# On the calculation and use of non-zero momentum correlators in lattice simulations.

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with the LSD collaboration

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# The Lattice Strong Dynamics Collaboration

The Lattice Strong Dynamics (LSD) Collaboration was formed in 2007, to pursue non-perturbative studies of strongly interacting theories likely to produce observable signatures at the Large Hadron Collider or other experiments searching for new physics beyond the Standard Model.

Membership:

Argonne National Laboratory: Xiao-Yong Jin, James Osborn
Boston University: Richard Brower, Claudio Rebbi, Evan Weinberg
Lawrence Livermore National Laboratory: Pavlos Vranas
RIKEN-BNL Research Center (RBRC): Enrico Rinaldi
University of Bern: David Schaich
University of California, Davis: Joseph Kiskis
University of Colorado: Anna Hasenfratz, Ethan Neil (joint w/RBRC), Oliver Witzel
University of Oregon: Graham Kribs
Yale University: Thomas Appelquist, Kimmy Cushman, George Fleming, Andy Gasbarro

# Calculation of scalar singlet correlators

The disconnected component involves a correlation of local operators:

$$D(t) = (1/N_x N_t) \sum_{\vec{x},\vec{y},t'} \langle \phi(\vec{x}, t') \phi(\vec{y}, t' + t) \rangle$$

with

$$\phi(\vec{x}, t) = \bar{\psi}(\vec{x}, t) \psi(\vec{x}, t)$$

(I will neglect normalization factors from now on.)

# Statistical fluctuations

Recap without normalization:

$$D(t) = \sum_{\vec{x},\vec{y},t'} \langle \phi(\vec{x},t')\phi(\vec{y},t'+t) \rangle$$

For large separation $\phi(\vec{x},t'), \phi(\vec{y},t'+t)$ are essentially uncorrelated, but still contribute to the noise.

As lattices become larger the problem becomes more severe,
e.g. with $N_x = 256$ for $t = 32$ the separation between
$\phi(\vec{x},t'), \phi(\vec{y},t'+t)$ ranges from 32 to $\sqrt{32^2 + 3 \times 128^2} = 224$

We may try to reduce the statistical noise by giving higher weight to the operators facing each other.

# Weighted correlators

Replace $D(t)$ with

$$D_G(t) = \sum_{\vec{x},\vec{y},t'} \langle \phi(\vec{x},t') G(\vec{y}) \phi(\vec{x}+\vec{y},t'+t) \rangle$$

where $G(\vec{y})$ is a weight function (e.g. a Gaussian) falling off with distance. This may reduce the statistical noise at the expense of the introduction of non-zero momentum components.

Computing the above correlation is too demanding in configuration space ($N_x^6$ correlations) but can be done in momentum space.

# Using momentum components

(normalizations are still neglected)

$$\phi(\vec{x},t) = \sum_{\vec{p}} \tilde{\phi}(\vec{p},t) e^{\imath \vec{p}\cdot\vec{x}}, \quad G(\vec{x}) = \sum_{\vec{p}} \tilde{G}(\vec{p}) e^{-\imath \vec{p}\cdot\vec{x}}$$

gives

$$D_G(t) = \sum_{\vec{x},\vec{y},\vec{p},\vec{q},\vec{r},t'} \tilde{G}(\vec{q}) \langle \tilde{\phi}^*(\vec{p},t') \tilde{\phi}(\vec{r},t'+t) \rangle e^{\imath[-\vec{q}\cdot\vec{y} - \vec{p}\cdot\vec{x} + \vec{r}\cdot(\vec{x}+\vec{y})]}$$

(where we used $\phi(\vec{x},t) = \phi^*(\vec{x},t)$)

# Using momentum components

Reprise:

$$D_G(t) = \sum_{\vec{x},\vec{y},\vec{p},\vec{q},\vec{r},t'} \tilde{G}(\vec{q}) \langle \tilde{\phi}^*(\vec{p},t') \tilde{\phi}(\vec{r},t'+t) \rangle e^{\imath[-\vec{q}\cdot\vec{y}-\vec{p}\cdot\vec{x}+\vec{r}\cdot(\vec{x}+\vec{y})]}$$

Crucial point: correlators are diagonal in momentum space:
this introduces a delta function forcing $\vec{r} = \vec{p}$ and giving

$$D_G(t) = \sum_{\vec{x},\vec{y},\vec{p},\vec{q},t'} \tilde{G}(\vec{q}) \langle \tilde{\phi}^*(\vec{p},t') \tilde{\phi}(\vec{p},t'+t) \rangle e^{\imath[-\vec{q}\cdot\vec{y}+\vec{p}\cdot\vec{y})]}$$

$$= \sum_{\vec{p},t'} \tilde{G}(\vec{p}) \langle \tilde{\phi}^*(\vec{p},t') \tilde{\phi}(\vec{p},t'+t) \rangle$$

where $\tilde{\phi}(\vec{p},t)$ is calculated from the traces $\phi(\vec{x},t) = (\bar{\psi}\psi)(\vec{x},t)$:

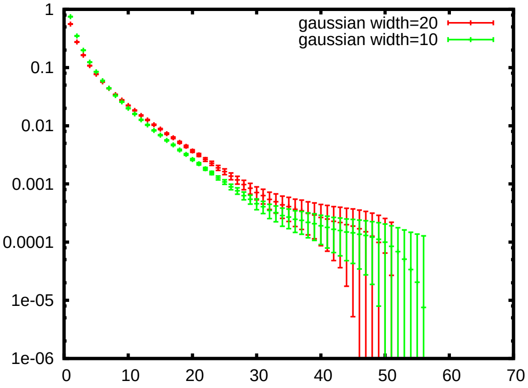
$$\tilde{\phi}(\vec{p},t) = \sum_{\vec{x}} \phi(\vec{x},t) e^{-\imath\vec{p}\cdot\vec{x}}$$

# Illustrations

All the results shown in the graphs that follow have been obtained using the traces and connected correlators calculated on SU(3), $64 \times 128$ configurations with $N_f = 8, \beta_F = 4.8, \beta_F/\beta_A = -0.25, am_f = 0.00125$, in the context of a detailed investigation of the SU(3) theory with 8 dynamical fermions done by the LSD collaboration (see Ethan Neil's talk today at 12PM and arXiv:1807.08411.)
The graphs show data for the scalar singlet correlator, given by $2D - C$, where $D$ is the disconnected correlator and $C$ is the connected correlator.

# Reduction of statistical fluctuations

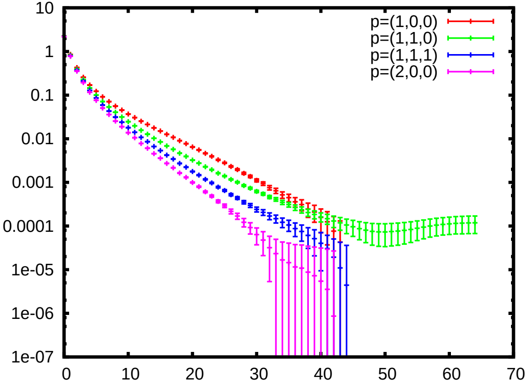
The fluctuations of the scalar singlet correlator are dominated by the very large fluctuations of the condensate, which should be subtracted. But the formalism we developed allows us to do the subtraction by simply omitting $\vec{p} = 0$ in $\sum_{\vec{p},t'} \tilde{G}(\vec{p})\langle\tilde{\phi}^*(\vec{p},t')\tilde{\phi}(\vec{p},t'+t)\rangle$:



# Focusing on the individual momentum components

Although the original motivation of the formalism was to study gaussian weighted correlators, it allows us to consider individual momentum components, which are easier to analyze.

The figure shows the four correlators with the lowest $|p|$, averaged over all component permutations.



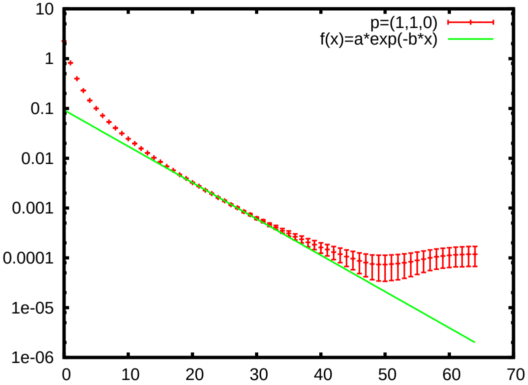
# A few results for definite momentum.

Please note: The fits in the following slides are for illustration purposes only. A proper fit of multiple momentum data has not been done yet. It may be done and published in a future paper by the LSD collaboration.

The results presented in arXiv:1807.08411 and by Ethan Neil, today at 12PM are based on a detailed, careful analysis of data with $\vec{p} = 0$ only.

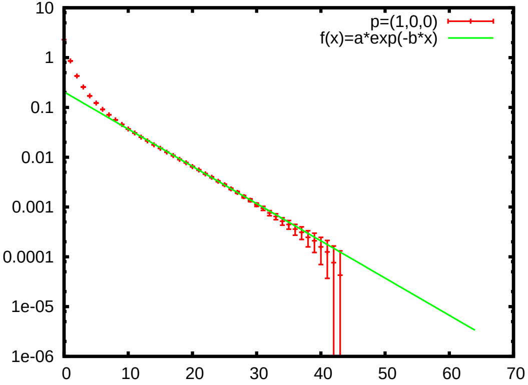
# Momentum component $\vec{p} = (1, 1, 0)$

A straightforward fit $f(t) = ae^{-bt}$, with uncorrelated errors, to the data with $\vec{p} = (1, 1, 0)$ for $20 \leq t \leq 30$ gives $b = 0.168$ corresponding to $am = 0.094$, in agreement with the value $am = 0.089(32)$ found in the LSD study for the scalar singlet ground state, which corresponds to $aE(\vec{p} = 1, 1, 0) = 0.168$.



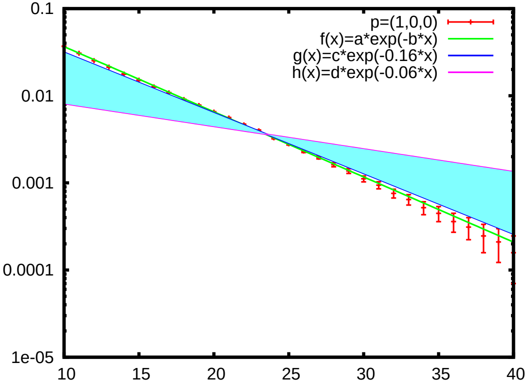
# Momentum component $\vec{p} = (1, 0, 0)$

However the same type of fit to the data with $\vec{p} = (1, 0, 0)$ gives $b = 0.172$ corresponding to $am = 0.120$, by the top of the range $am = 0.089 \pm 0.032$ found the the LSD study. This is compatible with a statistical fluctuation.



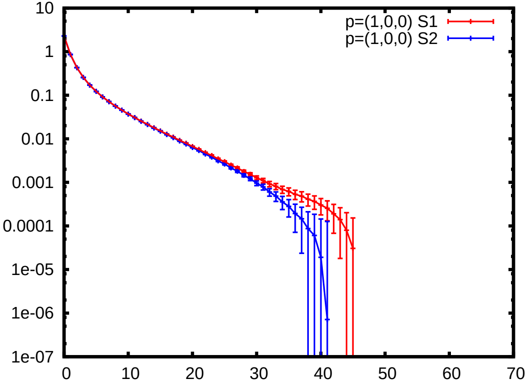
# Momentum component $\vec{p} = (1, 0, 0)$

The figure below shows fits to the coefficient only in the expression $f(t) = ae^{-bt}$ with the values of $b$ corresponding to the top and bottom of the range $am = 0.089 \pm 0.032$.



# Momentum component $\vec{p} = (1, 0, 0)$

The value of the energy coefficient observed for $\vec{p} = (1, 0, 0)$ may be due to a statistical fluctuation, as put into evidence by the correlators obtained from the configurations in two separate streams, shown below. This single data point does not invalidate the LSD result for the ground state mass $am = 0.089 \pm 0.032$, which has been obtained with a very thorough and careful analysis of the $\vec{p} = 0$ data.



# Conclusions

Calculating the momentum component of disconnected, as well as connected, correlators requires minimal extra computational effort. (It may involve the use of extra storage space, because the individual traces, and not just their sum, must be stored if the configurations space correlators are saved.)

But it can produce valuable additional insight into the spectrum of the system under study.