Recent progress on the QCD phase diagram

Sayantan Sharma

The Institute of Mathematical Sciences

July 24, 2018

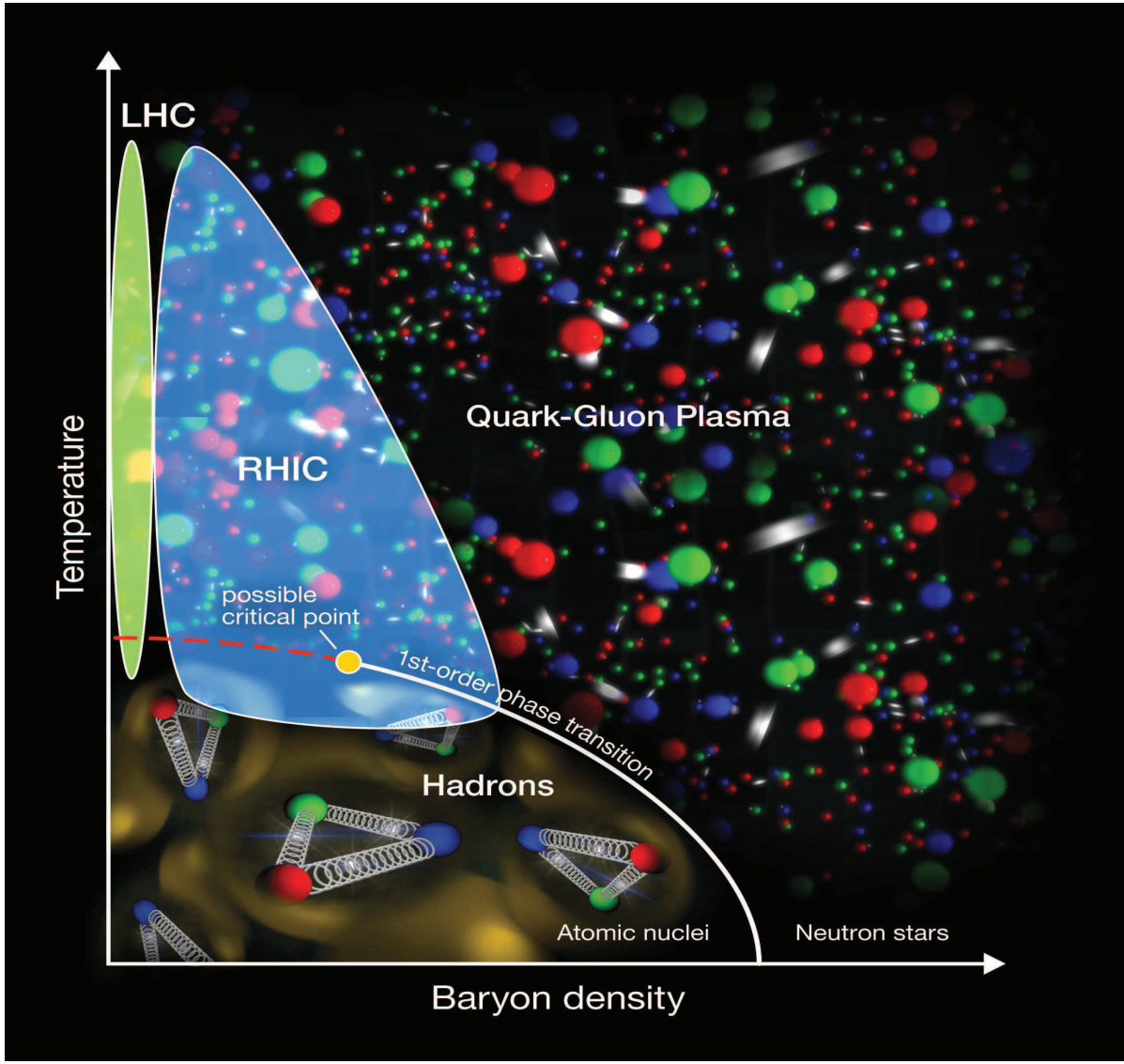
36th International Symposium on Lattice Field Theory 2018
Michigan State University, East Lansing

Outline

The QCD phase diagram: outstanding issues

- The QCD phase diagram is just beginning to be unraveled.
- Two underlying mechanisms: confinement and chiral symmetry breaking is not yet completely understood.

  [Schaefer and Shuryak, 96]
- Lattice techniques are allowing us to draw lines and points on this plot
- Even more exciting as it allowing us to understand deeper the microscopic mechanisms.



[Courtesy www.bnl.gov]

Towards understanding the phase diagram: key ingredients

- Symmetries and order parameters.

Towards understanding the phase diagram: key ingredients

- Symmetries and order parameters.
- Role of anomalies and its connection to topological properties of QCD

Towards understanding the phase diagram: key ingredients

- Symmetries and order parameters.
- Role of anomalies and its connection to topological properties of QCD
- Towards finite μ_B : Curvature of the chiral crossover transition and towards critical end-point.

Towards understanding the phase diagram: key ingredients

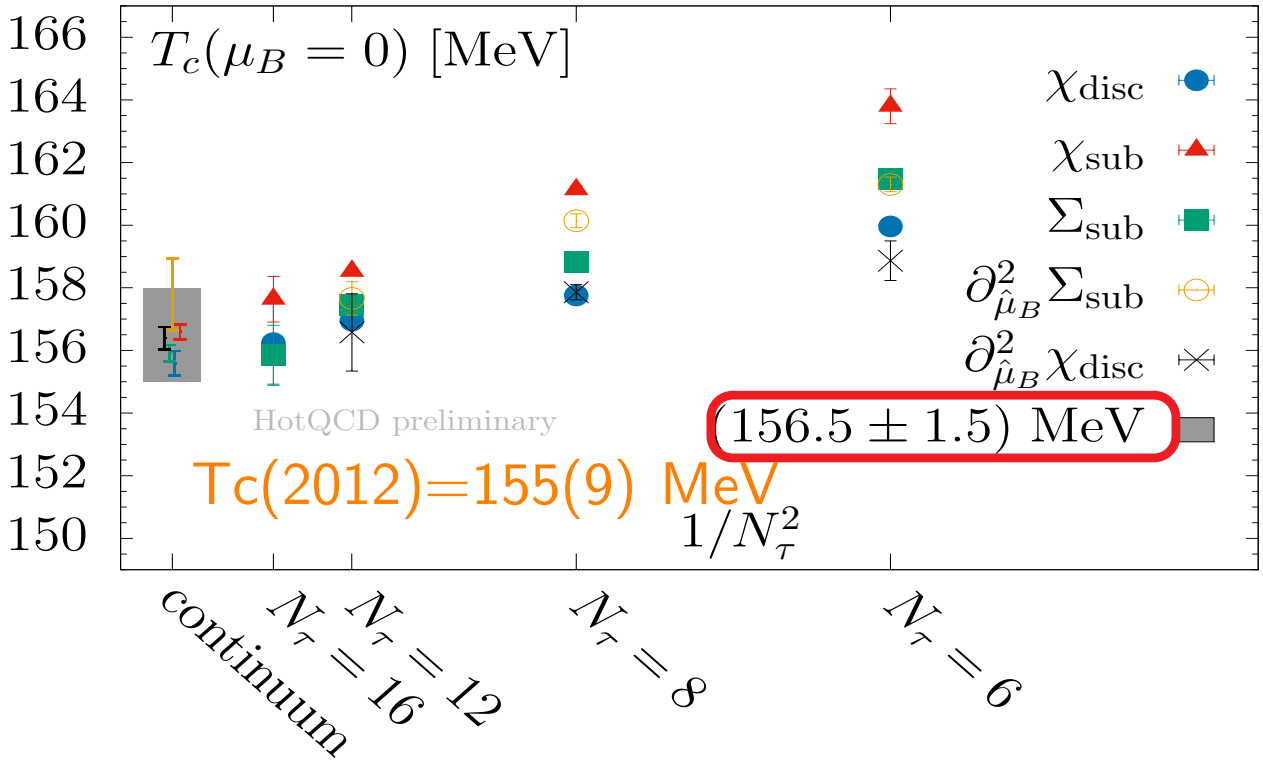
- Symmetries and order parameters.
- Role of anomalies and its connection to topological properties of QCD
- Towards finite μ_B : Curvature of the chiral crossover transition and towards critical end-point.
- Could not include updates on physics of heavy quarks, photon and di-lepton rates, viscosities, QCD in magnetic field, QCD at strong coupling, large N due to time constraint

  [See talks by A. Kumar on jet quenching parameter in gauge theory Thu, QCD in magnetic field by A. Tomiya, Wed 17:10, QCD near strong coupling by W. Unger, M. Klegrewe, hadron spectrum in QGP by T. Glesaaen, Fri, spectral functions by H-T. Ding, Fri, large N QCD, Hackett Thu 12:40, Thu, N=2 QCD Itou, Thu 9:50]

The phase diagram at $\mu_B = 0$

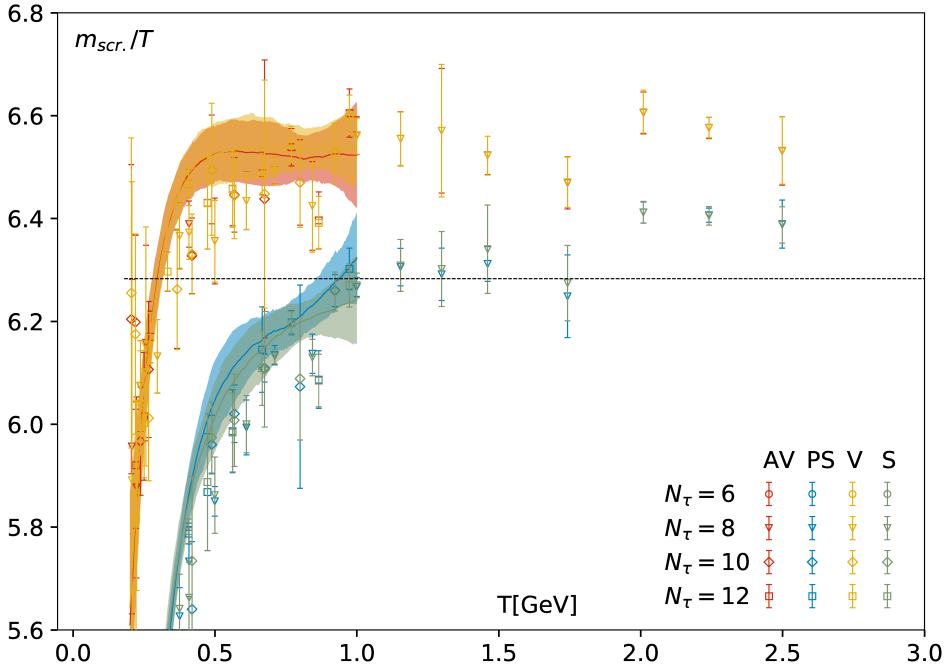
- For finite quark masses, no unique order parameter.
- Now well established that $\mu_B = 0$ chiral symmetry restoration occurs via crossover transition.

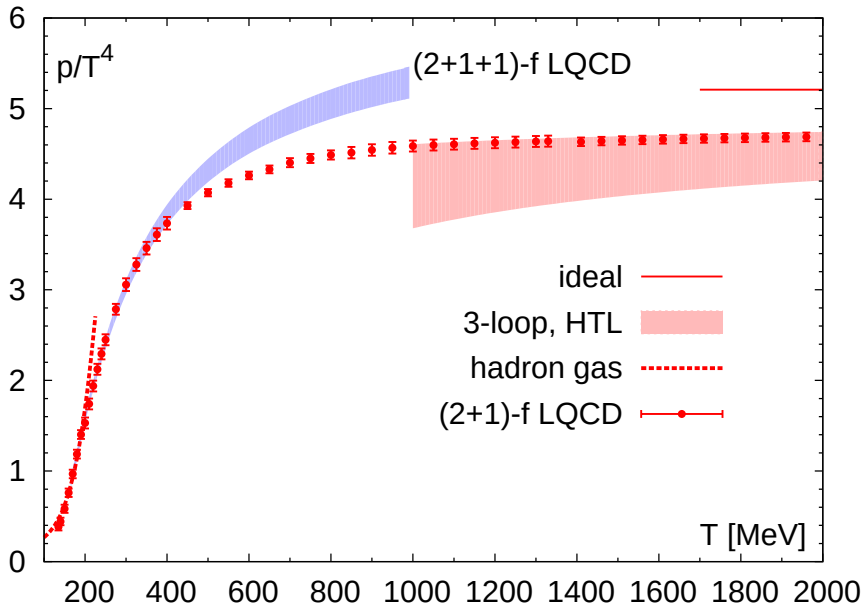
  [Budapest-Wuppertal collaboration, 1309.5258, HotQCD collaboration, Bazavov et. al, 1407.6387]
- However remnants of chiral symmetry are quite strong in observables. Important update in T_c from chiral observables [See talk by P. Steinbrecher, Wed 16:10]



The phase diagram at $\mu_B = 0$

- EoS is close to the perturbative behaviour for $T > 5\,T_c$ but close to the edge of the error band [See talk by J. Weber, Thurs 8:50]
- Screening masses of scalar/ pseudo-scalar excitations show deviation from perturbation theory [H. Sandmeyer et. al., HotQCD in prep]
- Dynamical effects of charm quarks included till 1 GeV $\rightarrow$ important EoS during cosmological evolution. [Borsanyi et. al, 1606.07494]

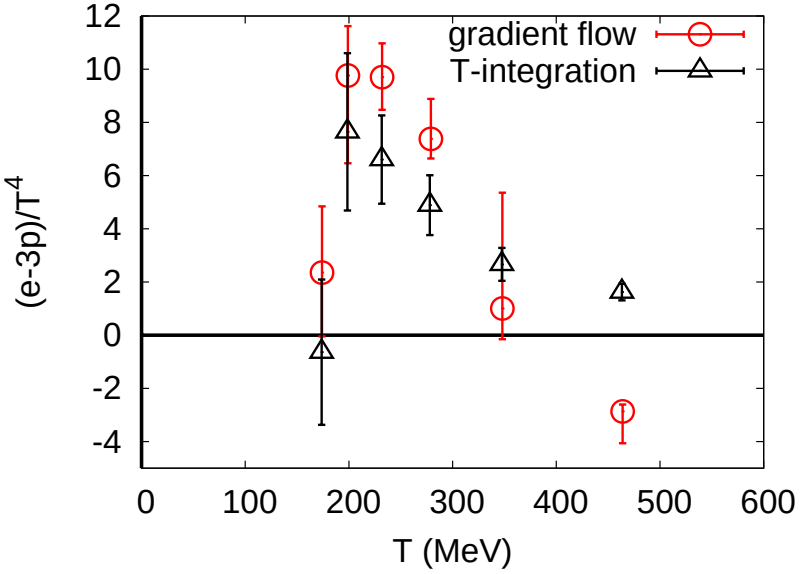


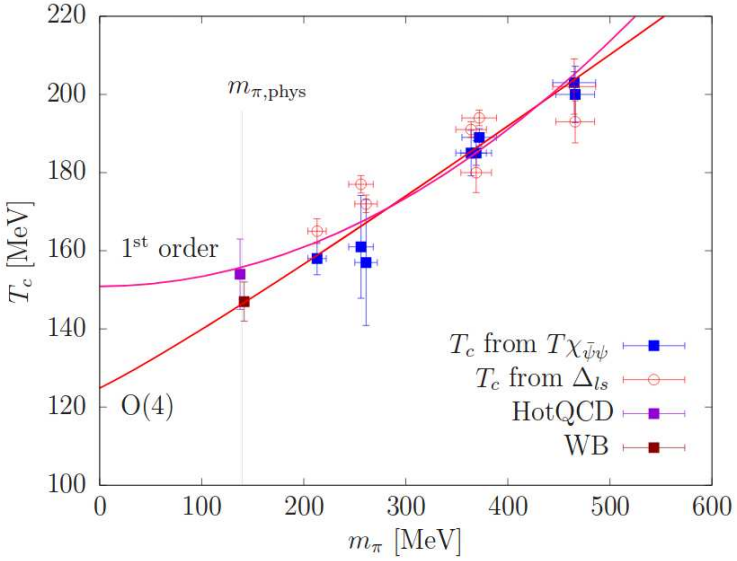


The phase diagram at $\mu_B = 0$

- Recent update EoS with Wilson fermions [WHOT QCD col., Phys.Rev.D95, 054502 (2017)] measurement of T_c from chiral observables, [ETM Collaboration, 1805.06001]
- Energy-Mom. tensor extracted using gradient flow. A peak in chiral susceptibility observed even with Wilson fermions at $m_\pi \sim 400$ MeV. New results on EM tensor correlators [See talk by Y. Taniguchi, Thurs 9:10, A. Baba, Thu 12:00].
- EM Tensor correlators calculated with better precision in pure glue

  [See talk by Shirogane, Hirakida, Thus Morn.]





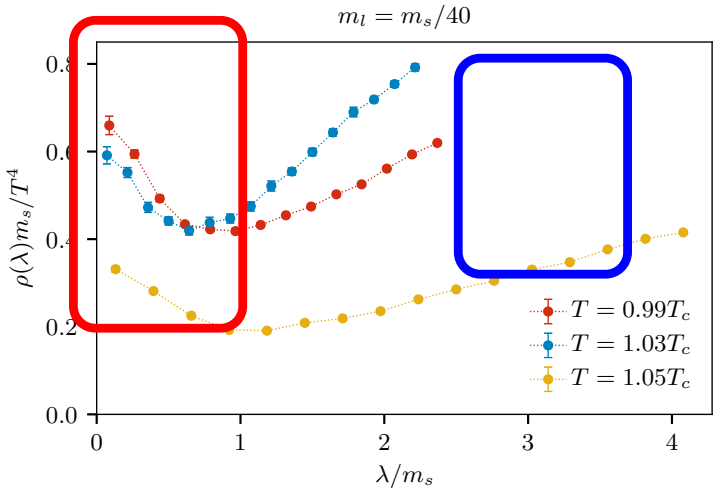
- Since $m_u, m_d << \Lambda_{QCD}$ is $U_L(2) \times U_R(2)$ a good symmetry of QCD?
- $U_L(2) \times U_R(2) \rightarrow SU(2)_V \times SU(2)_A \times U_B(1) \times U_A(1)$
- Is $U_A(1)$ effectively restored at T_c? $\rightarrow$ can change the universality class of the second order phase transition at $\mu_B = 0$ or first order?
  Either O(4) or $U_L(2) \times U_R(2)/U_V(2)$
  [Pisarski & Wilczek, 84, Butti, Pelissetto & Vicari, 03, 13, Nakayama & Ohtsuki, 15]
- New symmetries in high T? [Rohrhofer, Fri 17:50] Anderson Transition at finite T? [Holicki, Fri 15:20]

- $U_A(1)$ not an exact symmetry$\rightarrow$ what observables to look for?
- Degeneracy of the 2-point correlators [Shuryak, 94] $\rightarrow$ higher point correlation functions imp [Aoki, Fukaya & Taniguchi, 1209.2061]

$$\chi_\pi - \chi_\delta \overset{V\rightarrow\infty}{\rightarrow} \int_0^\infty d\lambda \frac{4m_f^2\ \rho(\lambda, m_f)}{(\lambda^2 + m_f^2)^2}$$

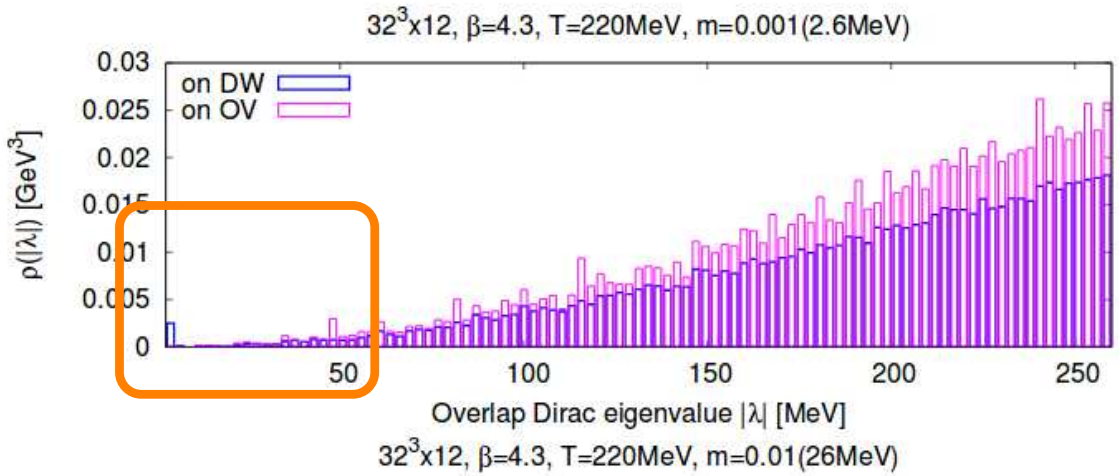
- Sufficient condition for restoration in chiral limit:
  $\rho(\lambda) \sim \lambda^3$ [Aoki, Fukaya & Taniguchi, 1209.2061]

Update on Eigenvalue spectrum of QCD Dirac operator

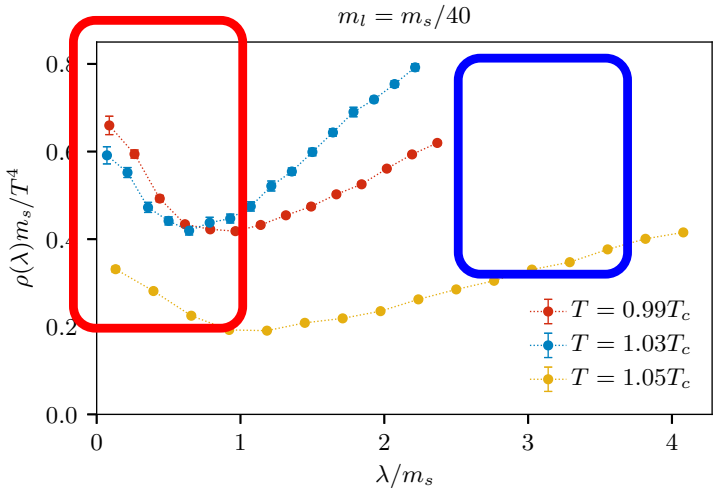


- $\rho(\lambda) \sim \lambda$
  for QCD spectrum with Highly improved Staggered quarks towards the chiral limit measured with overlap operator for $T \leq 1.1\,T_c$.
  [See talk by Lukas Mazur, Tues. 14:20]
- role of non-analyticities? Seem to be reduced but survive in the chiral limit with HISQ.
  [HotQCD collaboration, 1205.3535, V. Dick et. al. 1502.06190]

- Non-Analyticities sensitive to lattice cut-off effects. Reduces with lattice spacing. See talk by K. Suzuki, Tues. 14:00, also 1711.09239

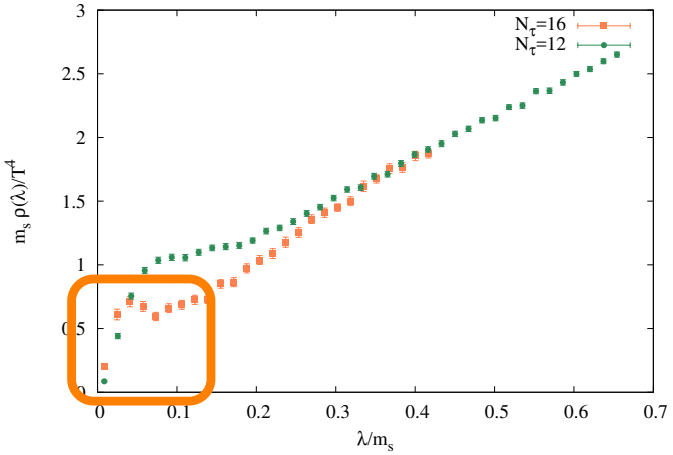


Update on Eigenvalue spectrum of QCD Dirac operator

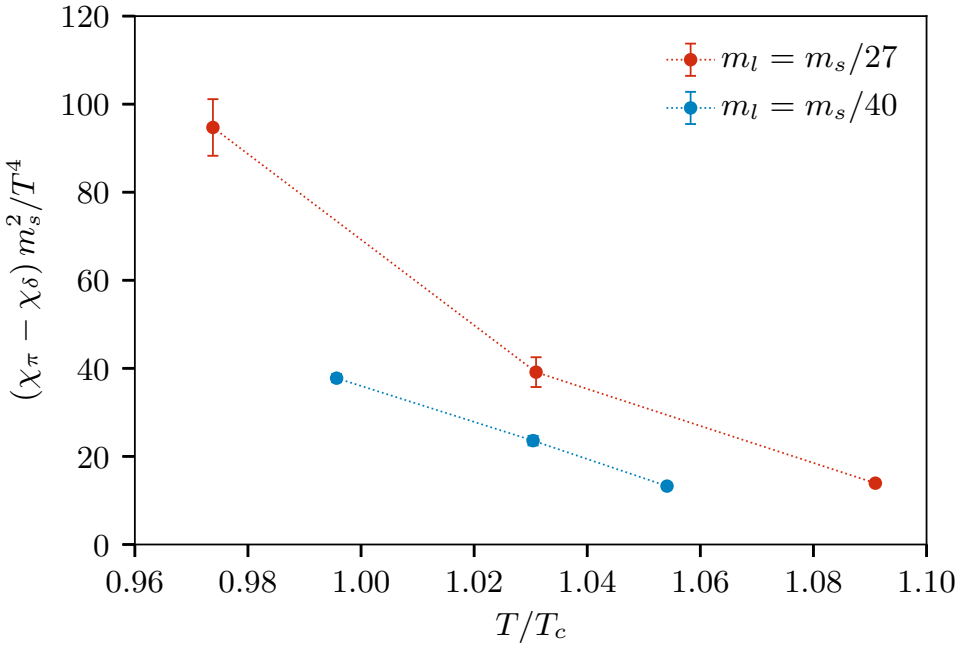


- $\rho(\lambda) \sim \lambda$
  for QCD spectrum with Highly improved Staggered quarks towards the chiral limit measured with overlap operator for $T \leq 1.1\,T_c$.
  [See talk by Lukas Mazur, Tues. 14:20]
- role of non-analyticities? Seem to be reduced but survive in the chiral limit with HISQ.
  [HotQCD collaboration, 1205.3535, V. Dick et. al. 1502.06190]

- Not due to partial quenching: HISQ spectrum on the finest lattices show such a peak $\rightarrow$ continuum limit needed to resolve this issue! [HotQCD in prep.]



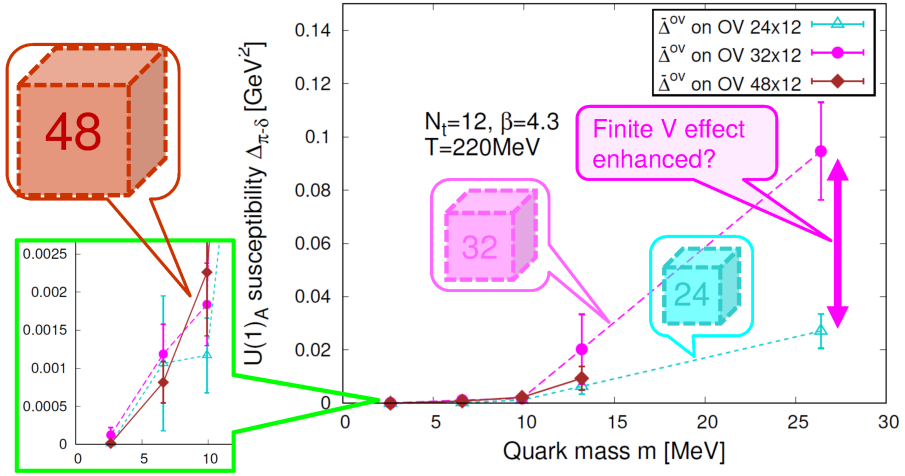
- Zero modes show strong lattice cut-off dependence [G. Cossu et. al, 13, A. Tomiya et. al, 15,16]. Will not contribute in thermodynamic limit!
- Non-analytic part still needs careful study. Analytic part of the spectrum strongly suggest that $U_A(1)$ is broken! [See talk by L. Mazur, Tues] [V. Dick, et. al, 1502.06190, 1602.02197, G. Cossu et. al., 1510.07395, K. Suzuki et. al. 1711.09239].
- New update on volume dependence [See talk by K. Suzuki, Tues.] $\longrightarrow$ in the chiral limit is vol. dep. milder?



- Zero modes show strong lattice cut-off dependence [G. Cossu et. al, 13, A. Tomiya et. al, 15,16]. Will not contribute in thermodynamic limit!
- Non-analytic part still needs careful study. Analytic part of the spectrum strongly suggest that $U_A(1)$ is broken! [See talk by L. Mazur, Tues] [V. Dick, et. al, 1502.06190, 1602.02197, G. Cossu et. al., 1510.07395, K. Suzuki et. al. 1711.09239].
- New update on volume dependence [See talk by K. Suzuki, Tues.] $\longrightarrow$ in the chiral limit is vol. dep. milder?

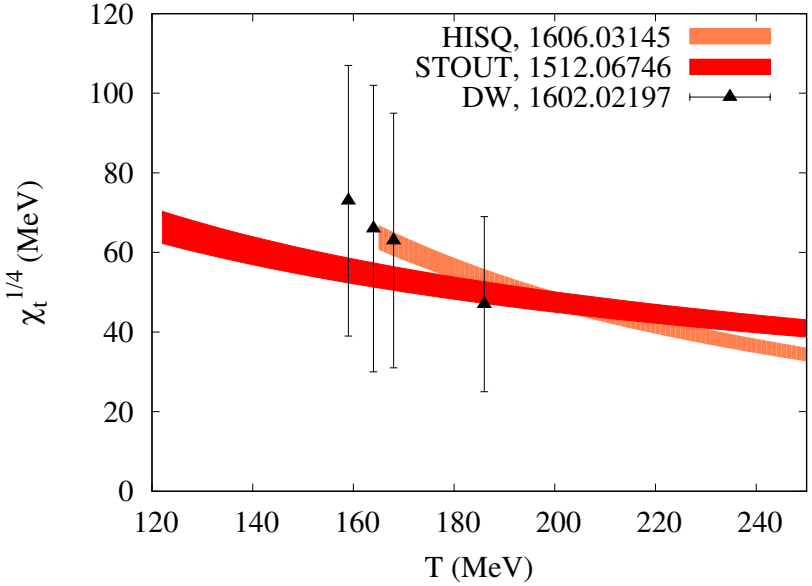
JLQCD, preliminary (2018)

$U(1)_A$ susceptibility (volume effect)



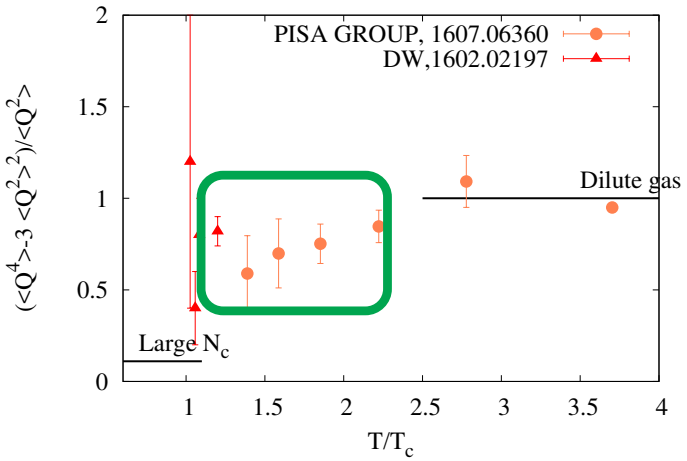
⇒For small m, V-dependence seems to be small

From Dirac spectrum to Topological fluctuations



- $\chi_t^{1/4} = AT^{-b}$.
- $b = 0.9 - 1.2$ for $T < 250$ MeV
- Different from dilute instanton gas: $b \sim 2$.
  [from continuum extrapolated results with HISQ. [P. Petreczky, et. al., 1606.03145]. Agrees well with independent study [Bonati et. al, 1512.06746] and with results with chiral fermions 1602.02197].
- χ_t is studied as a function of quark mass near T_c along with vol. dependence [See talk by Y. Aoki, Tues 14:40]

- Since θ is tiny, $F(\theta) = \frac{1}{2}\chi_t\theta^2\left(1 + b_2\theta^2 + \ldots\right)$.
  [L. D. Debbio, H. Panagopoulos, E. Vicari, 0407068]
- Strong non-Gaussianity in higher order expansions. What causes them?

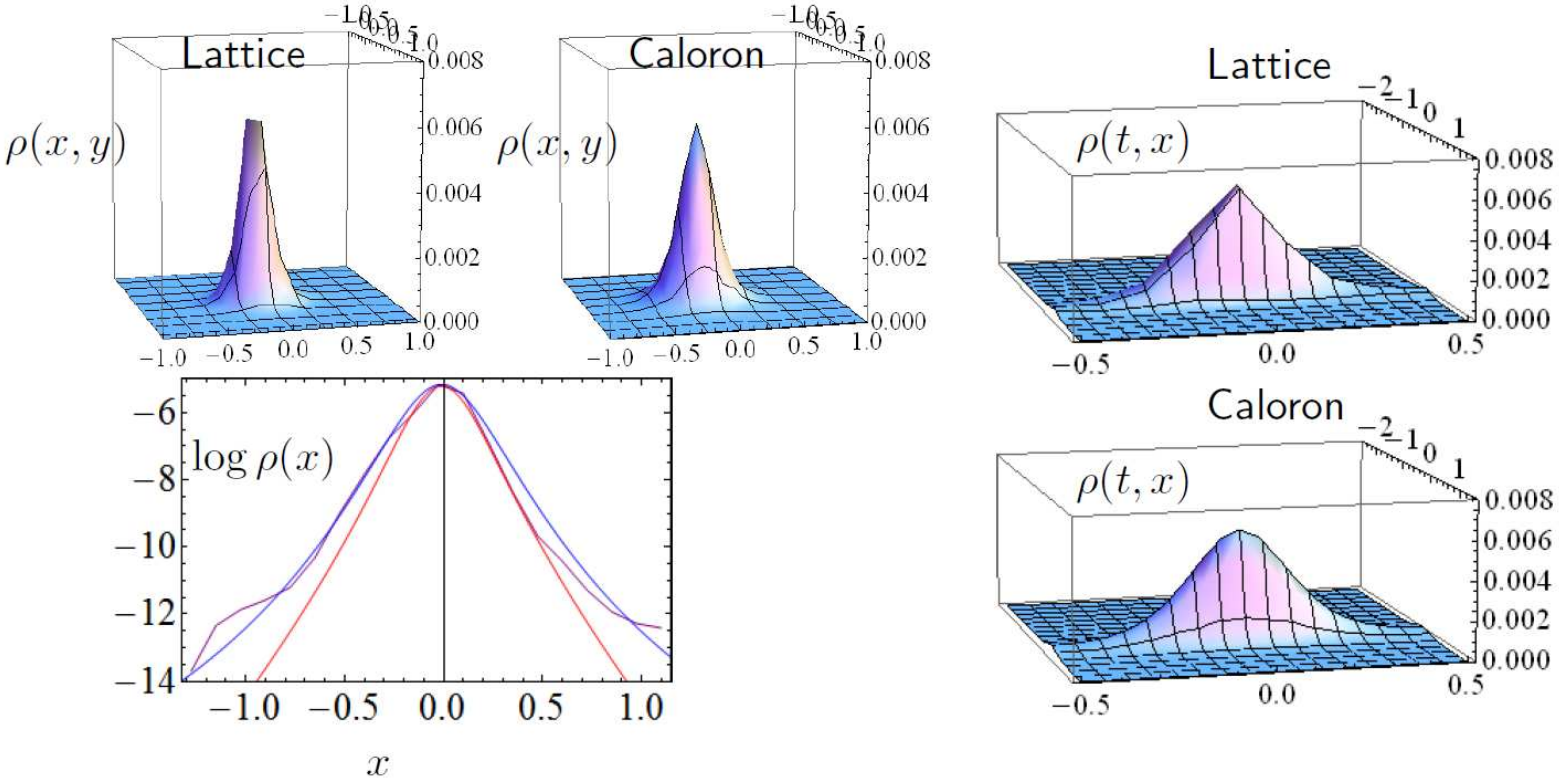


Towards interpreting these findings

- Going beyond the interacting instanton liquid? Can there be instanton-dyons present $\sim T_c$ due to non-trivial eigenvalues of Polyakov loop.
  Hints from over-improved cooling studies from the lattice
  [M. Ilgenfritz, M-Mueller Pruessker, et. al. 14, 15].
- Using twisted boundary conditions of the valence fermionic (overlap) operator can move the zero modes from one instanton-dyon to other.
  [See for more details in talk by R. Larsen, Tues 15:20]
  $\rightarrow$ fall off of density profiles at large distances can be a way to distinguish between them?

Towards interpreting these findings

- Anti periodic fermionic zero modes at $1.08T_c$ with Overlap Dirac Operator

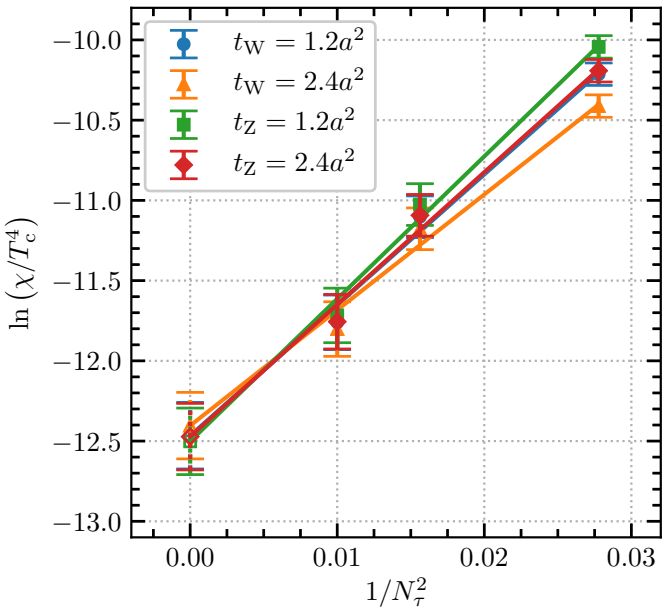


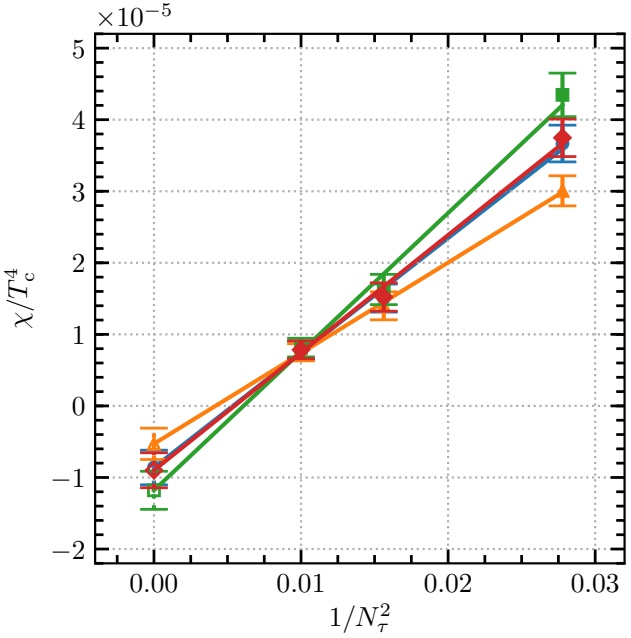
Improving topological tunneling at high temperatures

- High temperatures $\rightarrow$ topological tunneling becomes rarer. Similar to going to finer lattice spacings.

- New techniques developed : Reweighting ensembles with coarse grained definition of Q [C. Bonati & M. D'Elia, 1709.10034, P. T. Jahn, G. Moore, D. Robaina, 1806.01162] allows to go $T \sim 4\,T_c$ with $N_\tau = 10$ lattices with reasonable cost.

  [See talk by T. Jahn, Tues. 15:00]

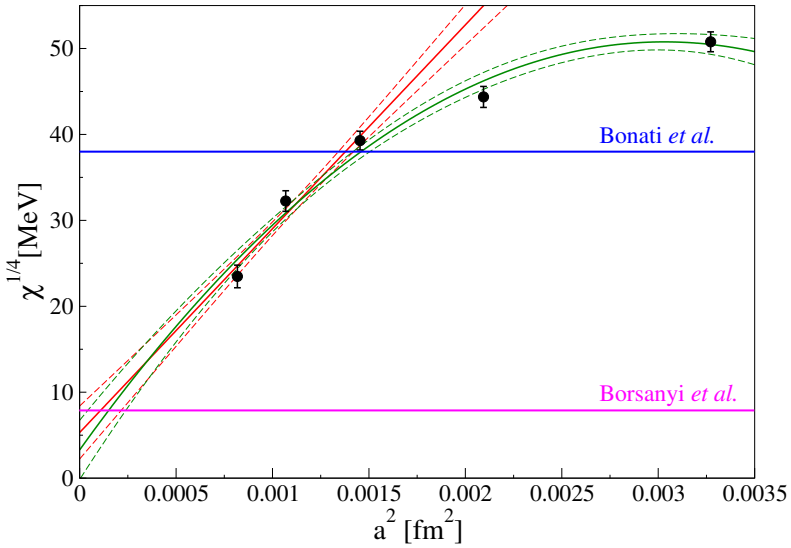




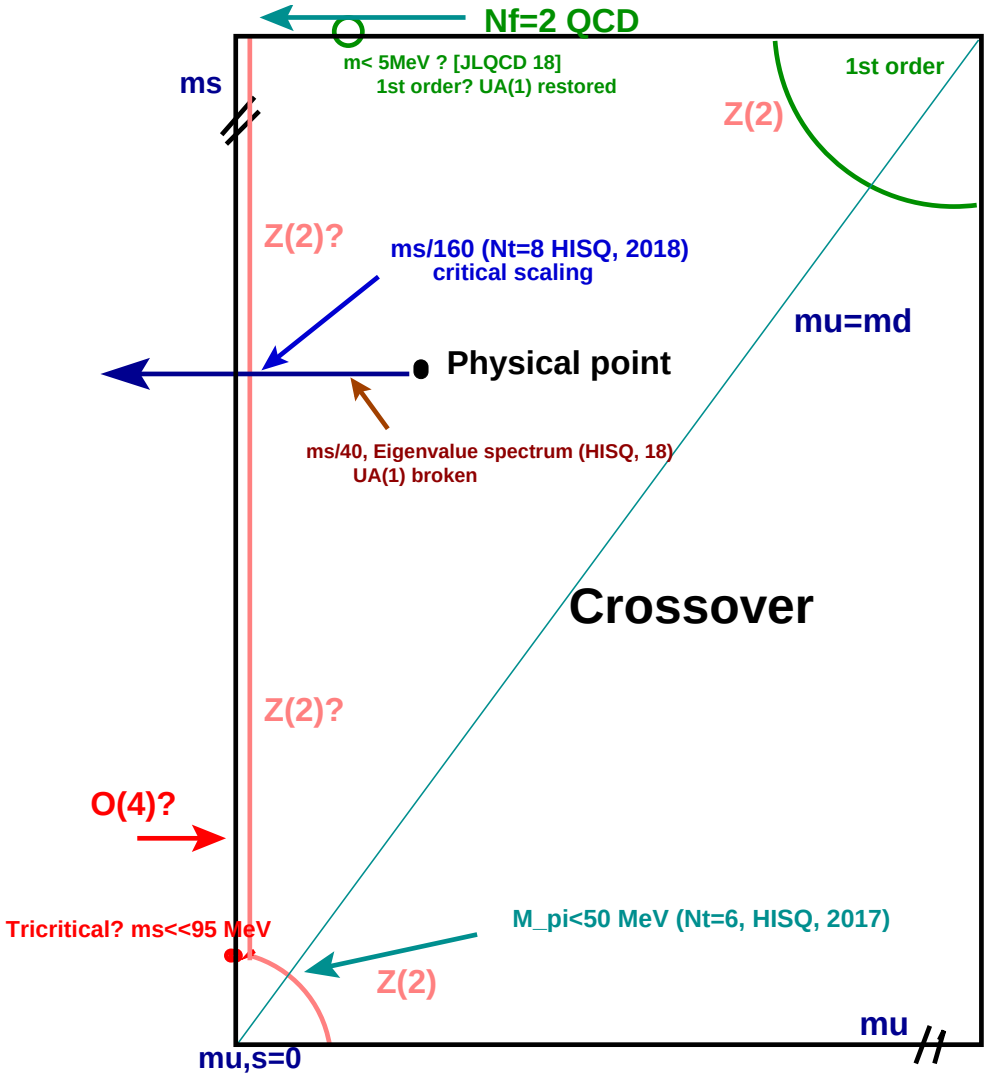
Improving topological tunneling at high temperatures

- High temperatures $\rightarrow$ topological tunneling becomes rarer. Similar to going to finer lattice spacings.
- Reweighting applied in full QCD improves Q measurement at high T $\rightarrow$ finite vol. dependence under control

  [C. Bonati et. al., 1807.07954, and see also 1709.10034]
- Many other techniques discussed : Metadynamics, Open boundary conditions.. [F. Sanfillipo et. al, Borsanyi et. al, 1606.07494, J. Frison et. al., 1606.07175]



Towards understanding the Columbia plot

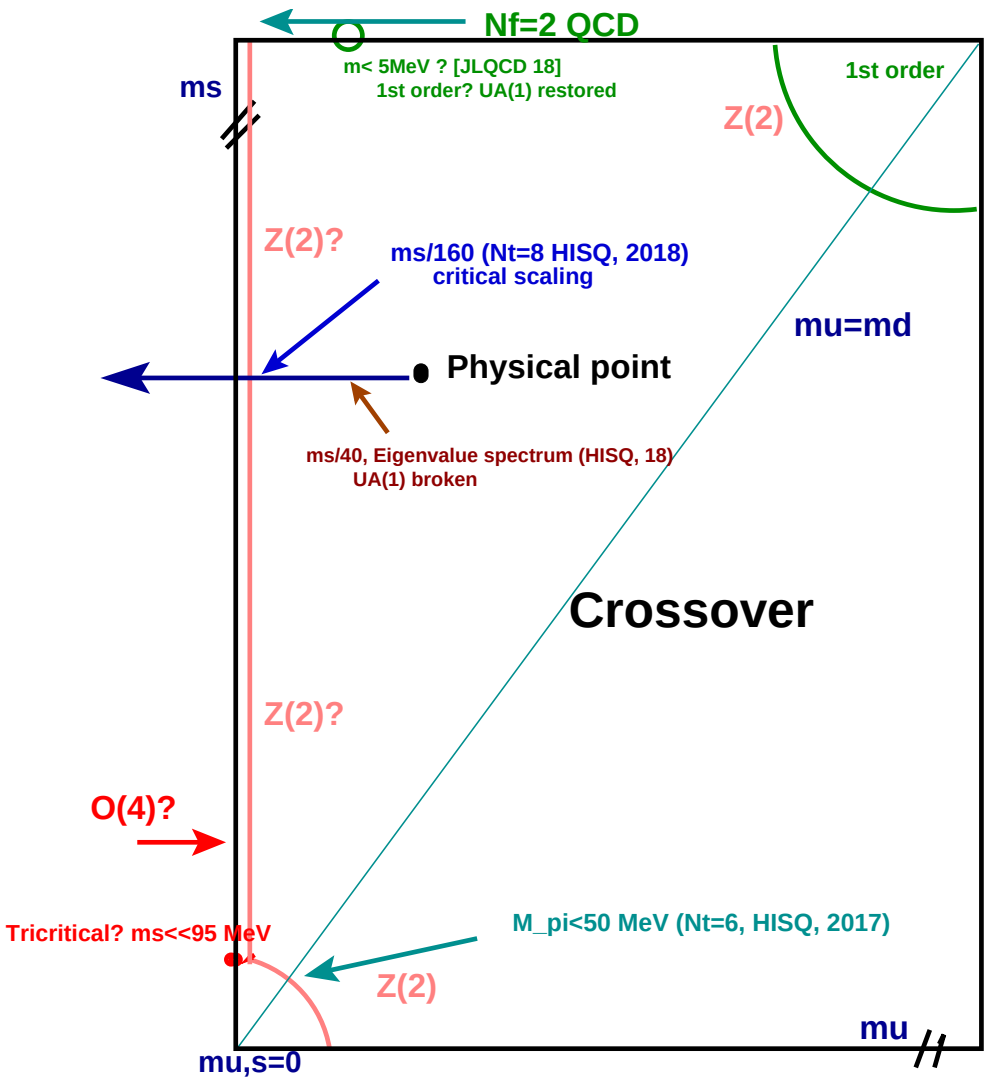


- Approaching chiral limit at fixed m_s
- $N_f = 2$ QCD updates with overlap valence on overlap sea via reweighting [See talk by K. Suzuki]
- HISQ eigenvalue spectrum for 2+1 QCD towards chiral limit

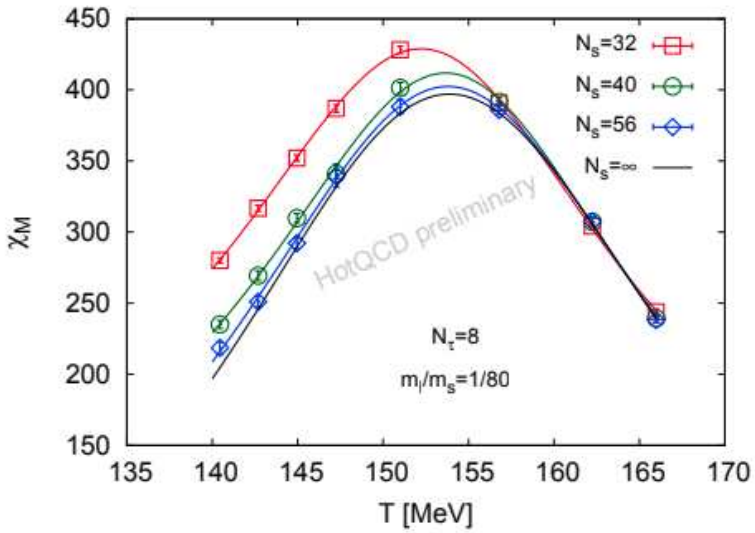
  [See talk by L. Mazur]
- From spectral density extract T_c, order of transition in $m_q \to 0$

  [See talk by G. Endrodi, Thurs. 11:40]

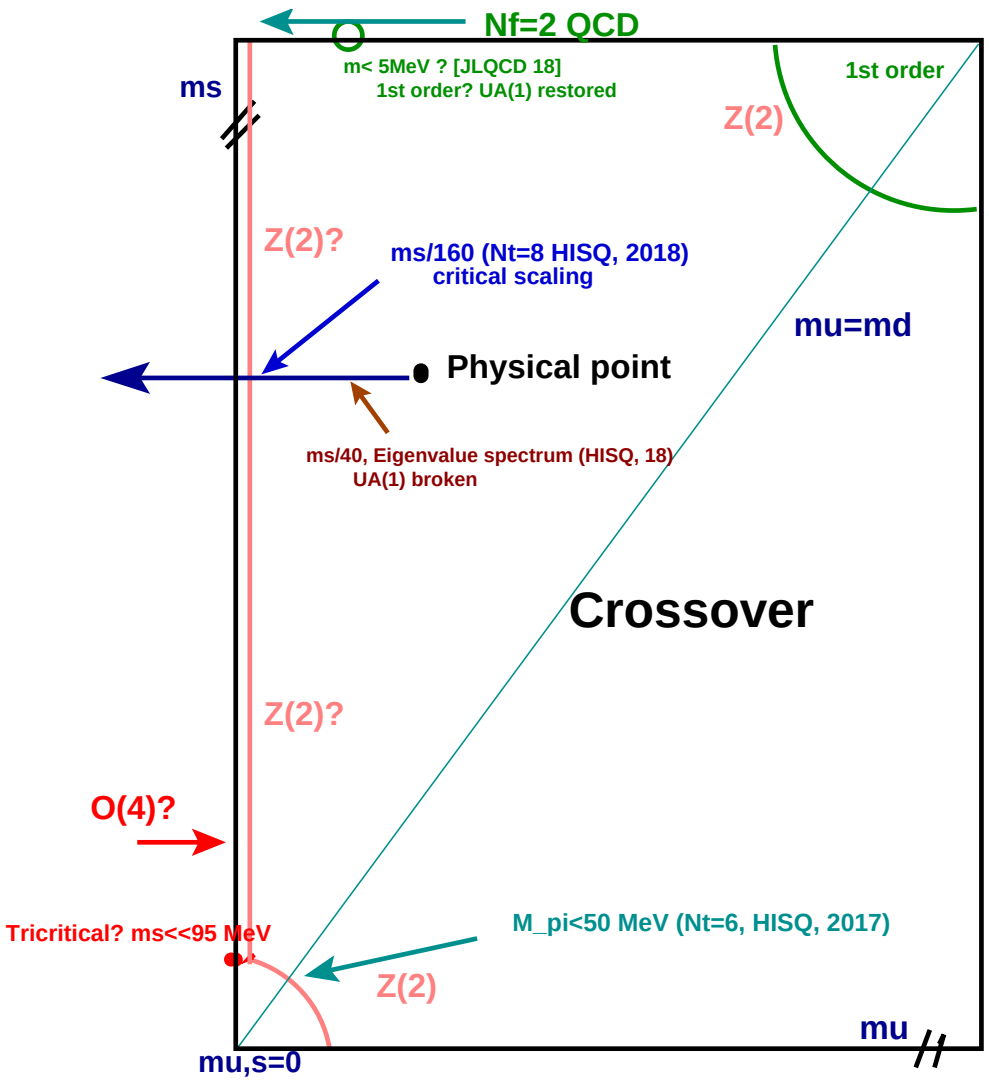
Towards understanding the Columbia plot



- Approaching chiral limit at physical m_s
- New: Scaling analysis of chiral condensate with Highly Improved Staggered quarks on finer lattices $N_\tau = 8, 12$.
  [See talk by Sheng-Tai Lee, Thurs. 11:20]
- Peak of χ_M decreases with volume ruling out 1st order transition for $m_\pi \geq 80$ MeV.

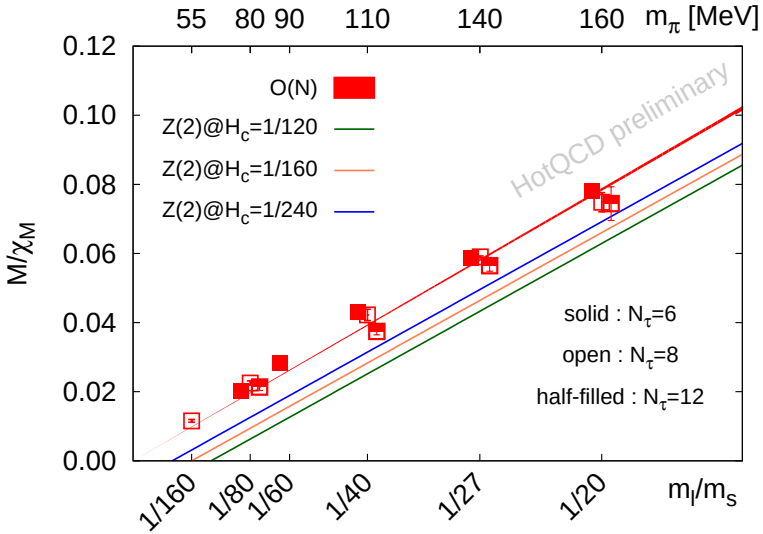


Towards understanding the Columbia plot

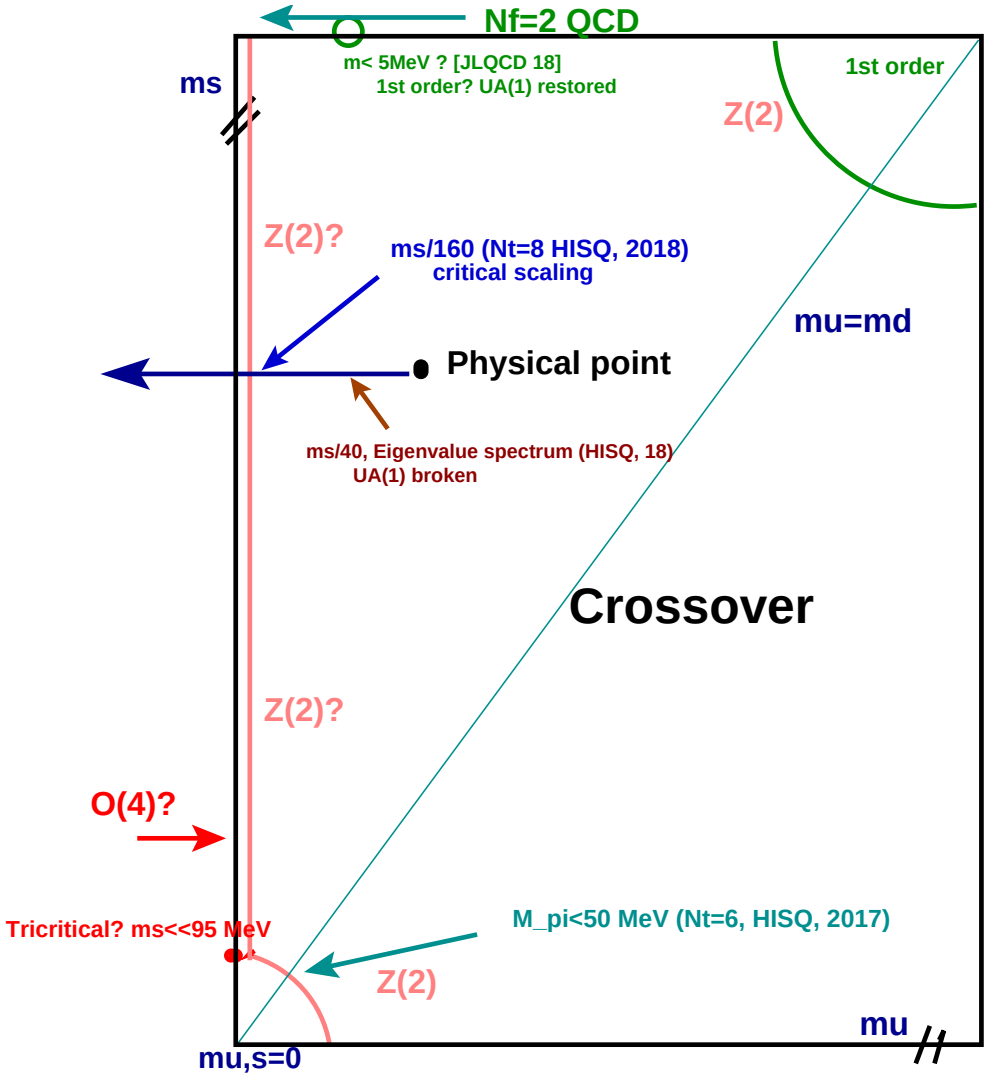


- Approaching chiral limit at physical m_s
- Scaling seems to be consistent with $O(2)$ rather than Z_2.

[A. Lahiri et. al., 1807.05727]

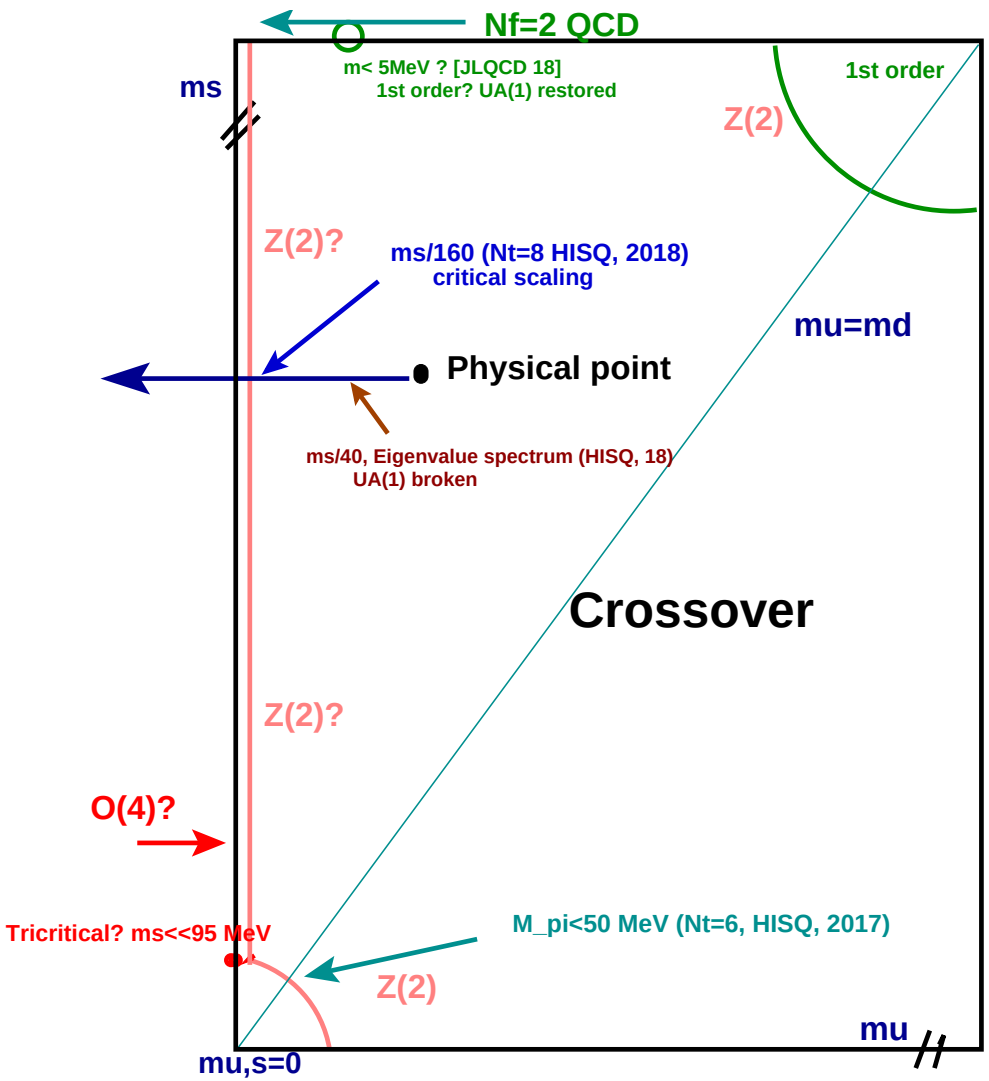


Towards understanding the Columbia plot



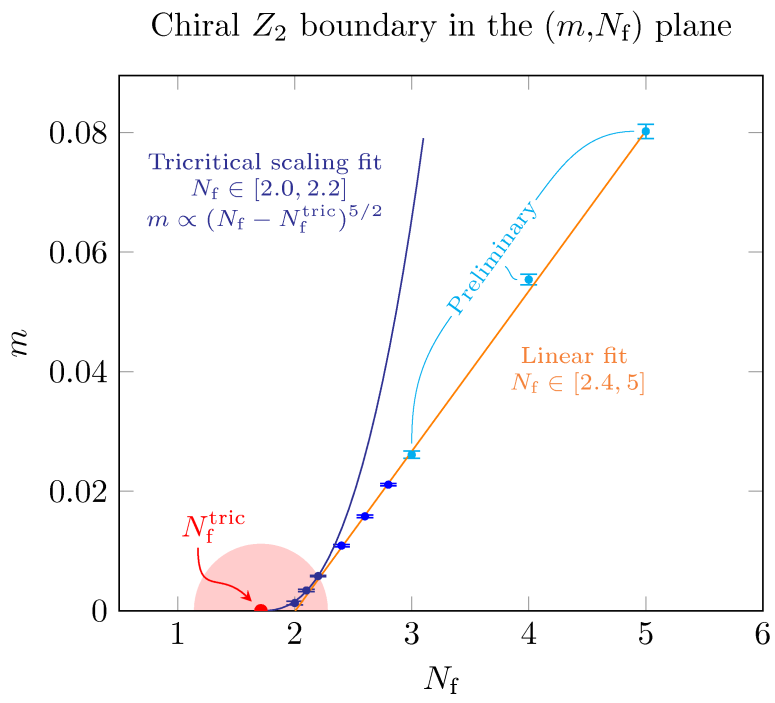
- Along $N_f = 3$ line
  - $N_f = 3$ QCD scaling analysis with HISQ [A. Bazavov et. al.,1701.03548]
  - Reweighting expansion with $2 + N_f$ flavors. [N. Yamada et. al, 1602.04595].
  - $N_f = 3$ QCD with Wilson fermions give $m_{PS} < 170$ MeV [X Jin et. al.,1706.01178]
  - The m_π^c could be extremely small for $N_f = 3, 4$ [de Forcrand & M. D'Elia, 1702.00330]
  - New update on $N_f = 4$ phase diagram with Wilson clover fermions [See talk by H. Ohno, Thurs. 12:20]
  - Very challenging! need to go to continuum limit..scope for new lattice techniques.

Towards understanding the Columbia plot

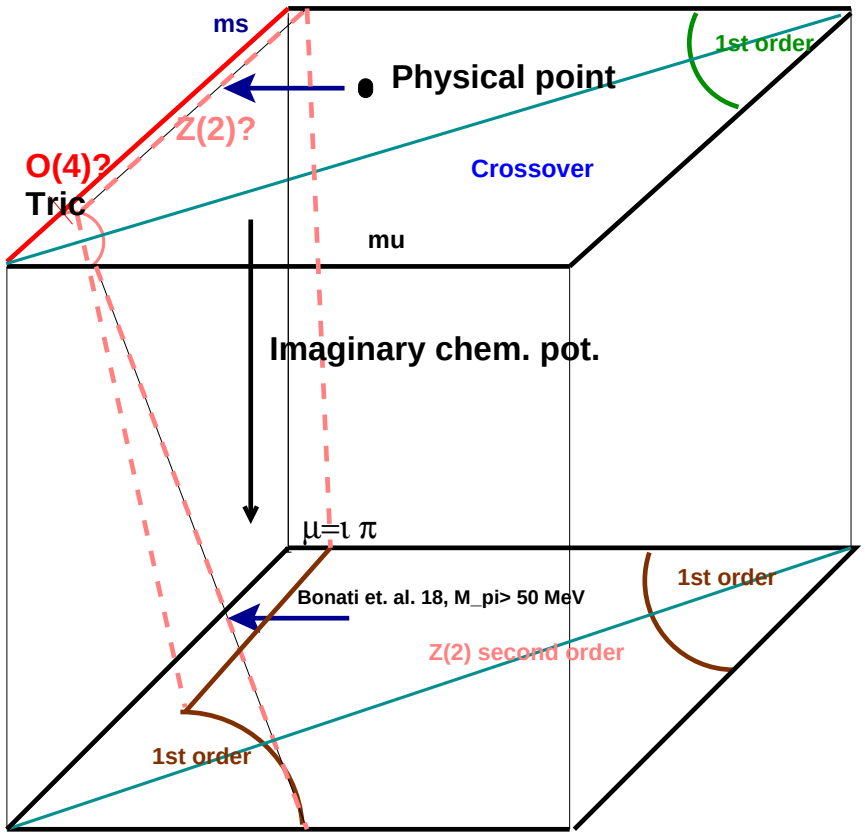


- N_f as a continuous parameter
- Upper bound on tricrit. scaling $N_f < 2 \rightarrow$ first order transition for $N_f = 2$? Check at finer lattices?

[See talk by F. Cuteri, Thurs. 11:00]

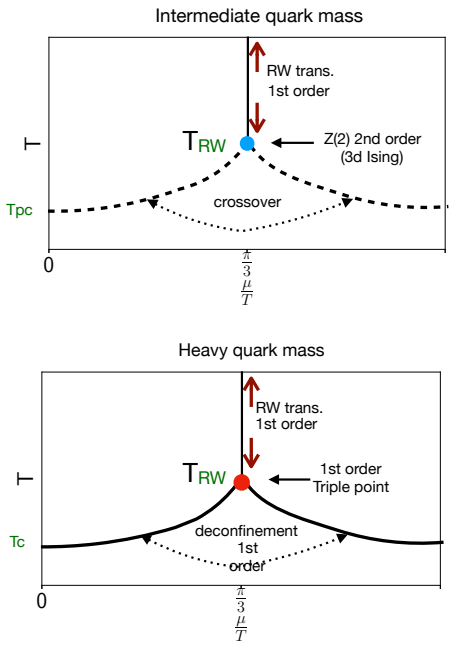


Adding a new axis to the Columbia plot: Imaginary μ

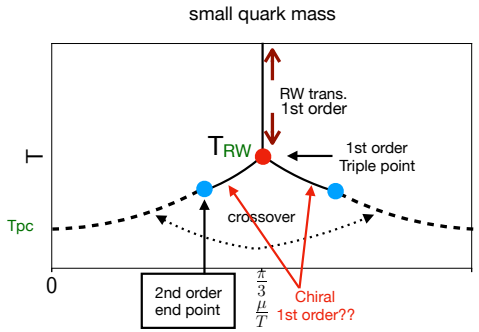


- For $\mu_B/T = i(2n+1)\pi$ an exact Z_2 symmetry. Spontaneously broken at Roberge-Weiss T_{RW}. Order parameter: ImL

[See talk by J. Goswami, Wed 16:50]

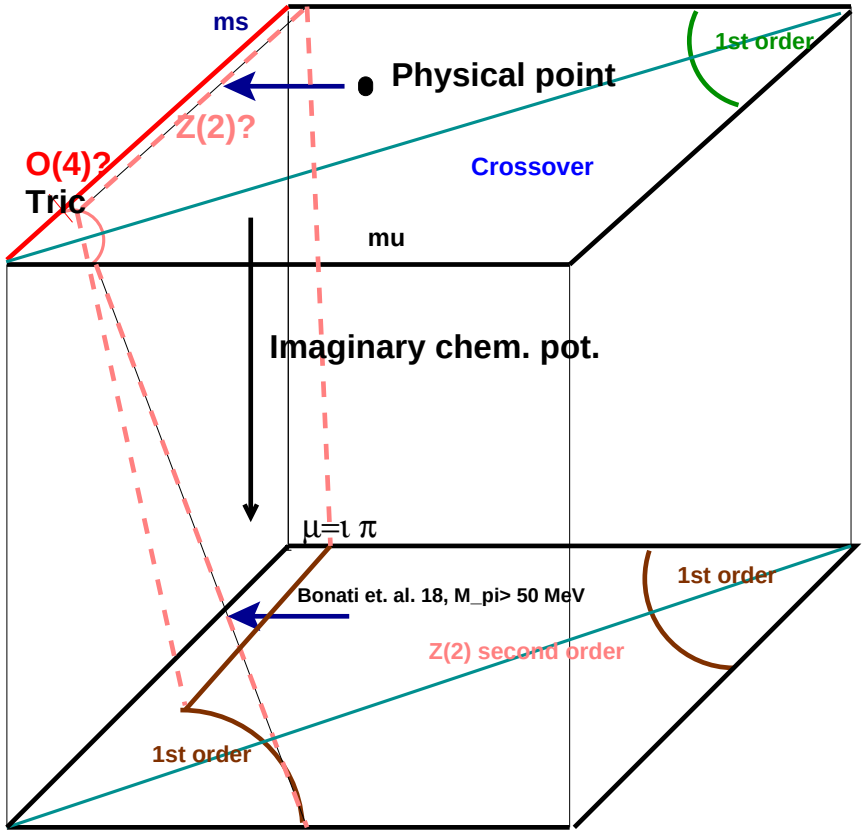


Conjectured phase diagrams in the imaginary chemical potential plane

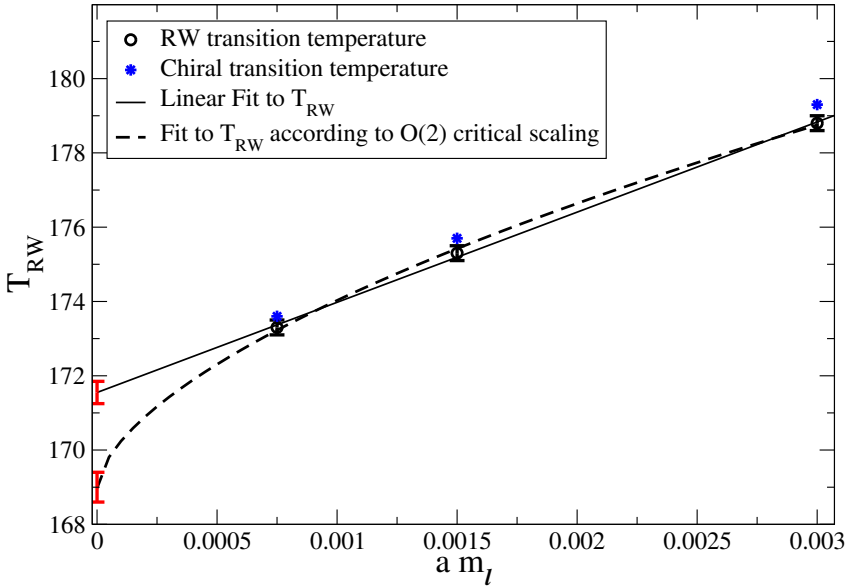


Different scenarios for different quark masses

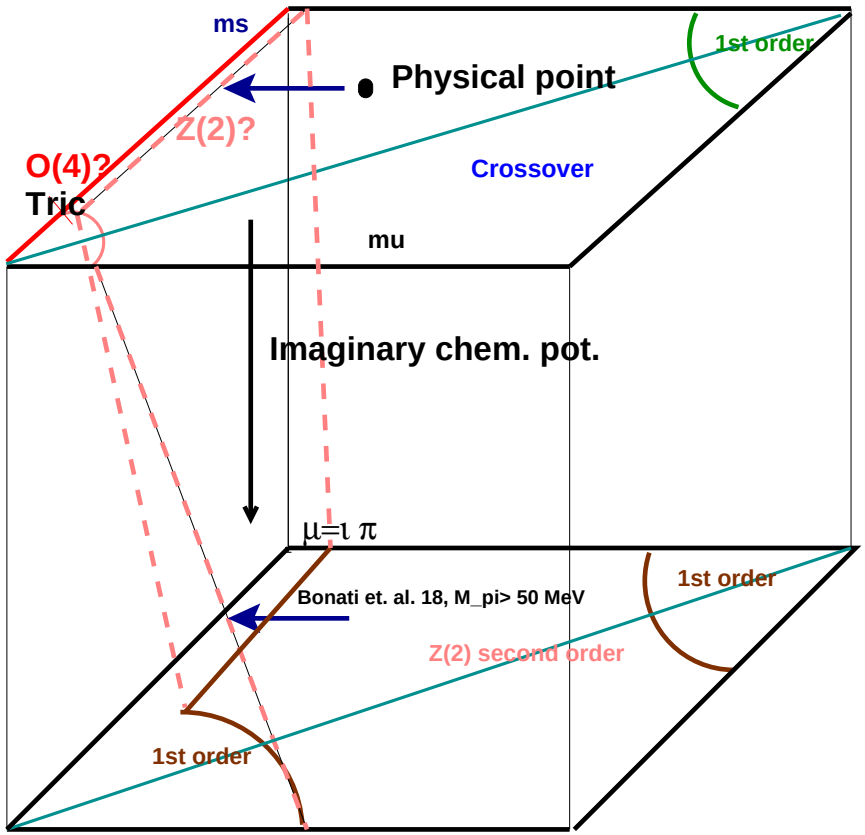
Adding a new axis to the Columbia plot: Imaginary μ



- $N_\tau = 4$ QCD with stout fermions, no sign of first order RW transition for $m_\pi > 50$ MeV. [C. Bonati et. al 1807.02106].
- Most plausibly the chiral and RW end-point occur at the same T?

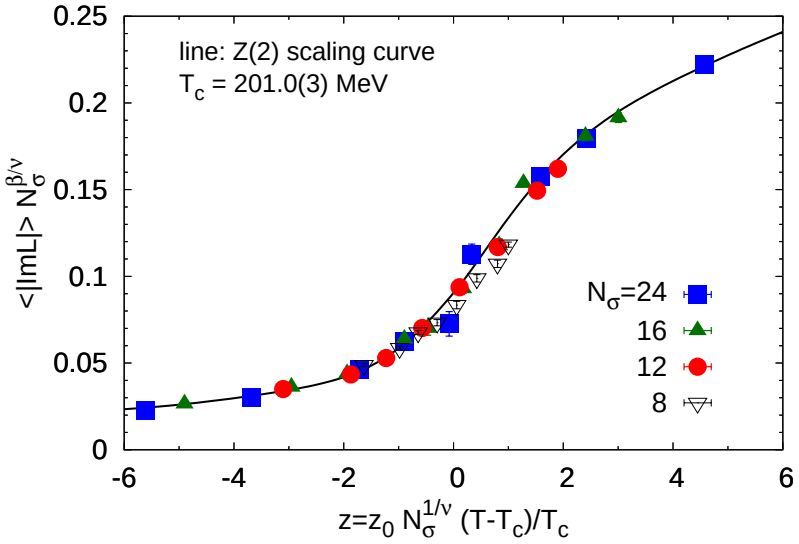


Adding a new axis to the Columbia plot: Imaginary μ

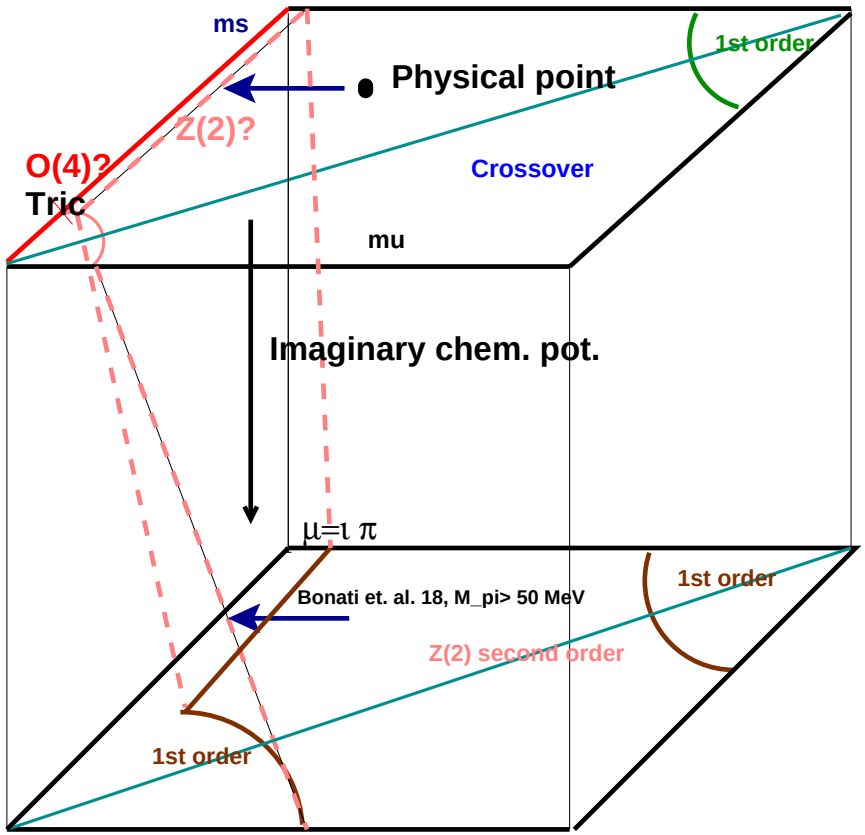


- Under Z_2, Re $L \to$ Re L , Im $L \to$ -Im L.
- Im L shows Z_2 scaling with HISQ fermions at $N_\tau = 4$! What about ReL?

[See talk by J. Goswami, Wed 16:50].

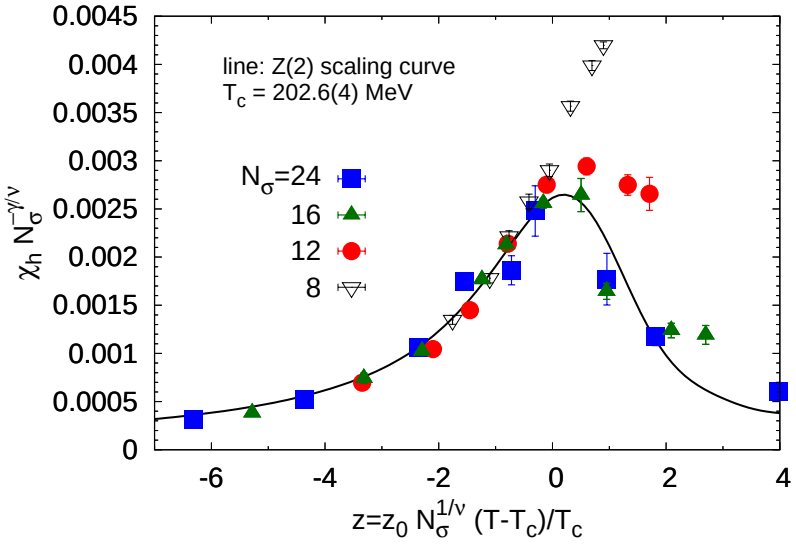


Adding a new axis to the Columbia plot: Imaginary μ



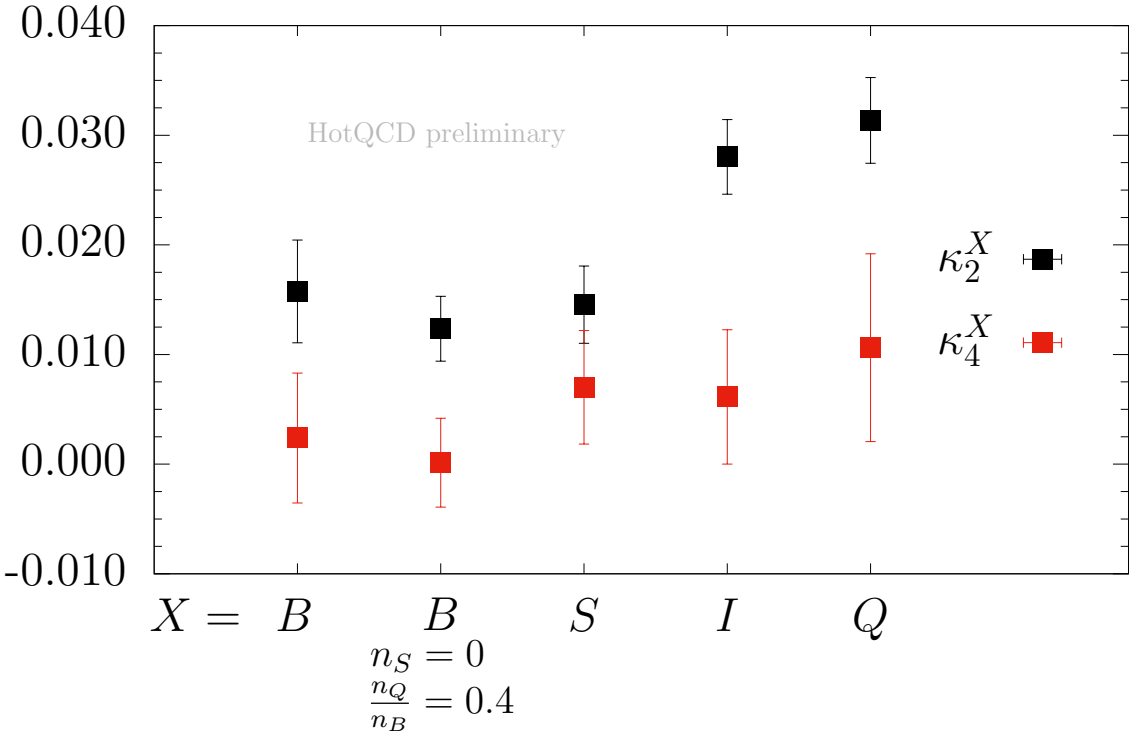
- Under Z_2, Re $L \to$ Re L , Im $L \to$ -Im L.
- Im L shows Z_2 scaling with HISQ fermions at $N_\tau = 4$! What about ReL?

[See talk by J. Goswami, Wed 16:50].



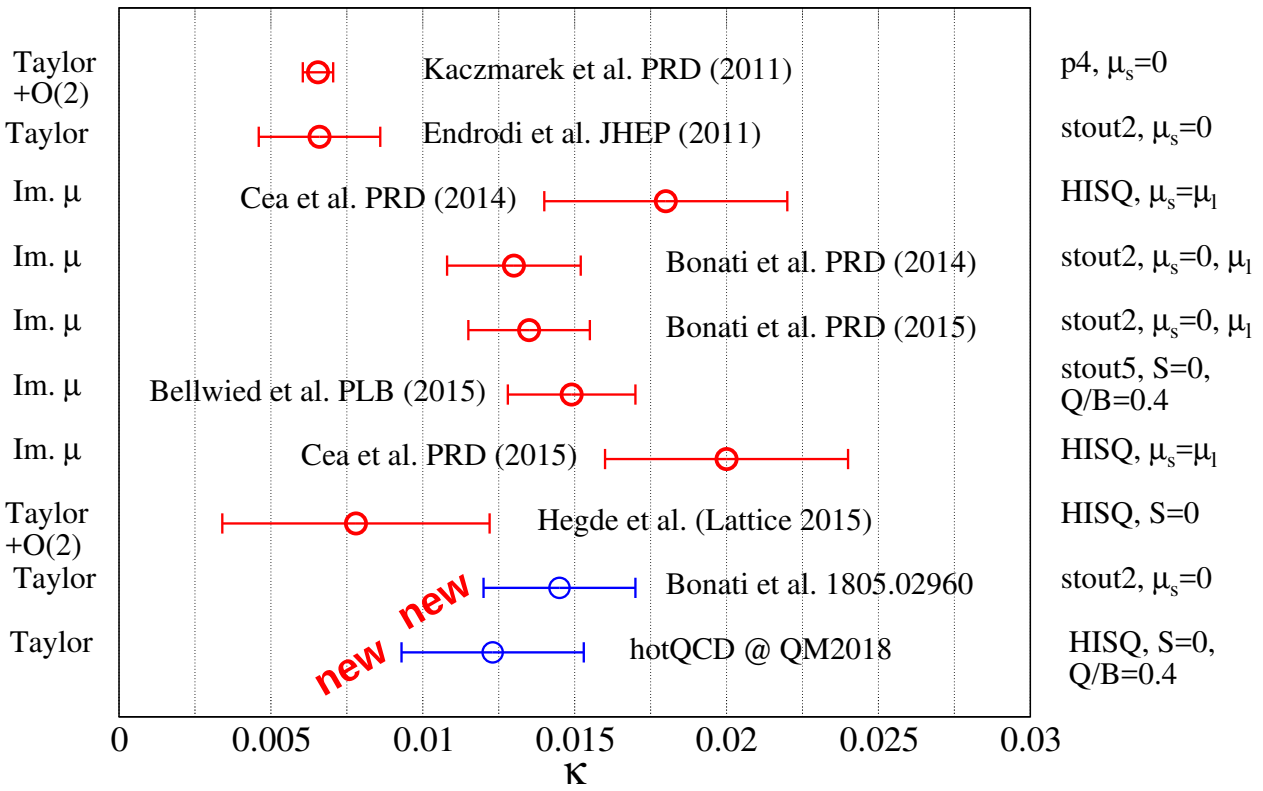
Curvature of the chiral crossover line

- $\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa_2 \frac{\mu_B^2}{T_c(0)^2} - \kappa_4 \frac{\mu_B^4}{T_c(0)^4}$
- For strangess neutral system, $\kappa_2 = 0.0120(20)$ with Taylor series and HISQ fermions. [HotQCD collaboration, 1807.05607, talk by P. Steinbrecher]



Curvature of the chiral crossover line

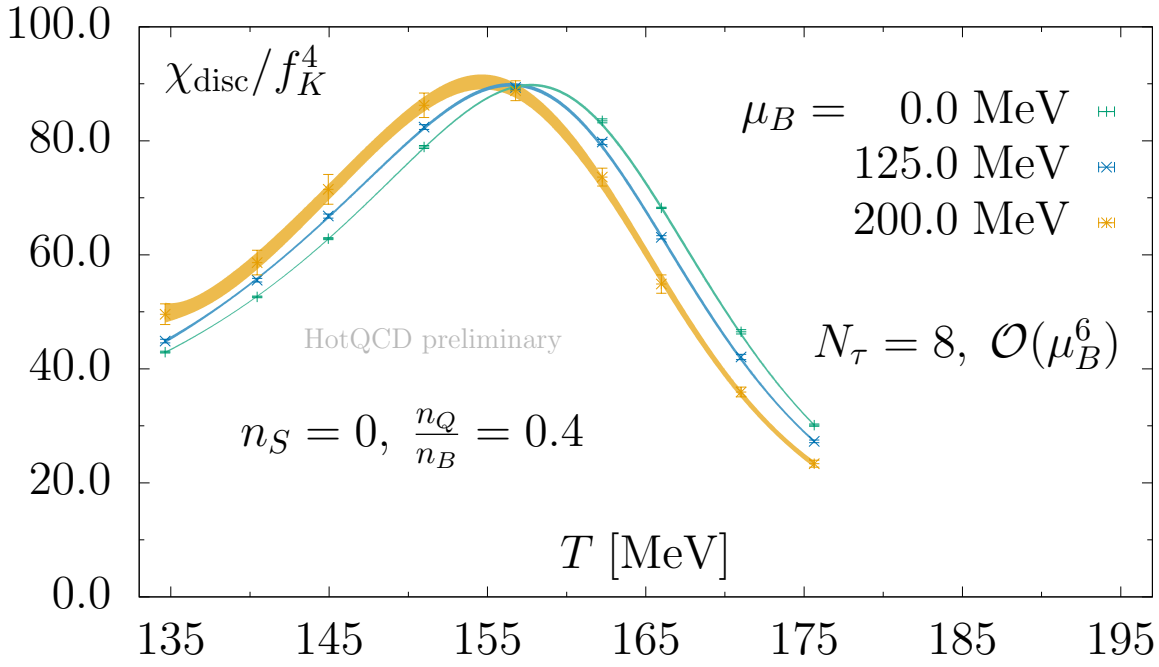
- $\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa_2 \frac{\mu_B^2}{T_c(0)^2} - \kappa_4 \frac{\mu_B^4}{T_c(0)^4}$
- Consistent with imaginary chemical potential method and stout fermions $\kappa_2 = 0.0135(20)$ [C. Bonati et. al., 1805.02960]
- removes earlier possible tension between two methods! [courtesy M. D'Elia QM 18]



Curvature of the chiral crossover line

- $\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa_2 \frac{\mu_B^2}{T_c(0)^2} - \kappa_4 \frac{\mu_B^4}{T_c(0)^4}$
- Chiral observables show little curvature as a function of $\mu_B < 250$ MeV.

  [HotQCD collaboration, 1807.05607]
- Need much higher order series in μ_B?



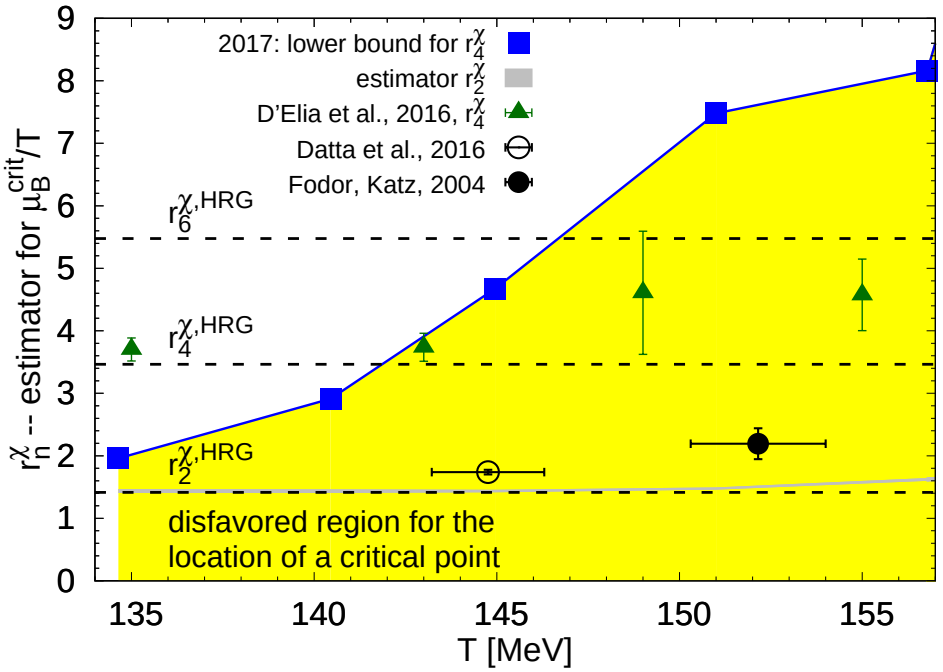
Critical-end point search from Lattice

- The Taylor series for $\chi_2^B(\mu_B)$ should diverge at the critical point for $N_f = 2$. On finite lattice χ_2^B peaks, ratios of Taylor coefficients equal, indep. of volume.
- The radius of convergence determines location of the critical point.

  [Gavai& Gupta, 03]
- Definition: $r_{2n} \equiv \sqrt{2n(2n-1)\left|\frac{\chi_{2n}^B}{\chi_{2n+2}^B}\right|}$.
  - Strictly defined for $n \to \infty$. How large n could be on a finite lattice?
  - Signal to noise ratio deteriorates for higher order χ_n^B.

Critical-end point search from Lattice

- Current bound for CEP: $\mu_B/T > 3$ for $135 \leq T \leq 150$ MeV [Bielefeld-BNL-CCNU, 1701.04325, update 2018].
- The r_n extracted by analytic continuation of imaginary μ_B data [D'Elia et. al., 1611.08285] consistent with this bound.
- Results with a lower bound? [Datta et. al., 1612.06673, Fodor and Katz, 04] $\rightarrow$ need to understand the systematics in these studies. Ultimately all estimates will agree in the continuum limit!



Summary and Outlook

- Lattice QCD allows us to calculate bulk thermodynamic quantities, χ_{top} with very high precision for a wide range of temp. with updated estimates on T_c.

Summary and Outlook

- Lattice QCD allows us to calculate bulk thermodynamic quantities, χ_{top} with very high precision for a wide range of temp. with updated estimates on T_c.
- Beginning to explore finite μ_B region with new results on the curvature of chiral crossover line.

Summary and Outlook

- Lattice QCD allows us to calculate bulk thermodynamic quantities, χ_{top} with very high precision for a wide range of temp. with updated estimates on T_c.
- Beginning to explore finite μ_B region with new results on the curvature of chiral crossover line.
- Latest bounds on the critical end-point LQCD data suggest $\mu_B(\text{CEP})/T > 3$ in the region $T = 145 - 150$ MeV.

Summary and Outlook

- Lattice QCD allows us to calculate bulk thermodynamic quantities, χ_{top} with very high precision for a wide range of temp. with updated estimates on T_c.
- Beginning to explore finite μ_B region with new results on the curvature of chiral crossover line.
- Latest bounds on the critical end-point LQCD data suggest $\mu_B(\text{CEP})/T > 3$ in the region $T = 145 - 150$ MeV.
- Lattice methods now give more insights on the Columbia plot $\rightarrow$ ultimately allow us to understand the phase diagram for $N_f = 2 + 1$ QCD.

Summary and Outlook

- Lattice QCD allows us to calculate bulk thermodynamic quantities, χ_{top} with very high precision for a wide range of temp. with updated estimates on T_c.
- Beginning to explore finite μ_B region with new results on the curvature of chiral crossover line.
- Latest bounds on the critical end-point LQCD data suggest $\mu_B(\text{CEP})/T > 3$ in the region $T = 145 - 150$ MeV.
- Lattice methods now give more insights on the Columbia plot $\rightarrow$ ultimately allow us to understand the phase diagram for $N_f = 2 + 1$ QCD.
- Increased sophistication towards understanding the fate of $U_A(1)$ towards the chiral limit for QCD $\rightarrow$ ultimately will lead to our understanding of the deeper relation between anomalies and underlying topology in QCD.