Towards a determination of the quark-chromo EDM with the gradient flow

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Introduction

- The quark-chromo EDM (qCEDM) is a dimension 5 operator parametrizing at low energy BSM contributions to a non-vanishing EDM.
- We discuss the implementation of the qCEDM with the gradient flow and show preliminary results for the flow-time dependence of the CP-violating mixing angle α_N induced by the qCEDM between nucleon states.
- These results are computed on the $N_f = 2+1$ lattices with Wilson-clover action.

L	$a[fm]$	κ	c_{SW}	Nconfs
$16^3 \times 32$	0.1215	0.13825	1.761	800
$20^3 \times 40$	0.0907	0.13700	1.715	790
$28^3 \times 56$	0.0685	0.13560	1.628	650

Definitions and Notations

- Neutron can be written as follows.

$$N_\gamma(y) = \epsilon^{abc} u^a_\alpha(y) \widetilde{C}_{\alpha\beta} d^b_\beta(y) d^c_\gamma(y) \tag{1}$$

$$\bar{N}_{\gamma'}(x) = \epsilon^{a'b'c'} \bar{d}^{a'}_{\alpha'}(x) \widetilde{C}_{\alpha'\beta'} \bar{u}^{b'}_{\beta'}(x) \bar{d}^{c'}_{\gamma'}(x) \tag{2}$$

- where a, b, c are color indices, α, β, γ are spin indices.
- $\widetilde{C} = C\gamma_5 = \gamma_4\gamma_2\gamma_5 = \gamma_3\gamma_1$, C is charge conjugation matrix.
- Note: The spin indices are swapped.

$$\underline{S}^{ab}_{\beta'\alpha}(x,y) \equiv \widetilde{C}_{\alpha\beta} S^{ab}_{\beta\alpha'}(x,y) \widetilde{C}_{\alpha'\beta'} \tag{3}$$

- quark Chromo EDM(qCEDM) operator is

$$O(w) = \sum_{f=u,d} [\bar{\psi}^f(w)]^k_\kappa \Gamma^{kl}_{\kappa\lambda}(w) [\psi^f(w)]^l_\lambda \tag{4}$$

$$\Gamma^{kl}(w) = \frac{1}{2}\sigma_{\mu\nu}\gamma_5 G^{kl}_{\mu\nu}(w)\,, \quad \sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu] \tag{5}$$

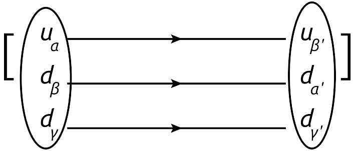
Contractions of two-point functions

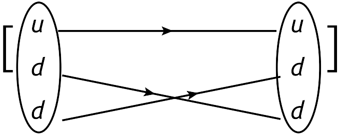
u-quark and one of d-quark form a diquark.

$$N_\gamma(y) = \epsilon^{abc} u^a_\alpha(y)\widetilde{C}_{\alpha\beta} d^b_\beta(y) d^c_\gamma(y) \tag{6}$$

$$\bar{N}_{\gamma'}(x) = \epsilon^{a'b'c'} \bar{d}^{a'}_{\alpha'}(x)\widetilde{C}_{\alpha'\beta'} \bar{u}^{b'}_{\beta'}(x) \bar{d}^{c'}_{\gamma'}(x) \tag{7}$$

$$G_2(P;y,x) = P_{\gamma'\gamma}\left\langle \left(\epsilon^{abc} u^a_\alpha(y)\widetilde{C}_{\alpha\beta} d^b_\beta(y) d^c_\gamma(y)\right)\left(\epsilon^{a'b'c'} \bar{d}^{a'}_{\alpha'}(x)\widetilde{C}_{\alpha'\beta'} \bar{u}^{b'}_{\beta'}(x) \bar{d}^{c'}_{\gamma'}(x)\right)\right\rangle \tag{8}$$





$(u^a_\alpha d^b_\beta d^c_\gamma)(\bar{u}^{b'}_{\beta'} \bar{d}^{a'}_{\alpha'} \bar{d}^{c'}_{\gamma'})$

$\epsilon^{abc}\epsilon^{a'b'c'} S^{(u)ab'}_{\alpha\beta'} \widetilde{C}_{\alpha\beta} S^{(d)ba'}_{\beta\alpha'} \widetilde{C}_{\alpha'\beta'} P_{\gamma'\gamma} S^{(d)cc'}_{\gamma\gamma'}$

$\epsilon^{abc}\epsilon^{a'b'c'} \mathrm{Tr}[S^{(u)ab'} \underline{S}^{(d)ba'}]\mathrm{Tr}[PS^{(d)cc'}]$

$(u^a_\alpha d^b_\beta d^c_\gamma)(\bar{u}^{b'}_{\beta'} \bar{d}^{a'}_{\alpha'} \bar{d}^{c'}_{\gamma'})$

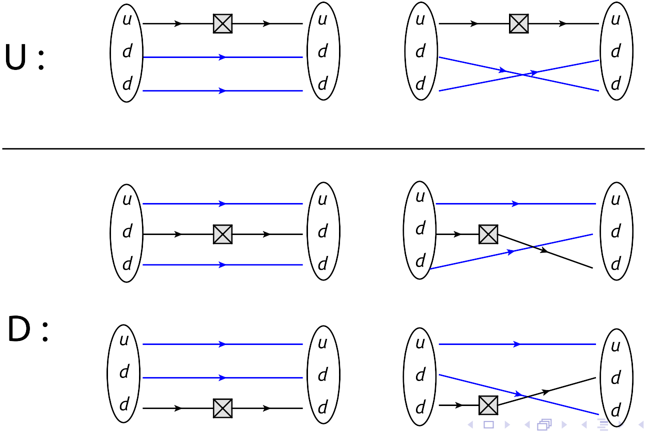
$\epsilon^{abc}\epsilon^{a'b'c'} S^{(u)ab'}_{\alpha\beta'} \widetilde{C}_{\alpha\beta} S^{(d)bc'}_{\beta\gamma'} P_{\gamma'\gamma} S^{(d)ca'}_{\gamma\alpha'} \widetilde{C}_{\alpha'\beta'}$

$\epsilon^{abc}\epsilon^{a'b'c'} \mathrm{Tr}[S^{(u)ab'} \underline{S}^{(d)ca'} \underline{PS}^{(d)bc'}]$

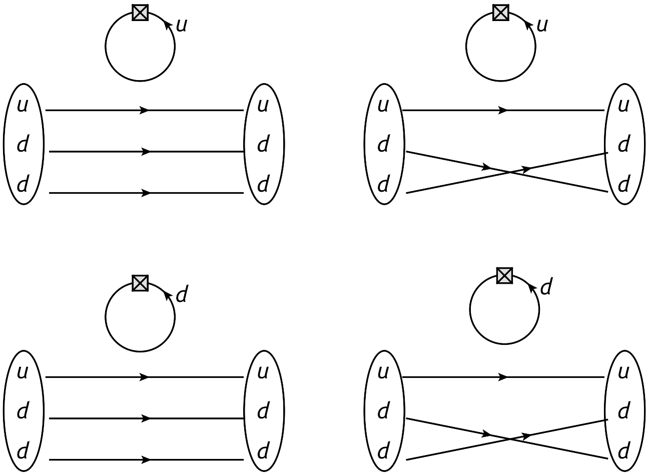
Three-point functions with qCEDM operator(connected diagrams)

$$G_3(P;y,w,x) = P_{\gamma'\gamma}\left\langle \left(\epsilon^{abc}u^a_\alpha(y)\widetilde{C}_{\alpha\beta}d^b_\beta(y)d^c_\gamma(y)\right)O(w)\left(\epsilon^{a'b'c'}\bar{d}^{a'}_{\alpha'}(x)\widetilde{C}_{\alpha'\beta'}\bar{u}^{b'}_{\beta'}(x)\bar{d}^{c'}_{\gamma'}(x)\right)\right\rangle \tag{9}$$

Note: the definitions of sequential propagators U and D are opposite to those of CHROMA.

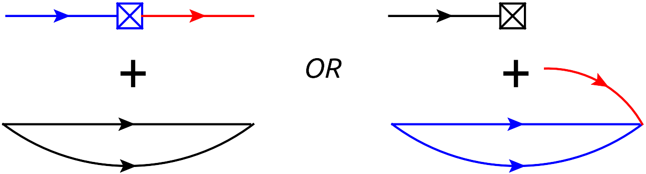


Three-point functions with qCEDM operator(disconnected diagrams)



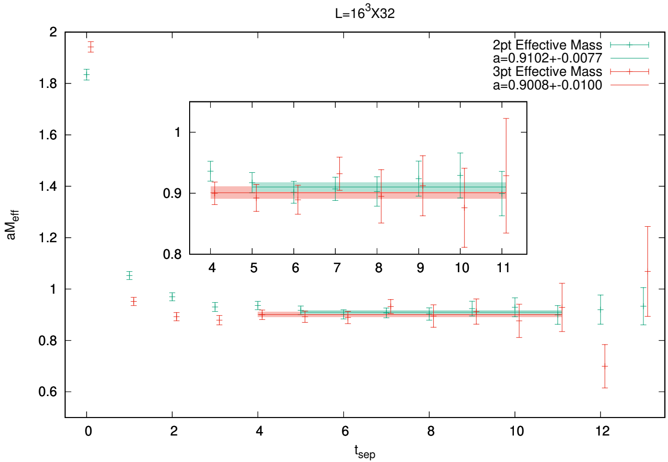
How to compute three-point functions

- There are two method to compute three-point functions.
- The blue object is a source for inversion to make red propagator.



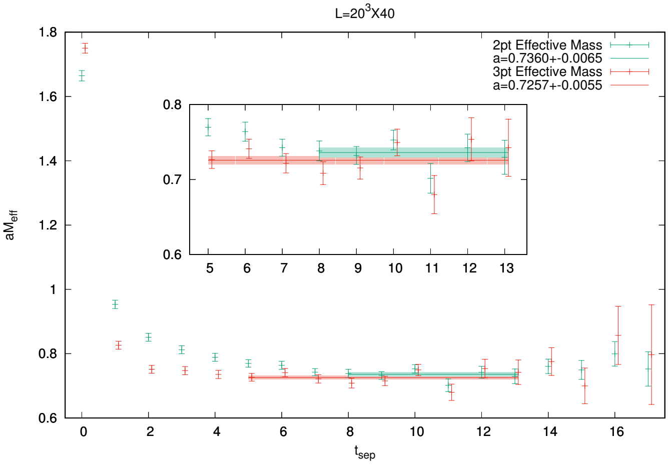
- "Through the operator" method
  - The number of inversions : $1 + t_{\text{op}}$
  - If we use the operator summed over t_{op}, the number of inversions is $1+1$.
  - U and D diagrams can be obtained separately without incresing the number of inversion.
- "Through the sink" method
  - The number of inversions : $1 + t_{\text{sink}}$

Effective Mass from 2pt and 3pt functions, $L = 16^3 \times 32$



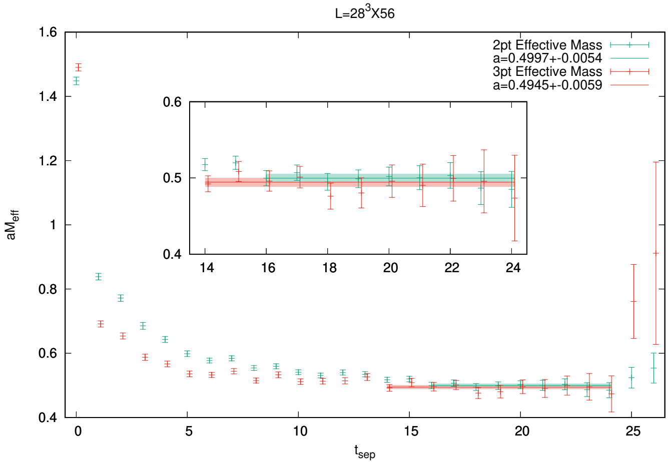
- $aM_{\text{eff}} = \log\left[\frac{G(t)}{G(t+1)}\right]$
- 3pt function reaches the plateau earlier.

Effective Mass from 2pt and 3pt functions, $L = 20^3 \times 40$



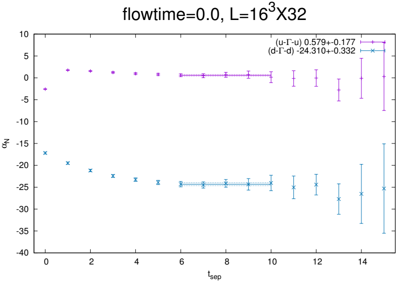
- $aM_{\text{eff}} = \log\left[\frac{G(t)}{G(t+1)}\right]$
- 3pt function reaches the plateau earlier.

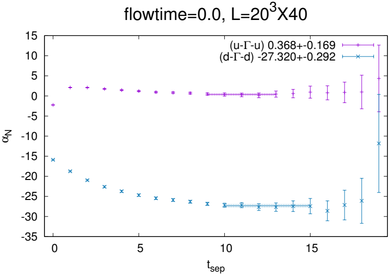
Effective Mass from 2pt and 3pt functions, $L = 28^3 \times 56$

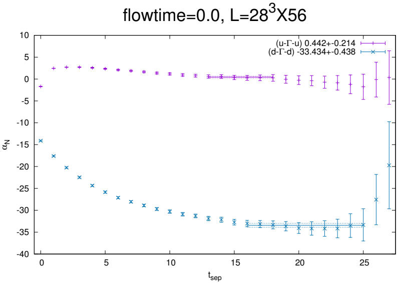


- $aM_{\text{eff}} = \log\left[\frac{G(t)}{G(t+1)}\right]$
- 3pt function reaches the plateau earlier.

α_N at zero flowtime







- $\alpha_N(t) = -\frac{\langle G_3(T^+\gamma_5,t)\rangle}{\langle G_2(T^+,t)\rangle}$, where $T^+ = \frac{1+\gamma_4}{2}$
- $(d - \Gamma - d)$ makes a dominant contribution to α_N

Gradient Flow

- Create new variable *flow time* t_f (of dimension 2) for the gauge fields to be extended too:

$$A_\mu(x) \to B_\mu(x, t_f) \qquad \text{with} \qquad B_\mu(x, 0) = A_\mu(x) \tag{10}$$

- Applying gradient flow to the sink and source of propagator. [Lüscher, 2013]

$$S(t, y; x) = \sum_v K(t, y; 0, v) S(v, x) \tag{11}$$

$$S(y; s, x) = \sum_w S(y, w) K(s, x; 0, w)^\dagger \,, \tag{12}$$

- K satisfies quark flow equation

$$(\partial_t - \Delta) K(t, x; s, y) = 0 \tag{13}$$

$$K(0, x; 0, y) = \delta_{xy} \tag{14}$$

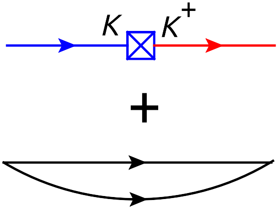
- $K^\dagger$ satisfies adjoint flow equation.

$$\xi(t; s, w) = \sum_x K(t, x; s, w)^\dagger \eta(x) \tag{15}$$

$$(\partial_s + \Delta) \xi(t; s, w) = 0 \tag{16}$$

$$\xi(t; t, w) = \eta(w) \tag{17}$$

How to compute three-point functions at finite flowtime ("Through the operator")

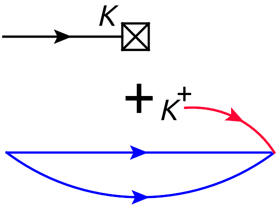


$$S_{\mathsf{seq}}(y,x)S(y,v)K(t;z,v)^{\dagger}\widetilde{\Gamma}(t,z)K(t,z;w)S(w,x)\,, \tag{18}$$

where $\widetilde{\Gamma}(t,z)$ is computed on flowed gauge.

- "Through Operator" method
  - Calculate propagator $S(w,x)$
  - Flow the sink
  - Multiply the flowed qCEDM operator
  - Apply adjoint flow operator
  - inversion for $S(y,x)$
  - Contraction with S_{seq}
- The number of inversions : $1+t_f$
- We can compute D and U diagrams separately without increasing the number of inversions

How to compute three-point functions at finite flowtime ("Through the sink")

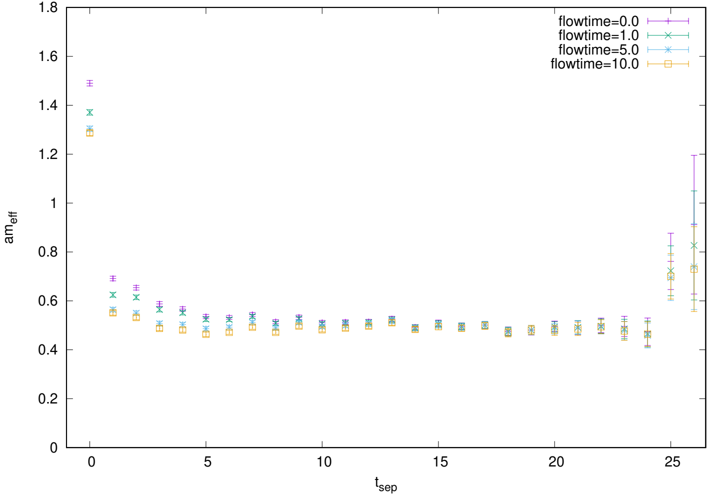


$$\gamma_5(K(t,z;v)S(v,y)\gamma_5 S_{\text{seq}}(y,x)^\dagger\gamma_5)^\dagger\gamma_5\widetilde{\Gamma}(t,z)K(t,z;w)S(w,x) \tag{19}$$

$$= S_{\text{seq}}(y,x)S(y,v)K(t,z;v)^\dagger\widetilde{\Gamma}(t,z)K(t,z;w)S(w,x) \tag{20}$$

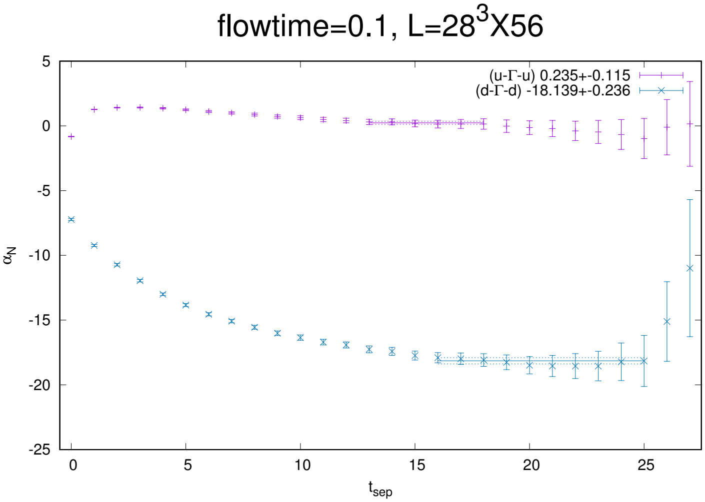
- The number of inversions : $1 + t_{\text{sink}}$
- If we compute U and D separately, the number of inversion is $1 + 2t_{\text{sink}}$

Effective Mass from 3-point function with qCEDM at finite flowtime ($L = 28^3 \times 56$)

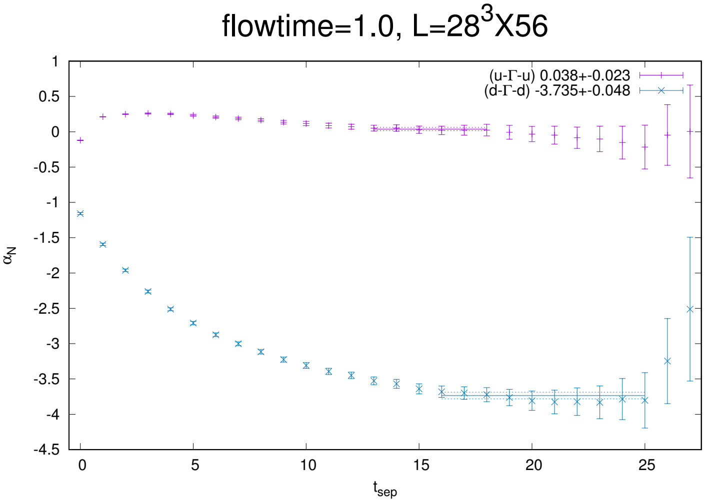


- flowing makes longer plateau

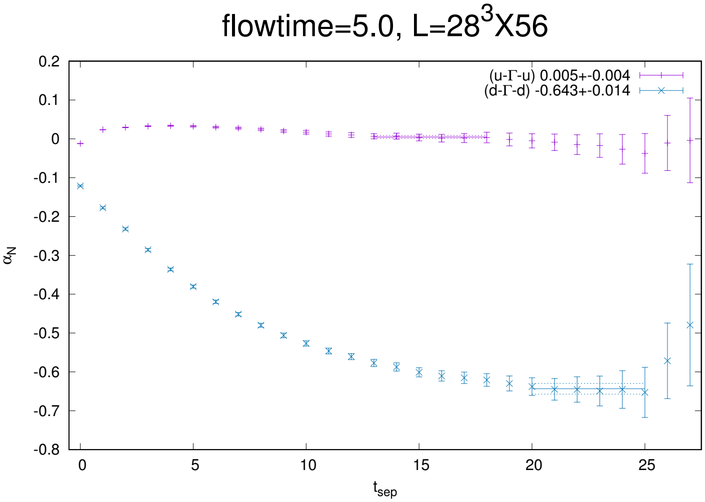
Flowtime dependence of α_N



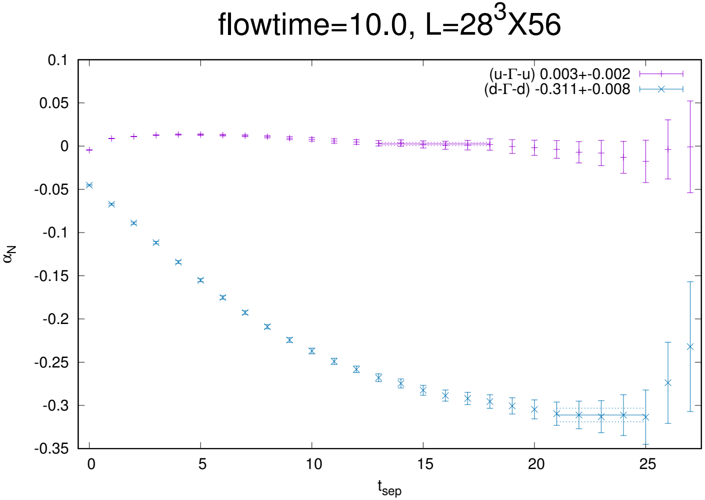
Flowtime dependence of α_N



Flowtime dependence of α_N

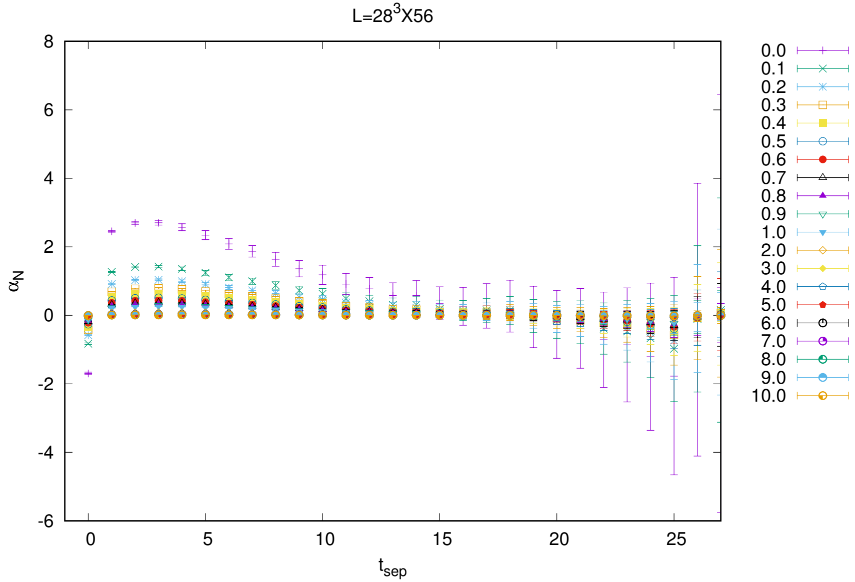


Flowtime dependence of α_N

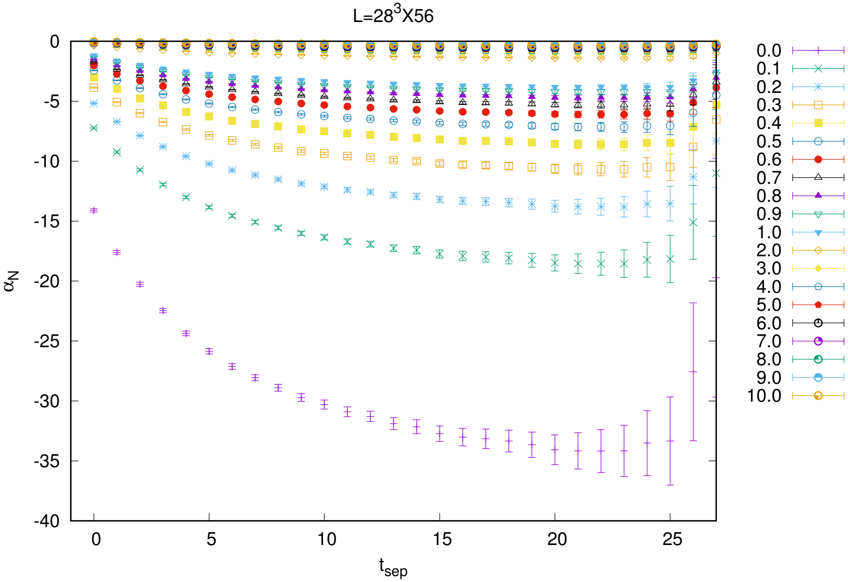


- Plateau is reduced as flowtime increases.
- α_N approches to zero for large flowtime.

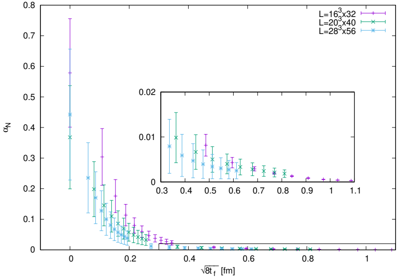
Flowtime dependence of α_N : U



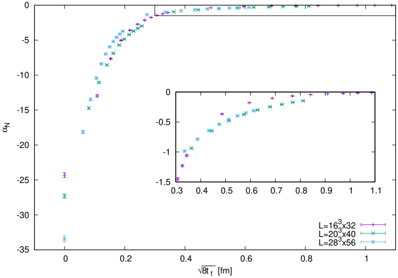
Flowtime dependence of α_N : D



Flowtime dependence of α_N



(c) (u-Γ-u)



(d) (d-Γ-d)

- $\sqrt{8t_f/a^2}$: root mean square of the smearing when the gradient flow is applied in lattice unit.
- $\sqrt{8t_f/a^2} < 1$: in lattice unit means there will be discretization effects because smearing radius is less than 1 lattice unit. But actually about < 4.

L	$a[fm]$	$4a$
$16^3 \times 32$	0.1215	0.4860
$20^3 \times 40$	0.0907	0.3628
$28^3 \times 56$	0.0685	0.274

Four-point functions

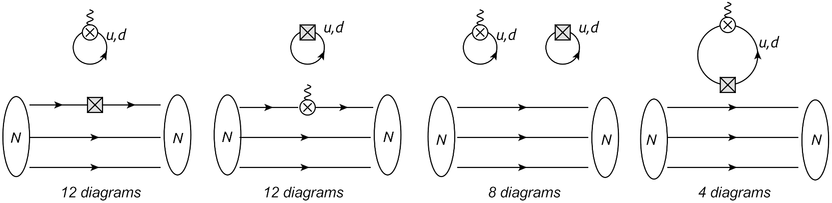
- Insert qCEDM operator and current between nucleon-nucleon states.

$$
\begin{aligned}
G_4(P; y, w, z, x) = P_{\gamma'\gamma} \Big\langle \Big(\epsilon^{abc} u^a_\alpha(y) \widetilde{C}_{\alpha\beta} d^b_\beta(y) d^c_\gamma(y) \Big) J(z) O(w) \\
\times \Big(\epsilon^{a'b'c'} \bar{d}^{a'}_{\alpha'}(x) \widetilde{C}_{\alpha'\beta'} \bar{u}^{b'}_{\beta'}(x) \bar{d}^{c'}_{\gamma'}(x) \Big) \Big\rangle ,
\end{aligned} \tag{21}
$$

where $J(z) = \sum_{f=u,d} q^f [\bar{\psi}^f(z)]^r_\rho \gamma_{\rho\sigma} [\psi^f(z)]^r_\sigma$ and q^f is a quark charge.

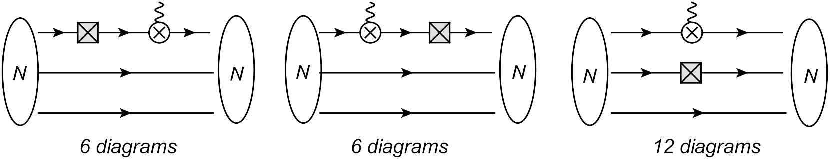
- The number of constractions is 60.

Diagrams of four-point functions (disconnected)



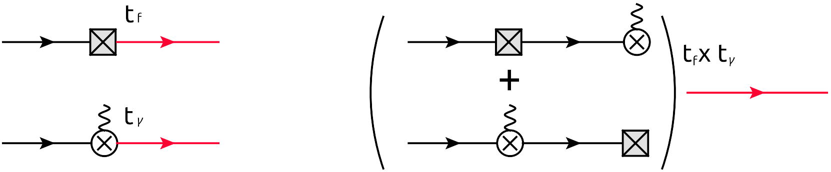
- 36 disconnected diagrams.
- 2-point function and 3-point functions with qCEDM can be reused.

Diagrams of four-point functions (connected)



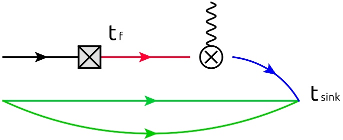
- 3-types of connected diagrams.
- 24 connected diagrams.

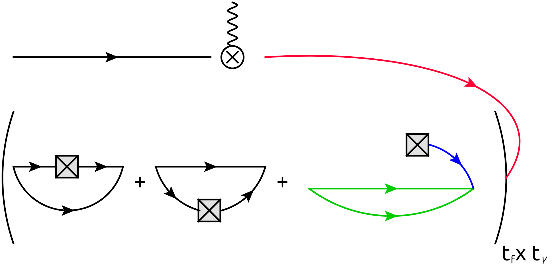
How to compute four-point functions(Through both operators)



- t_f : the number of flowtimes
- t_γ : the number of current insertion time (Euclidean)
- q_γ : the number of momenta transfer
- Total number of inversions : $1 + t_f + t_\gamma \times q_\gamma + \cancel{2}(t_f \times t_\gamma \times q_\gamma)$
- U and D diagrams can be obtained separately without increasing the number of inversions

How to compute four-point functions(Through the sink)





- t_{sink} : the number of sink time (Euclidean)
- p_{sink} : the number of momenta at sink
- Total number of inversions : $1 + t_f + t_{\text{sink}} \times p_{\text{sink}} + (t_f \times t_{\text{sink}} \times p_{\text{sink}})$
- If we compute U and D diagrams separately, the number of inversions will be $1 + t_f + 2t_{\text{sink}} \times p_{\text{sink}} + (2t_f \times t_{\text{sink}} \times p_{\text{sink}})$

Conclusion

- We compute 3-point function with quark-chromo EDM operator.
- We compare the number of inversions of "Through the operator" and "Through the sink" methods for 3-point and 4-point functions.
- Gradient flow removes the excited states quickly.
- $(d - \Gamma - d)$ makes a dominant contribution to α_N in neutron.
- We also present flowtime dependence of α_N.

Acknowledgement

- Thanks for supporting computing time for this work.