Universität Bielefeld

CRC-TR 211
Strong-interaction matter under extreme conditions

The fate of axial U(1) in 2+1 flavor QCD towards the chiral limit

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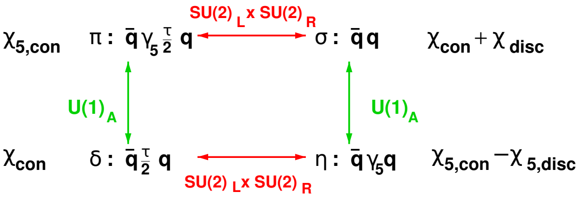
The $U_A(1)$ puzzle

- Nature of chiral phase transition for QCD with two light quark flavors is not yet completely resolved.
- Usually symmetries determine the order parameter across a phase transition.
- However, for 2 light quark flavors anomalous $U_A(1)$ may affect the nature of phase transition.
- How severely $U_A(1)$ is broken can be answered non-perturbatively.
- We want to investigate whether or not the anomalous $U_A(1)$ symmetry in the flavor sector is effectively restored along with the chiral symmetry.

Observables sensitive to $U_A(1)$ breaking

- $U_A(1)$ is not an exact symmetry $\Rightarrow$ no unique order parameter!
- Instead, look at n-point correlation functions which become degenerate upon $U_A(1)$ rotation.
  Start with 2-point correlation functions.



[A. Bazavov et al.,2012]

[see also Sheng-Tai Li, Thursday 11.20h]

$U_A(1)$ breaking and QCD eigenvalue density

- Observable of interest is [Shuryak, 1994]

$$\chi_\pi - \chi_\delta = \int d^4x \left[\langle i\pi^+(x) i\pi^-(0)\rangle - \langle i\delta^+(x) i\delta^-(0)\rangle\right]$$

- Equivalently study $\rho(\lambda, m_f)$ of the Dirac operator [Cohen, 1995, Hatsuda & Lee, 1995]

$$\chi_\pi - \chi_\delta \xrightarrow{V\to\infty} \int_0^\infty d\lambda \frac{4m_f^2 \rho(\lambda, m_f)}{(\lambda^2 + m_f^2)^2}$$

- Possible scenarios:
  - $\lim_{m_f\to 0} \rho(0, m_f) \to 0 \Rightarrow U_A(1)$ trivially restored.
  - $\lim_{\lambda\to 0} \rho(\lambda, m_f) = \delta(\lambda) m_f^\alpha$ with $1 < \alpha < 2 \Rightarrow U_A(1)$ broken.
  - $\lim_{m_f\to 0} \rho(\lambda, m_f) \sim \lambda^3 \Rightarrow U_A(1)$ restored.

More on the eigenvalue spectrum and $U_A(1)$ breaking

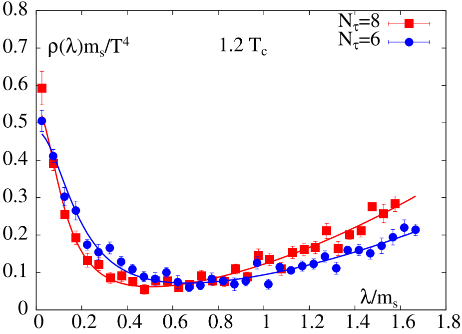
- Looking at higher n-point correlation functions is also important!
- $\rho(\lambda, m_f)$ has been investigated analytically using chiral Ward identities of n-point functions of scalar and pseudo-scalar currents, assuming $\rho(\lambda, m_f)$ to be analytic in m_f^2 [Aoki, Fukaya & Taniguchi, 2012].
- It was shown explicitly that $U_A(1)$ breaking is absent in upto six point correlation functions in the same scalar and pseudo-scalar sectors if $\rho(\lambda, m_f \to 0) \sim \lambda^3$.
- They found out that in the chiral limit $\rho(\lambda) \sim \lambda^n$ with $n > 2$.

Summary of the results till now

- Previous studies at almost physical quark masses:
  Infrared part has both, non-analytic and analytic in λ

  [Sayantan Sharma et al.,2015].

- $\rho(\lambda, m_f) \sim \lambda^2$ at $T = 1.2\,T_c \Rightarrow U_A(1)$ broken.



[Viktor Dick et al.,2015]

Setup

- Gauge ensembles where generated within the Highly Improved Staggered Quark discretization scheme (HISQ) with 2+1 quark flavor.
- We used the overlap Dirac operator to measure the low-lying eigenvalue spectrum.
- 98 eigenvalues per configuration.

m_s/m_l	$N_s^3 \times N_\tau$	β	T/T_c	#conf
27	$32^3 \times 8$	6.390	0.97	78
27	$32^3 \times 8$	6.445	1.03	80
27	$32^3 \times 8$	6.500	1.09	109
40	$32^3 \times 8$	6.390	0.99	213
40	$32^3 \times 8$	6.423	1.03	103
40	$32^3 \times 8$	6.445	1.05	94

Overlap Operator

- The overlap operator is given as,

$$D_{ov} = M\left[1 + \gamma_5 \mathrm{sgn}\left(\gamma_5 D_W(-M)\right)\right]$$

$$\mathrm{sgn}\left(\gamma_5 D_W(-M)\right) = \gamma_5 D_W(-M) / \sqrt{D_W^\dagger(-M) D_W(-M)}$$

  where D_W is the Wilson-Dirac operator with a negative mass parameter $M \in [0, 2)$.

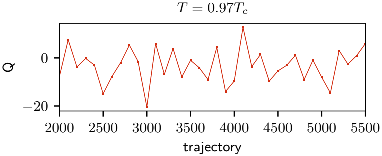
- Eigenvalues of $D_{ov}^\dagger D_{ov}$ are computed using the Kalkreuter-Simma(KS) Ritz algorithm.

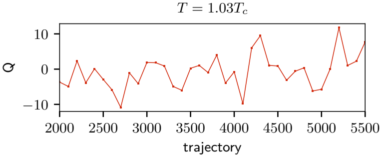
- Zero-modes were not measured. Only eigenvalues with positive or negative chirality have been measured.

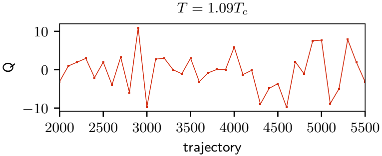
- Finally we perform a tuning of the valence overlap masses to the sea HISQ masses to get the renormalized observables.

Is topological tunneling sufficient?

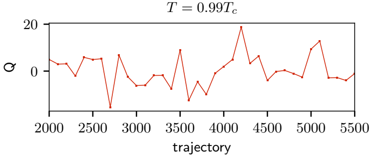
$m_l = m_s/27$:

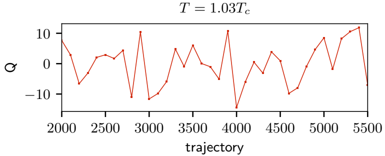


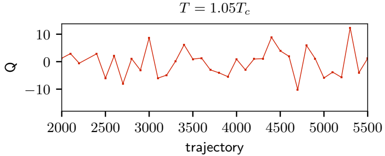




$m_l = m_s/40$:

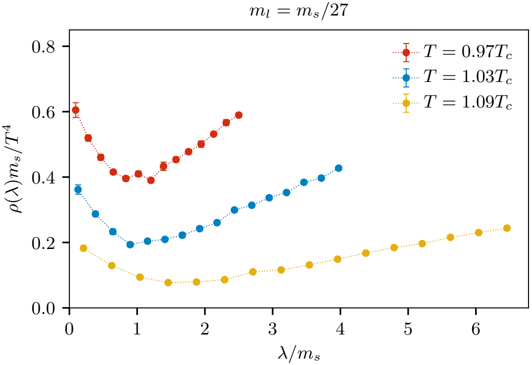






Dirac eigenvalue spectrum

- Analytic part goes as λ^γ with $\gamma \geq 1$.
- Non-analytic part reduces with temperature.
- Going towards smaller quark masses these features survive.



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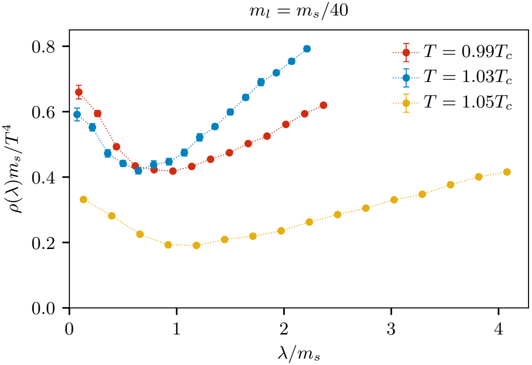

Renormalized $U_A(1)$ breaking parameter as a function of quark mass

- Valence overlap quark mass has been tuned to the HISQ sea quark masses by matching the renormalized quantity

$$\Delta = \frac{m_s\langle\bar{\Psi}\Psi\rangle_l - m_l\langle\bar{\Psi}\Psi\rangle_s}{T^4}$$

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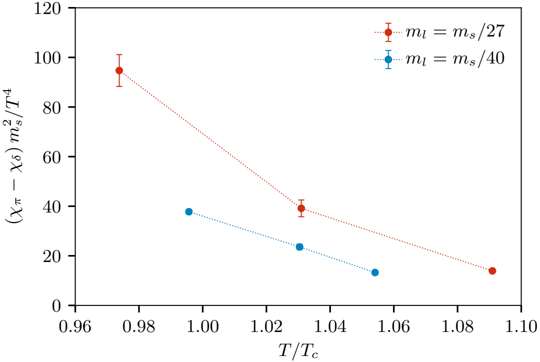

Conclusion

- $\rho(\lambda, m_f) \sim \lambda$ even when m_f reduces from $m_s/27$ to $m_s/40$ for $T/T_c < 1.1$.
- Non-analytic part also survives when we reduce the mass.
- Both of them contribute to the breaking of $U_A(1)$ above T_c after proper retuning of the valence quark masses and looking at renormalized quantities.
- We are looking at even smaller quark masses to check whether our conclusions survive in the chiral limit.