Renormalization Group Properties of Scalar Field Theory using Gradient Flow

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Can GF be related to RG?

- GF is used to define renormalized quantities: couplings, operators
- It smooths out short-distance fluctuations – blocking
- But it does not involve rescaling – must be modified
- This leads to predictions for anomalous dimensions of operators and has been successfully applied in a SU(3), Nf = 12 system (previous talk, arXiv:1806.01385)
- Another check: Test on a well-known system like 3d φ^4 theory:

$$S(\varphi) = \int \mathrm{d}^d x \left[\frac{1}{2}\varphi(-\Delta + m^2)\varphi + \frac{\lambda}{4!}\varphi^4\right]$$

Wilson-Fisher Fixed Point

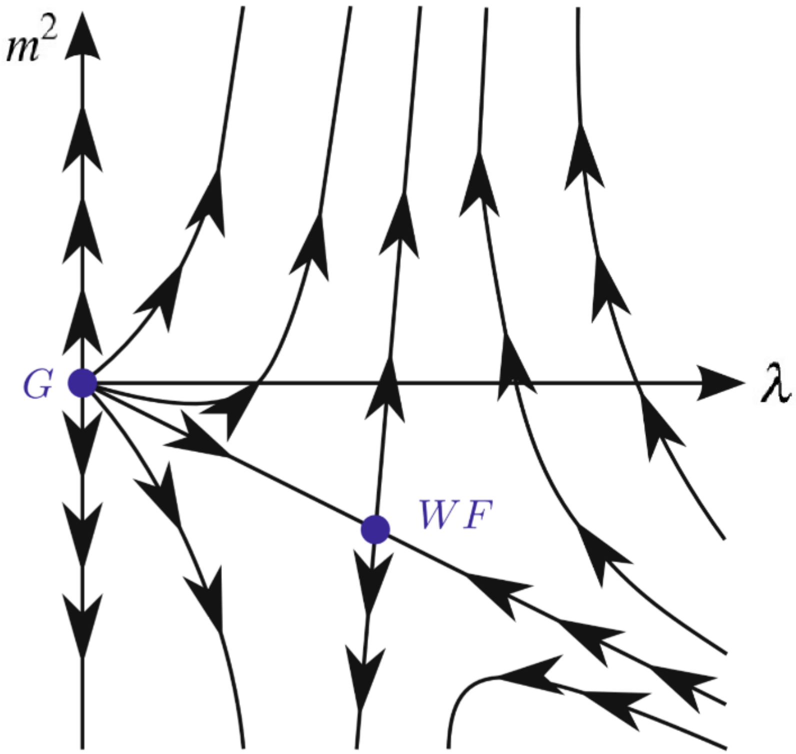
- The (conformal) IRFP of the 3D Ising universality class
- Relevant coupling: m^2
- Irrelevant couplings: $\lambda = g_4,\ g_6, \ldots$
- Exponents $\eta,\ \nu,\ \omega, \ldots$

$$\Delta_O = d_O + \gamma_O$$

$$\eta = 2\gamma_\phi = 0.036298(2)$$

$$\gamma_{\phi^2} = 2 - \nu^{-1} = 0.412625(10)$$

$$\gamma_{\phi^4} = 4 - d + \omega = 1.82966(9)$$

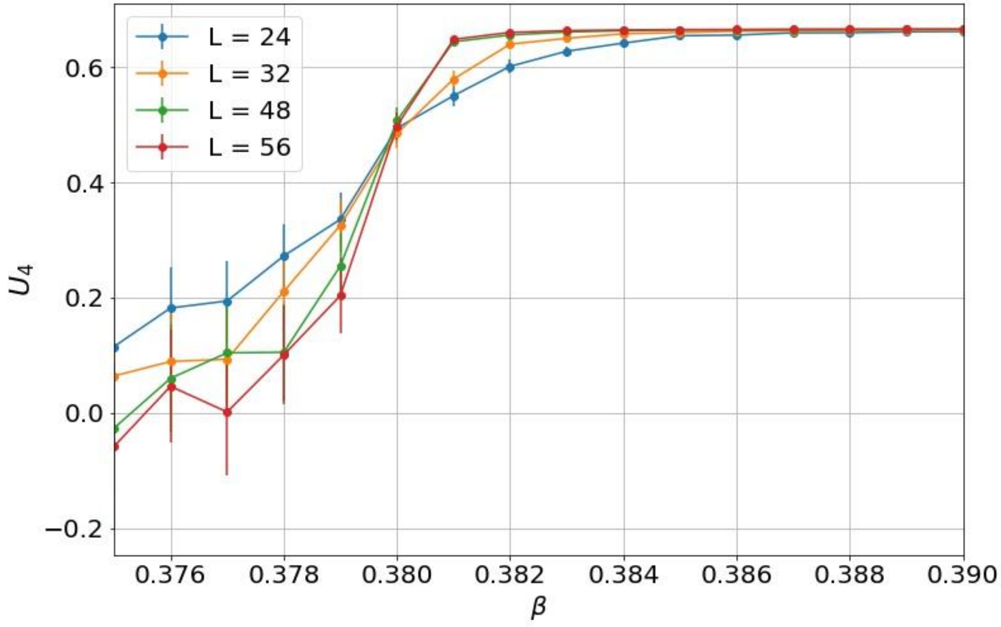


Adapted from Kopietz *et al.*, *Introduction to the Functional Renormalization Group (Springer 2010)*

The Binder Cumulant: Finding the Critical Surface

$$U_4 = 1 - \frac{\langle M^4 \rangle}{3\langle M^2 \rangle^2}$$

- U_4 takes a universal value at the critical point
- U_4 vs β curves for different volumes cross at the CP
- At $\lambda = 1.0,\ \beta_c \approx 0.38$



Completing GF to get GFRG

- Block spins are defined generally as

$$\varphi_b(n/b) = \frac{b^{\Delta_\phi}}{b^d} \sum_{\varepsilon} \varphi(n+\varepsilon)$$

- Meanwhile, the free GF equation is a heat equation: $\partial_t \phi_t = \Delta \phi_t$

$$\phi_t(n) = \frac{1}{(4\pi t)^{d/2}} \int \mathrm{d}^d\varepsilon \, \mathrm{e}^{-\varepsilon^2/4t} \varphi(n+\varepsilon)$$

- Identify the blocking radius

$$b \propto \sqrt{t}$$

- Finally, define the GFRG-blocked field as

$$\varphi_b(n/b) = b^{\Delta_\phi} \phi_t(n)$$

Correlator Ratios

- Standard correlator scaling law for scaling operators, and $n \gg b$ (see *Cardy, Amit*):

$$\langle O_b(0) O_b(n/b) \rangle = b^{2\Delta_O} \langle O(0) O(n) \rangle$$

- Then use the GFRG-blocked field definition, with $O_t = O(\phi_t)$, to get

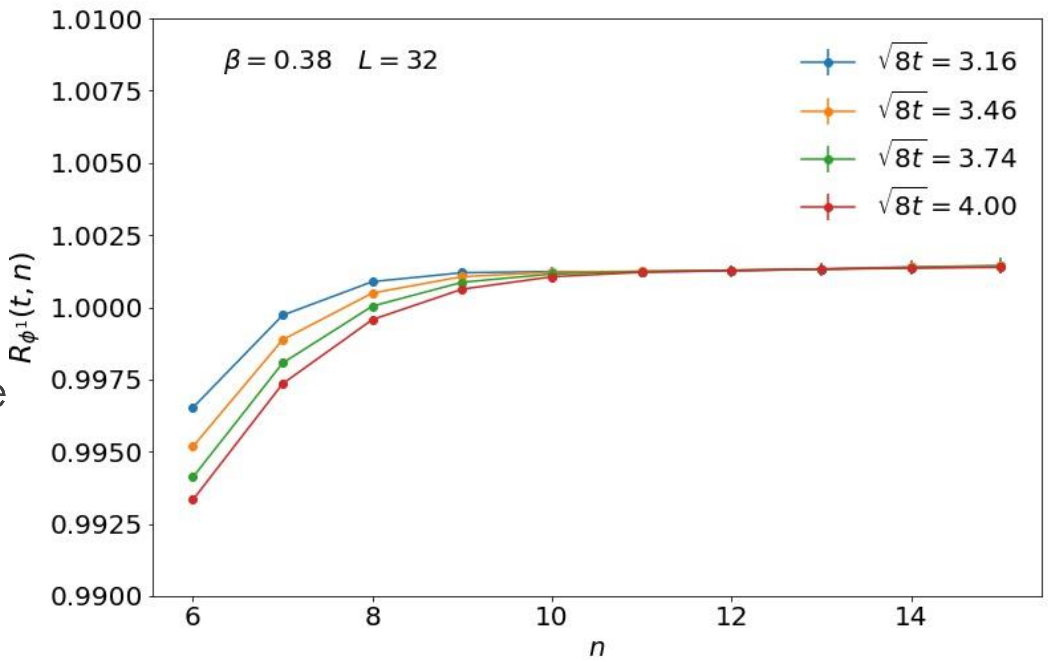
$$R_O(t,n) = \frac{\langle O_t(0) O_t(n) \rangle}{\langle O(0) O(n) \rangle} = t^{\gamma_O - n_O \gamma_\phi}$$

- Only valid near the FP at which the exponents are defined

$\varphi - \varphi$ Correlator

$$\frac{\langle \phi_t(0)\phi_t(n)\rangle}{\langle \phi_0(0)\phi_0(n)\rangle} = t^0 = 1$$

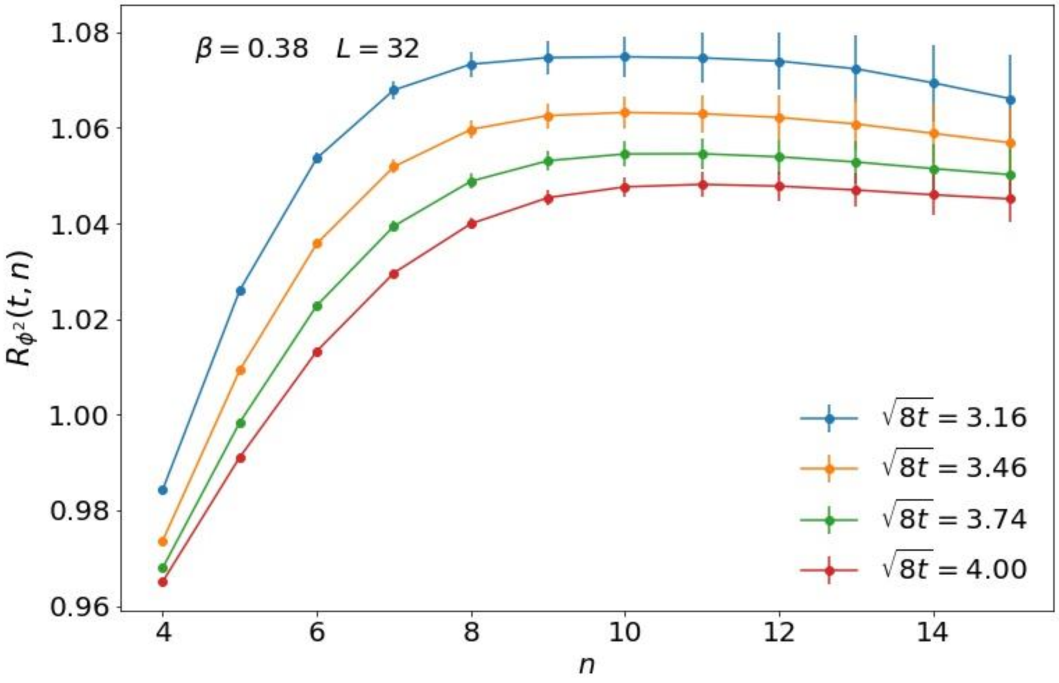
- Cannot measure the field anomalous dimension using the field itself!
- Can check for consistency by looking at t-dependence
- Begins to deviate for $n < 2(8t)^{1/2}$



$\varphi^2 - \varphi^2$ Correlator

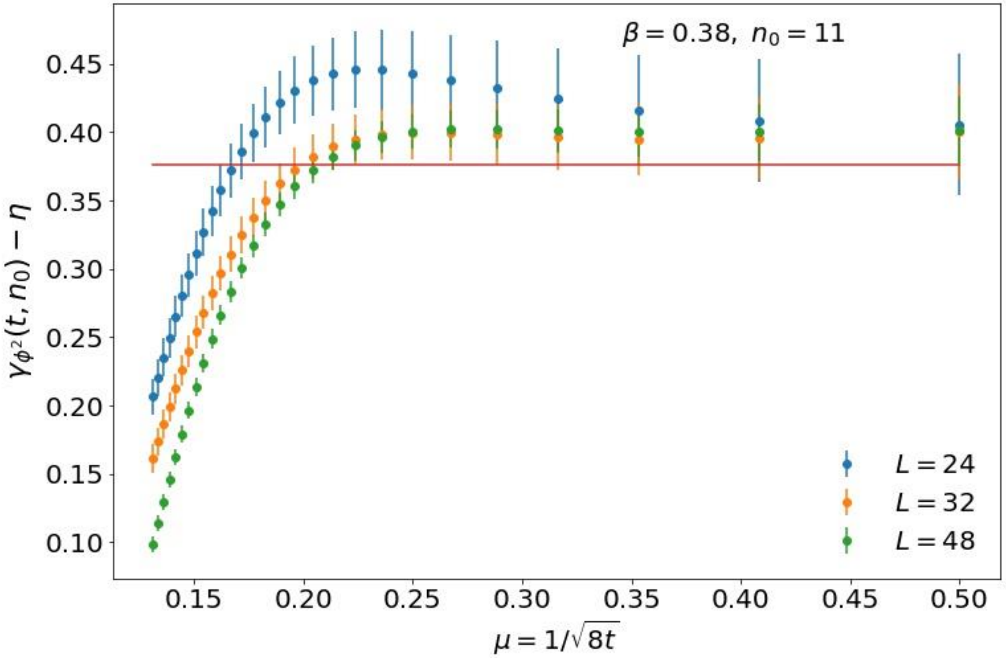
$$\frac{\langle \phi_t^2(0)\phi_t^2(n)\rangle}{\langle \phi_0^2(0)\phi_0^2(n)\rangle} = t^{\gamma_{\phi^2}-\eta}$$

- This gives the mass anomalous dimension
- Operator is not a scaling operator, but close



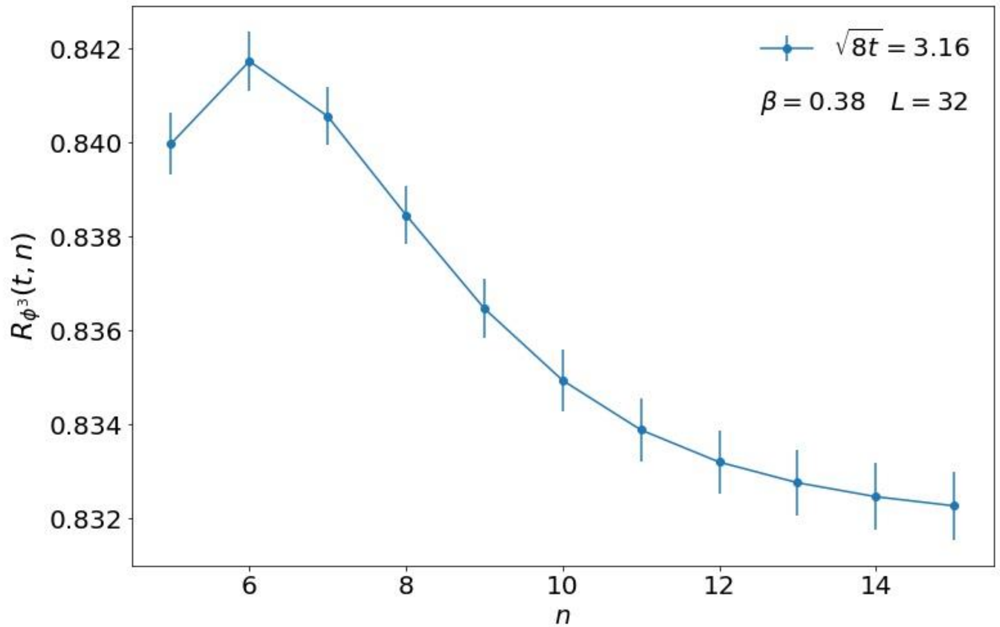
$\varphi^2 - \varphi^2$ Correlator

- The exponent approaches a reasonable value as L increases
- Expected $\gamma_{\phi^2} - \eta = 0.376327$



Higher Operators

- Ratios for the operators $\varphi^n,\ \ n > 2$, have more pronounced slopes at large distances
- The correlator ratio formula states that there should be no n-dependence, however
- Can be accounted for if higher powers are not pure scaling operators



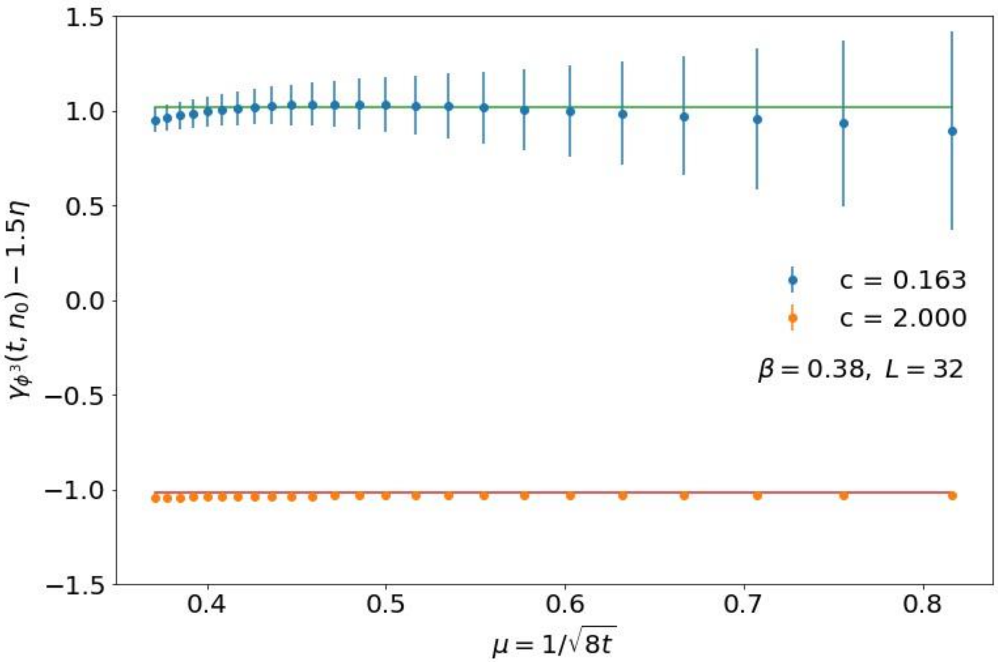
Finding Scaling Operators: φ^3 Exponent

- Suppose φ^3 mixes with φ:

$$O_3 = \phi^3 + ct^{-\Delta_\phi}\phi$$

- **Brute force approach**: sweep c values until a plateau is observed in ratio plot vs n
- For large c , should get an exponent of -1.036298
- Predicted to be (*Rychkov et al.,* arXiv:1505.00963)

$$\gamma_{\phi^3} = 1 + \eta/2 = 1.018149$$



Summary and Future Work

- GF supplemented by rescaling: GFRG
- This leads to a formula for exponents in terms of ratios of flowed correlators
- φ^2 and φ^3 exponents measured, but mixing is important: need to systematically find scaling operators
- Does not require matching volumes, but requires an infinite volume extrapolation
- Stress-energy tensor to find η: $R_{T_{\mu\mu}}(t) = t^{-2\eta}$

Backups…

Cardy-style correlator scaling

- Near the FP, couplings and operators scale (linearization)

$$\langle O_b(0)O_b(n/b)\rangle = b^{2(\Delta_O-d)}\sum_{\varepsilon\varepsilon'}\langle O(\varepsilon)O(n+\varepsilon')\rangle$$

- For large separation,

$$\langle O(\varepsilon)O(n+\varepsilon')\rangle \approx \langle O(0)O(n)\rangle$$

- So

$$\langle O_b(0)O_b(n/b)\rangle = b^{2\Delta_O}\langle O(0)O(n)\rangle$$

- At the FP, correlation length is infinite and the decay must be power law-like.

Field theory vs. simulation parameters

- Simulation action

$$S(\hat{\varphi}) = \sum_n \left[-\beta \sum_\mu \hat{\varphi}(n)\hat{\varphi}(n+\mu) + \hat{\varphi}^2(n) + \lambda(\hat{\varphi}^2(n) - 1)^2 - \lambda \right]$$

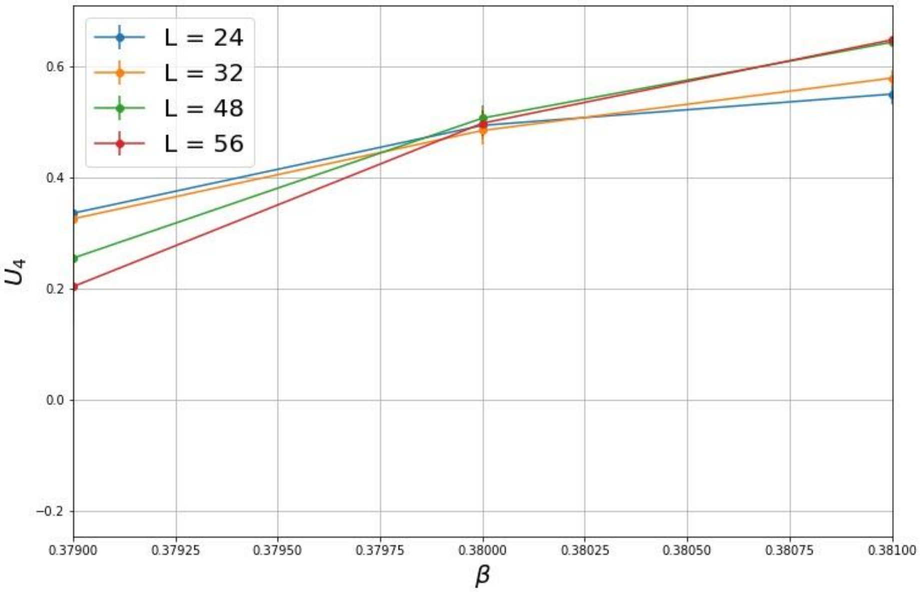
- Fields related by

$$\varphi(an) = a^{-d_\phi} \sqrt{\beta}\, \hat{\varphi}(n)$$

- Couplings related by

$$\frac{(ma)^2}{2} = \frac{1-2\lambda}{\beta} - d \qquad\qquad g = \frac{4!\lambda}{\beta^2} a^{d-4}$$

Zoomed Binder

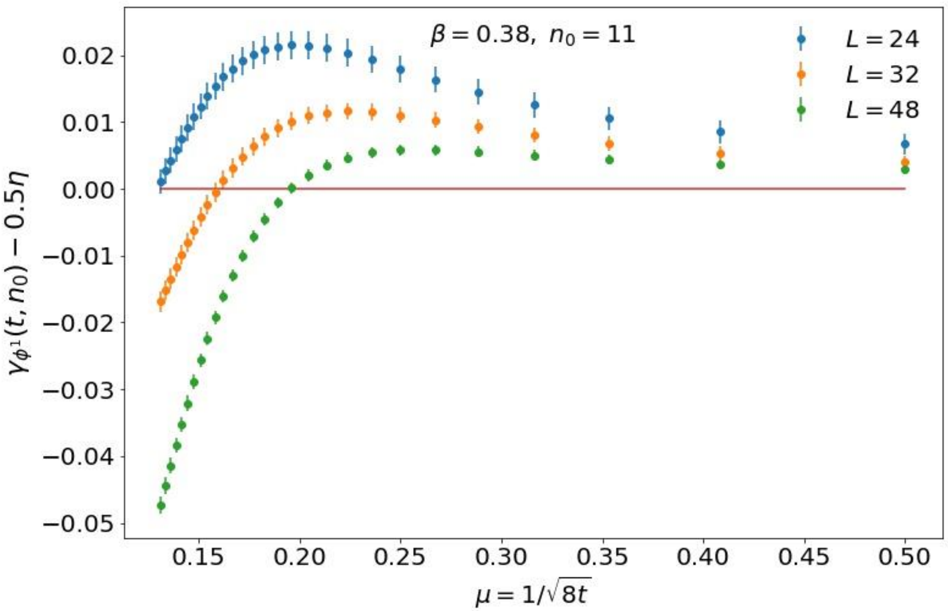


Simulation details

- Configurations generated with the Wolff cluster algorithm followed by radial Metropolis updates
- Ensemble size ~1900
- Gradient flow integration via 4^{th} order Runge-Kutta
- Flow measurements every $\Delta t = 0.25$
- Computed on the CU Beowulf cluster

$\varphi - \varphi$ Correlator

The "exponent" approaches zero as the volume increases



Renormalization Group: Generalities

Two main steps:

1. Integrate over high modes $\Lambda/b \leq p \leq \Lambda$, i.e. average over short distances
2. Rescale momenta by $b > 1$ to restore the cutoff

- Amounts to a mapping of couplings: $S(g) \rightarrow S(g')$
- Fixed-points: $S(g') = S(g) = S(g_*)$
- Scaling variables: $g' = b^{y_g} g$
- Scaling operators: $O' = b^{\Delta_O} O$
- Coupling scaling dimensions vs. operator scaling dimensions:

$$y_g = d - \Delta_O \quad \text{where} \quad \Delta_O = d_O + \gamma_O$$