Proton decay matrix element on the lattice at the physical point

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Introduction

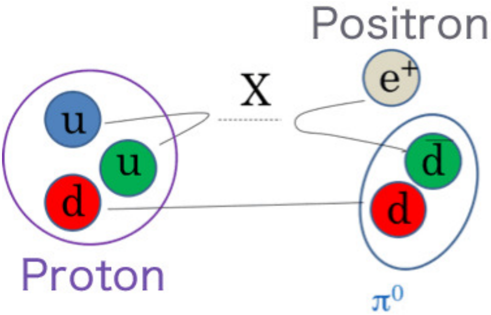


Figure: Proton decay schematic diagram (Hyper-k.org)

- GUT, SUSY-GUT: new interactions between quarks and leptons
- Nucleon stability
- Proton decay : Baryon number violation
  Is one of Sakharov's necessary conditions for Baryogenesis

Introduction

Experimental Effort

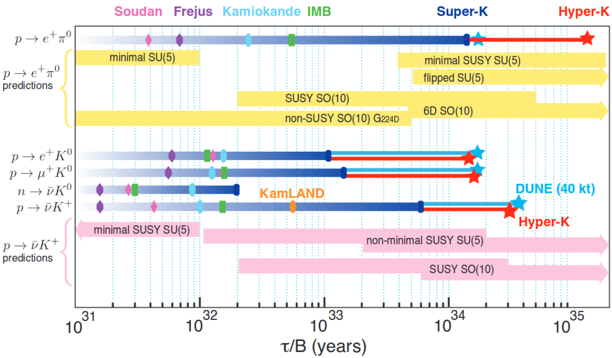


Figure: Proton decay schematic diagram [DUNE arXiv:1512.06148]

- Super-Kamiokande
- DUNE (Deep Underground Neutrino Experiment)

Introduction

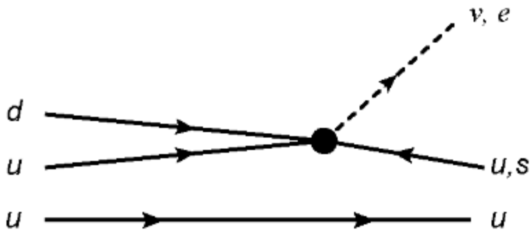


Figure: Proton decay effective diagram

$$
\begin{aligned}
&C^I \langle \ell, PS | [\bar{\ell} O_{\Gamma\Gamma'}] | N \rangle \;=\; C^I \bar{v}_\ell \langle PS | O_{\Gamma\Gamma'} | N \rangle \\
&\to C^I P_{\Gamma'} \left[W_0^{\Gamma\Gamma'}(q^2) - \frac{i\not{q}}{m_N} W_1^{\Gamma\Gamma'}(q^2) \right] u_N(p,s)
\end{aligned}
$$

, where C^I being the Wilson coefficient of I-th kind of operator.

Introduction

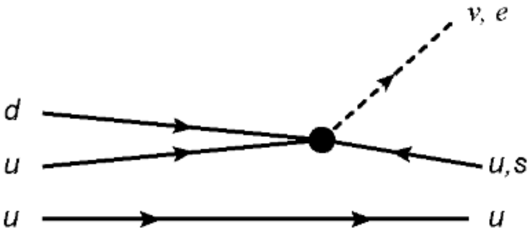


Figure: Proton decay schematic diagram

$$\Gamma(N \to P + \bar{\ell}) = \frac{m_N}{32\pi}[1 - (\frac{m_p}{m_N})^2)]^2 |\sum_I C^I W_0^I(N \to P)|^2$$

, where C^I being the Wilson coefficient of I-th kind of operator.

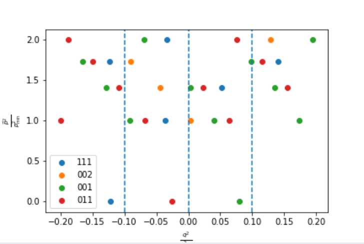
Lattice settings

- RBC/UKQCD generated $N_f = 2+1$ dynamic Domain wall Fermion, gauge action Iwasaki-DSDR
- Lattice size $24^3 \times 64 (L \sim 4.8 fm)$, $L_5 = 24$, $\beta = 1.633$, $m_\ell a = 0.00107, m_h a = 0.0850, m_{res} = 0.00228$
- $a^{-1} = 1.015$ GeV, $m_\pi = 139$ MeV, $m_K = 505$ MeV, $m_\pi L \sim 3.4$

- Deflated CG with 2000 Eigenvectors (basis 1000)
- Generated 32+1 AMA samples on 52 gauge configurations with 3 source-sink separation, i.e., $t_{sep} \in \{8, 9, 10\}$
- To meet the kinematic condition, chose the most suitable two sets of $\vec{p}$ for each meson.

Lattice settings

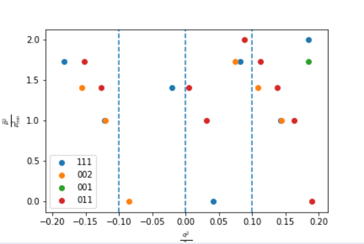
$p = p' + q$



- Physical $q^2 \sim 10^{-8} GeV^2$
- Chose [001] [011] for Kaons
- Chose [111] [002] for pions

Lattice settings

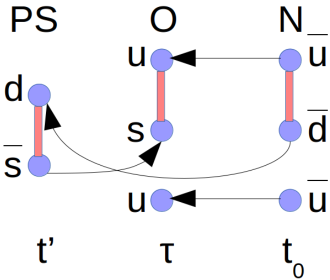
$p = p' + q$



- Physical $q^2 \sim 10^{-8} GeV^2$
- Chose [001] [011] for Kaons
- Chose [111] [002] for pions

Lattice settings

$\langle PS(t')|O_{\Gamma\Gamma'}(\tau)|N(t_0)\rangle$



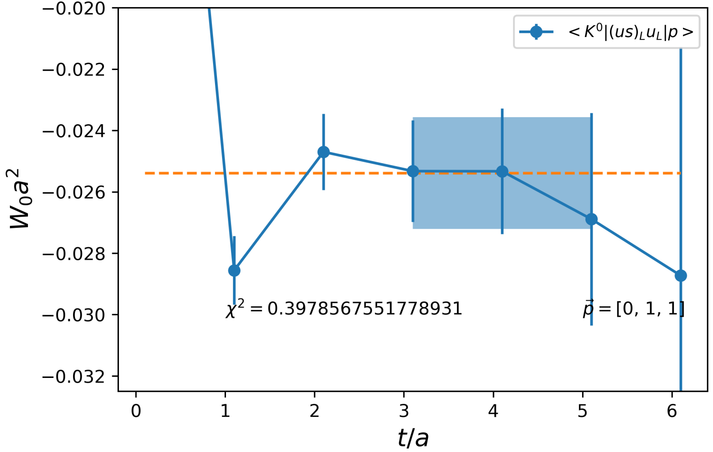
$$R_3^{\Gamma\Gamma'}(t',\tau,t_0;\vec{p'},q;P) = \frac{tr[PC_3^{\Gamma\Gamma'}(t',\tau,t_0;q)]}{C_{PS}(t',\tau;\vec{p'})tr[P_4C_N(\tau,t_0)]}\sqrt{Z_{PS}Z_N}$$

As $t'-\tau\to\infty$, $\tau-t'\to\infty$,

$$R_3 \longrightarrow P_{\Gamma'}\left[W_0^{\Gamma\Gamma'}(q^2) - \frac{i\not{q}}{m_N}W_1^{\Gamma\Gamma'}(q^2)\right]u_N(p,s)$$

Form Factors

$$W_0^{\Gamma\Gamma'} = R_3^{\Gamma\Gamma'}(t', t, t_0; \vec{p'}, q; P_4) - \frac{m_N - E_\pi}{q_j} R_3^{\Gamma\Gamma'}(t', t, t_0; \vec{p'}, q; iP_4\gamma_j)$$



Form Factors

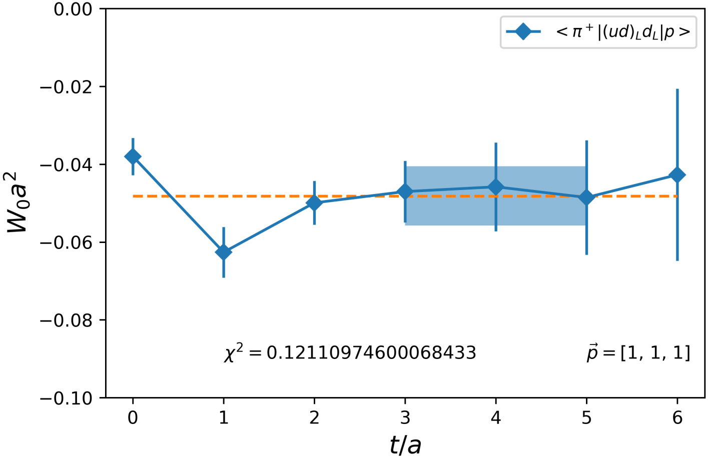


Figure: Pion Channel form factor with $\vec{p}_\pi = [111]$

Form Factors

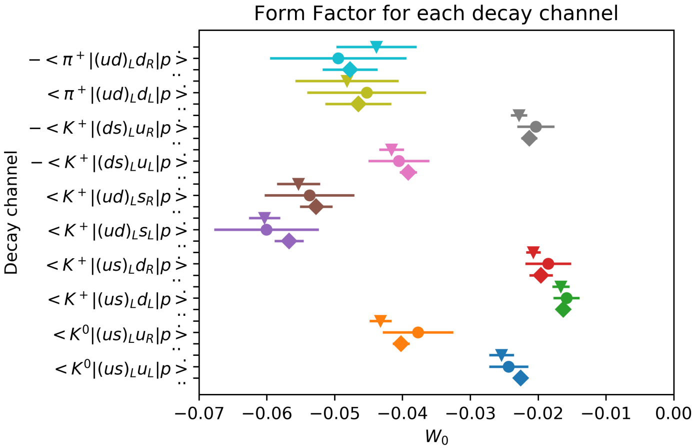


Figure: Form factors Channel by Channel with $\vec{p_\pi} = [111]$ $\vec{p_K} = [011]$

Form Factors

Interpolation to physical point

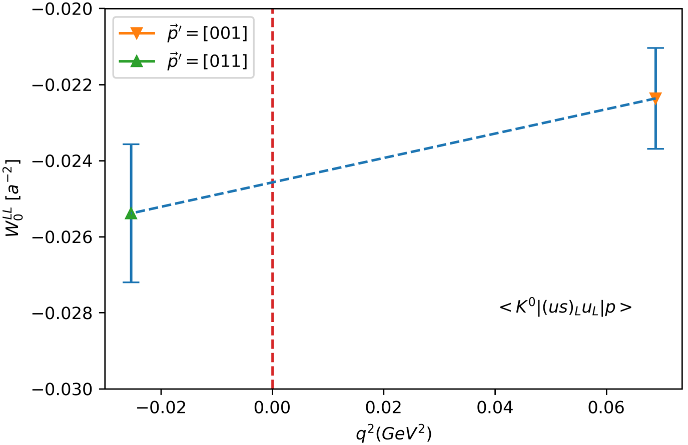


Figure: Form factors Across different $\vec{p}_K$

Consistency check with earlier study

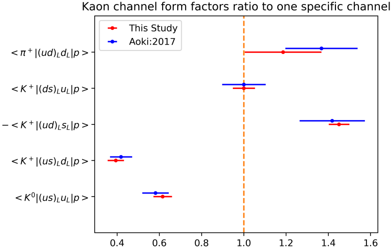


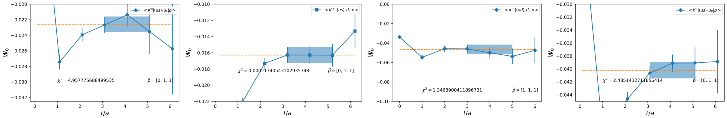
Figure: Normalized Form factors Comparison with Earlier study and our data at $t_{sep} = 10$

	Stat. [%] (This study)	Stat.[%] (Aoki:2017)	Chiral Extrapol.[%]	a^2 [%]	Δ_Z [%]
$\langle K^0 \vert (us)_L u_L \vert p\rangle$	5.1	3.5	3.1	5.0	8.1
$\langle K^+ \vert (us)_L d_L \vert p\rangle$	17	4.4	7.5	5.0	8.1
$\langle K^+ \vert (ud)_L s_L \vert p\rangle$	4.8	3.0	3.9	5.0	8.1
$-\langle K^+ \vert (ds)_L u_L \vert p\rangle$	4.1	2.8	2.8	5.0	8.1
$\langle \pi^+ \vert (ud)_L d_L \vert p\rangle$	15.8	3.4	5.7	5.0	8.1

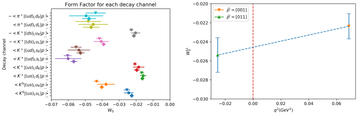
Summary

What is done

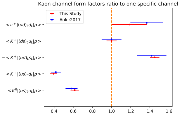
Preliminary bare form factor is extracted



Preliminary Analysis



Comparison with earlier study



Summary

Tasks Ahead

Multi-state fit(Excited states)
Renormalization
Increase samples(Esp. pion channel)
Different Lattice (Different volume, Different cutoff)
Indirect method checkup (using α, β)
Exploring New channels

References

Y. Aoki, E. Shintani, A. Soni
Proton decay matrix element on lattice, 2013

Hyper Kamiokande Collaboration

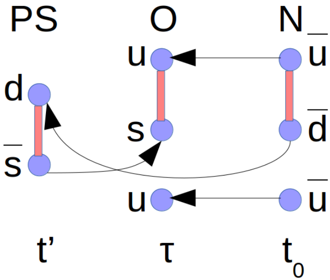
Y. Aoki, T. Izubuchi, E. Shintani, A. Soni
Improved lattice computation of proton decay matrix element, 2017

Deep Underground Neutrino Experiment
Conceptual Design Report

The End

Lattice settings

$\langle PS(t')|O_{\Gamma\Gamma'}(\tau)|N(t_0)\rangle$



$$C_{PS}(t) = \frac{Z_{PS}}{2m_{PS}} e^{-m_{PS}t}$$

$$\langle 0|J_{PS}|PS\rangle$$

$$= \sqrt{Z_{PS}}$$

Lattice settings

Effective mass plots

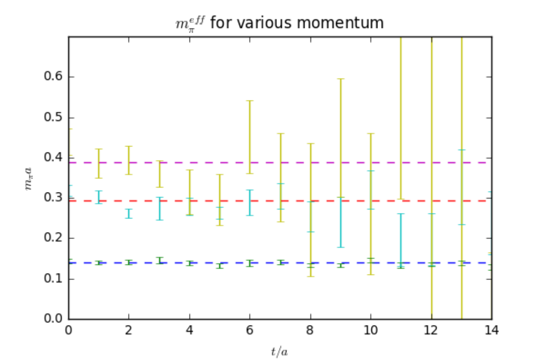


Figure: Pion mass dispersion with $\vec{p} = [000], [001], [011]$

Lattice settings

Effective mass plots

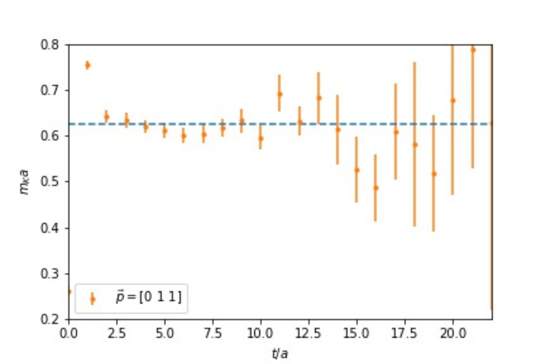
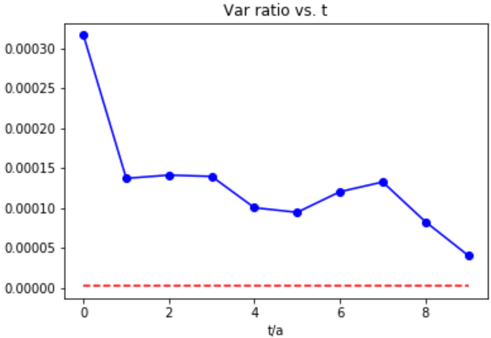


Figure: Kaon effective mass with $\vec{p} = [011]$

AMA method



- possible distribution of computation time to exact, approximate samples enhance statistics
- Should be easy to devise approximate sample for Domain Wall Fermion :zmobius
- the most cost efficient variance around $N_{sl}/N_{ex} = \sqrt{\frac{1}{1.5\cdot 10^{-4}}\cdot 11} \approx 270$ for both protons and pions.

Multi State Fit

Using two-state fit to exclude excited states contribution

$$C^{2pt}(t_f, t_i) = Tr[\mathcal{P}\sum_{\vec{x}}\langle 0|J_N(t,\vec{x})\bar{J}_N(0,\vec{0})|0\rangle]$$

$$C^{3pt}(t_f, \tau, t_i) = Tr[\mathcal{P}\sum_{\vec{x},\vec{x}'}\langle 0|J_N(t,\vec{x})\mathcal{O}(\tau,\vec{x}')\bar{J}_N(0,\vec{0})|0\rangle]$$

, where J_N, nucleon interpolator on the lattice, $\mathcal{P}$ spin projection operator.
Spectral decomposition:

$$C^{2pt}(t_f, t_i) = |\mathcal{A}_0|^2 e^{-M_0(t_f-t_i)} + |\mathcal{A}_1|^2 e^{-M_1(t_f-t_i)} + \dots$$

$$\begin{aligned} C^{3pt}(t_f, \tau, t_i) = |\mathcal{A}_0|^2\langle\mathcal{O}_{00}\rangle e^{-M_0(t_f-t_i)} + \mathcal{A}_0\mathcal{A}_1^*\langle\mathcal{O}_{01}\rangle e^{-M_0(t_f-t_i)} + \\ \mathcal{A}_1\mathcal{A}_0^*\langle\mathcal{O}_{10}\rangle e^{-M_0(t_f-t_i)} + |\mathcal{A}_1|^2\mathcal{O}_{11} e^{-M_1(t_f-t_i)} + \dots \end{aligned}$$

Multi State Fit

Using two-state fit to exclude excited states contribution

- Two-state fit to two point correlation function

$\mathcal{A}_0$	$M_0 a$	$\mathcal{A}_1$	$M_1 a$
0.00027891	-0.961095	0.640617	-2.278580756339

Low energy constant

Chiral Perturbation theory

- $\langle PS(p')|O^{\Gamma\Gamma'}(q)|N(p)\rangle$ to be approximated to chiral perturbation to $\langle 0|O^{\Gamma\Gamma'}(q)|N(p)\rangle \rightarrow$ Low Energy Constant (LEC)
- LEC to be calculated :
  $\langle 0|O^{LL}(q)|N(p)\rangle = \alpha P_L u_s$
  $\langle 0|O^{LR}(q)|N(p)\rangle = \beta P_L u_s$
  , where O being specifically $(ud)_\Gamma P_{\Gamma'} u$

!sic! Pending slides!