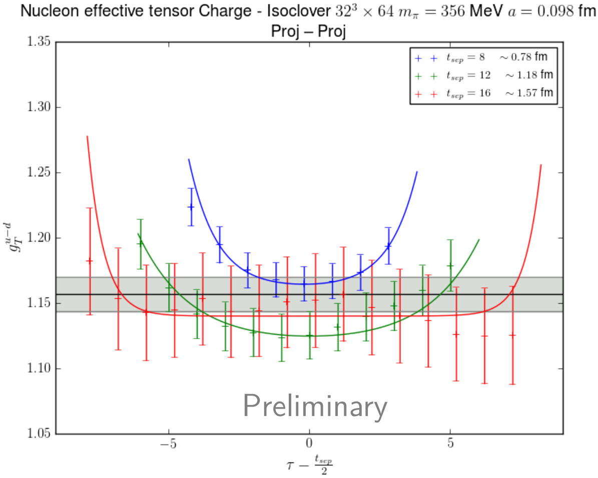
Determination of g_A - Projected Operator (w/o $t_{\text{sep}} = 16$)



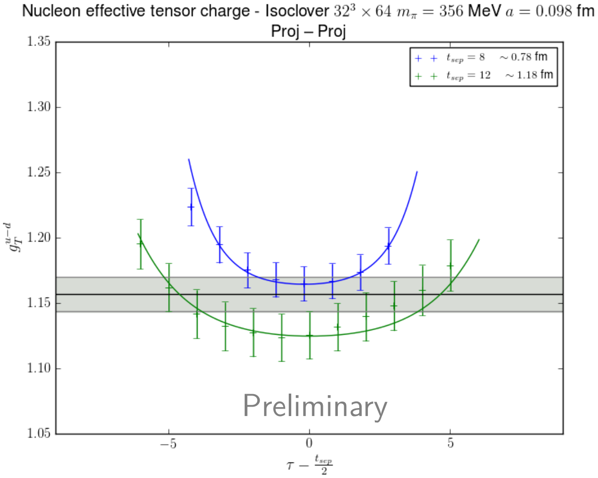
Determination of g_T - Local Operator



Determination of g_T - Projected Operator



Determination of g_T - Projected Operator (w/o $t_{\text{sep}} = 16$)



Closing Thoughts

- Success of Distillation in spectroscopy begs study when considering structure
- Variational method applied to an extended basis of Distilled interpolators
  - effective mass plateaus for much smaller source-sink separations ~ 0.6 fm $\rightarrow$ better statistics
  - clear separation of ground & excited states - c.f. projected 2-state fit
- $C_{\text{3pt}}\left(t_{\text{sep}}, \tau\right) / C_{\text{2pt}}\left(t_{\text{sep}}\right)$ plateaus
  - little src-snk dependence when using variationally optimized nucleon interpolators **noisy $t_{\text{sep}=16}$
- Considered forward-scattering of at rest nucleons
  - $\vec{p} \neq \vec{0}$ states - address, say, g_{A_i} and g_{A_4} [J. Liang, et al., arXiv/1612.04388.]
- $\mathcal{Q}^2$ axial/vector form factors? Other structure calculations?

BACKUPS

Principal Correlators - $t_0 = 2, 3$

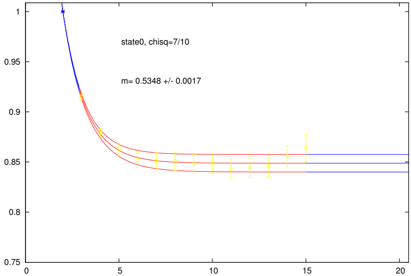


Figure 2: Ground-state principal correlator for $t_0 = 2$ and $t_Z = 5$.

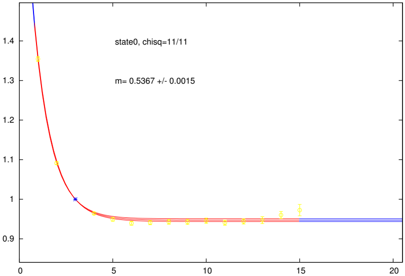


Figure 3: Ground-state principal correlator for $t_0 = 3$ and $t_Z = 5$.

Method for renormalized g_Γ Extraction

- Relation between continuum and lattice charges

$$g_\Gamma = Z_\Gamma g_\Gamma^{\text{lat}}$$

- Z_V set based on knowledge of continuum vector charge
  - i.e. $1 + \mathcal{O}\left(a^2\right) = Z_V g_V^{\text{lat}} \quad \Longrightarrow \quad Z_V = 1/g_V^{\text{lat}} + \mathcal{O}\left(a^2\right)$
- Utilized definition of arbitrary renormalized charge

$$g_\Gamma = \frac{Z_\Gamma}{Z_V}\frac{g_\Gamma^{\text{lat}}}{g_V^{\text{lat}}}$$

- Z_Γ/Z_V on isoclover lattices with $a = 0.094$ fm and $m_\pi = 280$ MeV
  - n.b. Z_Γ appears to be weakly dependent on pion mass, but depends on lattice spacing [B. Yoon, et. al., (2017) arXiv: 1611.07452v3]