Update on the improved lattice calculation of direct CP-violation in K decays

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Lattice 2018, East Lansing, MI, USA
July 26th 2018

COLUMBIA UNIVERSITY
IN THE CITY OF NEW YORK

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Motivation for studying $K \to \pi\pi$ Decays

- Likely explanation for matter/antimatter asymmetry in Universe, baryogenesis, requires violation of CP.
- Amount of CPV in Standard Model appears too low to describe measured M/AM asymmetry: tantalizing hint of new physics.
- Direct CPV first observed in late 90s at CERN (NA31/NA48) and Fermilab (KTeV) in $K^0 \to \pi\pi$:

$$\eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)}, \qquad \eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)}.$$

$$\mathrm{Re}(\epsilon'/\epsilon) \approx \frac{1}{6}\left(1 - \left|\frac{\eta_{00}}{\eta_{\pm}}\right|^2\right) = 16.6(2.3) \times 10^{-4} \quad \text{(experiment)}$$

measure of direct CPV

measure of indirect CPV

- In terms of isospin states:

ΔI=3/2 decay to I=2 final state, amplitude A_2

ΔI=1/2 decay to I=0 final state, amplitude A_0

$$\omega = \mathrm{Re}A_2/\mathrm{Re}A_0$$

$$A(K^0 \to \pi^+\pi^-) = \sqrt{\frac{2}{3}}A_0 e^{i\delta_0} + \sqrt{\frac{1}{3}}A_2 e^{i\delta_2},$$

$$A(K^0 \to \pi^0\pi^0) = \sqrt{\frac{2}{3}}A_0 e^{i\delta_0} - 2\sqrt{\frac{1}{3}}A_2 e^{i\delta_2}.$$

$$\Rightarrow \quad \epsilon' = \frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}}\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)$$

(δ_I are strong scattering phase shifts.)

- Small size of ε' makes it particularly sensitive to new direct-CPV introduced by many BSM models.

Summary of 2015 published result

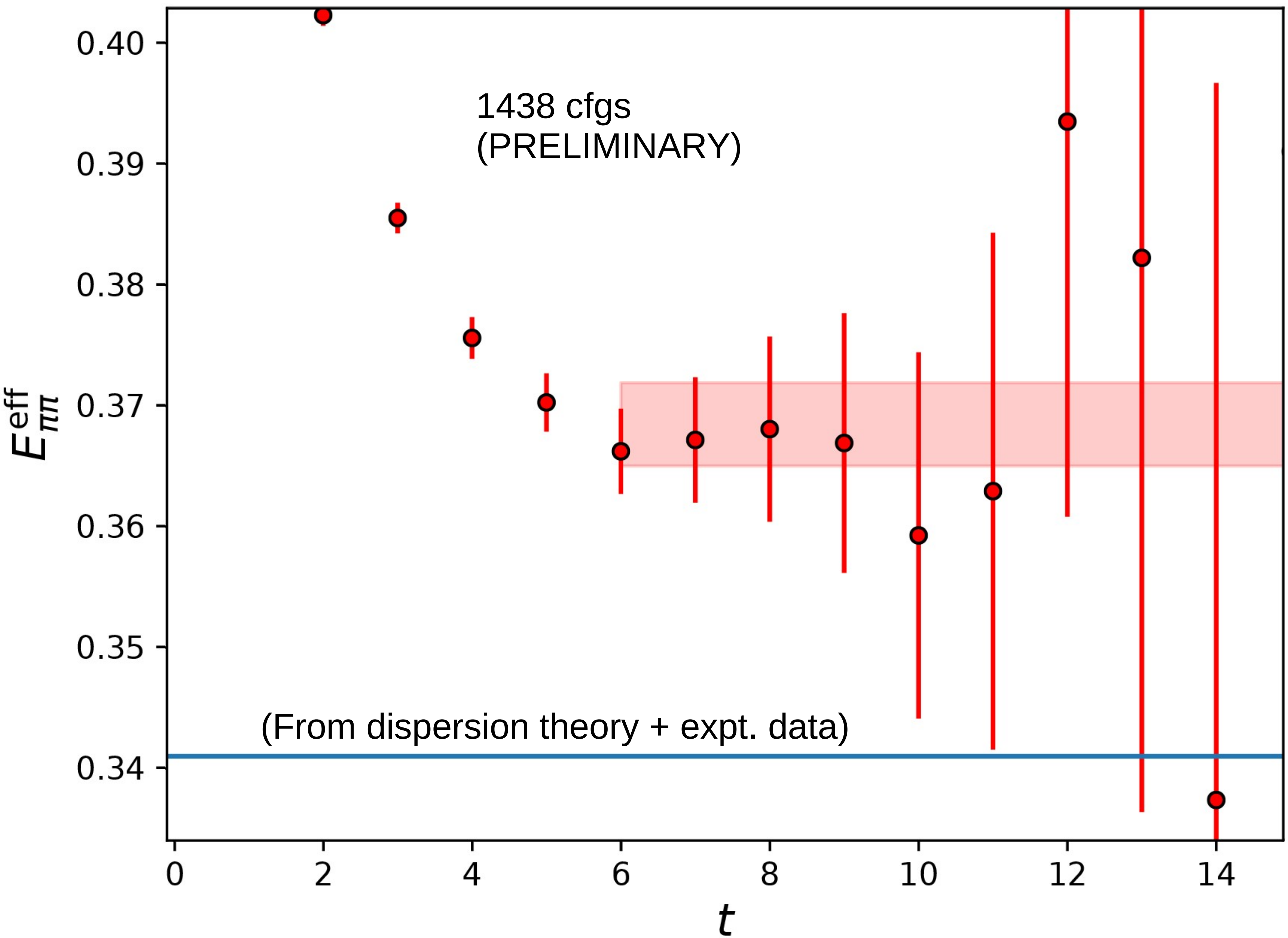
[Phys.Rev.Lett. 115 (2015) 21, 212001]

- A_2 previously computed on lattice precisely (12% total error)
  - [Phys.Rev. D91 (2015) no.7, 074502]
- Computed A_0 on 32^3x64 Mobius DWF ensemble with Iwasaki+DSDR gauge action. G-parity BCs in 3 directions to give physical kinematics.
- Single, coarse lattice with a^{-1}= 1.38 GeV but large physical volume to control FV errors.
- Re(A_0) and Re(A_2) from expt.
- Lattice values for Im(A_0), Im(A_2) and the phase shifts.

$$\boxed{\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right) = \begin{array}{ll} 1.38(5.15)(4.43)\times 10^{-4} & \text{(our result)} \\ 16.6(2.3)\times 10^{-4} & \text{(experiment)} \end{array}}$$

- Find reasonable consistency with experimental value (at 2.1σ level).
- Total error on is ~3x the experimental error.
- “This is now a quantity accessible to lattice QCD”!
- Focus since has been to improve statistics and reduce / improve understanding of systematic errors.

The ππ puzzle



Resolving the ππ puzzle

- Since 2015 publication have been working to resolve discrepancy between our lattice I=0 ππ phase shift ($\delta_0 = 23.8(4.9)(1.2)$) and that predicted by dispersion theory (~34^o). [RBC&UKQCD PRL 115 (2015) 21, 212001] [Colangelo *et al*, Nucl.Phys. B603 (2001) 125-179]

- Increased statistics from 216 to almost 1400 configurations, a 6.5x increase. Observed discrepancy becomes more significant.
- Alongside existing 1s hydrogen wavefunction pion source smearing we added a 2s form and a scalar (σ=ūd) ππ operator to the 2-pt function calculation.
- Also added 2s pion sources to K→ππ calculation.

- While 2s data appears too noisy, combined fits (or GEVP) to ππ→ππ, σ→ππ and σ→σ correlators result in considerably lower ground-state energy: [cf. T.Wang, prev. talk]

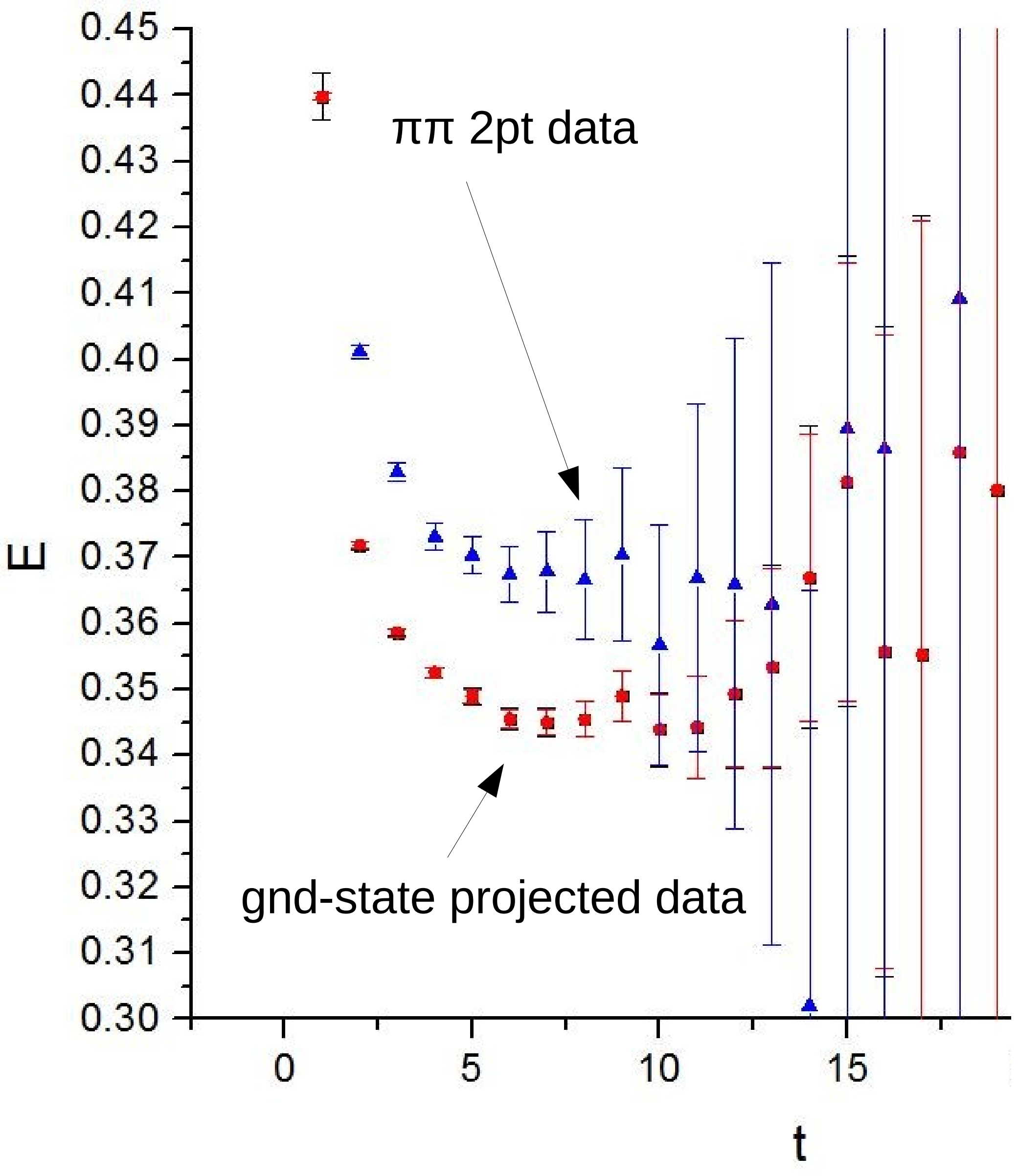
508(5) MeV [1386 cfgs] from ππ→ππ alone
vs
483(1) MeV [501 cfgs] from sim. fit of all 3 correlators.

- Strong new evidence for nearby excited finite-volume ππ state. Indeed such a state with E ~ 770 MeV is predicted by dispersion theory.

Implications for K→ππ and resolution

- Despite vast increase in statistics, *this state cannot be resolved based on the time dependence using only a single ππ operator.*
- Possibly a significant underestimate of excited state systematic in K→ππ calculation that can only be resolved by adding additional operators.
- In response we have **expanded the scope of the calculation**:
    - Added K→σ matrix elements. This involved significant work in both deriving the Wick contractions and in implementing/optimizing the parallel code.
    - Added more pion momenta.
      Previously we computed only zero-momentum ππ-states with pion momenta in the set $(\pm1,\pm1,\pm1)\pi/L$ (8-total).
      We have now added 24 new momenta: $(\pm3,\pm1,\pm1)\pi/L$ + perms.
- Result is **3x increase in the number of S-wave ππ operators in K→ππ**
- Using sim. fits / GEVP to 2-pt function data can then determine appropriate linear comb. of these 3 sets of matrix elements that best projects onto the ground-state.
- Also added ππ 2pt functions with non-zero total ππ momenta. Will allow calculation of phase shift at several (smaller) additional center-of-mass energies.
    - Additional points that can be compared to dispersive result / experiment
    - Improve ~11% systematic on Lellouch-Luscher factor associated with slope of phase shift.

Effect of projecting 2pt data onto ground-state using existing data (c/o T.Wang)



Expect even better
ground-state projection
with new higher-momentum
operators in upcoming analysis

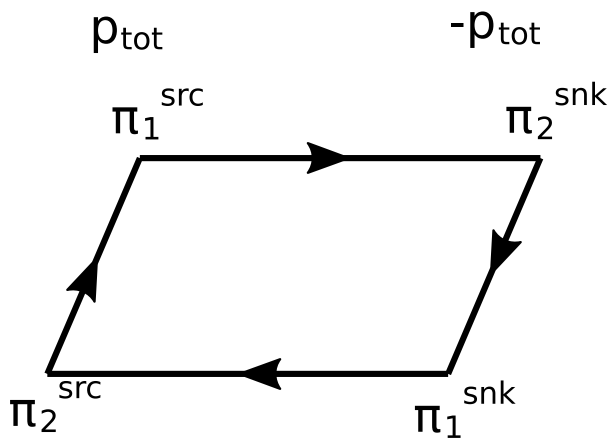
Scaling of ππ contraction timing

- On 512-nodes of BG/Q, computing the 8x8=64 ππ contractions with 0 total ππ momentum takes 13.6 mins.
- However: 32 pion momenta, computing all contractions with
  $p_{\pi\pi}$=(0, 0, 0) , (±2π/L, 0, 0) , (±2π/L, ±2π/L, 0) , (±2π/L, ±2π/L, ±2π/L) + perms
  Number rises to 7848 contractions : ~27.8 *hours* on the ππ contractions alone!
- To make tractable take advantage of symmetries.
  Take care to use only those that do not significantly affect statistical error.
- To determine symmetries to use, we studied our ππ data including 121 cfgs of new data at non-zero ππ momentum computed using saved meson fields.
- Examined:

Parity: exchange $\vec{p} \to -\vec{p}$

Axis permutation: global interchange of momentum components
(GPBC in 3 dirs so all spatial dirs equivalent)

“Auxiliary diagram” symmetry:

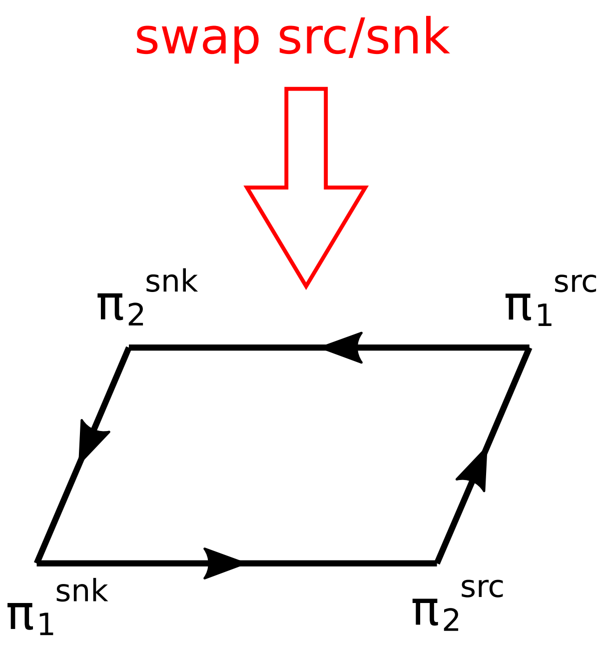


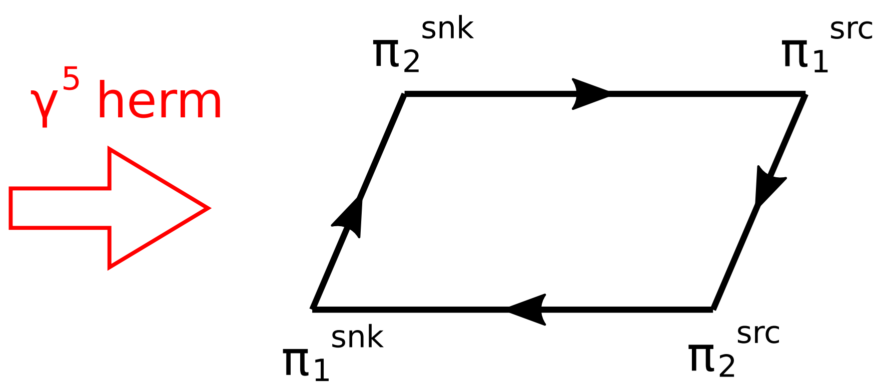
Source/sink timeslice interchange coupled with γ^5 hermiticity relates $\pi\pi$ correlators (after temporal folding/config avg):

$$\langle C(\vec{p}_1^{\,\mathrm{src}}, \vec{p}_1^{\,\mathrm{snk}}, \vec{p}_{\mathrm{tot}})\rangle \equiv$$

$$\langle C(-\vec{p}_{\mathrm{tot}} - \vec{p}_1^{\,\mathrm{snk}}, \vec{p}_{\mathrm{tot}} - \vec{p}_1^{\,\mathrm{src}}, -\vec{p}_{\mathrm{tot}})\rangle$$

[Parity + aux.diag together preserve p_{tot}]





$p_{tot} = 0$

- Observe all symmetries individually well realized and do not significantly affect statistical error.

Quantity	Orig (64 diags)	Pty+perm (10 diags)	Aux+pty+perm (8 diags)
$\|A_{\pi\pi}\|^2$	0.1609(22)	0.1615(24)	0.1601(24)
$E_{\pi\pi}$	0.3686(33)	0.3690(36)	0.3672(36)
$C_{\pi\pi}$	$3(10) \times 10^{-5}$	$3(10) \times 10^{-5}$	$-1(11) \times 10^{-5}$
χ^2/dof	1.30(57)	1.25(54)	1.10(52)

- 8x reduction in #correlators for base pion momentum set!

$p_{tot} = (\pm 2,0,0)\pi/L$ + perms

- Applied globally, utilizing parity for 2x reduction in diags does not affect error, but axis permutation does: suggests (2,0,0), (0,2,0) and (0,0,2) largely uncorrelated.

Quantity	Orig (96 diags)	Pty (48 diags)	Pty+perm (16 diags)
$\|A_{\pi\pi}\|^2$	0.3466(41)	0.3462(42)	0.3478(64)
$E_{\pi\pi}$	0.3869(23)	0.3869(23)	0.3879(40)
$C_{\pi\pi}$	$2(1) \times 10^{-4}$	$2(1) \times 10^{-4}$	$4(2) \times 10^{-4}$

- Take second column and allow parity, axis perm and aux. diag. to relate the 48 diags:

Quantity	Pty (48 diags)	Pty+perm+aux (21 diags)
$\|A_{\pi\pi}\|^2$	0.3462(42)	0.3464(41)
$E_{\pi\pi}$	0.3869(23)	0.3868(23)
$C_{\pi\pi}$	$2(1) \times 10^{-4}$	$2(1) \times 10^{-4}$

- 4.5x reduction in #diagrams with no observed increase in errors.
- Similar picture observed for $(\pm 2,\pm 2,0)\pi/L$ and $(\pm 2,\pm 2,\pm 2)\pi/L$:
  Different orientations (up to parity) largely uncorrelated but applying symmetries for fixed p_{tot} leaves errors unchanged.

- <u>For our extended calculation</u>
  Using parity to exclude 1/2 of diags with $p_{tot} \neq 0$: 7848 diags → 4436
  Then applying symmetries with fixed p_{tot} :

p^{tot}	Total	pty	pty+perm	pty+perm+aux
$(0,0,0)$	1024	512	102	62
$(2,0,0)$	1200	1200	654	357
$(2,2,0)$	768	768	408	228
$(-2,2,0)$	768	768	384	216
$(2,2,2)$	169	169	33	21
$(-2,2,2)$	507	507	267	153
Total	4436			1037

- Overall 7.6x reduction in diagram count, reducing time (pre-optimization) to 3.7 hours.

BG/Q Timings and Status

Description	Time (hours)
Light quark Lanczos	5.78
Light quark A2A vectors	4.48
Heavy quark A2A vectors	2.68
Gauge fix	0.31
Kaon 2pt function	0.44
Kaon WW meson fields	0.08
$K \to \sigma$ contractions	0.67
Sigma 2pt function	0.02
Light-light 1s pion meson fields	5.23
$\pi\pi \to \sigma$ 2pt function	0.06
$K \to \pi\pi$ contractions	7.02
Pion 2pt function	0.01
$\boxed{\pi\pi \text{ 2pt contractions}}$	1.51
Configuration total	29.10

- Currently running on 3x 512-node partitions of BG/Q at BNL
- Timing per configuration ~29 hours
- ππ contraction time only 1.5 hours after utilizing symmetries and code optimizations.
- Currently have measured 44 configurations (as of last night)
- New data can be combined with existing 1400 configs using super-jackknife procedure
- **Expect to be able to start serious analysis when ~100 configs, i.e. within the month.**

Conclusions

- Inclusion of additional scalar $\pi\pi$ operator in order to attempt to understand discrepancy with dispersion theory reveals nearby excited state.
- State unresolvable with just single operator, even with 6.5x more statistics.
- Suggests excited state systematic on published $K\rightarrow\pi\pi$ calculation significantly underestimated.
- In response added scalar operator and 24 additional pion momenta to $K\rightarrow\pi\pi$ calc, **increasing # of S-wave $\pi\pi$ operators in $K\rightarrow\pi\pi$ by 3x**
- Using 2pt data will ascertain appropriate linear combination that best projects onto ground state.
- New pion momenta and inclusion of non-zero CoM $\pi\pi$ momenta in 2pt calculation required utilization of symmetries to make computationally tractable.
- Generating 5 new measurements every 2 days on 3x 512-node BG/Q machines.

We hope to have enough new data to begin serious analysis within the next few weeks

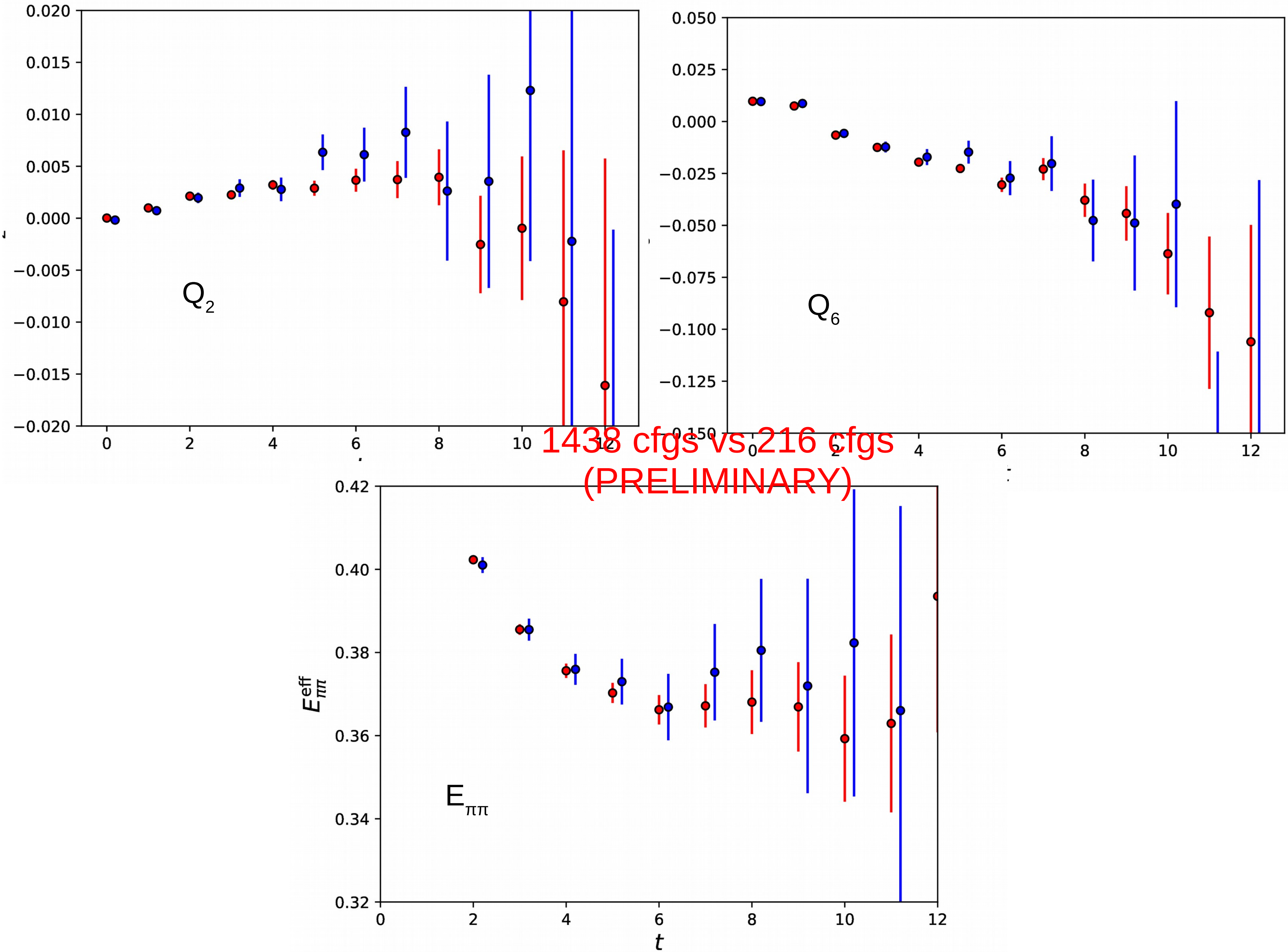
Thank you!

Statistics increase

- Original goal was a 4x increase in statistics over 216 configurations used in 2015 analysis.
- 4x reduction in configuration generation time obtained via algorithmic developments (exact one-flavor implementation)
- Large-scale programme performed involving many machines:

Source	Determinant computation	Independent configs.
Blue Waters	RHMC	34+18+4+3
KEKSC	RHMC	106
BNL	RHMC	208
DiRAC	RHMC	151
KEKSC	EOFA	275+215
BNL	EOFA	245
		1259 total

- Measurements performed using IBM BG/Q machines at BNL and the Cori computer (Intel KNL) at NERSC largely complete.
- Including original data, now have 6.7x increase in statistics!



1438 cfgs vs 216 cfgs
(PRELIMINARY)
Q2
Q6
Eππ
$E_{\pi\pi}^{\mathrm{eff}}$
t

Systematic error improvements

Description	Error	Description	Error
Finite lattice spacing	12%	Finite volume	7%
Wilson coefficients	12%	Excited states	$\leq 5\%$
Parametric errors	5%	Operator renormalization	15%
Unphysical kinematics	$\leq 3\%$	Lellouch-Lüscher factor	11%
Total (added in quadrature)			27%

[RBC&UKQCD PRL 115 (2015) 21, 212001]

NPR+Wilson Coefficients

- NPR error large due to use of 1-loop PT to match to MSbar at low, 1.53 GeV renormalization scale.
- Since 2015 have improved NPR error 15% → 8% (preliminary) by increasing scale to 2.29 GeV using step-scaling procedure. [PoS LATTICE2016 (2016) 308]
- Inclusion of dim.6 gauge-invariant operator G_1 which mixes with Q_i under renormalization, effects demonstrated to be %-scale as expected.

[G. McGlynn arxiv:1605.08807]

Do not expect significant improvement in Wilson coeffs. error as dominated by use of PT to cross the charm threshold (1.29 GeV).

- Working on circumventing this by computing 3→4 flavor matching non-perturbatively.
- Requires $\mu \ll m_c$. At these low energies, MOM-scheme NPR severely hampered by increased mixing with tower of gauge-noninvariant operators.
- Circumvent using position-space NPR which does not require gauge fixing.

Discretization error

- Currently have results only on single lattice with coarse lattice spacing a^{-1}=1.38(1) GeV.
- Require second lattice spacing. Going to finer lattice requires more lattice sites; prohibitively expensive for current gen. computers.
- Promising alternative is to go to a coarser lattice spacing, $a^{-1} \sim 1.0$ GeV. Preliminary studies suggest discretization errors remain under control.

[EPJ Web Conf. 175 (2018) 02006]

Related projects on the horizon:

- Performing calculation taking advantage of modern multi-operator techniques to fit excited-state ππ contributions directly, without G-parity BCs.
- Laying the groundwork for non-perturbatively computing the effects of isospin breaking and electromagnetism. [EPJ Web Conf. 175 (2018) 13016]
- Study of complete, non-perturbative calculation of Wilson coefficients

[EPJ Web Conf. 175 (2018) 13014, arXiv:1711.05768]