Pion Form Factor

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LATTICE 18
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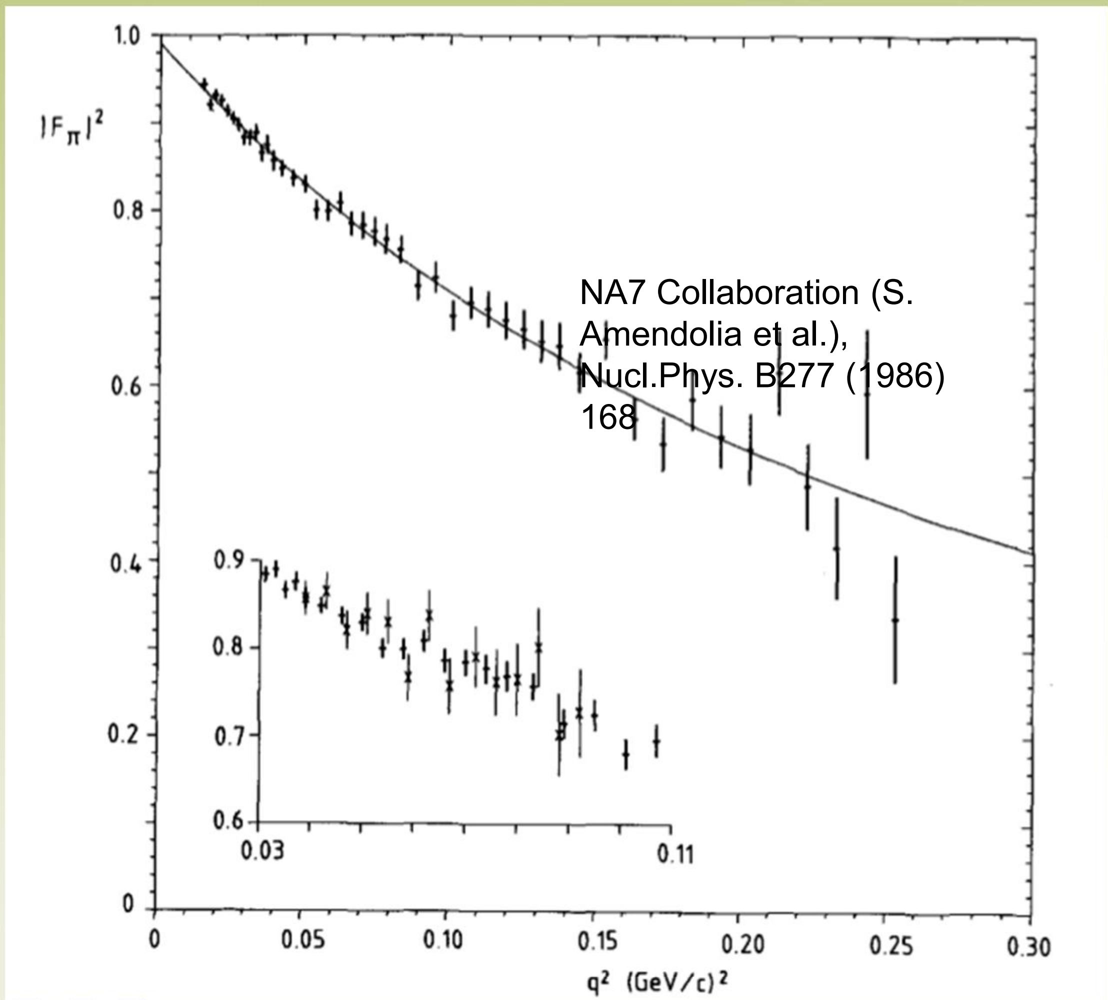
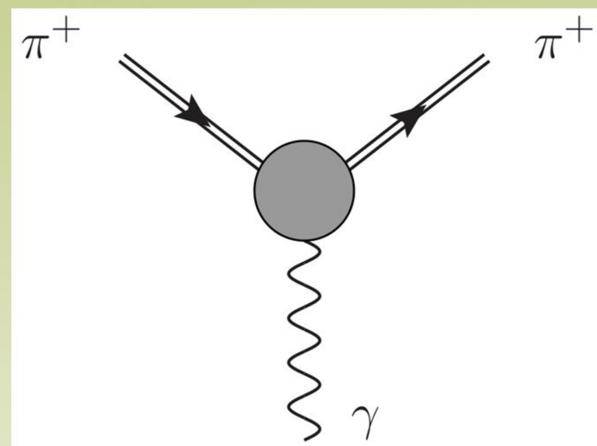
Motivation





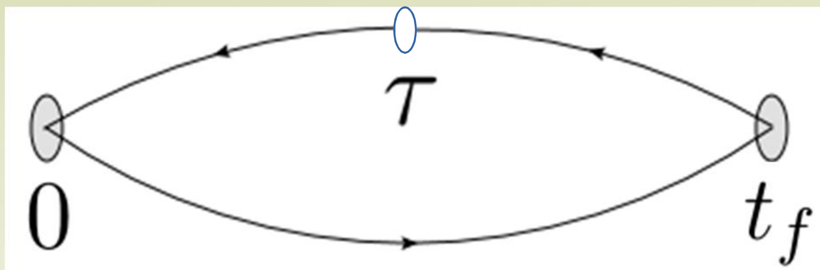
Mean Square Radius	Group
0.42(10)	ESCHRICH
0.439(8)	AMENDOLIA
0.440(30)	DALLY
0.452(11)	PDG

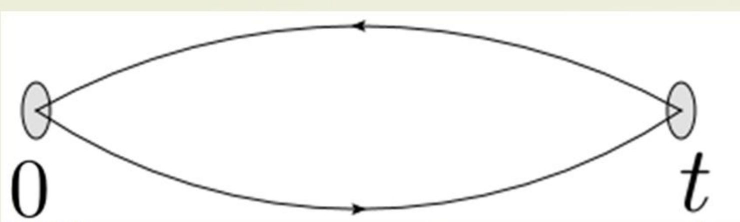
$$\langle \pi^+(\boldsymbol{p}_f)|\, V_\mu(x)\, |\pi^+(\boldsymbol{p}_i)\rangle = (p_f + p_i)_\mu f_{\pi\pi}(q^2)$$

Pion Form Factor

$$C_{3pt}(\tau, t_f, \boldsymbol{p}_i, \boldsymbol{p}_f) = \sum_{x_f, z} e^{-i\boldsymbol{p}_f\cdot(\boldsymbol{x}_f - \boldsymbol{z})} e^{i\boldsymbol{q}\cdot\boldsymbol{z}} \langle \mathrm{T}[\chi_{\pi^+}(x_f, t_f) V_\mu(z,\tau) \chi^\dagger_{\pi^+}(0,0)] \rangle$$
$$\approx \frac{m^2 Z_{p_i} Z_{p_f}}{E(p_i)E(p_f)} e^{-E(p_i)\tau - E(p_f)(t_f-\tau)} \langle \pi(p_f) | V_\mu(0) | \pi(p_i) \rangle$$
$$+ C_1 e^{-E^1(p_i)\tau - E(p_f)(t_f-\tau)} + C_2 e^{-E(p_i)\tau - E^1(p_f)(t_f-\tau)}$$

$$\boldsymbol{q} = \boldsymbol{p}_f - \boldsymbol{p}_i$$





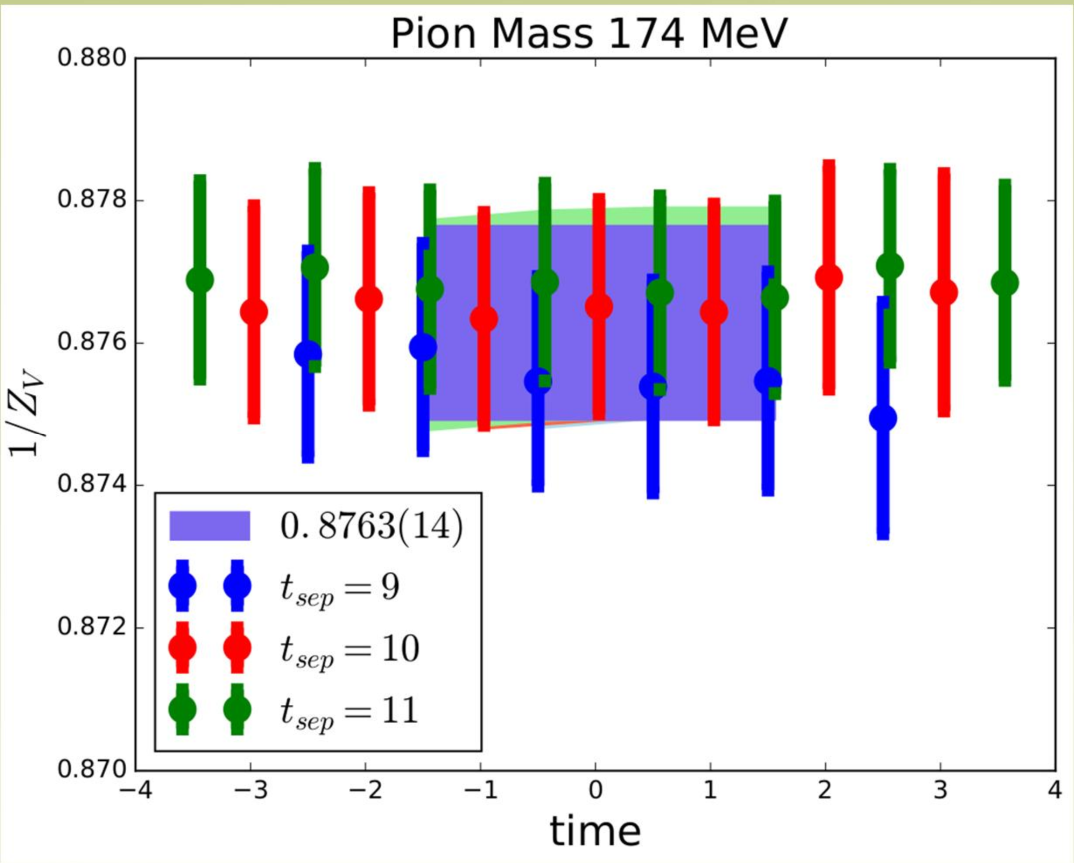
$$\chi_{\pi^+}(x_f, t_f) = \bar{u}(x,t)\gamma_5 d(x,t)$$
$$\chi^\dagger_{\pi^+}(x_f, t_f) = -\bar{d}(x,t)\gamma_5 u(x,t)$$
$$V_\mu(x) = \bar{u}(x)\gamma_\mu u(x)$$

$$C_{2pt}(t, p) = \sum_x e^{-i\boldsymbol{p}\cdot\boldsymbol{x}} \langle \mathrm{T}[\chi_{\pi^+}(x,t)\chi^\dagger_{\pi^+}(0,0)] \rangle$$
$$\approx \frac{m|Z_p|^2}{E(p)} (e^{-E(p)t} + e^{-E(p)(T-t)})$$
$$+ \frac{m|Z^1_p|^2}{E^1(p)} (e^{-E^1(p)t} + e^{-E^1(p)(T-t)})$$

Simulation Details

- Lattices
  - 24I--Domain Wall Lattice, $24^3 \times 64$, $a = 0.11$ fm, Pion 337 MeV
  - 32ID--Domain Wall Lattice, $32^3 \times 64$, $a = 0.143$ fm, Pion 171 MeV
  - Overlap Fermion with several valence quark masses
- Sources and Sinks
  - Grid Source at time 0 with momentum $p_i = q$
  - Stochastic Sinks with t_f with momentum $p_f = 0$
  - Local vector current at time τ
- Fitting Strategy
  - Joint fit of C_{2pt} and C_{3pt} with excited state

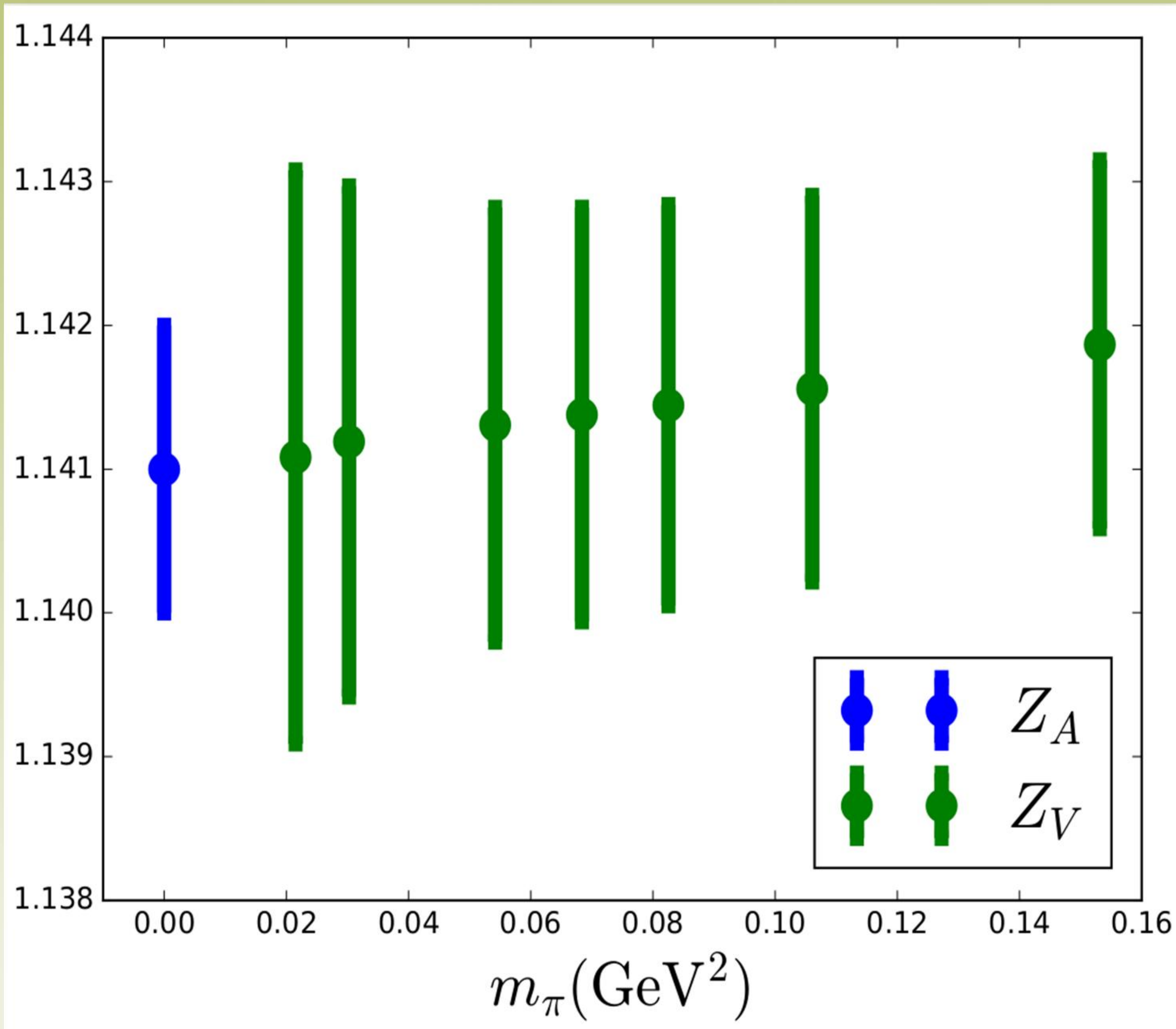
Zero Momentum Transfer



$$\frac{1}{Z_V} = \langle \pi(p_f) | \, V_\mu(0) \, | \pi(p_f) \rangle$$

By adding excited state to two point and three point, joint fit has been done with χ^2 to be around 0.6

Z_V and Z_A Compare



$$Z_A = \frac{2m_q\langle 0|P|\pi\rangle}{m_\pi\langle 0|A_4|\pi\rangle}$$

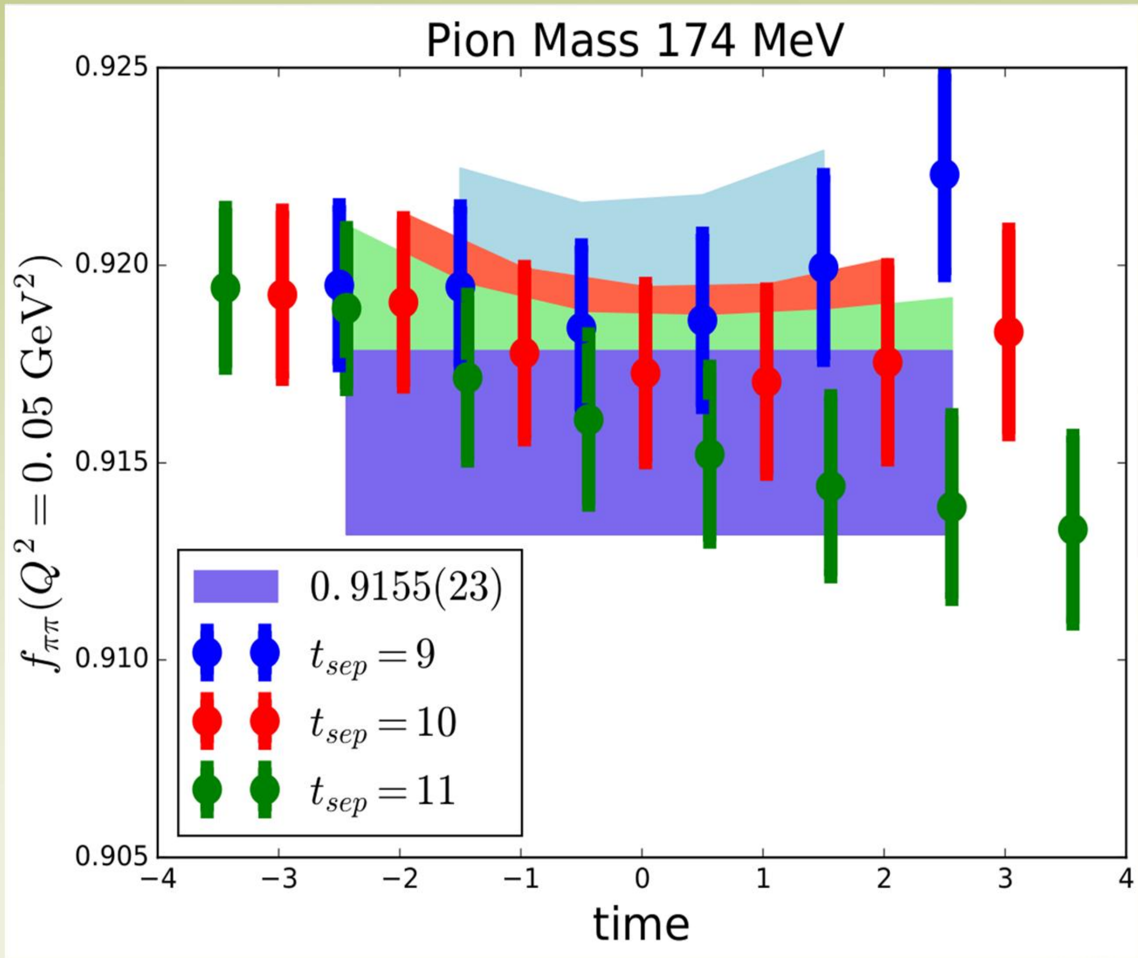
$$Z_V = \frac{1}{\langle \pi^+(0)|\, V^\mu(0)\, |\pi^+(0)\rangle}$$

Jian Liang, Yi-Bo Yang, Terrence Draper, Ming Gong, Keh-Fei Liu, arXiv:1806.08366

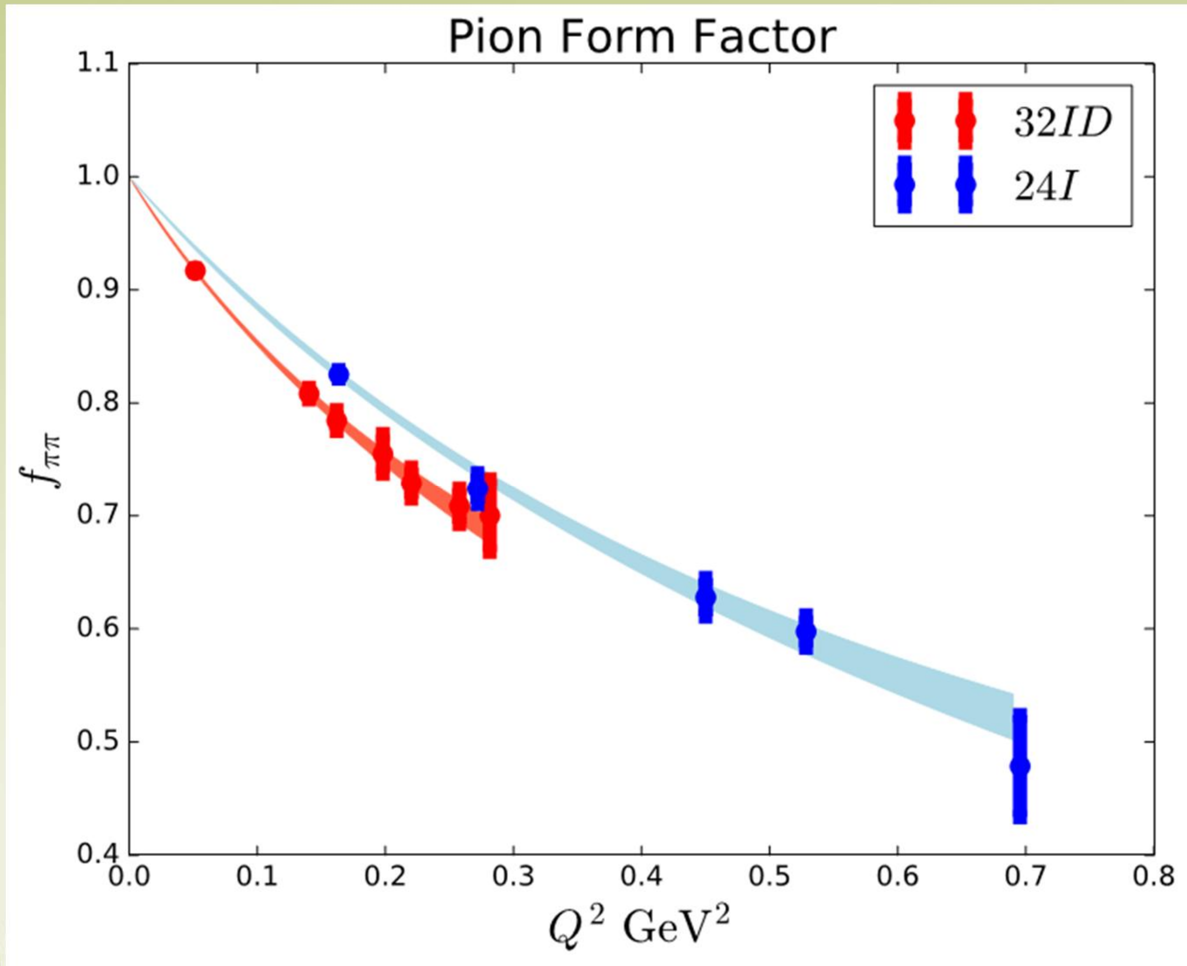
32ID $f_{\pi\pi}(Q^2 = 0.05\ GeV^2)$

$$f_{\pi\pi}(Q^2) = \frac{1}{E_i + E_f} \frac{\langle \pi(p_f) | V_\mu(0) | \pi(p_i) \rangle}{\langle \pi(p_f) | V_\mu(0) | \pi(p_f) \rangle}$$

With small momentum transfer, we get stable results at small pion mass



Z-expansion Fitting

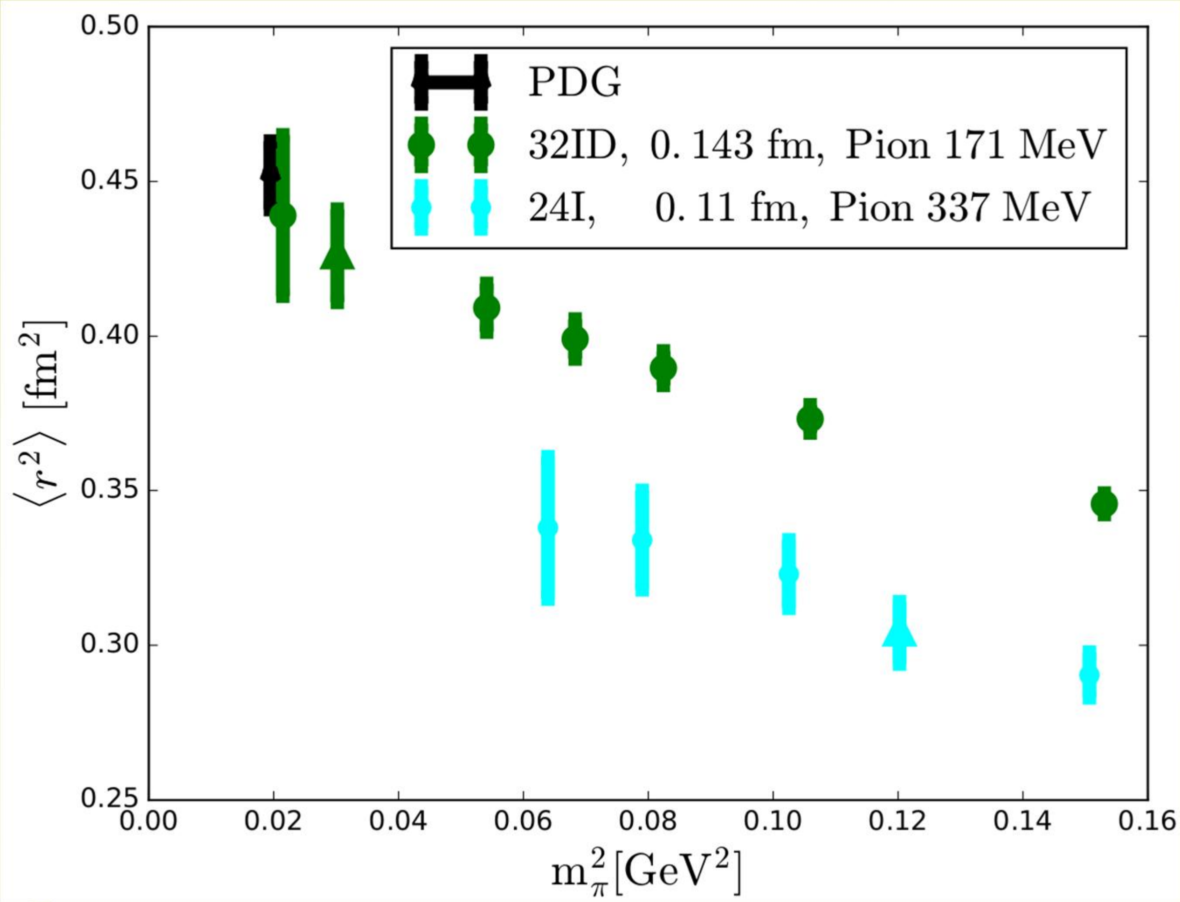


$$f_{\pi\pi}(Q^2) = 1 + \sum_{k=1}^{k_{max}} a_k z^k$$

$$z(t, t_{cut}, t_0) = \frac{\sqrt{t_{cut} - t_0} - \sqrt{t_c ut} - t_0}{\sqrt{t_{cut} - t_0} + \sqrt{t_{cut}} - t_0}$$

$$t = -Q^2, t_{cut} = 4\pi^2$$

Pion Radius from 24I and 32ID

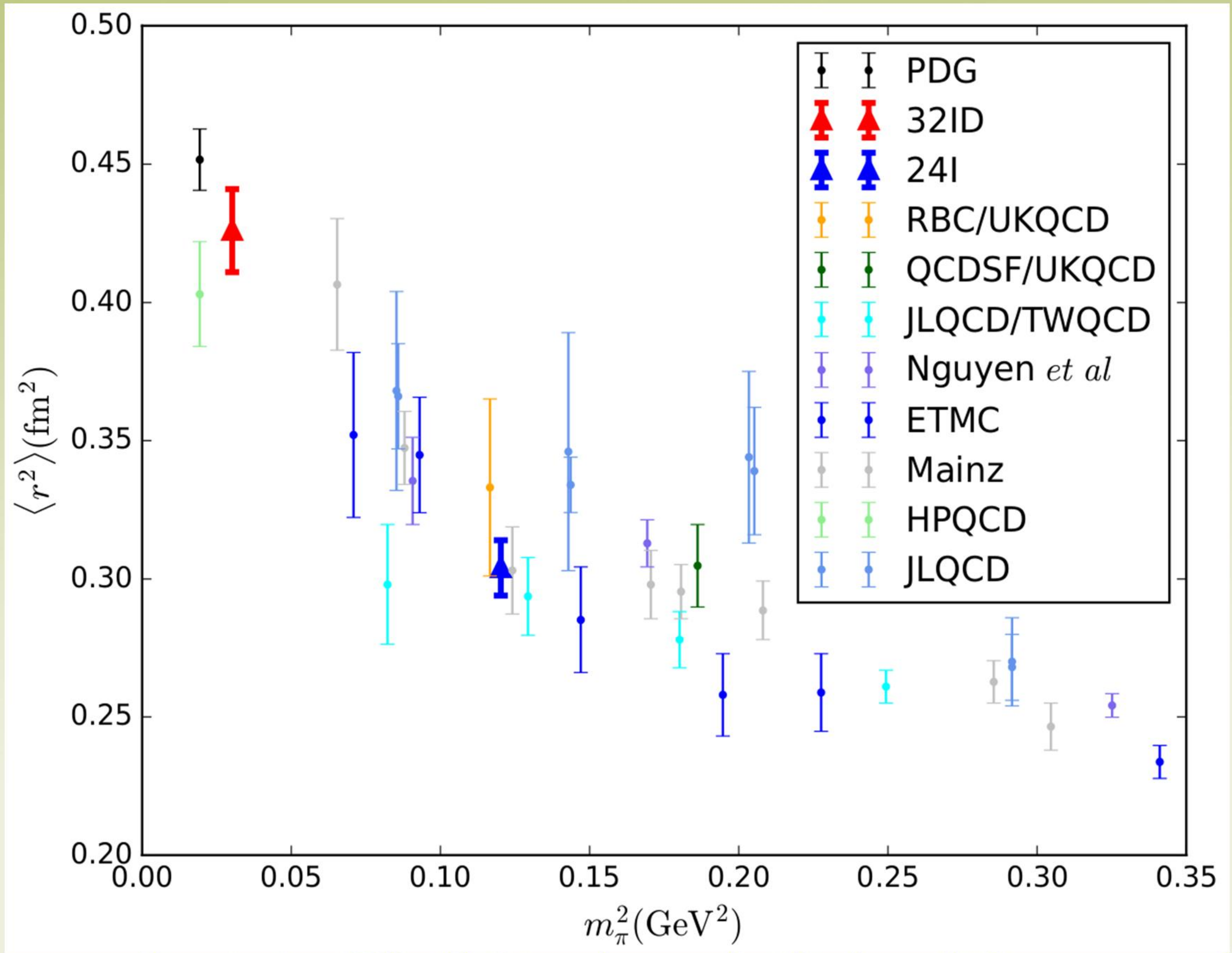


$$\langle r^2 \rangle = 6 \frac{d}{dQ^2} f_{\pi\pi}(Q^2)$$

Strong valence pion mass dependence observed here

Need more lattices to explain the difference between 24I and 32ID

Pion Radius



J. Koponen, et. al., Phys. Rev. D 93, 054503 (2016), arXiv:1511.07382
JLQCD, Phys. Rev. D 93, 034504 (2016), arXiv:1510.06470
Bastian B. Brandt, International Journal of Modern Physics E, Vol. 22 (2013) 1330030, arXiv: 1310.6389

Summary

- We have a preliminary result of pion form factor with sea pion mass 171 MeV and 337 MeV
- 32ID result agrees with experiment after extracted to physical pion mass point
- Production with different lattice spacing and sea pion masses are needed to control and understand varies systematics
- Momentum smearing source could be used to approach large momentum in further production