HISQ 2+1+1 light quark hadronic vacuum polarization at the physical point

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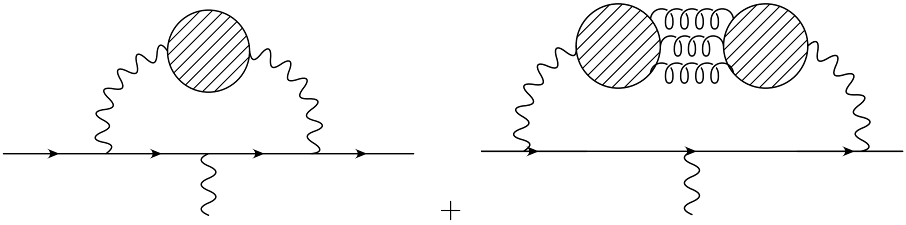
Outline I

HVP contribution to muon g-2 [Blum, 2003, Lautrup et al., 1971]



Using lattice QCD and continuum, ∞-volume pQED

$$a_\mu(\mathrm{HVP}) = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dq^2\, f(q^2)\, \hat{\Pi}(q^2)$$

$f(q^2)$ is known, $\hat{\Pi}(q^2)$ is subtracted HVP, $\hat{\Pi}(q^2) = \Pi(q^2) - \Pi(0)$, computed directly on Euclidean space-time lattice

$$\Pi^{\mu\nu}(q) = \int d^4x\, e^{iqx} \langle j^\mu(x) j^\nu(0)\rangle \qquad j^\mu(x) = \sum_i Q_i \bar{\psi}(x)\gamma^\mu\psi(x)$$

$$= \Pi(q^2)(q^\mu q^\nu - q^2\delta^{\mu\nu})$$

Time-momentum representation Bernecker-Meyer 2011

Interchange order of FT and momentum integrals

$$\Pi(q^2) - \Pi(0) = \sum_t \left(\frac{\cos qt - 1}{q^2} + \frac{1}{2} t^2 \right) C(t)$$

$$C(t) = \frac{1}{3} \sum_{x,i} \langle j_i(x) j_i(0) \rangle$$

$$w(t) = 2\alpha^2 \int_0^\infty \frac{d\omega}{\omega} f(\omega^2) \left[\frac{\cos \omega t - 1}{(2 \sin \omega/2)^2} + \frac{t^2}{2} \right]$$

$$a_\mu^{\mathrm{HVP}} = \sum_t w(t) C(t)$$

(note double subtraction)

Staggered Dirac operator M

Sum of hermitian and anti-hermitian parts, so it satisfies (even-odd ordering)

$$M\begin{pmatrix} n_o \\ n_e \end{pmatrix} = \begin{pmatrix} m & M_{oe} \\ M_{eo} & m \end{pmatrix}\begin{pmatrix} n_o \\ n_e \end{pmatrix} = (m + i\lambda_n)\begin{pmatrix} n_o \\ n_e \end{pmatrix} \tag{1}$$

and

$$\begin{pmatrix} m & -M_{oe} \\ -M_{eo} & m \end{pmatrix}\begin{pmatrix} m & M_{oe} \\ M_{eo} & m \end{pmatrix}\begin{pmatrix} n_o \\ n_e \end{pmatrix} = \tag{2}$$

$$\begin{pmatrix} m^2 - M_{oe}M_{eo} & 0 \\ 0 & m^2 - M_{eo}M_{oe} \end{pmatrix}\begin{pmatrix} n_o \\ n_e \end{pmatrix} = (m^2 + \lambda_n^2)\begin{pmatrix} n_o \\ n_e \end{pmatrix} \tag{3}$$

Compute eigenvectors $n_{o(e)}$, $m^2 + \lambda^2$ of preconditioned Dirac operator

Staggered Dirac operator M

Eigenvectors of preconditioned operator are eigenvectors of M with squared magnitude eigenvalues, construct the even part from odd,

$$n_e = \frac{-i}{\lambda_n} M_{eo} n_o.$$

eigenvalues come in $\pm$ pairs: If (n_o, n_e) is an eigenvector with eigenvalue λ, then

$$(-1)^x \psi(x) = (-n_o, n_e)$$

is also an eigenvector with eigenvalue $-\lambda$.

$$\begin{pmatrix} m & M_{oe} \\ M_{eo} & m \end{pmatrix} \begin{pmatrix} -n_o \\ n_e \end{pmatrix} = (m - i\lambda_n) \begin{pmatrix} -n_o \\ n_e \end{pmatrix}, \qquad (4)$$

Thus we can construct pairs of eigenvectors with $\pm i\lambda$ for each λ^2, n_o!

HVP using spectral decomposition of M^{-1}

Use conserved current

$$J^{\mu}(x) \;\; = \;\; -\frac{1}{2}\eta_{\mu}(x)\left(\bar{\chi}(x+\hat{\mu})U^{\dagger}_{\mu}(x)\chi(x) \; + \; \bar{\chi}(x)U_{\mu}(x)\chi(x+\hat{\mu})\right)$$

and spectral decomposition of propagator

$$M^{-1}_{x,y} \;\; = \;\; \sum_{n}^{N_{(\mathrm{low})}} \frac{\langle x|n\rangle\langle n|y\rangle}{m+i\lambda_n} + \frac{\langle x|n_-\rangle\langle n_-|y\rangle}{m-i\lambda_n} \tag{5}$$

(6)

HVP using spectral decomposition of M^{-1}

$$
\begin{aligned}
4J_\mu(t_x)J_\nu(t_y) &= \sum_{m,n}\sum_{\vec{x}} \frac{\langle m|x+\mu\rangle U^\dagger_\mu(x)\langle x|n\rangle}{\lambda_m} \sum_{\vec{y}} \frac{\langle n|y\rangle U_\nu(y)\langle y+\nu|m\rangle}{\lambda_n} \\
&+ \sum_{\vec{x}} \frac{\langle m|x\rangle U_\mu(x)\langle x+\mu|n\rangle}{\lambda_m} \sum_{\vec{y}} \frac{\langle n|y\rangle U_\nu(y)\langle y+\nu|m\rangle}{\lambda_n} \\
&+ \sum_{\vec{x}} \frac{\langle m|x+\mu\rangle U^\dagger_\mu(x)\langle x|n\rangle}{\lambda_m} \sum_{\vec{y}} \frac{\langle n|y+\nu\rangle U^\dagger_\nu(y)\langle y|m\rangle}{\lambda_n} \\
&+ \sum_{\vec{x}} \frac{\langle m|x\rangle U_\mu(x)\langle x+\mu|n\rangle}{\lambda_m} \sum_{\vec{y}} \frac{\langle n|y+\nu\rangle U^\dagger_\nu(y)\langle y|m\rangle}{\lambda_n}
\end{aligned}
$$

λ_n shorthand for $m \pm i\lambda_n$, need to construct the matrices (meson fields)

$$
(\Lambda_\mu(t))_{n,m} = \sum_{\vec{x}} \langle n|x\rangle U_\mu(x)\langle x+\mu|m\rangle(-1)^{(m+n)x+m}
$$

(order eigenvectors $\lambda_0, -\lambda_0, \lambda_1, -\lambda_1, \ldots, -\lambda_{2N_{\mathrm{low}}}$)

Outline I

HISQ 2+1+1 physical point ensembles MILC [Bazavov et al., 2017]

m_π (MeV)	a (fm)	size	L (fm)	$m_\pi L$	meas (approx-corr-lma)
133	0.12224(31)	$48^3 \times 64$	5.87	3.9	26-26-26
130	0.08786(26)	$64^3 \times 128$	5.62	3.7	18-18-40
135	0.05662(18)	$96^3 \times 192$	5.44	3.7	14-22-18

Noise reduction: AMA+LMA RBC/UKQCD [Blum et al., 2013, Giusti et al., 2004, DeGrand and Schaefer, 2005]

All mode averaging (AMA) combined with full volume low mode averaging (LMA) can be very effective in reducing statistical errors for HVP (C. Lehner RBC/UKQCD [Blum et al., 2018])

AMA $$\langle O \rangle = \langle O \rangle_{\rm exact} - \langle O \rangle_{\rm approx} + \frac{1}{N}\sum_i \langle O_i \rangle_{\rm approx}$$

$\langle O_i \rangle_{\rm approx}$ constructed from props with N_{low} exact low modes, sloppy CG

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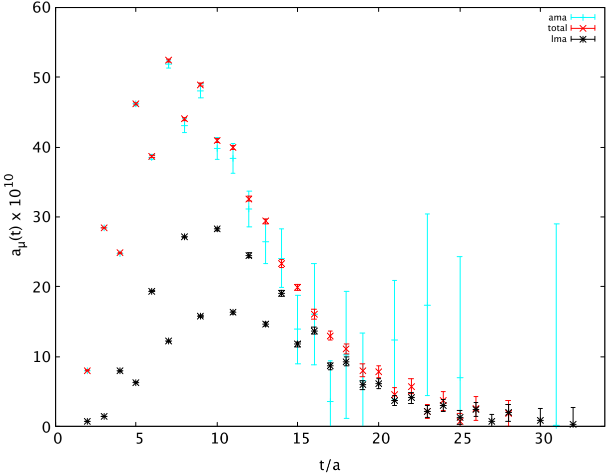
$$\begin{array}{llcl} \textbf{AMA} & \langle O \rangle & = & \langle O \rangle_{\rm exact} - \langle O \rangle_{\rm approx} + \frac{1}{N} \sum_i \langle O_i \rangle_{\rm approx} \\ +\textbf{LMA} & & - & \frac{1}{N} \sum_i \langle O_i \rangle_{\rm LM} + \frac{1}{V} \sum_i \langle O_i \rangle_{\rm LM} \end{array}$$

$\langle O_i \rangle_{\rm approx}$ constructed from props with $N_{\rm low}$ exact low modes, sloppy CG

Noise reduction: AMA+LMA RBC/UKQCD [Blum et al., 2013, Giusti et al., 2004, DeGrand and Schaefer, 2005]



- $4^3 \times 4 = 256$ approx props
- 8 exact props
- 3000($\times$2) Low modes of HISQ Dirac operator (2000($\times$2) for 96^3)

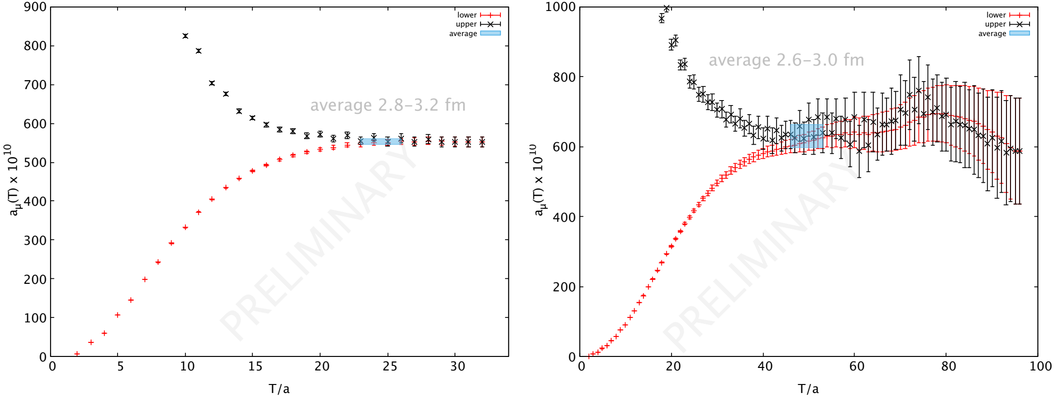
(integrand is plotted)

Huge reduction in statistical error at long distance from full volume low mode average

c.f. [Blum et al., 2018]

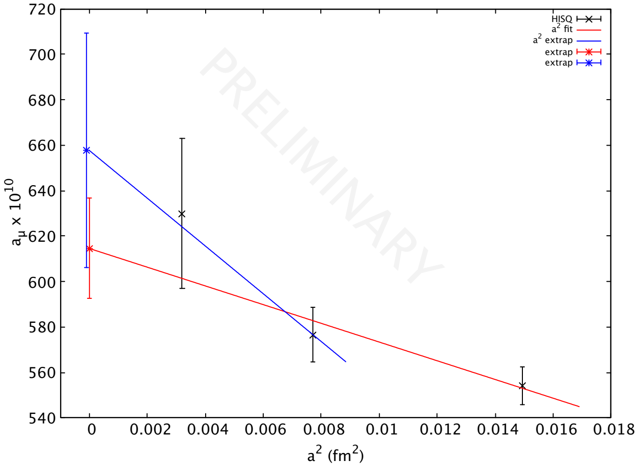
Bounding method RBC/UKQCD [Blum et al., 2018], BMW [Borsanyi et al., 2018]

Lower: $C(t) = 0, \; t > T$ (BMW choice)
Upper: $C(t) = C(T)e^{-E_0(t-T)}, \; E_0 = 2\sqrt{m_\pi^2 + (2\pi/L)^2}$



(total a_μ for choice of T is plotted)

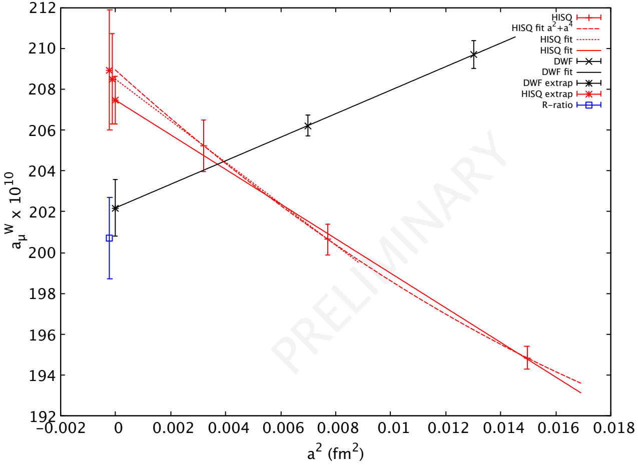
Continuum limit at finite volume ($L \approx 5.5$ fm)



- $a_\mu + b\, a^2$
- possible a^4 error
- poor statistics on 96^3 point

Window method RBC/UKQCD [Blum et al., 2018] comparison with DWF and R-ratio

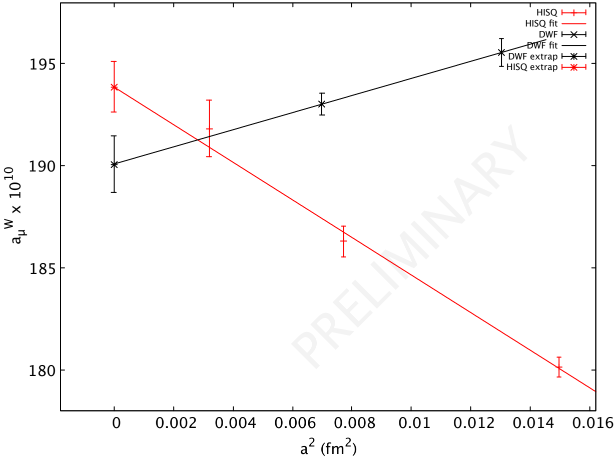
$$a_\mu^W = \sum C(t) w(t) (\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)), \quad \Theta(t, t', \Delta) = 0.5(1 + \tanh((t - t')/\Delta))$$



- allows precise comparison of continuum limit
- $t_0 = 0.4,\ t_1 = 1.0,\ \Delta = 0.15$ fm
- all points physical
- all $L \approx 5.5$ fm
- difference is 2-3 σ: lattice spacing, statistics may be responsible

Window method RBC/UKQCD [Blum et al., 2018] comparison with DWF and R-Ratio

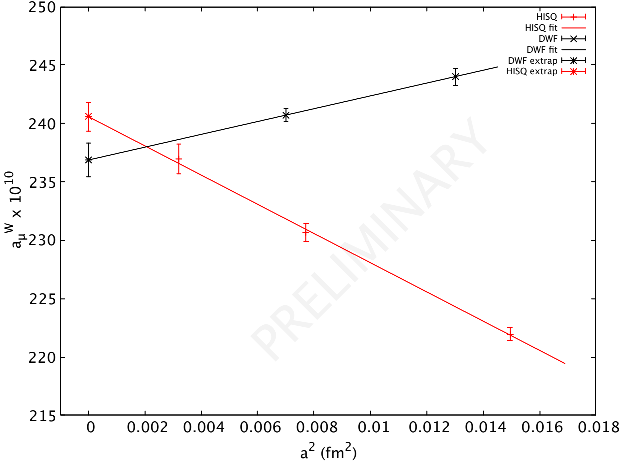
$$a_\mu^W = \sum C(t) w(t) (\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)), \quad \Theta(t, t', \Delta) = 0.5(1 + \tanh((t - t')/\Delta))$$



- allows precise comparison of continuum limit
- $t_0 = 0.4,\ t_1 = 1.0,\ \Delta = 0.3$ fm
- all physical points
- all $L \approx 5.5$ fm
- difference is $\sim 2\sigma$

Window method RBC/UKQCD [Blum et al., 2018] comparison with DWF and R-Ratio

$$a_\mu^W = \sum C(t)w(t)(\Theta(t,t_0,\Delta) - \Theta(t,t_1,\Delta)), \quad \Theta(t,t',\Delta) = 0.5(1+\tanh((t-t')/\Delta))$$



- allows precise comparison of continuum limit
- $t_0 = 0.2,\ t_1 = 1.0,\ \Delta = 0.15$ fm
- all physical points
- all $L \approx 5.5$ fm
- difference is $\sim 2\sigma$

Outline I

Summary/Outlook

Lattice calculations of HVP important for muon g-2 SM v. Exp comparion

- Physical point 2+1+1 HISQ HVP calculation on large lattices
- AMA+LMA very effective for achieving small statistical error (*c.f.* RBC/UKQCD)
- u+d quark connected contribution only

Important to compare different lattice calculations

- Window method by RBC/UKQCD allows precise comparisons in continuum limit
- small difference with DWF in continuum limit, R-ratio ($\lesssim 0.7\%$ of total a_μ)
- need to understand differences
- future
    - improve statistics on 0.06 fm, 96^3 ensemble
    - compare with other staggered calculations

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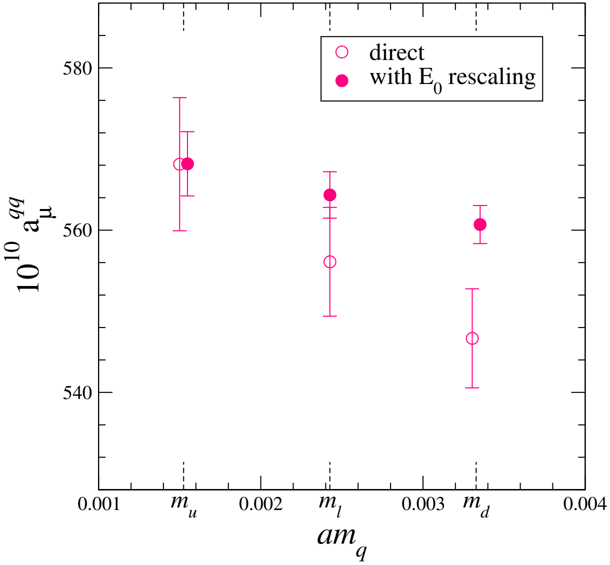
Light quark mass dependence of a_μ FnalHpqcdMilc[Chakraborty et al., 2018]



- strong isospin breaking study
- $m_\pi = 135$ MeV
- $a = 0.15$ fm
- change in a_μ for 130 MeV pion is negligible $\sim -2 \times 10^{-10}$