Continuum extrapolation of the critical endpoint in 4-flavor QCD with Wilson-Clover fermions

Hiroshi Ohno[1]
in collaboration with
Y. Kuramashi[1,2], Y. Nakamura[2] and S. Takeda[3]
[1]CCS, University of Tsukuba, [2]RIKEN, [3]Kanazawa University

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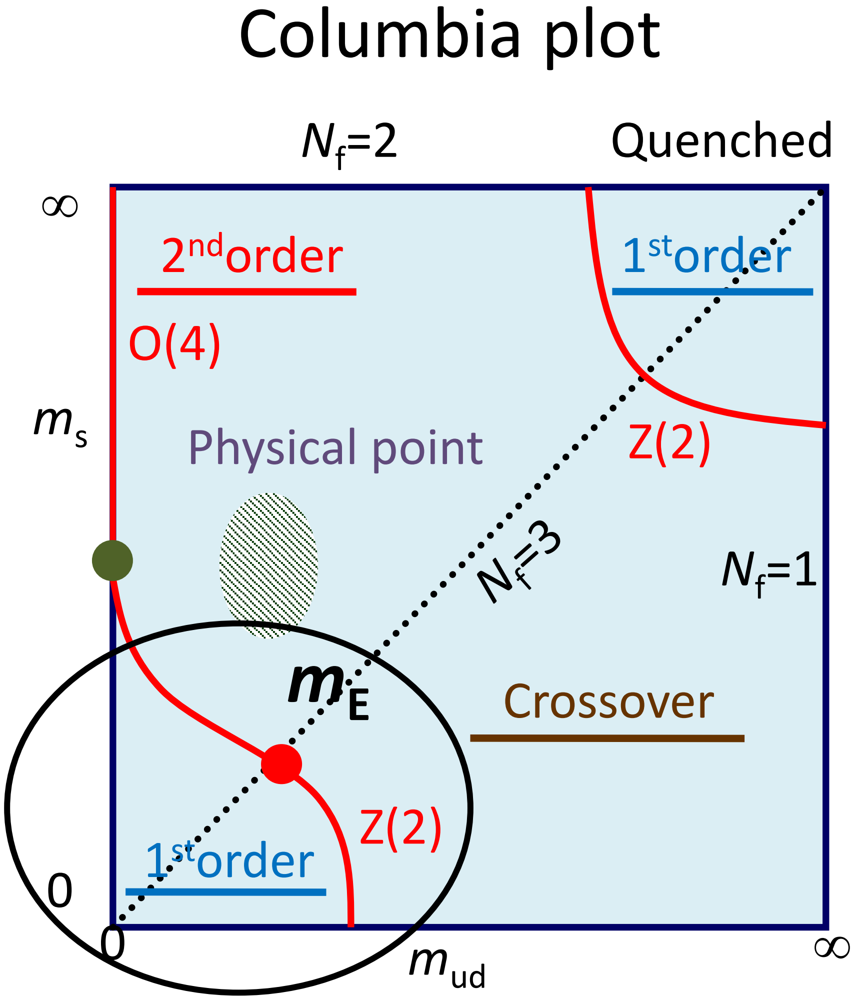
Critical endpoint in 3-flavor QCD

R. D. Pisarski and F. Wilczek, PRD 29 (1984) 338

- Phase transition of 3-flavor QCD
 - expected to be **1st order** at $m = 0$
 - **2nd order** critical endpoint at $m = m_E$
 - **crossover** $m > m_E$

- **The location of m_E is still not conclusive!**



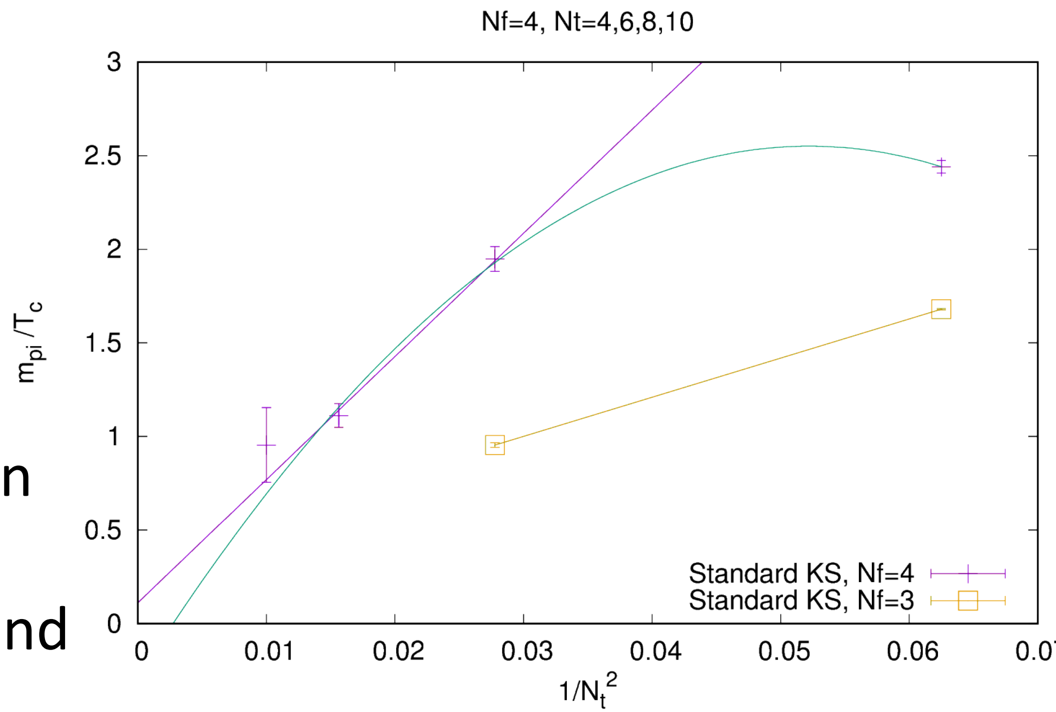
Lattice studies so far

Action	N_t	m^E_π	Ref.
Staggered, standard	4	290 MeV	Karsch et al ’01, Liao ‘01
Staggered, p4	4	67 MeV	Karsch et al ‘04
Staggered, standard	6	150 MeV	de Forcrand et al ‘07
Staggered, HISQ	6	< 50 MeV	Ding et al ‘17
Staggered, stout	4-6	~ 0	Varnhorst ‘14
Wilson, standard	4	< 670 MeV	Iwasaki et al, ‘96
Wilson, clover	6-8	300 MeV	Nakamura et al, ‘14
Wilson, clover	4-10	< 170 MeV	Jin et al, ‘17

m_E gets smaller for finer lattices and more improved actions.
There is clear difference between staggered and Wilson results.

A staggered result in 4-flavor QCD

- 4-flavor QCD
  - 1^{st} order phase transition is expected at $m = 0$
  - Stronger phase transition compared to $N_f = 3$
    → larger m_E
    → less expensive computation
  - No rooting issue
  - A good analogue to understand 3-flavor results



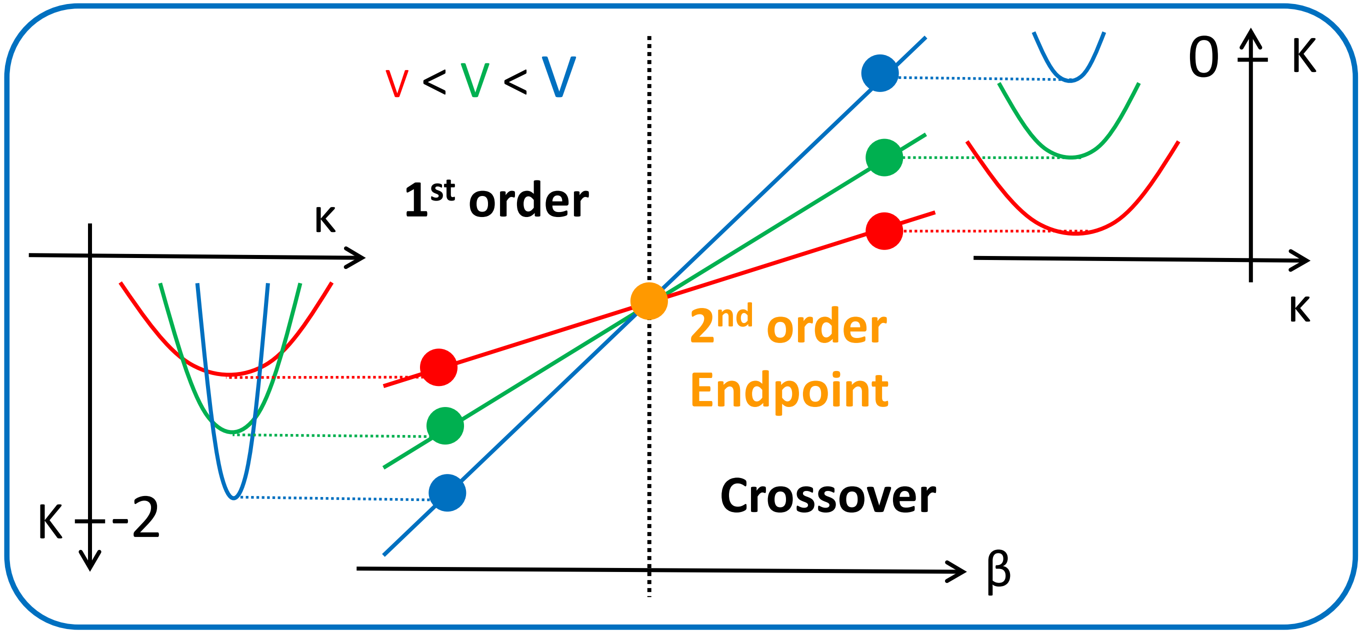
P. de Forcrand and M. D'Elia, PoS LATTICE2016 (2017) 081

- **Similar behavior as $N_f = 3$**
  - **quite small m^E_π in the continuum limit**
    → important to crosscheck also with Wilson-type quarks

Simulation setup

- Iwasaki gauge + $O(a)$-improved Wilson quarks with 4 degenerate flavors
  - **C_{sw} has been non-perturbatively determined in $N_f = 4$**
- 3 different cutoffs towards the continuum limit
  - $N_t = 4$, 6 and 8
- 2-3 different β values at each N_t
  - $\beta = 1.60$, 1.61 and 1.62 at $N_t = 4$
  - $\beta = 1.67$, 1.68 and 1.69 at $N_t = 6$
  - $\beta = 1.66$ and 1.67 at $N_t = 8$
- Chiral condensate and its cumulants
  - Up to 4th order, i.e. susceptibility, skewness and kurtosis
  - Traces, e.g. $\mathrm{Tr}D^{-n}$ for n = 1, 2, 3 and 4, measured with 10 noises
- Multi-ensemble reweighting for κ
  - to improve signals
- Zero temperature simulations also have been done on a $16^3 \times 32$ lattice for scale setting (t_0 and m_{PS})

Kurtosis intersection method



Critical endpoint : No volume dependence

↓

Searching for an intersection

An example of cumulants at $N_t = 4$

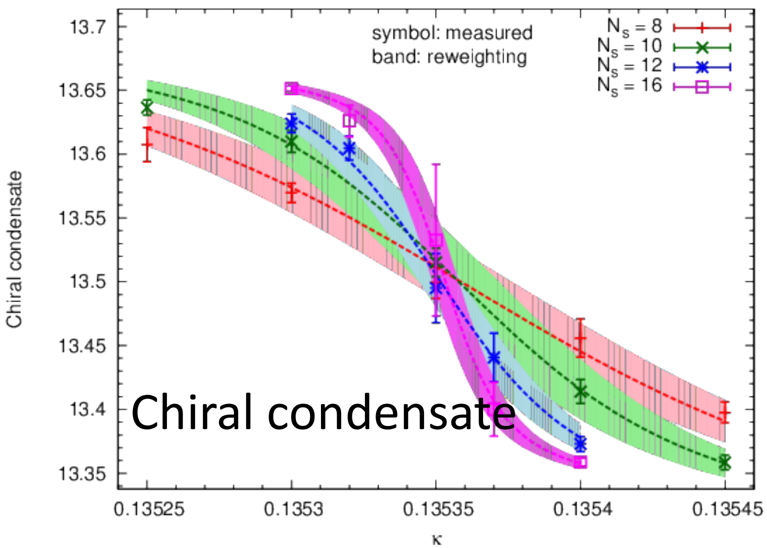
$\beta = 1.61$

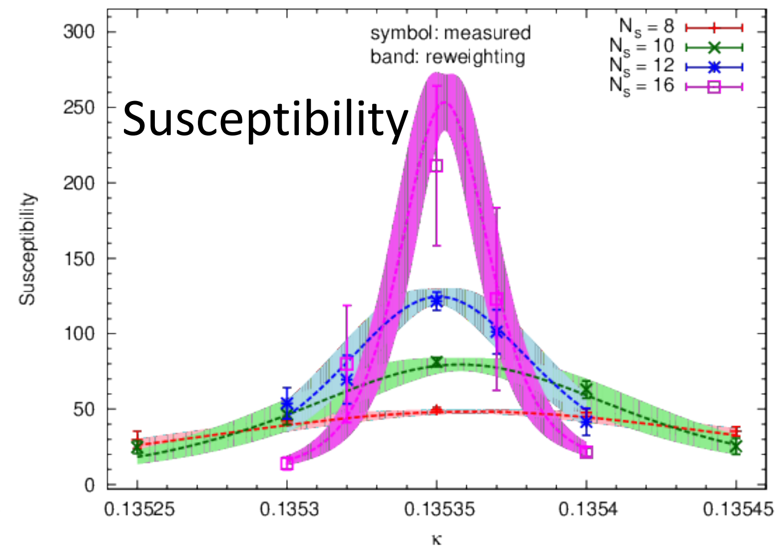
$N_s = 8$

$N_s = 10$

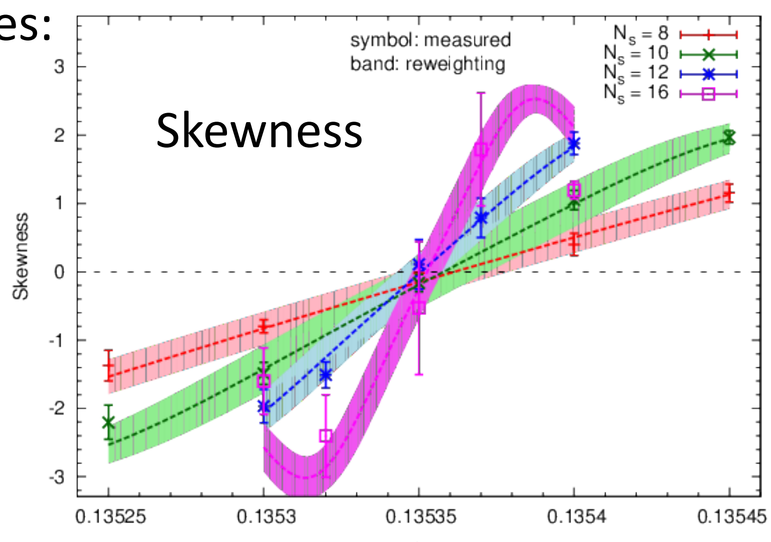
$N_s = 12$

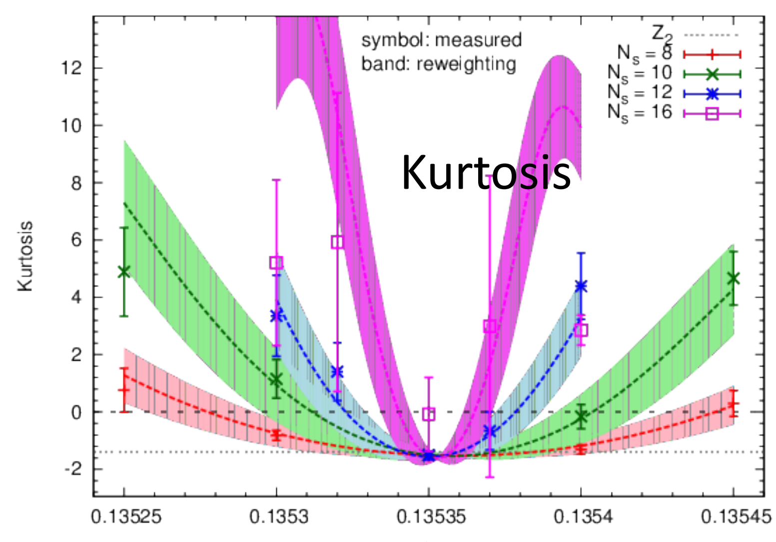
$N_s = 16$





dotted curves: reweighting





An example of cumulants at $N_t = 6$

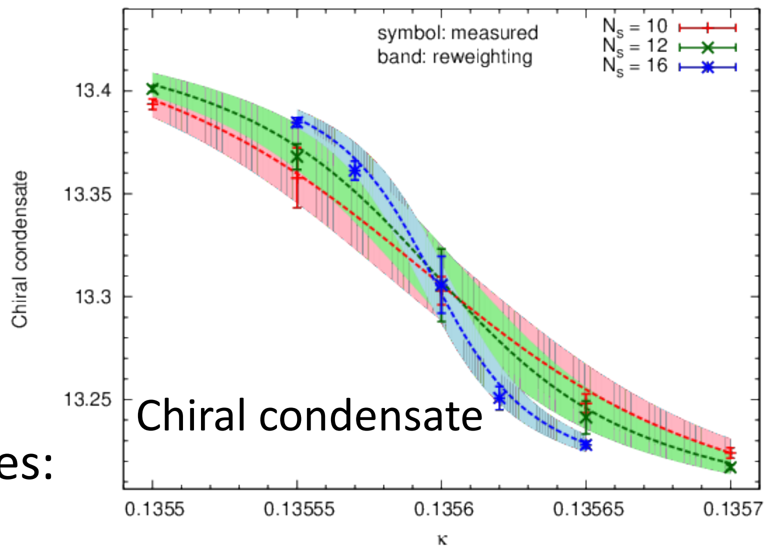
$\beta = 1.68$

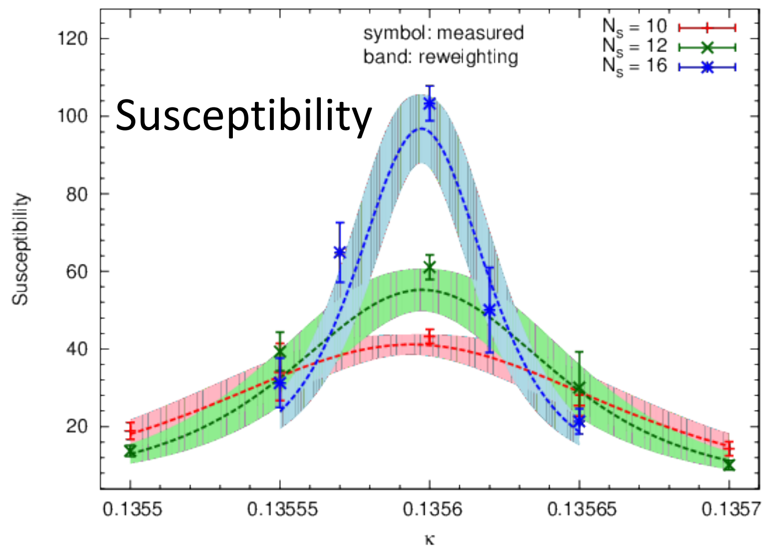
$N_s = 10$

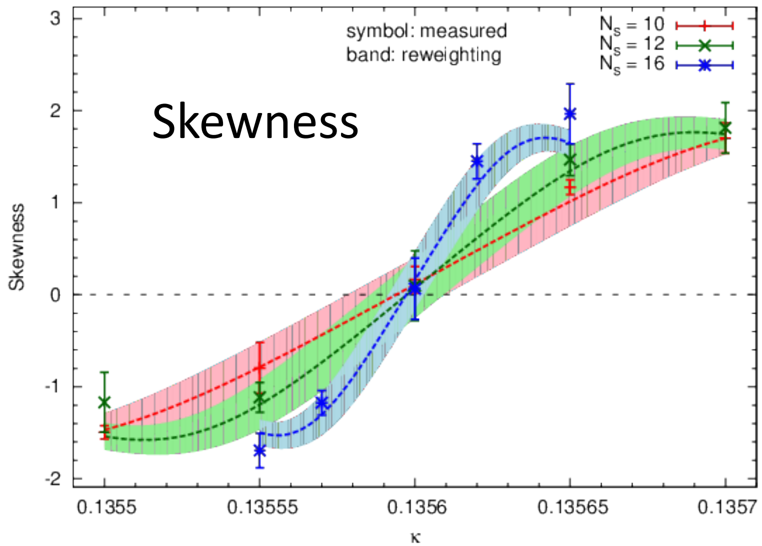
$N_s = 12$

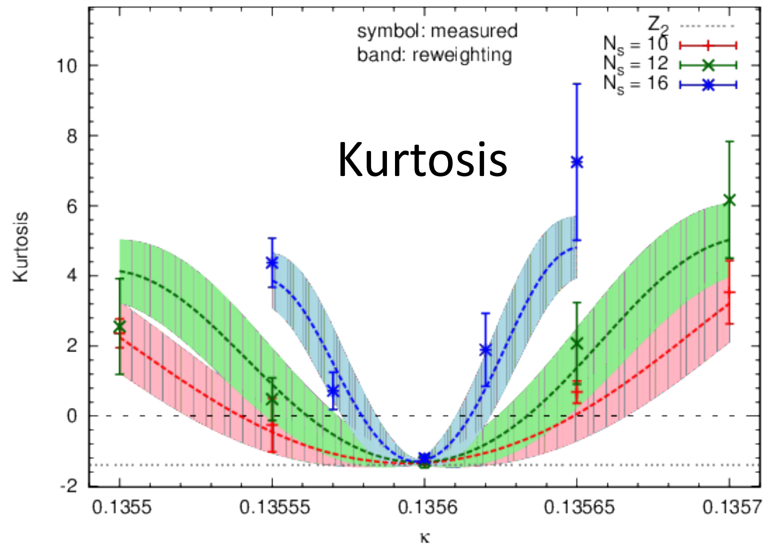
$N_s = 16$

dotted curves: reweighting









An example of cumulants at $N_t = 8$

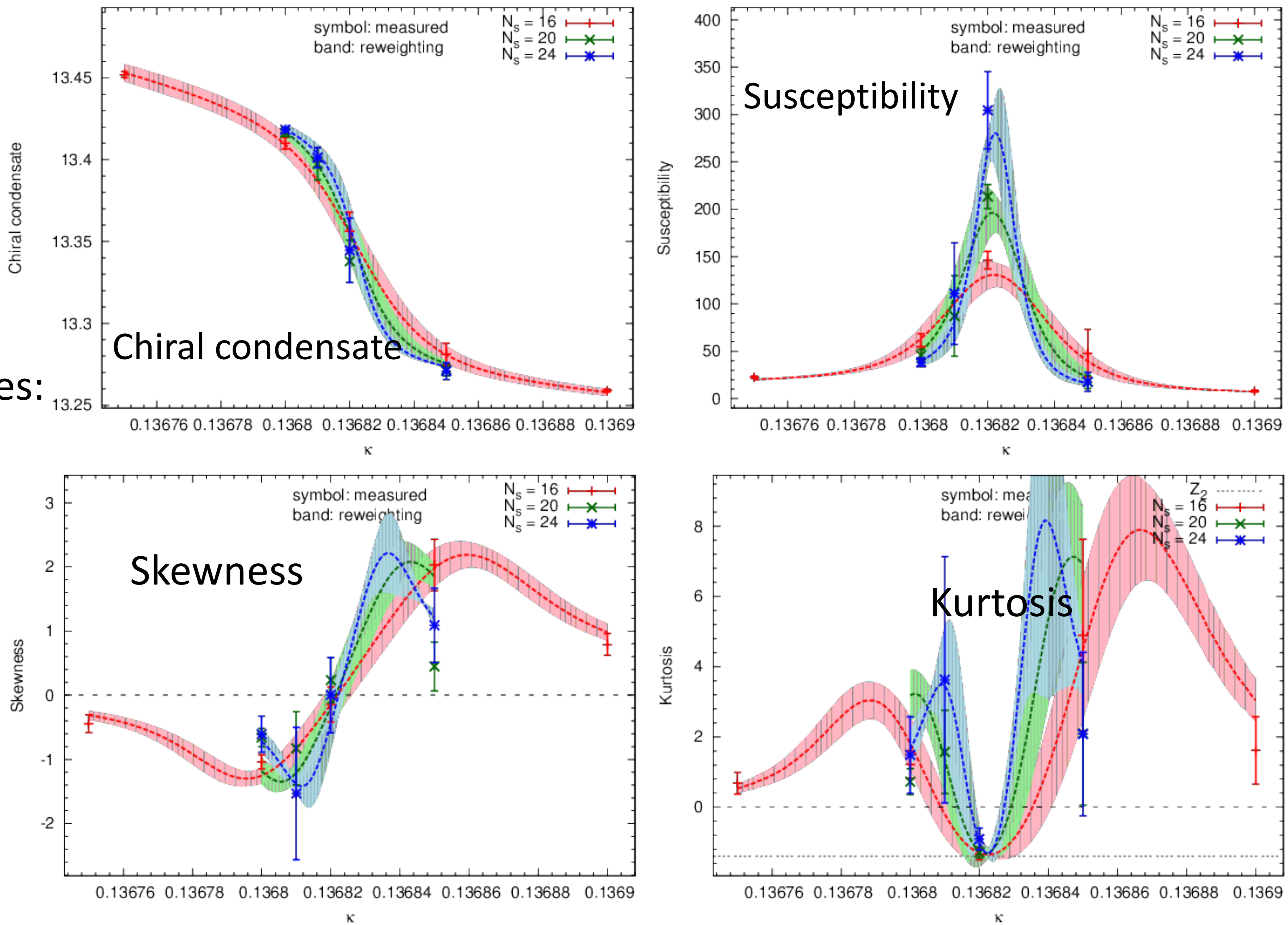
$\beta = 1.66$

$N_s = 16$

$N_s = 20$

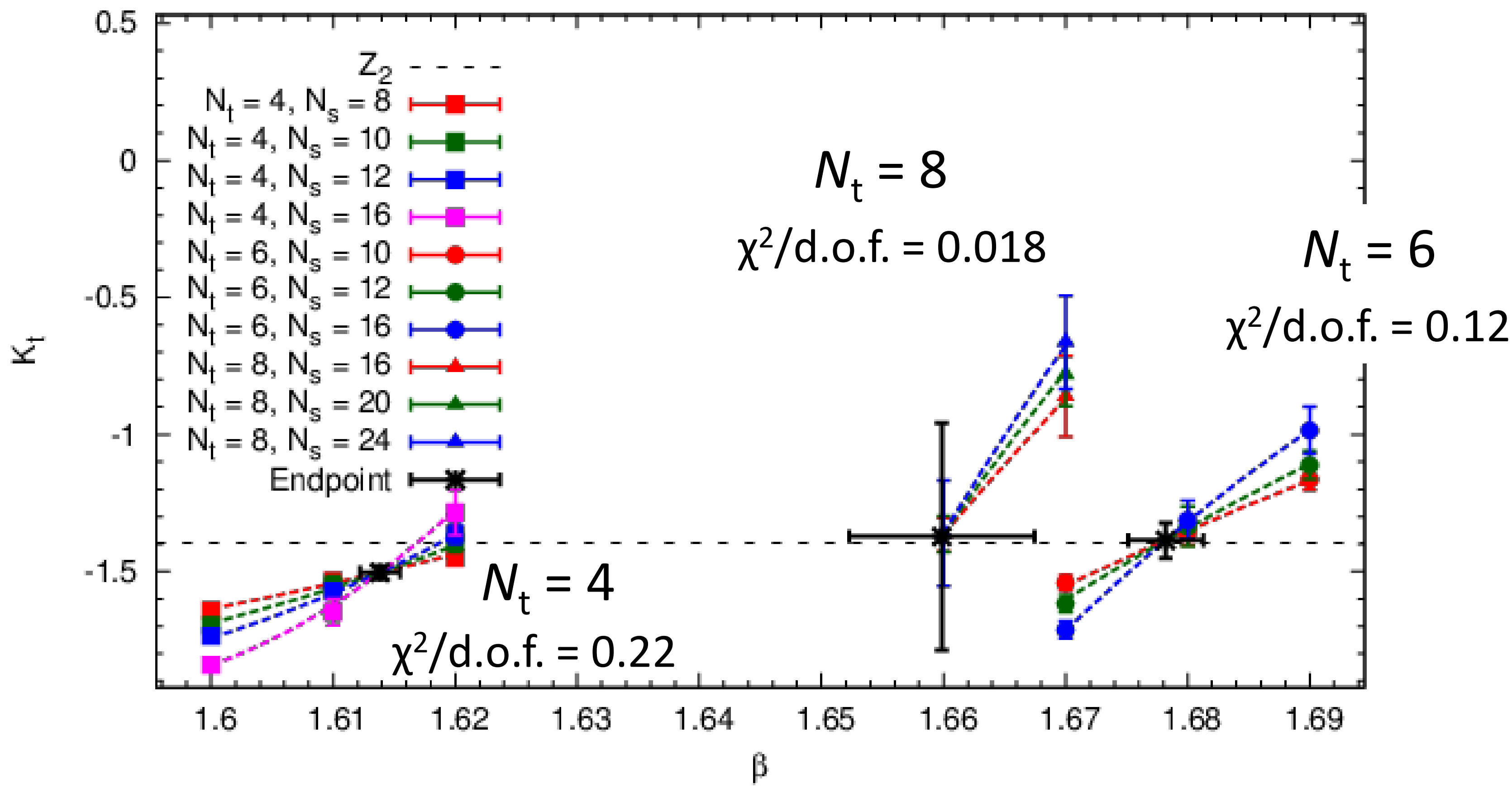
$N_s = 24$

dotted curves: reweighting



Results of kurtosis intersections

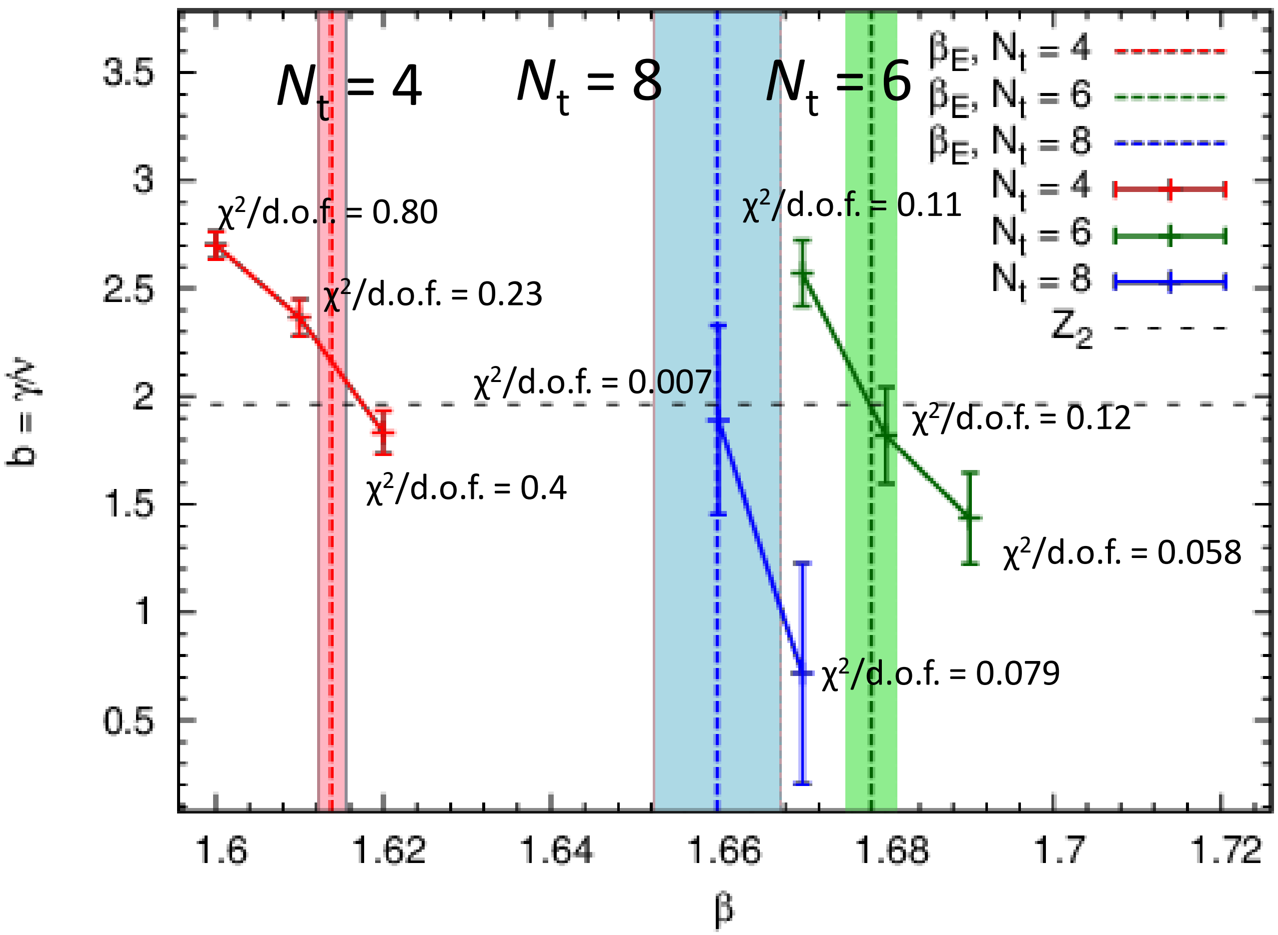
Fit ansatz: $K_t(\beta) = K_E + aN_s^{1/\nu}(\beta - \beta_E)$



K_E slightly deviates from that of Z_2 at $N_t = 4$ while it is consistent with Z_2 at $N_t = 6$ and 8.

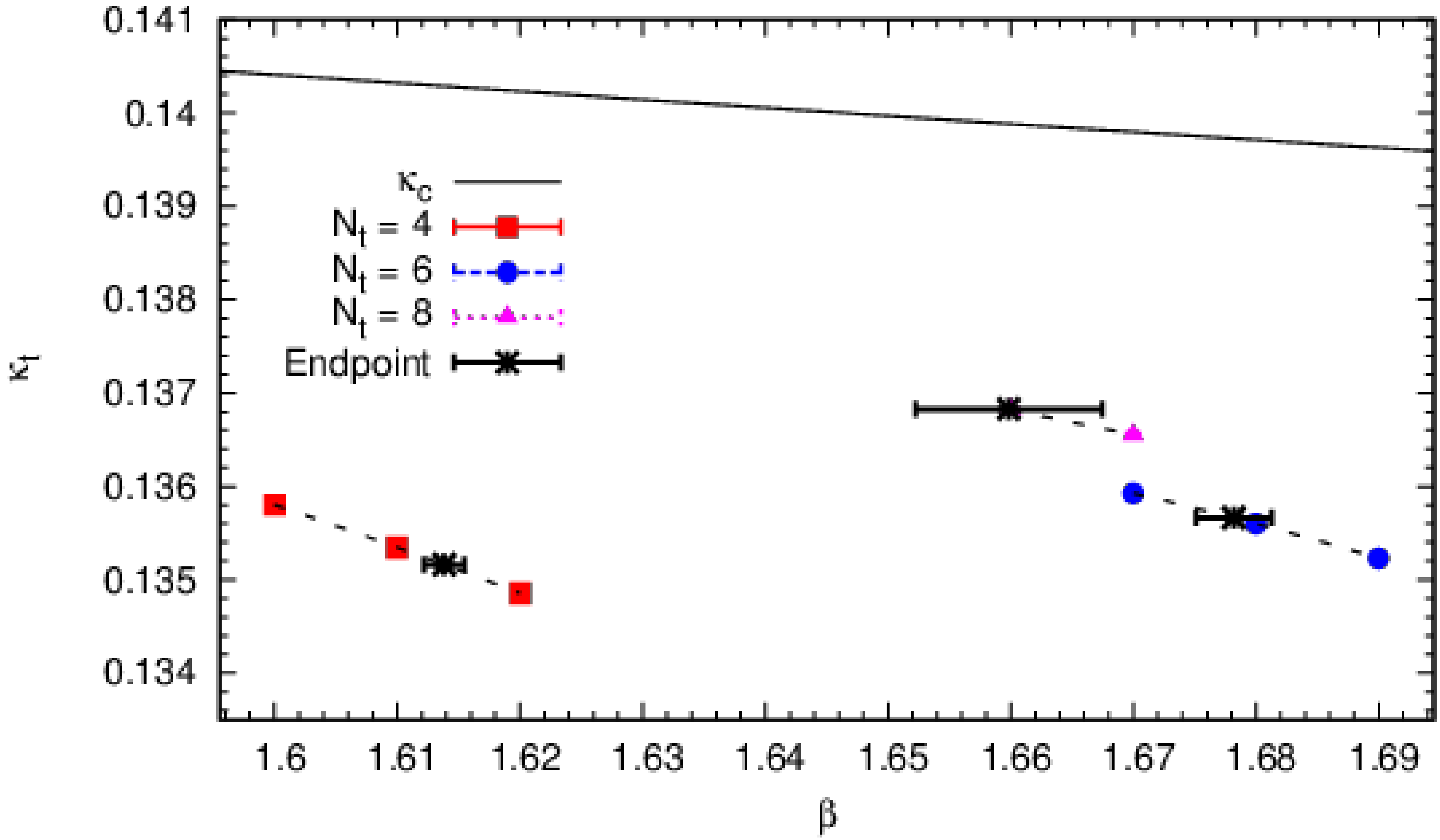
Further universality check with susceptibilities

Fit ansatz: $\chi_t(N_s) = aN_s^b$

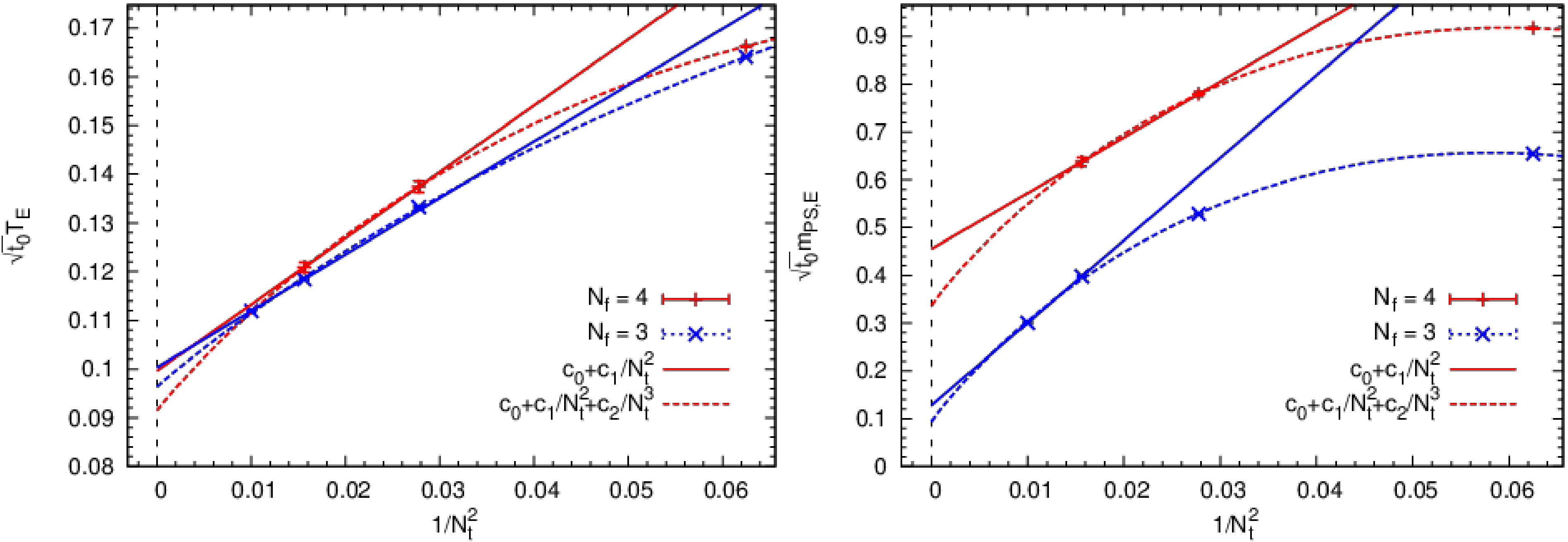


b at the endpoint is larger than that of Z_2 at N_t = 4 while it is consistent with Z_2 at N_t = 6 and 8.

Critical endpoints in N_f = 4 QCD



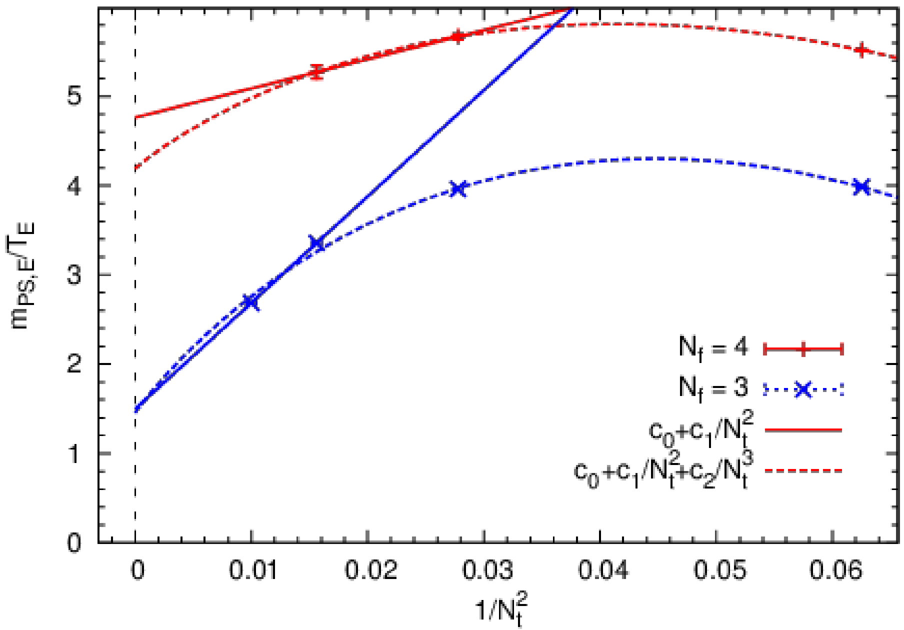
Cutoff dependence of critical endpoints



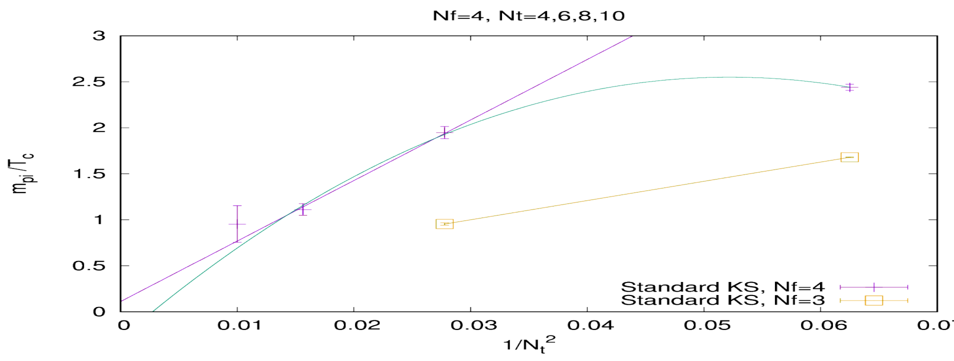
N_f = 3 results: X.-Y. Jin *et al.*, Phys.Rev. D96 (2017) no.3, 034523

N_f = 4 results have similar cutoff dependence as those of N_f = 3.
$m_{PS,E}$ in N_f = 4 is larger than that in N_f = 3.

Comparison with staggered results



Staggerd results:
P. de Forcrand and M. D'Elia,
PoS LATTICE2016 (2017) 081



$m_{PS,E}/T_E$ for N_f = 4 Wilson might remain finite even in a → 0, which is different from $m_{PS,E}/T_E \simeq 0$ for N_f = 4 staggered.

Difference between Wilson and staggered is not due to the rooting.

Summary and outlook

- Critical endpoints in $N_f = 4$ QCD has been studied.
- $N_f = 3$ and $N_f = 4$ have a similar scaling behavior in $a \to 0$.
- Larger $m_{PS,E}$ has been observed in $N_f = 4$ than in $N_f = 3$.
- Wilson and staggered results are different from each other even in $N_f = 4$, which indicates that the difference does not come from the rooting.

- Future plans:

  Further studies for more reliable continuum extrapolations, e.g. simulations with $N_t = 10$.

End

Backup slides

Cumulants at $N_t = 4$, $\beta = 1.60$

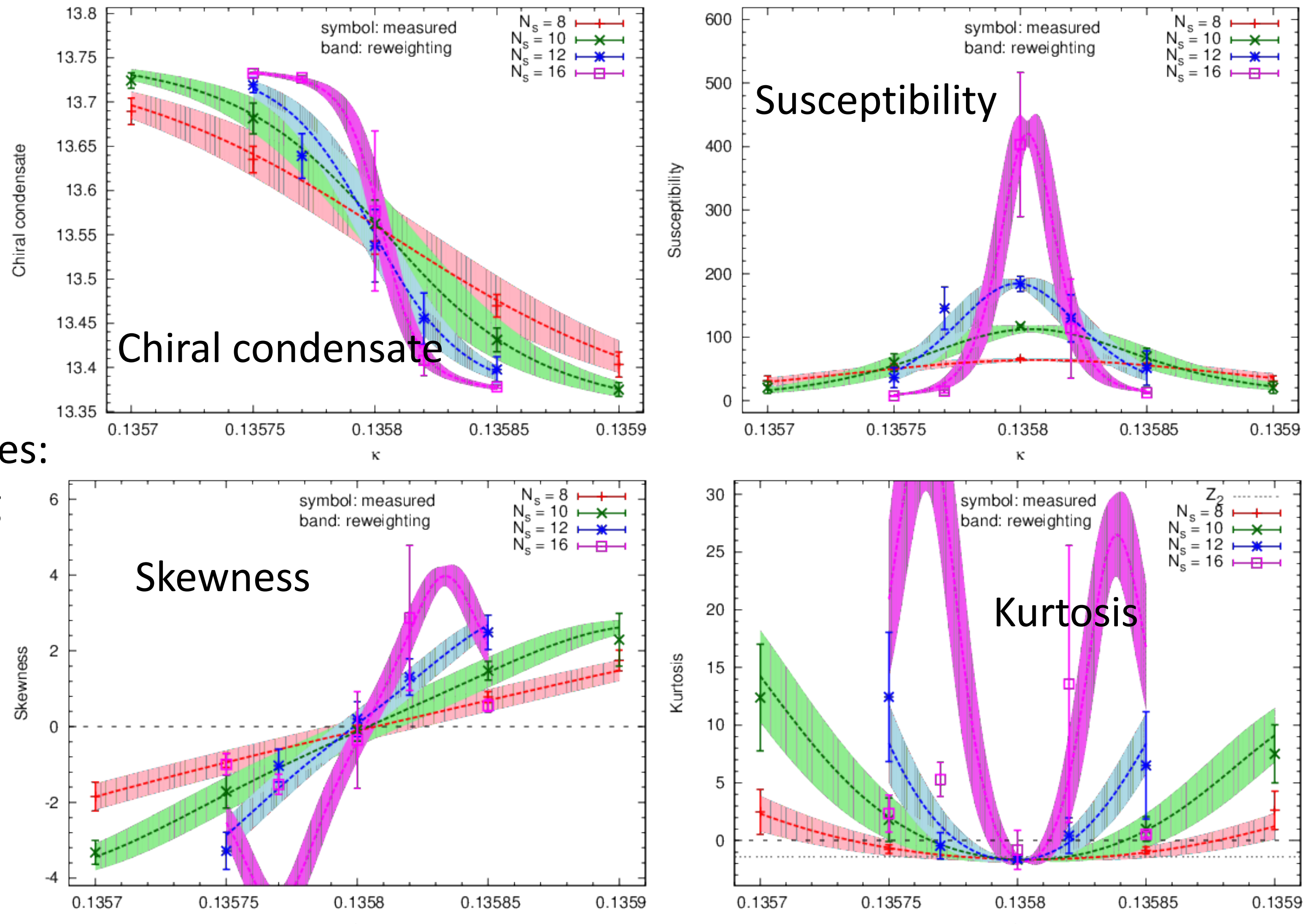
$N_s = 8$

$N_s = 10$

$N_s = 12$

$N_s = 16$

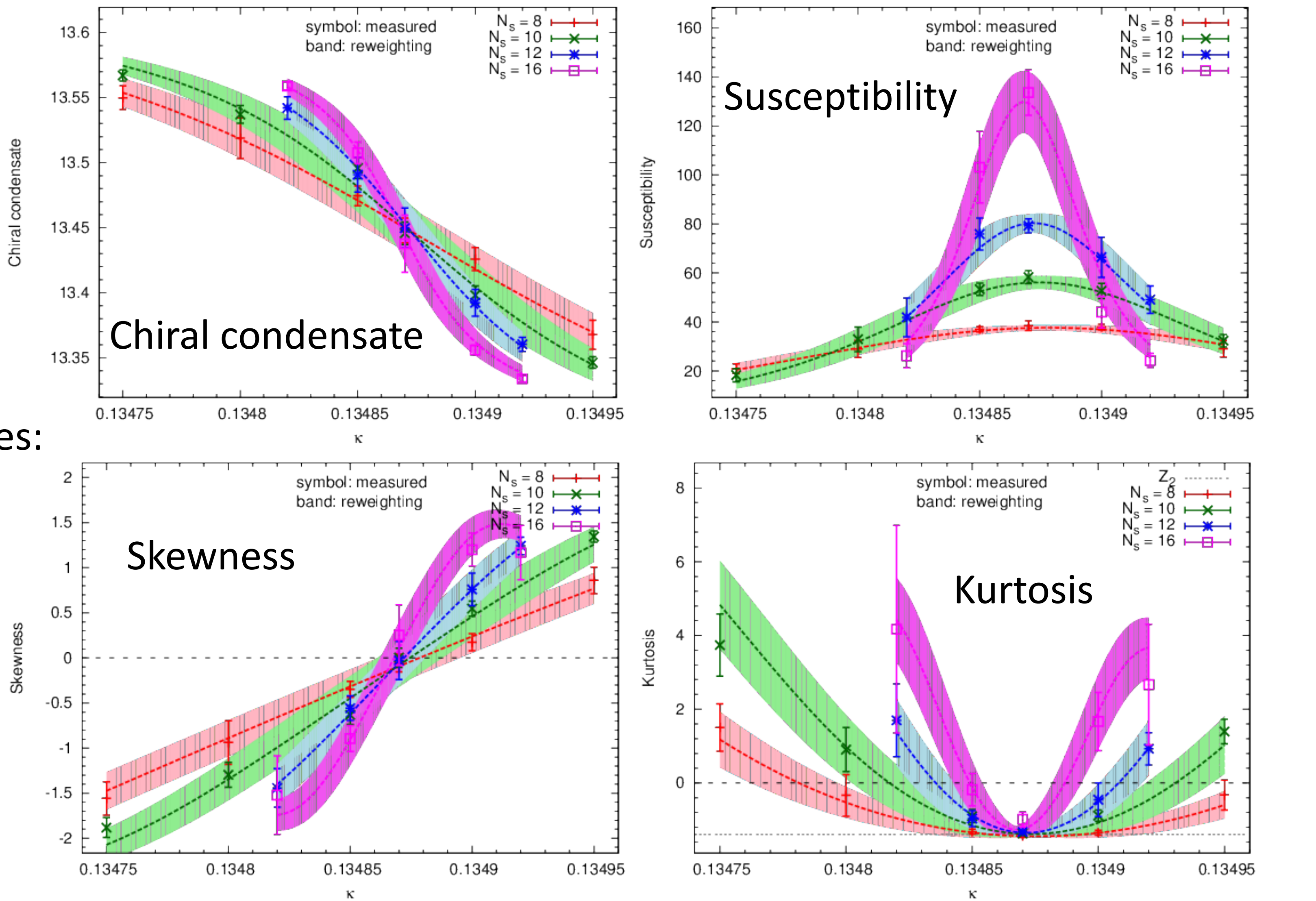
dotted curves: reweighting



Cumulants at $N_t = 4$, $\beta = 1.62$

$N_s = 8$
$N_s = 10$
$N_s = 12$
$N_s = 16$

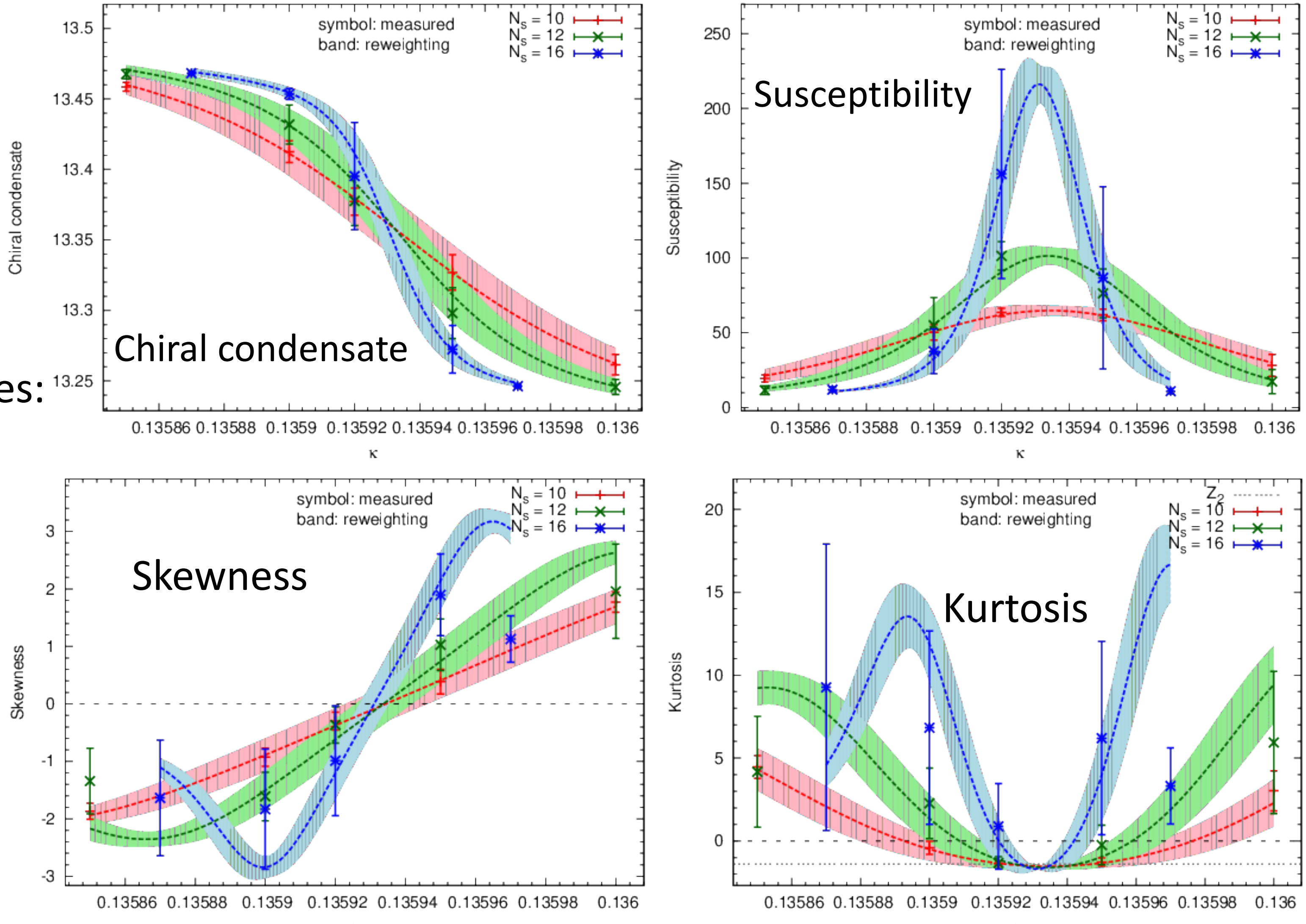
dotted curves: reweighting



Cumulants at N_t = 6, β = 1.67

N_s = 10
N_s = 12
N_s = 16

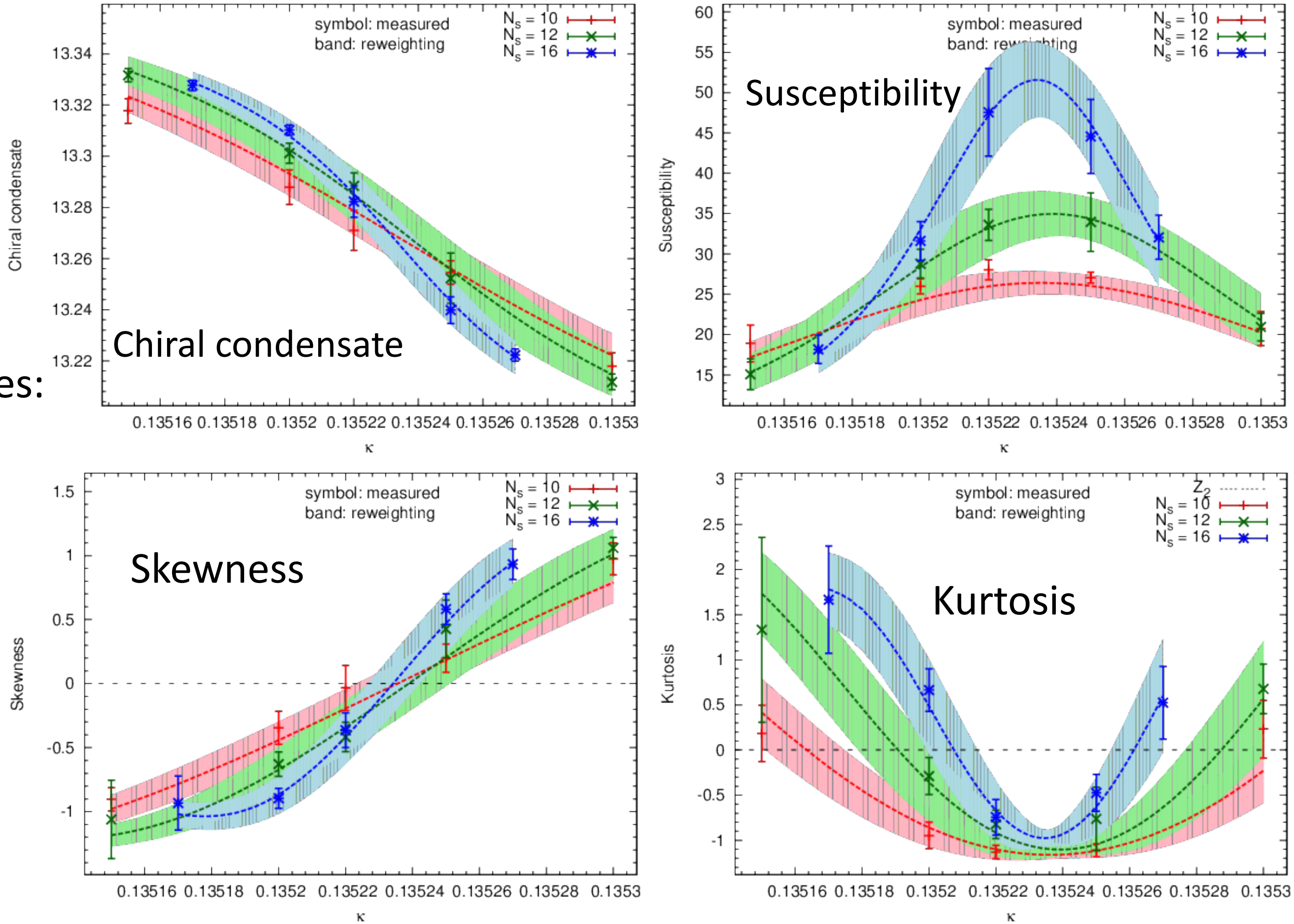
dotted curves: reweighting



Cumulants at $N_t = 6$, $\beta = 1.69$

$N_s = 10$
$N_s = 12$
$N_s = 16$

dotted curves: reweighting



Cumulants at $N_t = 8$, $\beta = 1.67$

$N_s = 16$
$N_s = 20$
$N_s = 24$

dotted curves: reweighting