Coupled channel scattering of vector and scalar charmonium resonances on the lattice

Stefano Piemonte

for the RQCD collaboration

G. Bali, S. Collins, D. Mohler, M. Padmanath, S. Prelovsek, S. Weishaeupl

Universität Regensburg

27 July 2018

Charmonium bound states and resonances on the lattice

What is the nature of charmonium $\bar{c}c$ resonances and exotic "XYZ" states near decay-threshold?

Our first study is focused on the 1^{--} and 0^{++} channels. In the 0^{++} channel the interest is on the first excited state of the χ_{c0} ($X^*(3860)$?) and on the $X(3915)$.

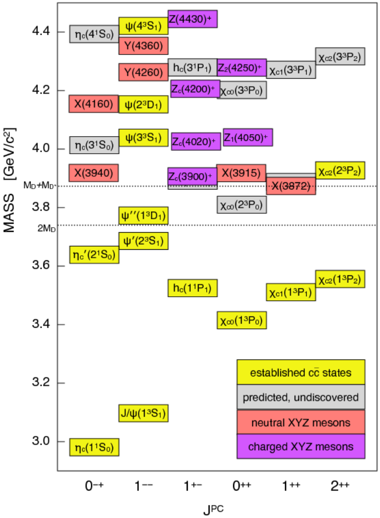


Figure: Spectrum of the charmonium states including exotic resonances. [Picture from hep-ex/1511.01589]

The scalar charmonium meson sector

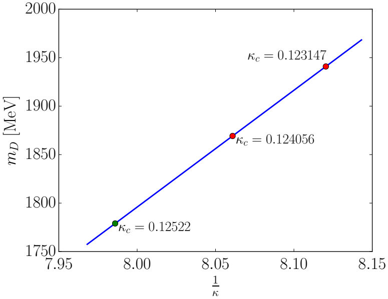
The scalar charmonium meson sector beyond the ground state is puzzling:

1. $X^*(3860)$: discovered by Belle (2017) is a possible candidate for an excited state of the χ_{c0}. [Phys. Rev. D 95, 112003 (2017)]
2. $X(3915)$: discovered by Belle (2004) in $J/\psi\omega$ decays, later confirmed by BaBar (2007). [Phys. Rev. Lett. 94(2005), 182002; Phys. Rev. Lett. 101 (2008) 082001; Phys. Rev. Lett. 104(2010) 092001]
   - Possible alternative interpretation of experimental data as 2^{++} state [Phys. Rev. Lett. 115, 022001 (2015)]
   - OZI rule allows decays for an excited $\bar{c}c$ state in $\bar{D}D$ (not seen) but not in $J/\psi\omega$. [Phys. Rev. D 86 (2012), 091501, Phys. Rev. D 91(2015), 057501]
   - Possible $\bar{D}_s D_s$ molecule: not seen in $\eta\eta_c$ channel. [Phys. Rev. D 91, 114014 (2015)]
   - Possible $cs\bar{c}\bar{s}$ tetraquark: decays to $J/\psi\omega$ explained in terms of the $\omega - \phi$ mixing. [Eur. Phys. J. C77 (2017) 78]
3. Other possible resonances?

Lattice and distillation methods

We study the charmonium spectrum on the U101 and H105 CLS ensembles, $m_\pi = 280$ MeV, $m_K = 467$ MeV, $a = 0.0854$ fm and $V = 24^3 \times 128$ and $V = 32^3 \times 96$. We employ full (U101) and stochastic (H105) distillation method [Phys. Rev. D 80: 054506 (2009)].

Charm quark mass: we use $\kappa_c = 0.123147$, corresponding to a D meson mass m_D equal to 1966(8) MeV, and $\kappa_c = 0.12522$ corresponding to $m_D = 1789(6)$ MeV.



Charmonium spectrum determined in rest and moving frames (lattice directions $\hat{n} = \{0,0,0\}, \{1,0,0\}, \{1,1,0\}$), from single meson and multi-hadron correlators. Diagrams with disconnected charm loops neglected.

Spin identification of lattice energy levels

On the lattice, the spin J is not a good quantum number: states with many possible J can contribute to a given lattice irreducible representation. We identify the continuum spin by studying the spectrum of $\bar{c}c$-operators with up to two covariant derivatives transforming in different lattice discrete groups.

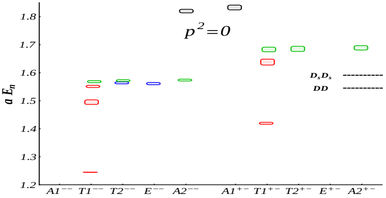
A state is identified according to the criteria [Phys. Rev. D 82 034508 (2010), Phys Rev. D 85 014507 (2012)]:

A Degeneracies of energies: states with a certain J^P are expected to appear in different irreps and corresponding energies should be to be nearly degenerate.

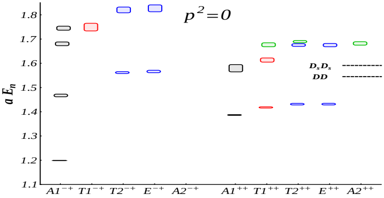
B Relative magnitude and close degeneracy of Z factors: a state $|J^P\rangle$ is expected to have similar Z-factors for a given operator $O^{J,P}$ subduced to different irreps.

Spin identification of lattice energy levels

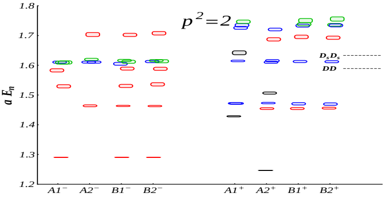
Charmonium spectrum in rest and moving frames:



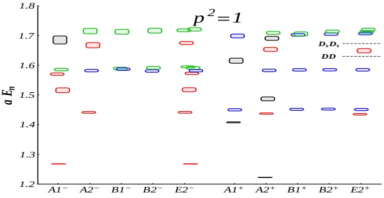
(a) O_h, $C = -$, $k_c = 0.12522$



(b) O_h, $C = +$, $k_c = 0.12522$



(c) Dic_2, $k_c = 0.12522$



(d) Dic_4, $k_c = 0.12522$

- $J = 0$: black
- $J = 3$: green
- $J = 1$: red
- $J = 2$: blue

Lüscher formula

The scattering properties are encoded in the phase shift, that can be computed from the energy levels dependence on the size L of the lattice box.

In the continuum case, a system of two non-interacting identical particles has energy

$$E = 2\sqrt{m^2 + p^2}\,.$$

The momentum p is quantized in units of $2\pi/L$ as $\hat{n}$

$$\hat{n} = \{n_x, n_y, n_z\}\,.$$

In the interacting case a similar relation holds, but the quantization of momentum is related to the phase shift by the Lüscher formula. [M. Lüscher, *Signatures of unstable particles in finite volume*, Nuclear Physics B Volume 364, Issue 1, 14 October 1991, Pages 237-251]

$-\pi < \arg(\boldsymbol{n}^2 - q^2) \leqslant \pi$ is adopted). An efficient way to evaluate the zeta functions numerically is described in ref. [2], appendix C.

A.2. DISCUSSION OF THE ANGLE $\phi(q)$

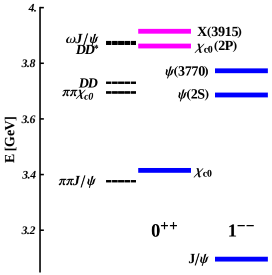
For all $q \geqslant 0$, the angle $\phi(q)$ is determined by

$$\tan\phi(q) = -\frac{\pi^{3/2}q}{\mathcal{Z}_{00}(1;q^2)}, \qquad \phi(0) = 0, \tag{A.3}$$

and the requirement that it depends continuously on q. For $q^2 \geqslant 0.1$, $\phi(q)$ is very nearly equal to πq^2 (see table A.1). The numbers listed there are exact to the precision stated. For intermediate values of q^2, quadratic interpolation will in general yield sufficiently accurate results.

The vector channel

In the vector channel, we expect two bound states, the J/ψ and the $\psi(2S)$, and a resonance just above the $\bar{D}D$ decay threshold, the $\psi(3770)$.



In the elastic approximation, we fit our energy levels to a polynomial form

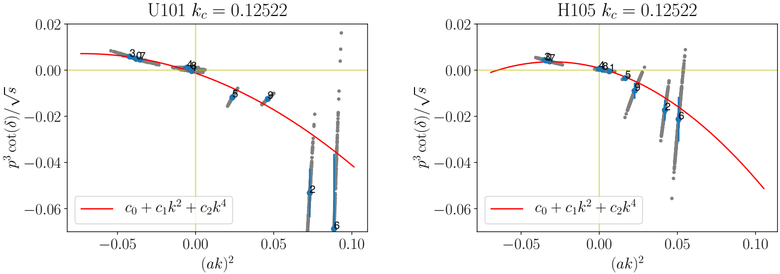
$$\frac{p^3 \cot(\delta)}{\sqrt{s}} = c_0 + c_1 s + c_2 s^2 \,,$$

chosen to describe the presence of a bound state and a resonance.

We use the package TwoHadronsInBox for fitting our energy levels to the Lüscher formula [Nucl. Phys. B924, 477-507 (2017)].

The vector channel

The fits in the elastic scattering approximation are performed for each ensemble independently. The lowest energy level, corresponding to the J/ψ, is excluded from the fits.



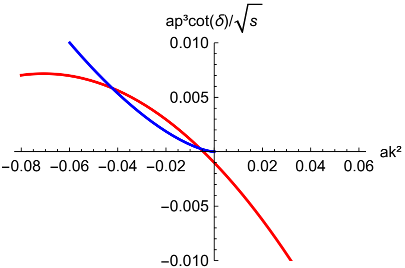
The points correspond to 0-2 $\rightarrow$ Rest frame, 3-6 $\rightarrow$ P=100 moving frame, 7-9 $\rightarrow$ P=110 moving frame.

The vector channel

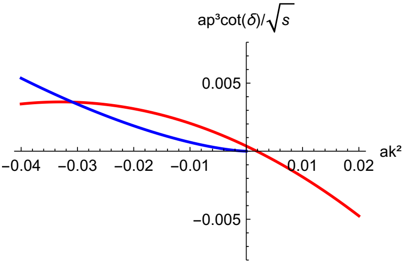
A bound state appears as a pole of the T matrix on the real axis below the scattering threshold in the first Riemann sheet. Such a pole appears if the condition

$$p^3 \cot(\delta) = -k^2\sqrt{-k^2} \tag{1}$$

is fulfilled.



(g) U101, $k_c = 0.12522$

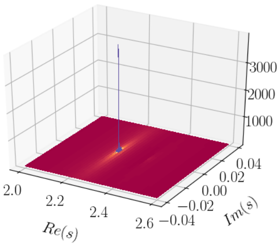


(h) H105, $k_c = 0.12522$

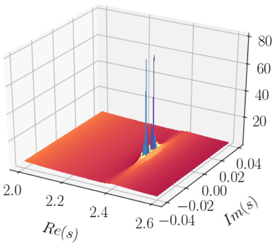
Exponentially suppressed finite volume effects are not negligible in the U101!

The vector channel

The T matrix in the s-complex plane has two complex conjugated pairs of poles in the second sheet corresponding to the $\psi(3770)$ resonance.



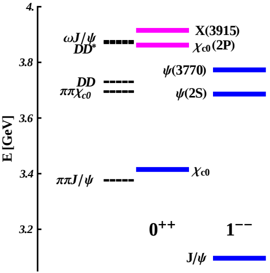
(i) H105, $k_c = 0.12522$



(j) H105, $k_c = 0.12522$

The scalar channel

In the scalar channel, the experimental situation above the $\bar{D}D$ scattering threshold is still not completely clear.



The resonances which have been predicted/observed are close to many different thresholds, we therefore need to consider the coupled channel Lüscher formalism. The $\bar{D}D$ and $\bar{D}_sD_s$ coupled channel in s-wave is considered, therefore T and K are 2×2 matrices.

A possible parametrization for the $\tilde{K}$-matrix could be for instance:

$$\tilde{K}^{-1} = \begin{bmatrix} c_{11}s + b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix}$$

The scalar channel

Fit of the energy levels using the *determinant-residual method*.
First check: are the predicted energy levels from the box-quantization condition matching *without extra-levels* the number of our observed energy levels?

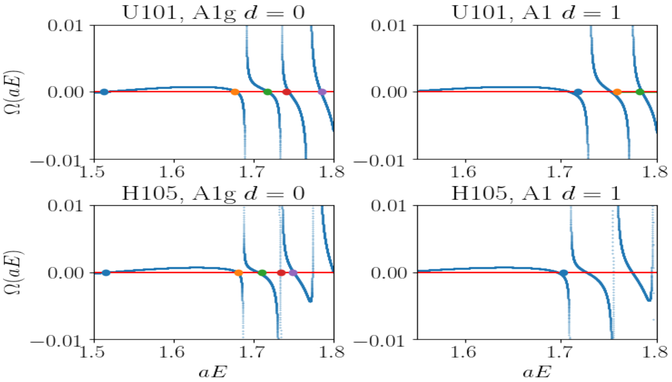
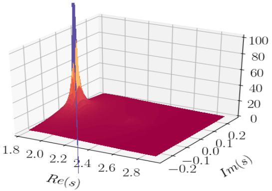


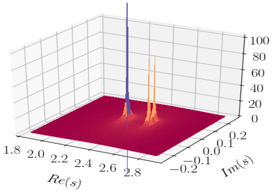
Figure: Ω function for $\kappa_c = 0.12315$

The scalar channel

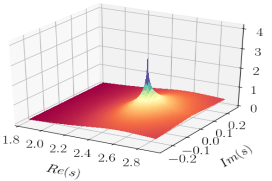
Very preliminary plots of the T matrix in the s-complex planes for $k_c = 0.12522$:



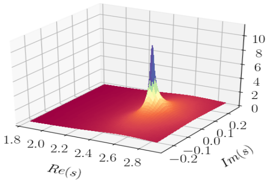
(a) sheet $(1,1)$



(b) sheet $(2,1)$



(c) sheet $(2,2)$



(d) sheet $(1,2)$

Conclusions

- The properties of the resonance $\psi(3770)$ can be extracted in the elastic approximation from our 32^3 lattice box
- Coupled channel analysis required for the study of the scalar resonances.
- Coupled channel fits are able to capture well the structure of our observed lattice energy levels.
- Preliminary plots of the T matrix show a rich structure in the complex plane in the scalar sector.

$\rightarrow$ Stay tuned, new results will be coming soon!

Thank you for the attention!