Supersymmetry and conformal theories on the lattice from $\mathcal{N} = 1$ super Yang-Mills towards super QCD

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1. Supersymmetric Yang-Mills theory and SQCD on the lattice
2. Mixed representation composite Higgs model
3. Supersymmetric QCD

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Why study SUSY on the lattice?

1. BSM physics: Supersymmetric particle physics requires breaking terms based on an unknown non-perturbative mechanism.
   $\Rightarrow$ need to understand non-perturbative SUSY
2. Supersymmetry is a general beautiful theoretical concept: (Extended) SUSY simplifies theoretical analysis and leads to new non-perturbative approaches.
   $\Rightarrow$ need to bridge the gap between "beauty" and **"reality"**

Why study (near) conformal theories on the lattice?

1. BSM physics: Composite Higgs / walking Technicolour scenarios, walking behaviour allows large scale separation with light scalar bound state
2. Theoretical questions: What is the conformal window? What non-QCD-like behaviour of a strongly interacting theory is possible? What is the effective field theory description for a walking theory?

$\mathcal{N} = 1$ super Yang-Mills theory

Supersymmetric Yang-Mills theory:

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \bar{\lambda} (\not{D} + m_g) \lambda$$

- supersymmetric counterpart of Yang-Mills theory; but in several respects similar to QCD
- λ Majorana fermion in the adjoint representation
- SUSY transformations: $\delta A_\mu = -2i\bar{\lambda}\gamma_\mu \varepsilon$, $\delta\lambda = -\sigma_{\mu\nu} F_{\mu\nu} \varepsilon$

What has been investigated so far:

- SU(2) and SU(3): SUSY Ward-identities and particle spectrum
  $\Rightarrow$ Talk by H. Gerber
- Indications for SUSY continuum limit and multiplet formation in SU(2) and SU(3) SYM.
- finite temperature SU(2) SYM
  $\Rightarrow$ SU(3) SYM: talk by C.Lopez
- compacitfied SYM: Witten index and absence of any deconfinement transition (continuity)

$\Rightarrow$ nearly concluded studies of SYM for SU(2) and SU(3)

Conformal window: adjoint QCD with different N_f

- near conformal behaviour with a constant mass ratios for $N_f > 1/2$
- range of N_f completed with $N_f = 3/2$ (Talk by P. Scior)

Theory	**scalar particle**	γ_* small β	γ_* larger β
$N_f = 1/2$ SYM	part of multiplet	–	–
$N_f = 1$ adj QCD	light	0.92(1)	0.75(4)*
$N_f = 3/2$ adj QCD	light	0.50(5)*	0.38(2)*
$N_f = 2$ adj QCD	light	0.376(3)	0.274(10)

($*$ preliminary)

$\Rightarrow$ Near conformal lattice data for a range of theories starting at smaller N_f than expected from perturbative analysis.

Going beyond $\mathcal{N} = 1$ SYM: SQCD

- add $N_c \oplus \bar{N}_c$ chiral matter superfield
- SYM + quarks ψ and squarks Φ_i with covariant derivatives, mass terms and

$$
\begin{aligned}
& i\sqrt{2}g\bar{\lambda}^a \left(\Phi_1^\dagger T^a P_+ + \Phi_2 T^a P_-\right)\psi \\
& - i\sqrt{2}g\bar{\psi}\left(P_- T^a \Phi_1 + P_+ T^a \Phi_2^\dagger\right)\lambda^a \\
& \frac{g^2}{2}\left(\Phi_1^\dagger T^a \Phi_1 - \Phi_2^\dagger T^a \Phi_2\right)^2 .
\end{aligned}
$$

Why we consider SQCD

- natural extension of supersymmetric Yang-Mills theory
- relation to possible extensions of the standard model
- earlier studies of lattice formulation: perturbative [Costa, Panagopoulos], tuning [Giedt, Veneziano]

SQCD analysis of Seiberg et al.:

- $N_f < N_c$ No vacuum
- $N_f = N_c$ confinement and chiral symmetry breaking
- $\frac{3}{2}N_c < N_f < 3N_c$ infrared fixed point (duality)

Like other SUSY theories beyond $\mathcal{N} = 1$ SYM: conformal or near conformal behaviour

Why we should better not consider SQCD

- large space of tuning parameters [Giedt] ($O(10)$ parameters)
- just test the mismatch
- might need formulation with Ginsparg-Wilson fermions
- still test it with Wilson fermions
- complex Pfaffian
- related to bosonic symmetry transforming $\mathrm{Pf} \to \mathrm{Pf}^*$
- not well behaved chiral limit:
  - either near conformal
  - test near conformal scenario in a related theory
  - or unstable vacuum
  - test with $N_f = 1$ SQCD

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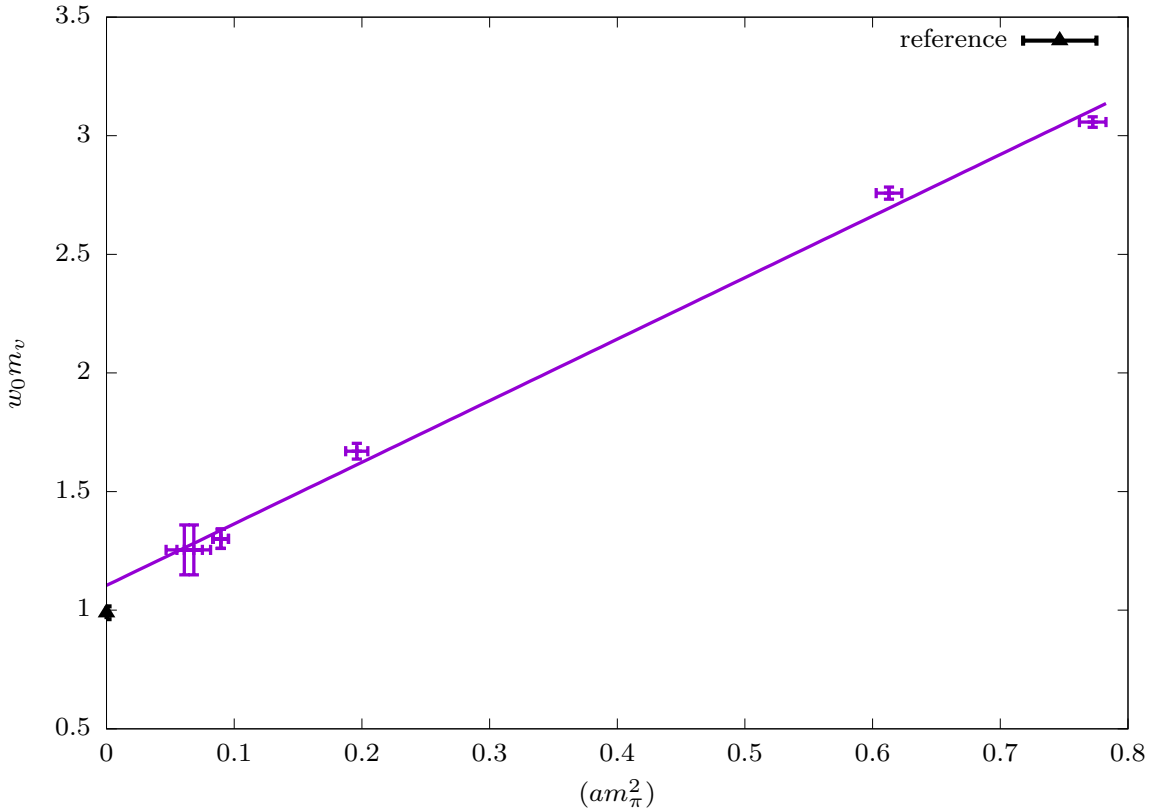
Ultra Minimal Walking Technicolour

- suggested composite Higgs model [Ryttov,Sannino]: $N_f = 1$ in adjoint + $N_f = 2$ in fundamental representation of SU(2)
- lattice studies indicate near conformal behaviour at lower N_f for the adjoint representation

$N_f = 1/2$ adjoint + $N_f = 2$ in fundamental

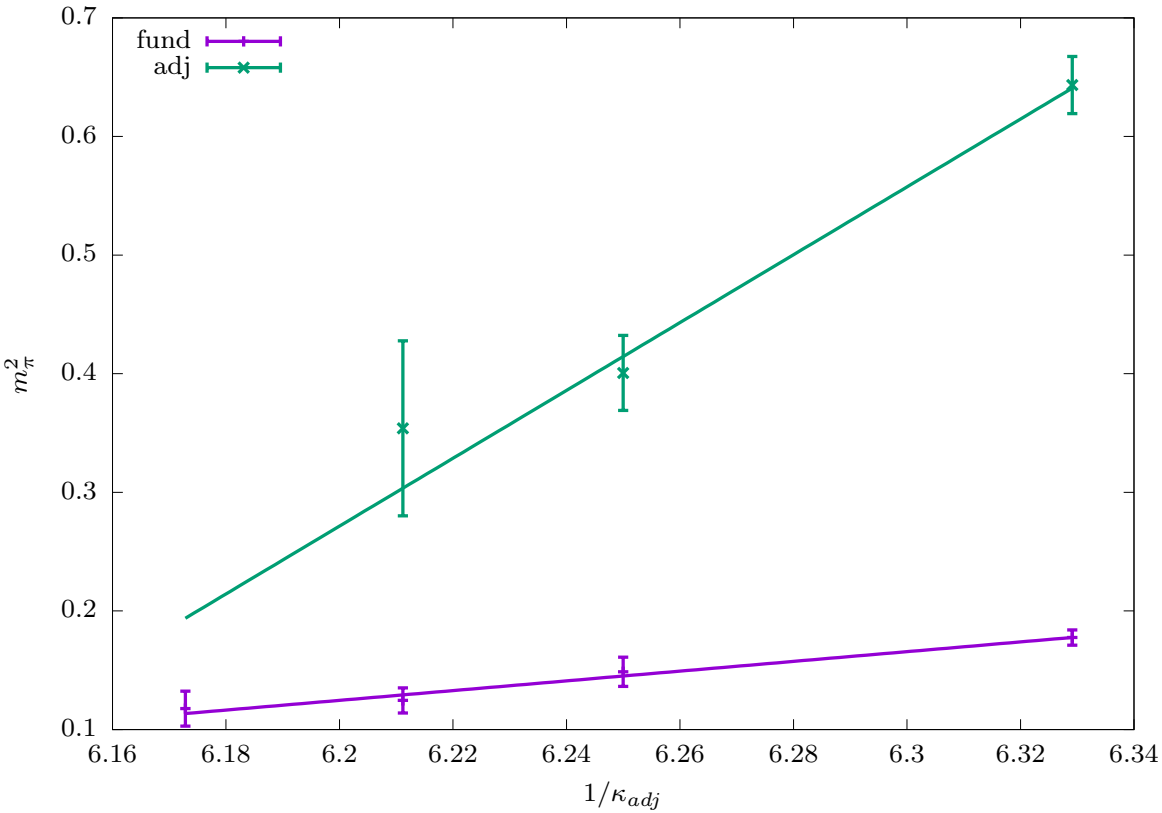
- expectations: close to conformal, but still walking
- ideal candidate for a check of effective theories
- SQCD without scalars

Cross check in pure $N_f = 2$ SU(2) fundamental theory



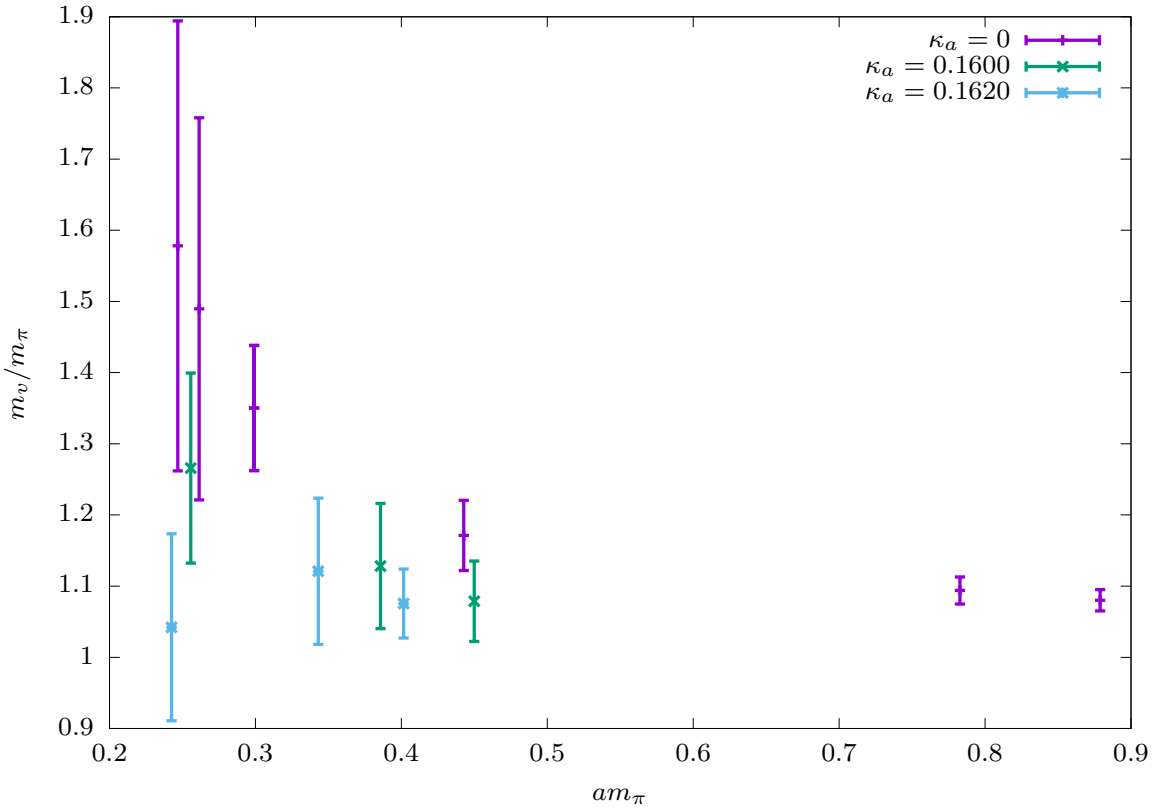
- reasonable agreement with recent (continuum extrapolated) results [Arthur,Drach,Hansen,Hietanen,Pica,Sannino]
- larger β to avoid possible bulk transition (SU(2) $N_f = 1$ adjoint)

First investigations in mixed representation setup: tuning



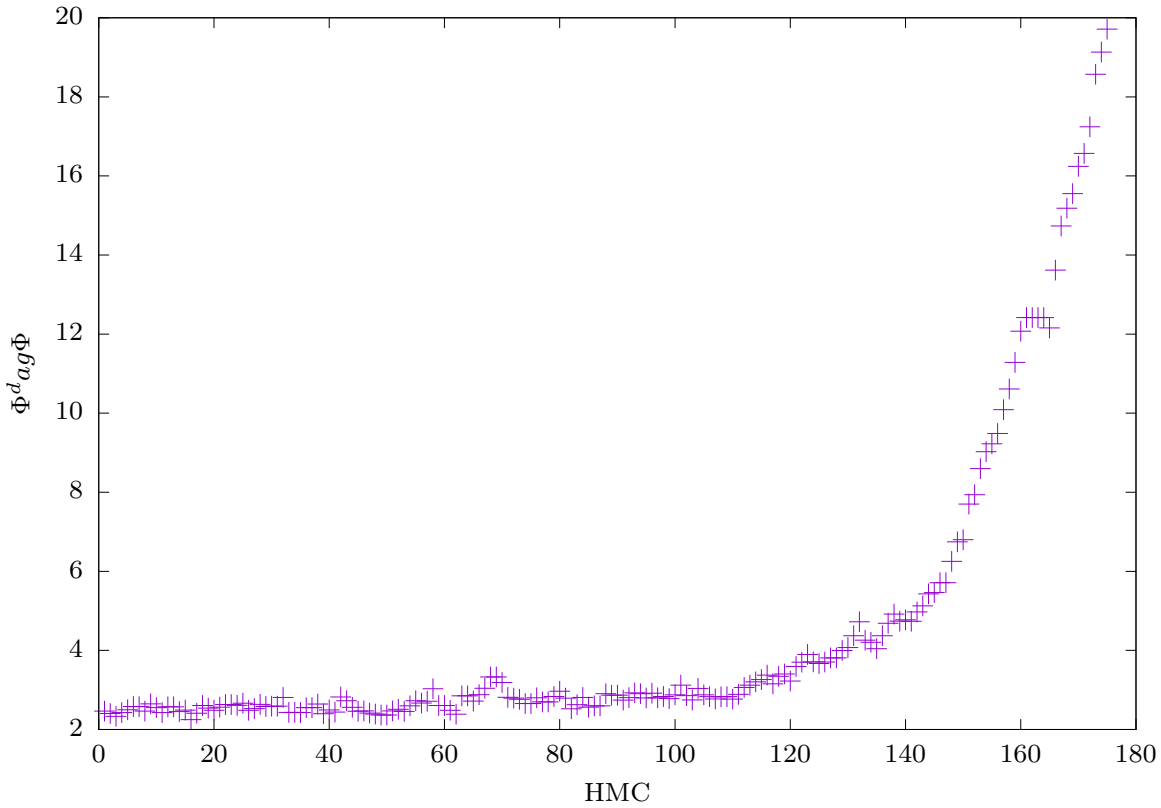
- one-loop improved Wilson clover fermions: tuning of fundamental and adjoint not independent

First investigations in mixed representation setup



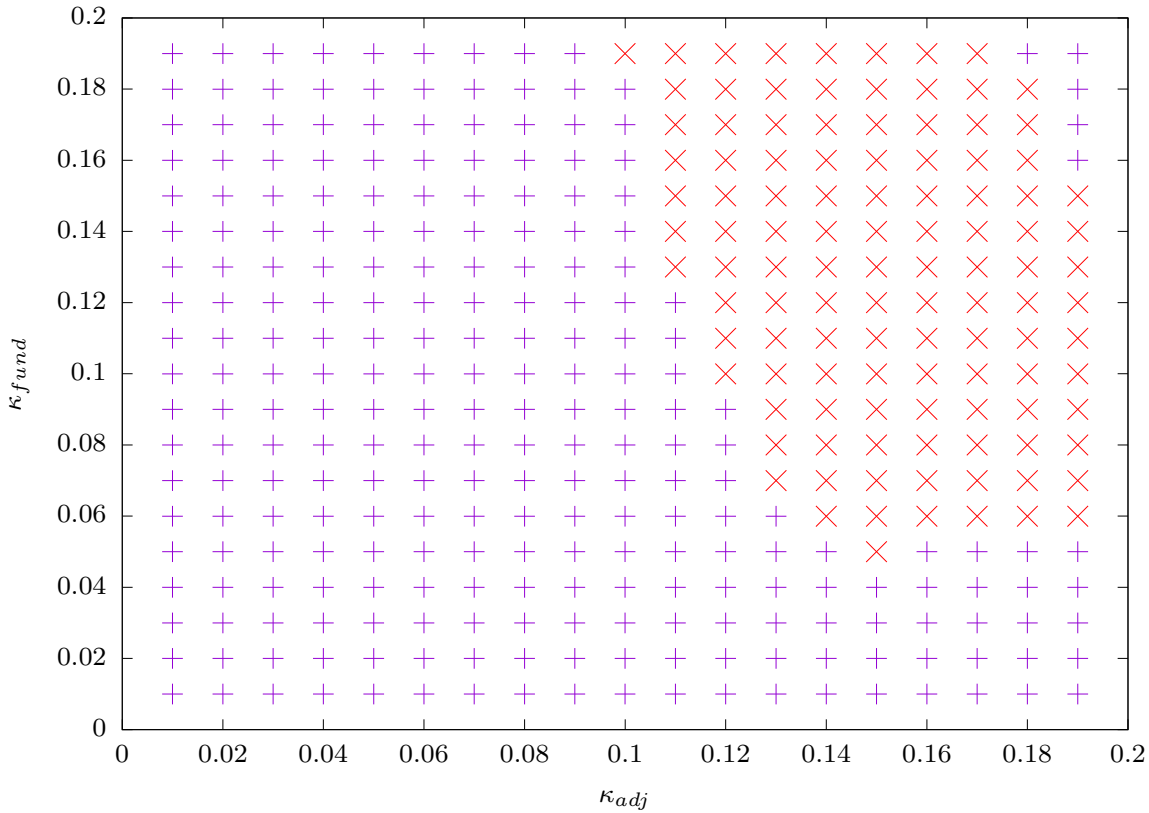
- adjoint flavour drives theory towards near conformal behaviour

$N_f = 1$ SU(2) SQCD vacuum



- the expected instability when going chiral

$N_f = 1$ SU(2) SQCD vacuum



- constraint phase diagram for the parameter tuning
- simulations with an $O(g^0)$ SUSY action

Conclusions

- SYM finished, new challenge theories with scalars like SQCD
- challenging tuning problem
- other challenges: conformal behaviour, vacuum structure
- two approaches for our investigations:
  - study of related mixed representation theory
  - simulations of $N_f = 1$ SQCD and search for non-perturbative tuning conditions
- Requires analysis in a regime where SUSY is restored in SYM (at least $24^3 \times 48$ lattice with unimproved action)