

# 36<sup>th</sup> Annual International Symposium on Lattice Field Theory

## RENORMALIZATION OF D≤6 CP-VIOLATING OPERATORS IN PERTURBATIVE QCD

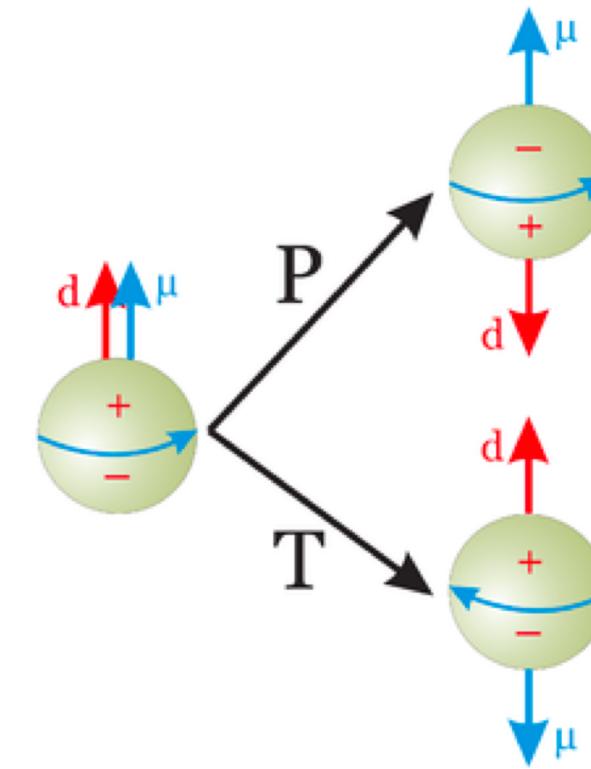
MATTHEW D. RIZIK IN COLLABORATION WITH CHRISTOPHER MONAHAN AND ANDREA SHINDLER

MSU • NSCL • FRIB



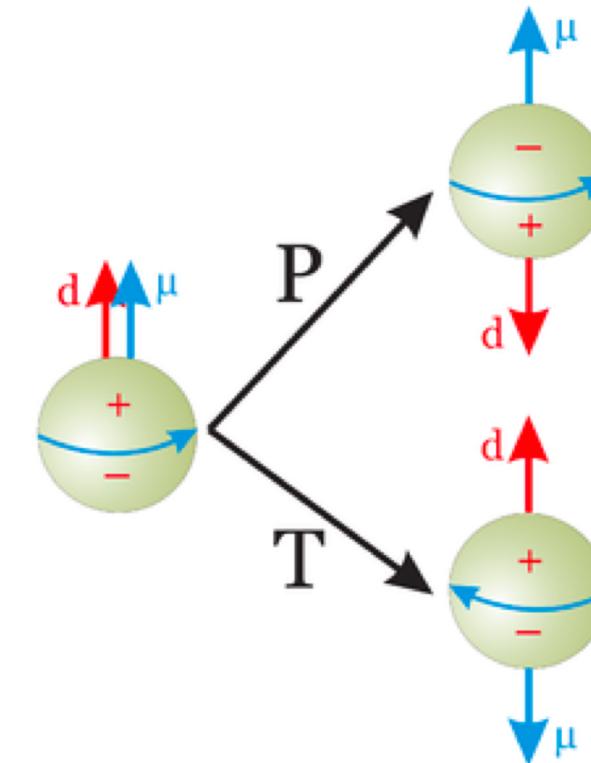
# MOTIVATION

- Search for large sources of CP-violation



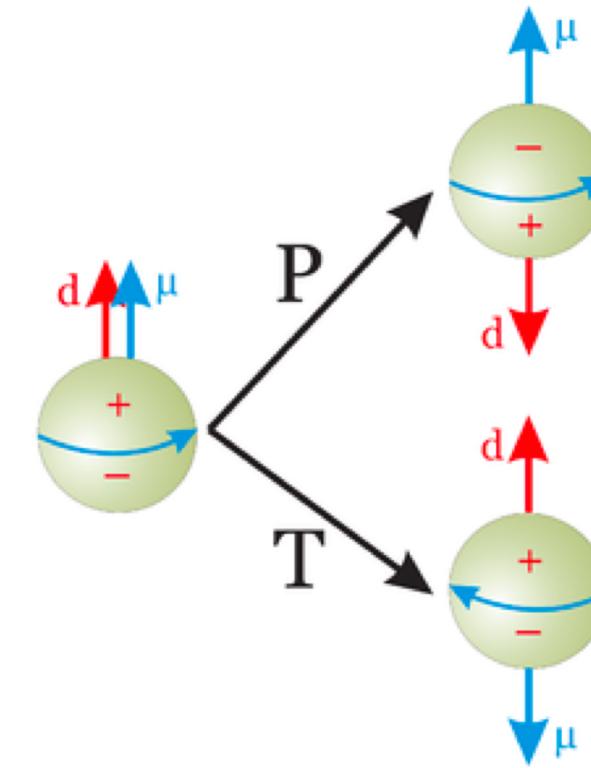
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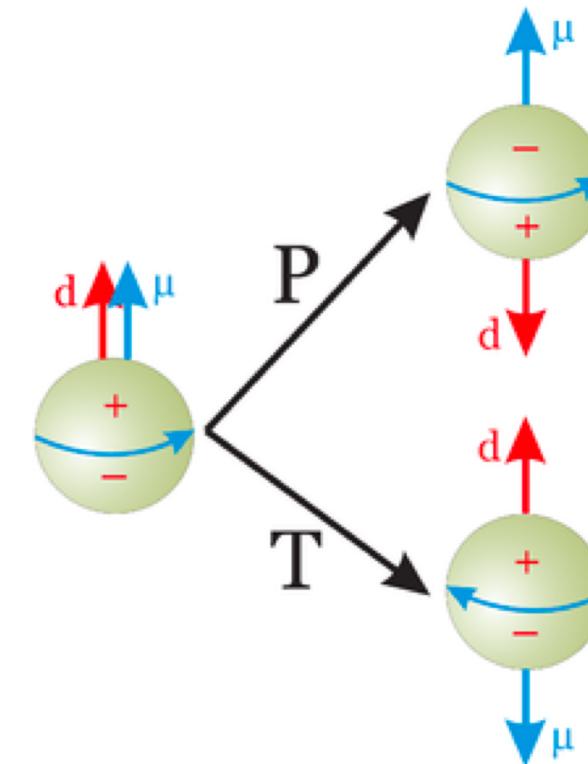
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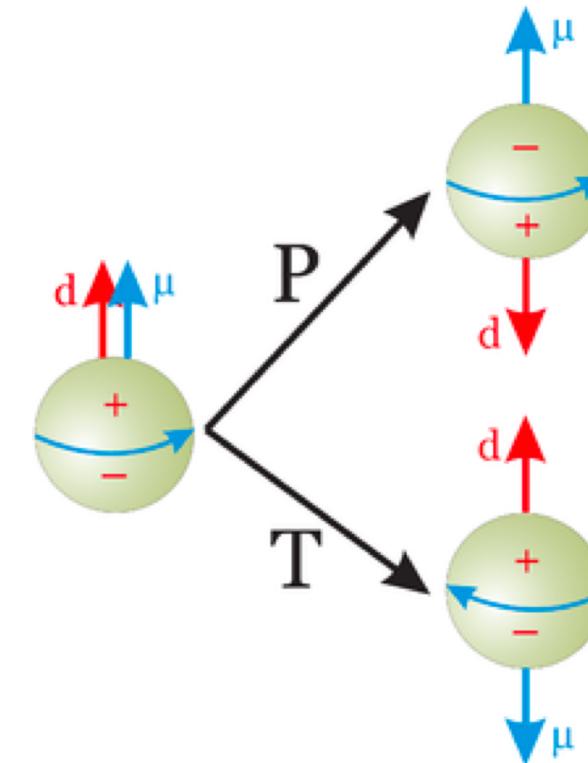
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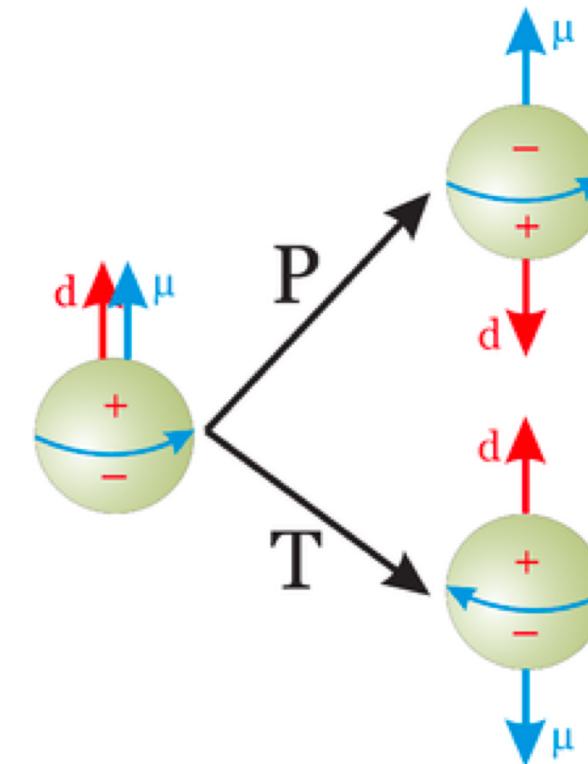
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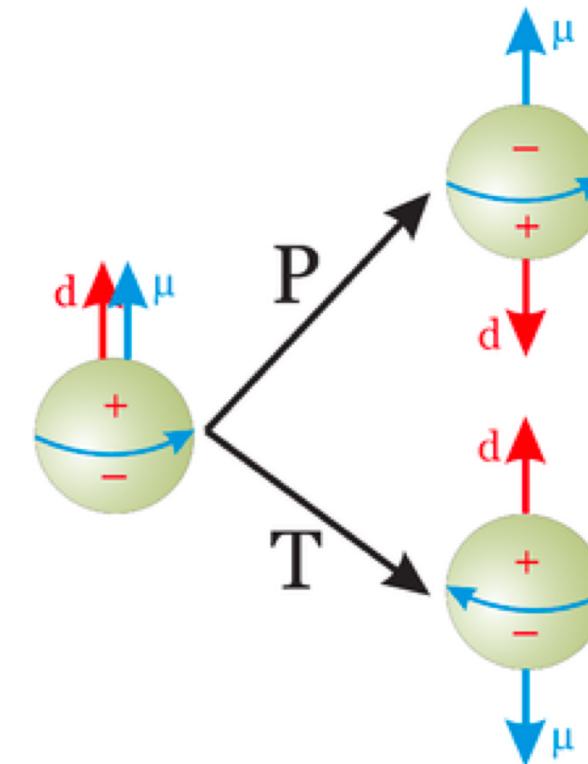
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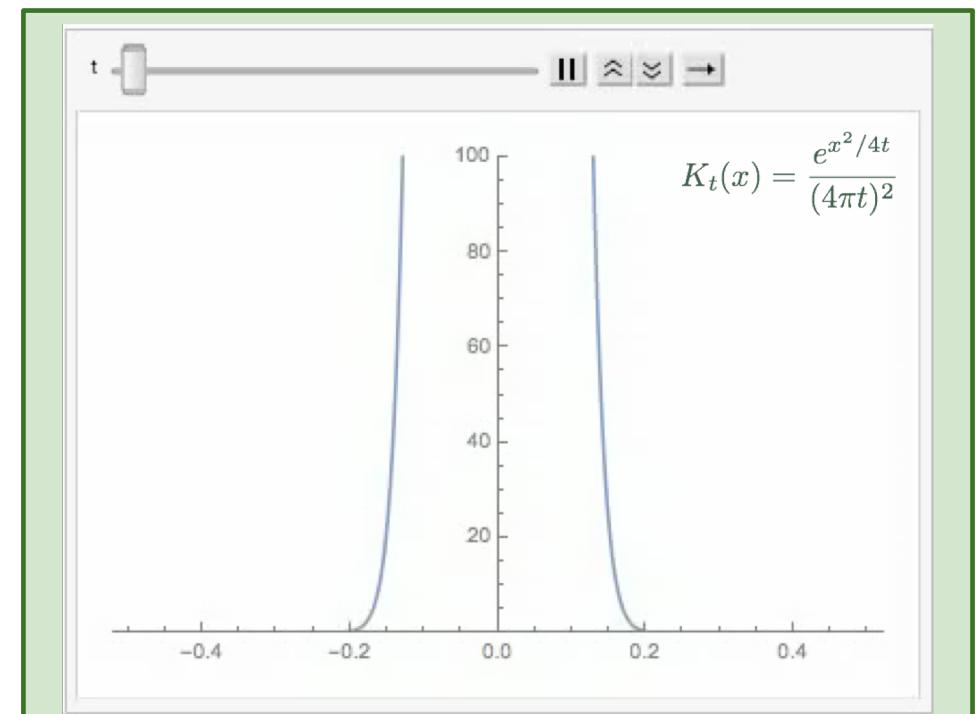
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- Gradient flow!



# YANG-MILLS GRADIENT FLOW: SYSTEMATICS

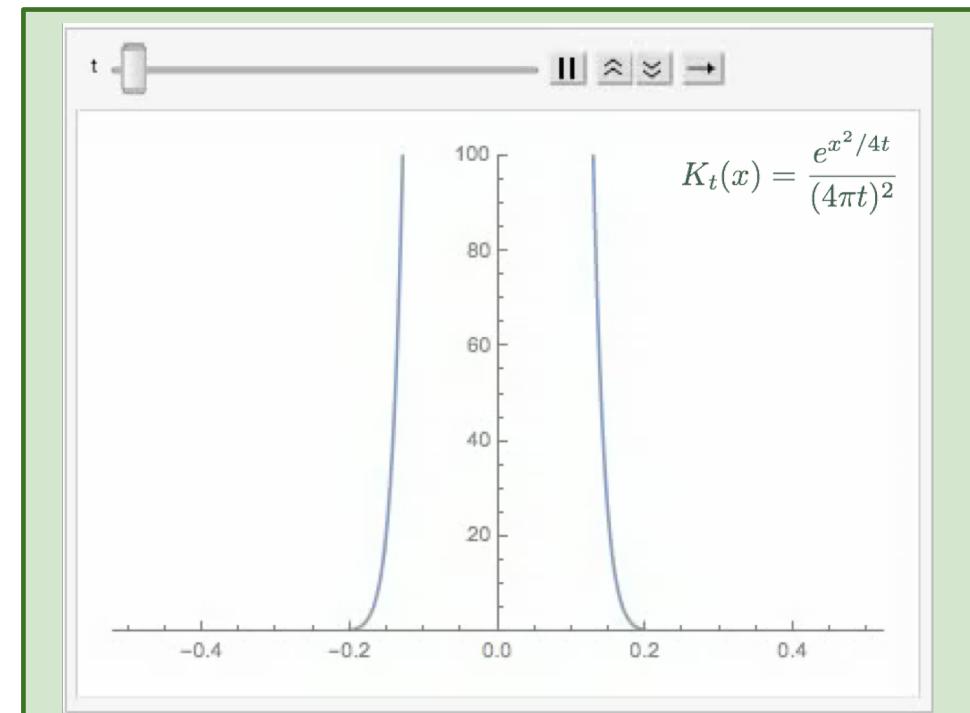
- $\partial_t B_\mu = D_\nu G_{\nu\mu} + \alpha_0 D_\mu \partial_\nu B_\nu; \quad B_\mu|_{t=0} = A_\mu$   
 $G_{\mu,\nu} = [D_\mu, D_\nu] \quad D_\mu = \partial_\mu + [B_\mu, \cdot] \quad [\text{Lüscher, Weisz, 2010}]$



Demonstration of the bulk field smearing as flow time increases

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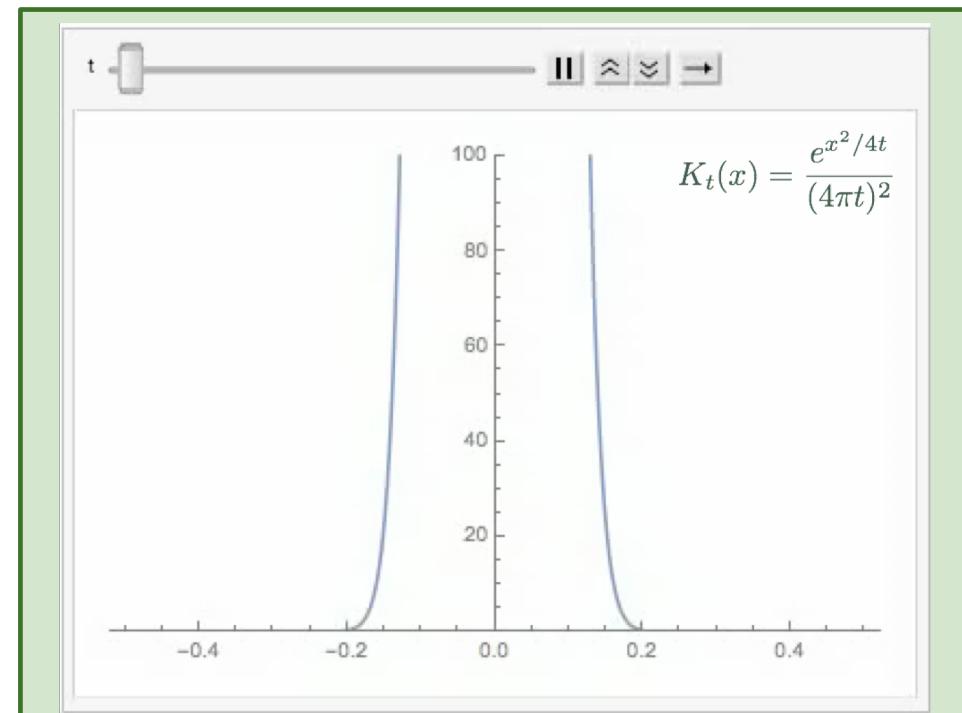


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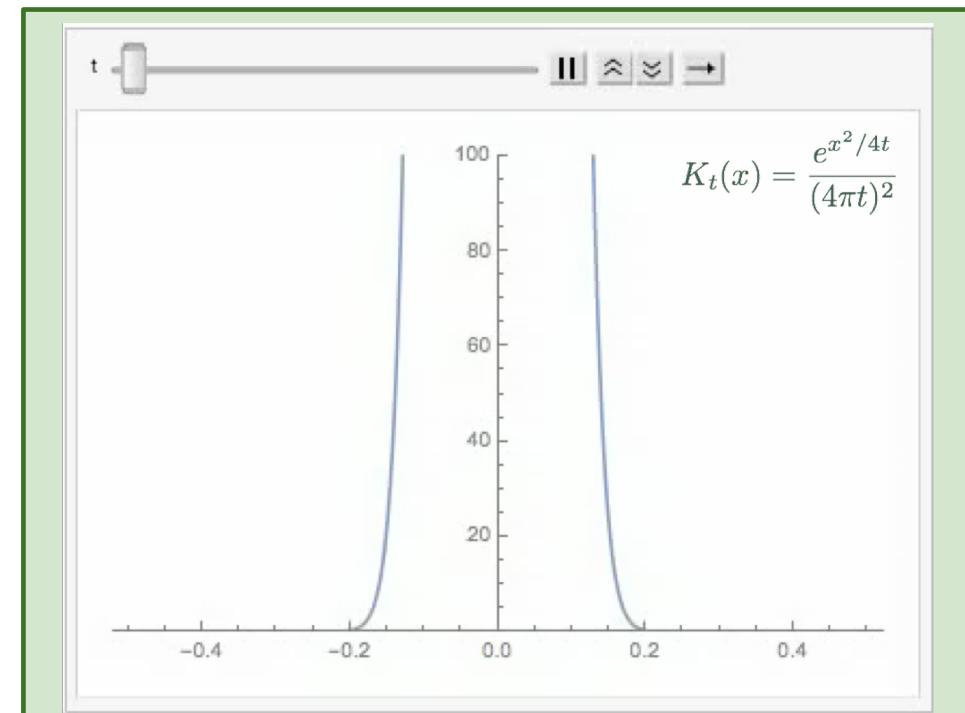
$$B_\mu^{(1)}(x; t) = \int d^4y K_t(x-y) A_\mu(y) \quad \frac{z}{\sqrt{(4\pi t)^3}} \sim \mathcal{N}(0, 2t)$$



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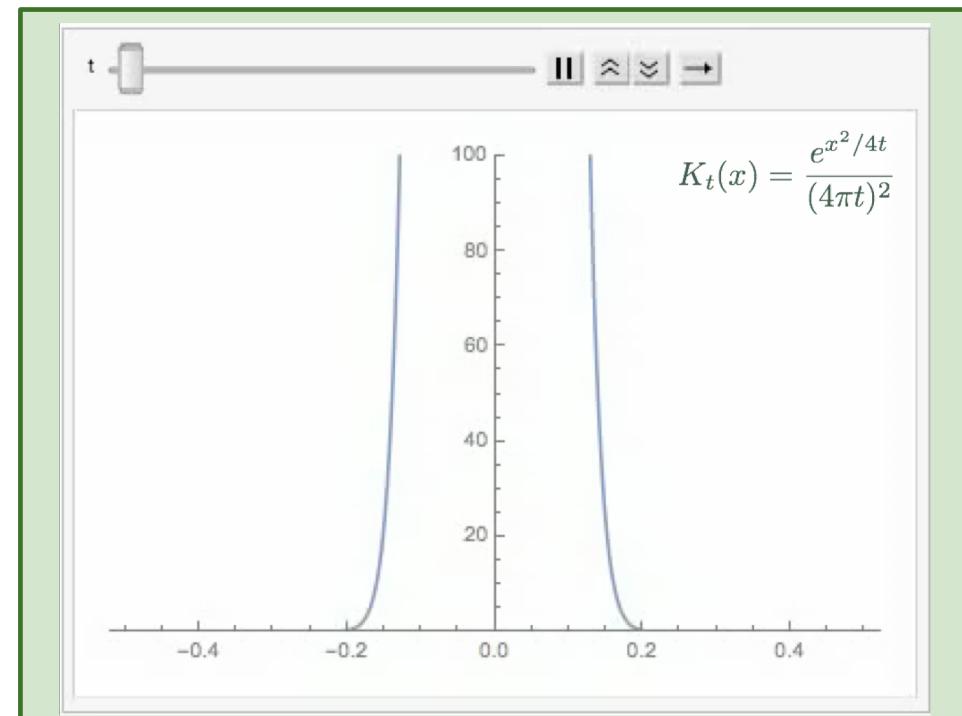
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- Lattice dynamics defined by

$$\partial_t U_t(x, \mu) = -g^2 (\partial_{x,\mu} S_W[U_t]) U_t(x, \mu)$$

can be integrated at arbitrary values of flow time, so that energy and regularization scales disentangle.



Demonstration of the bulk field smearing as flow time increases

# YANG-MILLS GRADIENT FLOW: SYSTEMATICS

$$\begin{aligned} B_\alpha^a(t, p) = & \tilde{K}_t(p)_{\alpha\mu} \tilde{A}_\mu^a(p) + \frac{1}{2} \int_0^t ds \tilde{K}_t(p)_{\alpha\mu_0} \int_{q_1, q_2} (2\pi)^D \delta^{(D)}(p + q_1 + q_2) X^{(2,0)}(p, q_1, q_2)_{\mu_0\mu_1\mu_2}^{ab_1b_2} \tilde{K}_s(q_1)_{\mu_1\nu_1} \tilde{K}_s(q_2)_{\mu_2\nu_2} \tilde{B}_{\nu_1}^{b_1}(q_1) \tilde{B}_{\nu_2}^{b_2}(q_2) \\ & + \frac{1}{6} \int_0^t ds \tilde{K}_t(p)_{\alpha\mu_0} \int_{q_1, q_2, q_3} (2\pi)^D \delta^{(D)}(p + q_1 + q_2 + q_3) X^{(3,0)}(p, q_1, q_2, q_3)_{\mu_0\mu_1\mu_2\mu_3}^{ab_1b_2b_3} \tilde{K}_s(q_1)_{\mu_1\nu_1} \tilde{K}_s(q_2)_{\mu_2\nu_2} \tilde{K}_s(q_3)_{\mu_3\nu_3} \tilde{B}_{\nu_1}^{b_1}(q_1) \tilde{B}_{\nu_2}^{b_2}(q_2) \tilde{B}_{\nu_3}^{b_3}(q_3) \end{aligned}$$

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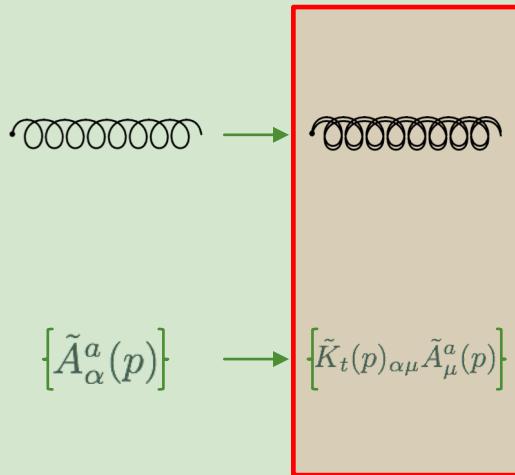
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$[\tilde{A}_\alpha^a(p)]$

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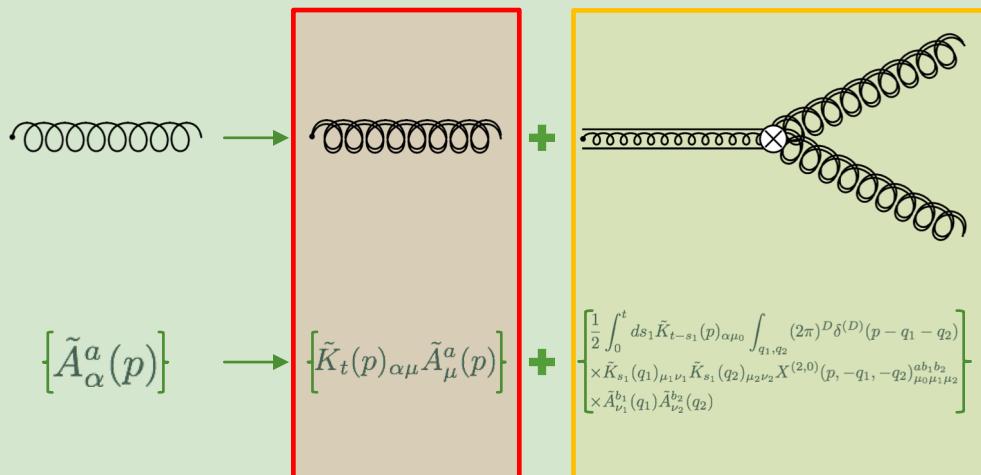
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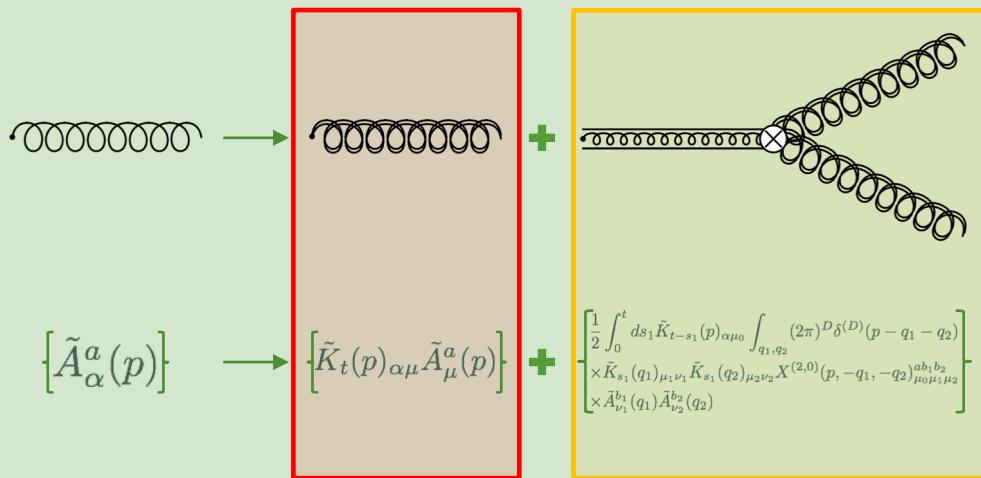
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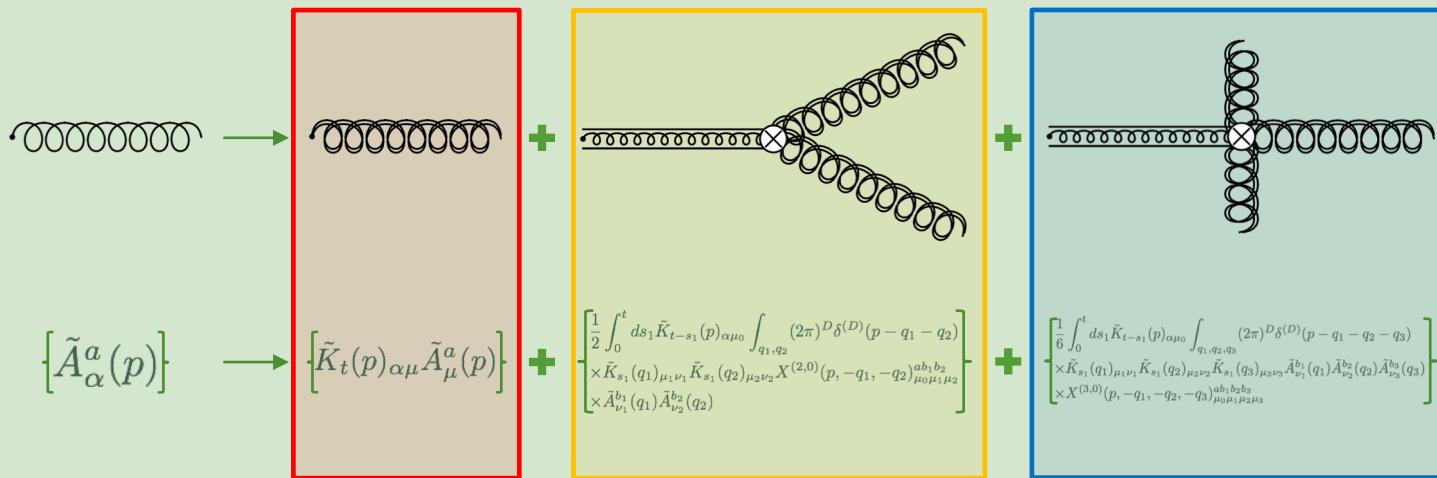
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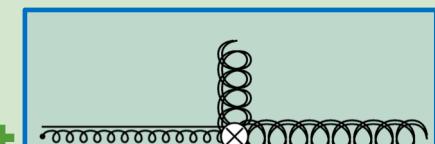
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$\left[ \tilde{A}_\alpha^a(p) \right] \rightarrow \left[ \tilde{K}_t(p)_{\alpha\mu} \tilde{A}_\mu^a(p) \right] + \left[ \frac{1}{2} \int_0^t ds_1 \tilde{K}_{t-s_1}(p)_{\alpha\mu_0} \int_{q_1, q_2} (2\pi)^D \delta^{(D)}(p - q_1 - q_2) \right. \right.$

$\times \tilde{K}_{s_1}(q_1)_{\mu_1\nu_1} \tilde{K}_{s_1}(q_2)_{\mu_2\nu_2} X^{(2,0)}(p, -q_1, -q_2)_{\mu_0\mu_1\mu_2}^{ab_1b_2}$

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\end{aligned}$$

Diagrammatic representation:

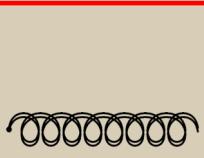
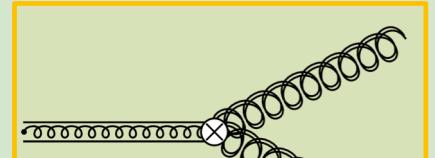
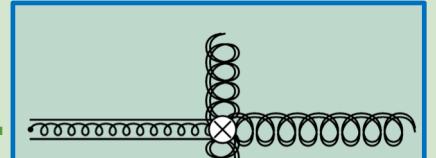
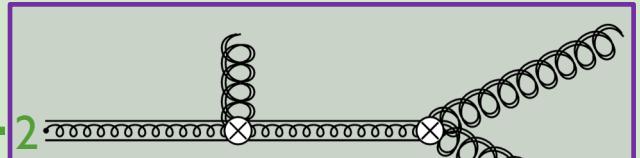
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Equations corresponding to the diagrams:

- $\tilde{A}_\alpha^a(p)$
- $\frac{1}{2} \int_0^t ds \tilde{K}_t(p)_{\alpha\mu_0} \int_{q_1, q_2} (2\pi)^D \delta^{(D)}(p - q_1 - q_2) X^{(2,0)}(p, -q_1, -q_2)_{\mu_0\mu_1\mu_2}^{ab_1b_2}$
- $\frac{1}{6} \int_0^t ds \tilde{K}_t(p)_{\alpha\mu_0} \int_{q_1, q_2, q_3} (2\pi)^D \delta^{(D)}(p - q_1 - q_2 - q_3) X^{(3,0)}(p, -q_1, -q_2, -q_3)_{\mu_0\mu_1\mu_2\mu_3}^{ab_1b_2b_3}$
- $\frac{1}{4} \int_0^t ds_1 \tilde{K}_{t-s_1}(p)_{\alpha\mu_0} \int_{q_1, q_2} (2\pi)^D \delta^{(D)}(p - q_1 - q_2 - q_3) \times \tilde{K}_{s_1}(q_1)_{\mu_1\nu_1} \tilde{K}_{s_1}(q_2)_{\mu_2\nu_2} X^{(2,0)}(p, -q_1, -q_2)_{\mu_0\mu_1\mu_2}^{ab_1b_2}$
- $+ 2 \left[ \frac{1}{6} \int_0^t ds_1 \tilde{K}_{t-s_1}(p)_{\alpha\mu_0} \int_{q_1, q_2, q_3} (2\pi)^D \delta^{(D)}(p - q_1 - q_2 - q_3) \times \tilde{K}_{s_1}(q_1)_{\mu_1\nu_1} \tilde{K}_{s_1}(q_2)_{\mu_2\nu_2} \tilde{K}_{s_1}(q_3)_{\mu_3\nu_3} \tilde{A}_{\nu_1}^{b_1}(q_1) \tilde{A}_{\nu_2}^{b_2}(q_2) \tilde{A}_{\nu_3}^{b_3}(q_3) \times X^{(3,0)}(p, -q_1, -q_2, -q_3)_{\mu_0\mu_1\mu_2\mu_3}^{ab_1b_2b_3} \right]$
- $+ 2 \left[ \frac{1}{4} \int_0^t ds_1 \tilde{K}_{t-s_1}(p)_{\alpha\mu_0} \int_{q_1, q_2} (2\pi)^D \delta^{(D)}(p - q_1 - q_2) \times \int_0^{s_1} ds_2 \tilde{K}_{s_1-s_2}(q_2)_{\mu_2\nu_2} \int_{r_1, r_2} (2\pi)^D \delta^{(D)}(q_2 - r_1 - r_2) \times X^{(2,0)}(p, -q_1, -q_2)_{\mu_0\mu_1\mu_2}^{ab_1b_2} X^{(2,0)}(q_2, -r_1, -r_2)_{\rho_0\rho_1\rho_2}^{b_2c_1c_2} \times \tilde{K}_{s_1}(q_1)_{\mu_1\nu_1} \tilde{K}_{s_2}(r_1)_{\rho_1\sigma_1} \tilde{K}_{s_2}(r_2)_{\rho_2\sigma_2} \tilde{A}_{\nu_1}^{b_1}(q_1) \tilde{A}_{\rho_1}^{c_1}(r_1) \tilde{A}_{\rho_2}^{c_2}(r_2) \right]$

# YANG-MILLS GRADIENT FLOW: SYSTEMATICS

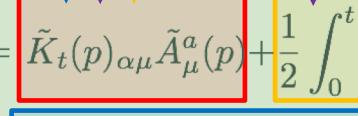
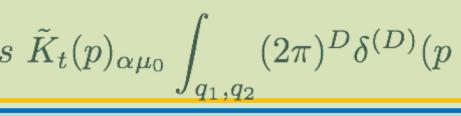
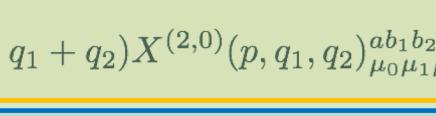
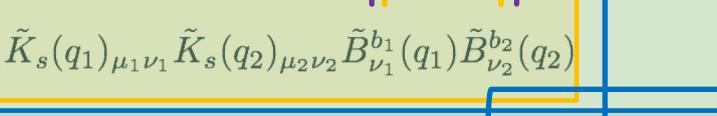
$$\begin{aligned}
B_\alpha^a(t, p) = & \tilde{K}_t(p)_{\alpha\mu} \tilde{A}_\mu^a(p) + \frac{1}{2} \int_0^t ds \tilde{K}_t(p)_{\alpha\mu_0} \int_{q_1, q_2} (2\pi)^D \delta^{(D)}(p + q_1 + q_2) X^{(2,0)}(p, q_1, q_2)_{\mu_0\mu_1\mu_2}^{ab_1b_2} \tilde{K}_s(q_1)_{\mu_1\nu_1} \tilde{K}_s(q_2)_{\mu_2\nu_2} \tilde{B}_{\nu_1}^{b_1}(q_1) \tilde{B}_{\nu_2}^{b_2}(q_2) \\
& + \frac{1}{6} \int_0^t ds \tilde{K}_t(p)_{\alpha\mu_0} \int_{q_1, q_2, q_3} (2\pi)^D \delta^{(D)}(p + q_1 + q_2 + q_3) X^{(3,0)}(p, q_1, q_2, q_3)_{\mu_0\mu_1\mu_2\mu_3}^{ab_1b_2b_3} \tilde{K}_s(q_1)_{\mu_1\nu_1} \tilde{K}_s(q_2)_{\mu_2\nu_2} \tilde{K}_s(q_3)_{\mu_3\nu_3} \tilde{B}_{\nu_1}^{b_1}(q_1) \tilde{B}_{\nu_2}^{b_2}(q_2) \tilde{B}_{\nu_3}^{b_3}(q_3)
\end{aligned}$$

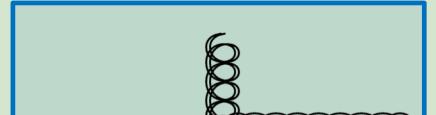
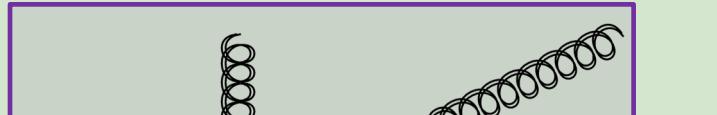
$\left[ \tilde{A}_\alpha^a(p) \right] \rightarrow \left[ \tilde{K}_t(p)_{\alpha\mu} \tilde{A}_\mu^a(p) \right] + \left[ \frac{1}{2} \int_0^t ds_1 \tilde{K}_{t-s_1}(p)_{\alpha\mu_0} \int_{q_1, q_2} (2\pi)^D \delta^{(D)}(p - q_1 - q_2) \times \tilde{K}_{s_1}(q_1)_{\mu_1\nu_1} \tilde{K}_{s_1}(q_2)_{\mu_2\nu_2} X^{(2,0)}(p, -q_1, -q_2)_{\mu_0\mu_1\mu_2}^{ab_1b_2} \times \tilde{A}_{\nu_1}^{b_1}(q_1) \tilde{A}_{\nu_2}^{b_2}(q_2) \right]$ 
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 $+ \left[ \frac{1}{4} \int_0^t ds_1 \tilde{K}_{t-s_1}(p)_{\alpha\mu_0} \int_{q_1, q_2} (2\pi)^D \delta^{(D)}(p - q_1 - q_2) \times \int_0^{s_1} ds_2 \tilde{K}_{s_1-s_2}(q_2)_{\mu_2\rho_2} \int_{r_1, r_2} (2\pi)^D \delta^{(D)}(q_2 - r_1 - r_2) \times X^{(2,0)}(p, -q_1, -q_2)_{\mu_0\mu_1\mu_2}^{ab_1b_2} X^{(2,0)}(q_2, -r_1, -r_2)_{\rho_0\rho_1\rho_2}^{b_2c_1c_2} \times \tilde{K}_{s_1}(q_1)_{\mu_1\nu_1} \tilde{K}_{s_2}(r_1)_{\rho_1\sigma_1} \tilde{K}_{s_2}(r_2)_{\rho_2\sigma_2} \tilde{A}_{\nu_1}^{b_1}(q_1) \tilde{A}_{\rho_1}^{c_1}(r_1) \tilde{A}_{\rho_2}^{c_2}(r_2) \right]$ 
 $+ 2 \left[ \frac{1}{4} \int_0^t ds_1 \tilde{K}_{t-s_1}(p)_{\alpha\mu_0} \int_{q_1, q_2} (2\pi)^D \delta^{(D)}(p - q_1 - q_2) \times \int_0^{s_1} ds_2 \tilde{K}_{s_1-s_2}(q_2)_{\mu_2\rho_2} \int_{r_1, r_2} (2\pi)^D \delta^{(D)}(q_2 - r_1 - r_2) \times X^{(2,0)}(p, -q_1, -q_2)_{\mu_0\mu_1\mu_2}^{ab_1b_2} X^{(2,0)}(q_2, -r_1, -r_2)_{\rho_0\rho_1\rho_2}^{b_2c_1c_2} \times \tilde{K}_{s_1}(q_1)_{\mu_1\nu_1} \tilde{K}_{s_2}(r_1)_{\rho_1\sigma_1} \tilde{K}_{s_2}(r_2)_{\rho_2\sigma_2} \tilde{A}_{\nu_1}^{b_1}(q_1) \tilde{A}_{\rho_1}^{c_1}(r_1) \tilde{A}_{\rho_2}^{c_2}(r_2) \right]$

# YANG-MILLS GRADIENT FLOW: SYSTEMATICS

$$\begin{aligned}
B_\alpha^a(t, p) = & \tilde{K}_t(p)_{\alpha\mu} \tilde{A}_\mu^a(p) + \frac{1}{2} \int_0^t ds \tilde{K}_t(p)_{\alpha\mu_0} \int_{q_1, q_2} (2\pi)^D \delta^{(D)}(p + q_1 + q_2) X^{(2,0)}(p, q_1, q_2)_{\mu_0\mu_1\mu_2}^{ab_1b_2} \tilde{K}_s(q_1)_{\mu_1\nu_1} \tilde{K}_s(q_2)_{\mu_2\nu_2} \tilde{B}_{\nu_1}^{b_1}(q_1) \tilde{B}_{\nu_2}^{b_2}(q_2) \\
& + \frac{1}{6} \int_0^t ds \tilde{K}_t(p)_{\alpha\mu_0} \int_{q_1, q_2, q_3} (2\pi)^D \delta^{(D)}(p + q_1 + q_2 + q_3) X^{(3,0)}(p, q_1, q_2, q_3)_{\mu_0\mu_1\mu_2\mu_3}^{ab_1b_2b_3} \tilde{K}_s(q_1)_{\mu_1\nu_1} \tilde{K}_s(q_2)_{\mu_2\nu_2} \tilde{K}_s(q_3)_{\mu_3\nu_3} \tilde{B}_{\nu_1}^{b_1}(q_1) \tilde{B}_{\nu_2}^{b_2}(q_2) \tilde{B}_{\nu_3}^{b_3}(q_3)
\end{aligned}$$



$\left[ \tilde{A}_\alpha^a(p) \right] \rightarrow \left[ \tilde{K}_t(p)_{\alpha\mu} \tilde{A}_\mu^a(p) \right] + \left[ \frac{1}{2} \int_0^t ds_1 \tilde{K}_{t-s_1}(p)_{\alpha\mu_0} \int_{q_1, q_2} (2\pi)^D \delta^{(D)}(p - q_1 - q_2) \right. \right.$ 

$$\left. \times \tilde{K}_{s_1}(q_1)_{\mu_1\nu_1} \tilde{K}_{s_1}(q_2)_{\mu_2\nu_2} X^{(2,0)}(p, -q_1, -q_2)_{\mu_0\mu_1\mu_2}^{ab_1b_2} \right. \\
\left. \times \tilde{A}_{\nu_1}^{b_1}(q_1) \tilde{A}_{\nu_2}^{b_2}(q_2) \right] + \left[ \frac{1}{6} \int_0^t ds_1 \tilde{K}_{t-s_1}(p)_{\alpha\mu_0} \int_{q_1, q_2, q_3} (2\pi)^D \delta^{(D)}(p - q_1 - q_2 - q_3) \right. \\
\left. \times \tilde{K}_{s_1}(q_1)_{\mu_1\nu_1} \tilde{K}_{s_1}(q_2)_{\mu_2\nu_2} \tilde{K}_{s_1}(q_3)_{\mu_3\nu_3} \tilde{A}_{\nu_1}^{b_1}(q_1) \tilde{A}_{\nu_2}^{b_2}(q_2) \tilde{A}_{\nu_3}^{b_3}(q_3) \right. \\
\left. \times X^{(3,0)}(p, -q_1, -q_2, -q_3)_{\mu_0\mu_1\mu_2\mu_3}^{ab_1b_2b_3} \right] + 2 \left[ \frac{1}{4} \int_0^t ds_1 \tilde{K}_{t-s_1}(p)_{\alpha\mu_0} \int_{q_1, q_2} (2\pi)^D \delta^{(D)}(p - q_1 - q_2) \right. \\
\left. \times \int_0^{s_1} ds_2 \tilde{K}_{s_1-s_2}(q_2)_{\mu_2\rho_2} \int_{r_1, r_2} (2\pi)^D \delta^{(D)}(q_2 - r_1 - r_2) \right. \\
\left. \times X^{(2,0)}(p, -q_1, -q_2)_{\mu_0\mu_1\mu_2}^{ab_1b_2} X^{(2,0)}(q_2, -r_1, -r_2)_{\rho_0\rho_1\rho_2}^{b_2c_1c_2} \right. \\
\left. \times \tilde{K}_{s_1}(q_1)_{\mu_1\nu_1} \tilde{K}_{s_2}(r_1)_{\rho_1\sigma_1} \tilde{K}_{s_2}(r_2)_{\rho_2\sigma_2} \tilde{A}_{\nu_1}^{b_1}(q_1) \tilde{A}_{\rho_1}^{c_1}(r_1) \tilde{A}_{\rho_2}^{c_2}(r_2) \right]$$

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B_\alpha^a(t, p) = & \tilde{K}_t(p)_{\alpha\mu} \tilde{A}_\mu^a(p) + \frac{1}{2} \int_0^t ds \tilde{K}_t(p)_{\alpha\mu_0} \int_{q_1, q_2} (2\pi)^D \delta^{(D)}(p + q_1 + q_2) X^{(2,0)}(p, q_1, q_2)_{\mu_0\mu_1\mu_2}^{ab_1b_2} \tilde{K}_s(q_1)_{\mu_1\nu_1} \tilde{K}_s(q_2)_{\mu_2\nu_2} \tilde{B}_{\nu_1}^{b_1}(q_1) \tilde{B}_{\nu_2}^{b_2}(q_2) \\
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\end{aligned}$$

Diagrammatic representation:

- $\tilde{A}_\alpha^a(p)$  →
- $\tilde{A}_\alpha^a(p)$  →  $\tilde{K}_t(p)_{\alpha\mu} \tilde{A}_\mu^a(p)$  →
- +  $\left[ \frac{1}{2} \int_0^t ds_1 \tilde{K}_{t-s_1}(p)_{\alpha\mu_0} \int_{q_1, q_2} (2\pi)^D \delta^{(D)}(p - q_1 - q_2) \right] \times \tilde{K}_{s_1}(q_1)_{\mu_1\nu_1} \tilde{K}_{s_1}(q_2)_{\mu_2\nu_2} X^{(2,0)}(p, -q_1, -q_2)_{\mu_0\mu_1\mu_2}^{ab_1b_2}$
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- +  $2 \left[ \frac{1}{4} \int_0^t ds_1 \tilde{K}_{t-s_1}(p)_{\alpha\mu_0} \int_{q_1, q_2} (2\pi)^D \delta^{(D)}(p - q_1 - q_2) \right] \times \int_0^{s_1} ds_2 \tilde{K}_{s_1-s_2}(q_2)_{\mu_2\nu_2} \int_{r_1, r_2} (2\pi)^D \delta^{(D)}(q_2 - r_1 - r_2) \times X^{(2,0)}(p, -q_1, -q_2)_{\mu_0\mu_1\mu_2}^{ab_1b_2} X^{(2,0)}(q_2, -r_1, -r_2)_{\rho_0\rho_1\rho_2}^{b_2c_1c_2}$
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$\mathcal{O}(g_0^3)$

# WEINBERG OPERATOR

- $$W = \frac{1}{3} d_W g_s f^{abc} G_{\mu\rho}^a G_{\nu\rho}^b \tilde{G}_{\mu\nu}^c$$

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- At next-to-leading order in coupling:

$$W^{(0)}(t) + W^{(1)}(t) = \sum_{i=Q,q,W} \left[ c_i^{(0)}(t) + c_i^{(1)}(t) \right] \left[ O_i^{(0)}(0) + O_i^{(1)}(0) \right] + \mathcal{O}(t)$$

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  - ⇒ Extract coefficients to desired order with variable external states.

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- Expansion is probe-independent
  - ⇒ Extract coefficients to desired order with variable external states.
- Example:

*Extract the NLO coefficient for the topological charge density*

# WEINBERG OPERATOR

$$W(t) = c_Q(t)Q(0) + c_q(t)q(0) + c_W(t)W(0) + \dots$$

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## WEINBERG OPERATOR

$$W(t) = c_Q(t)Q(0) + c_q(t)q(0) + c_W(t)W(0) + \dots$$

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$$\begin{aligned} W^{(0)}(t) &= C_Q^{(0)}(t)Q^{(0)}(0) + C_q^{(0)}(t)q^{(0)}(0) \\ &\quad + C_W^{(0)}(t)W^{(0)}(0) \end{aligned}$$

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$$\begin{aligned} \langle 0 | W^{(1)}(t) | AAA \rangle &= \langle 0 | c_Q^{(0)}(t)Q^{(1)}(0) | AAA \rangle + \langle 0 | c_Q^{(1)}(t)Q^{(0)}(0) | AAA \rangle \\ &\quad + \langle 0 | c_q^{(0)}(t)q^{(1)}(0) | AAA \rangle + \langle 0 | c_W^{(0)}(t)W^{(1)}(0) | AAA \rangle \end{aligned}$$

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- $\mathcal{O}(g_0^0)$  part  $\Rightarrow c_Q^{(0)}(t) = 0$

# WEINBERG OPERATOR

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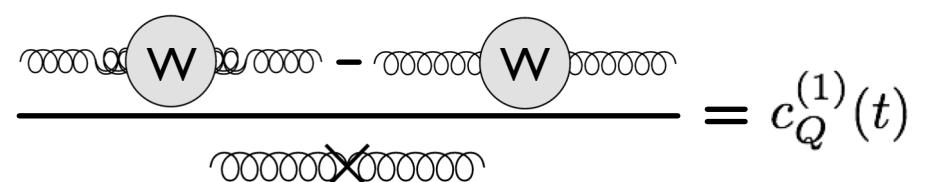
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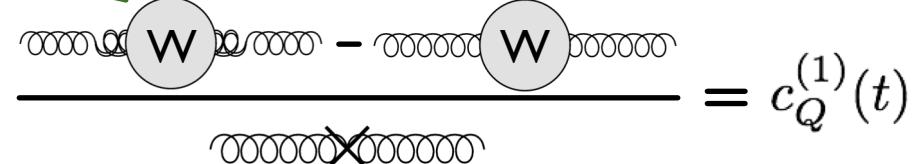
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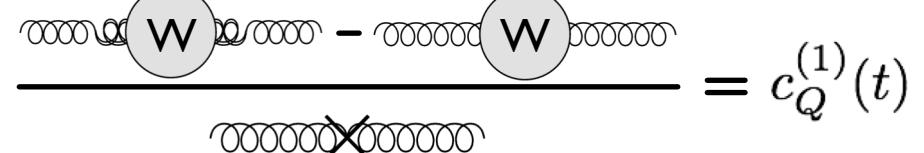
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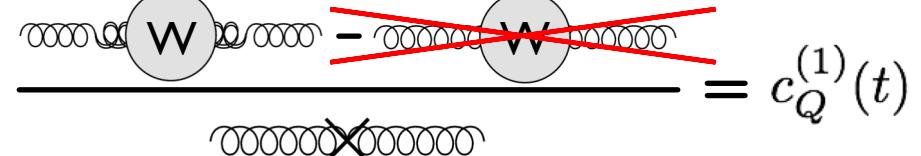
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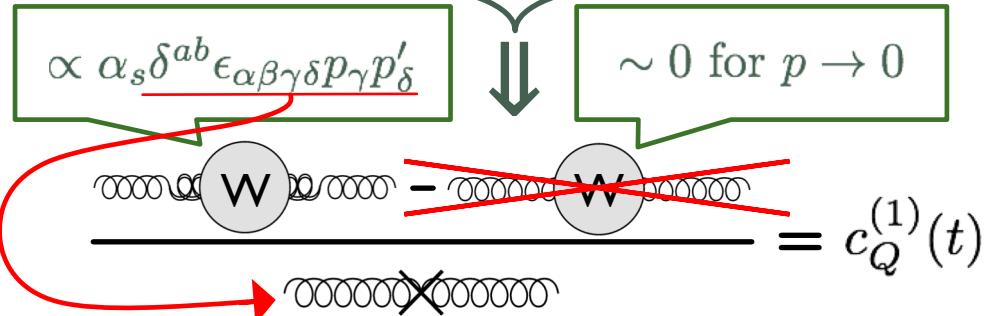
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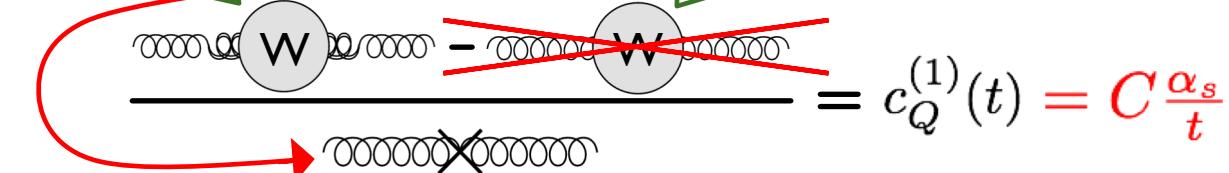
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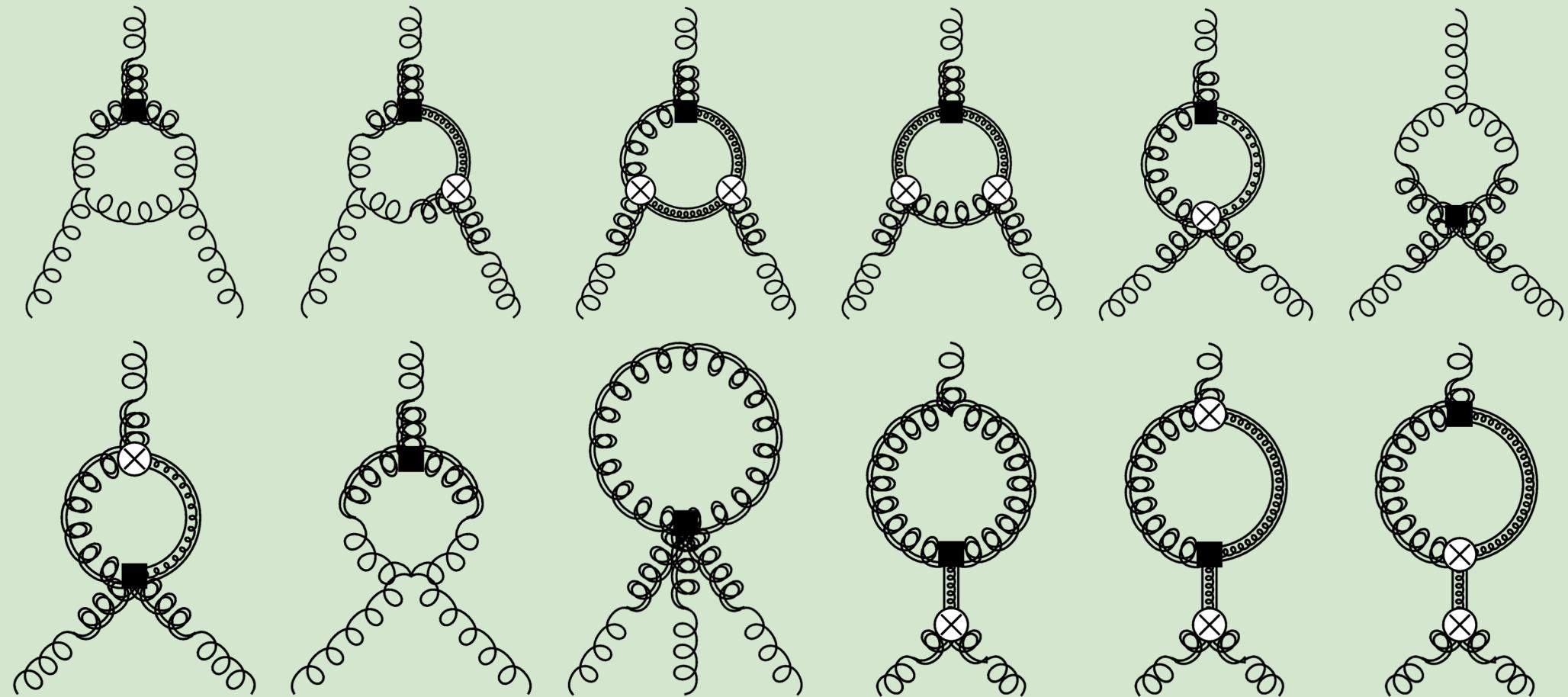
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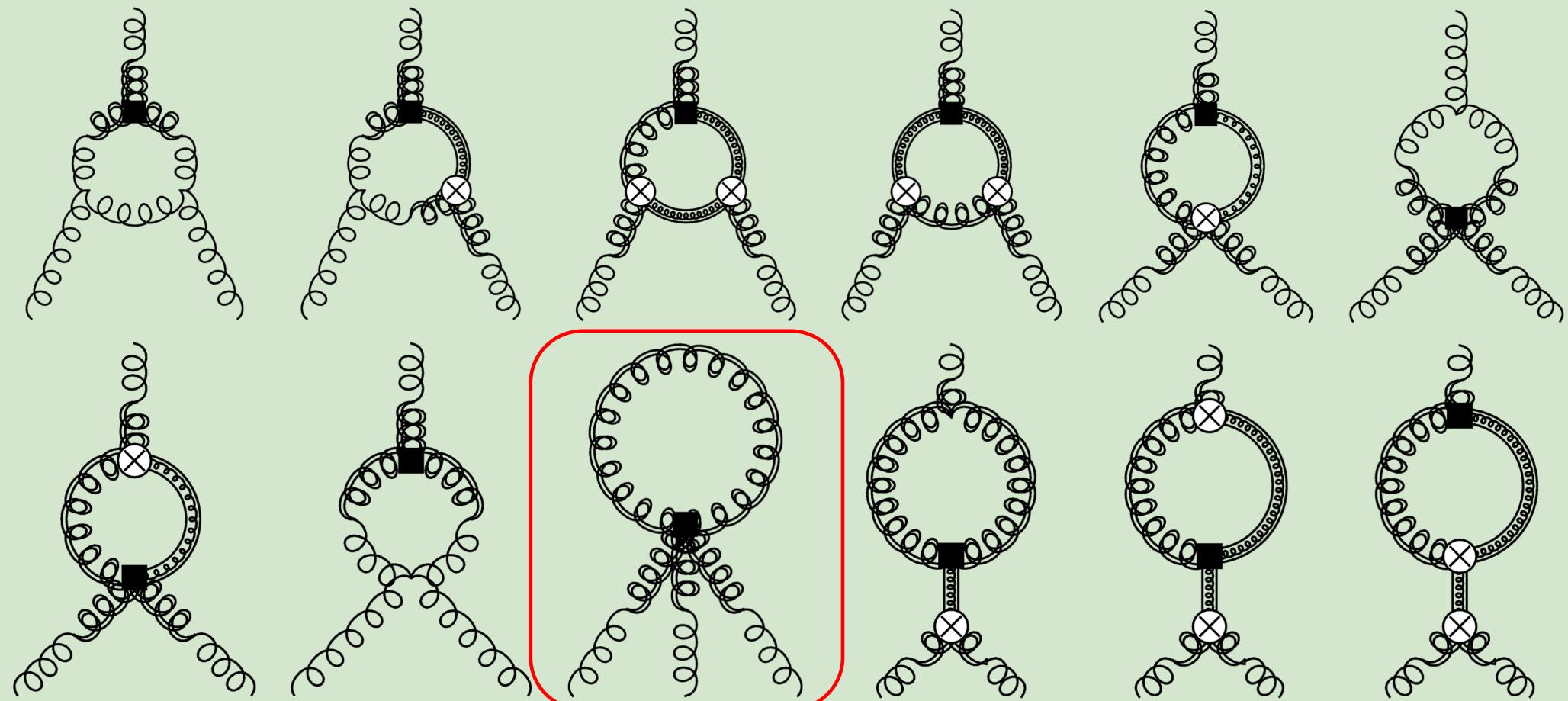


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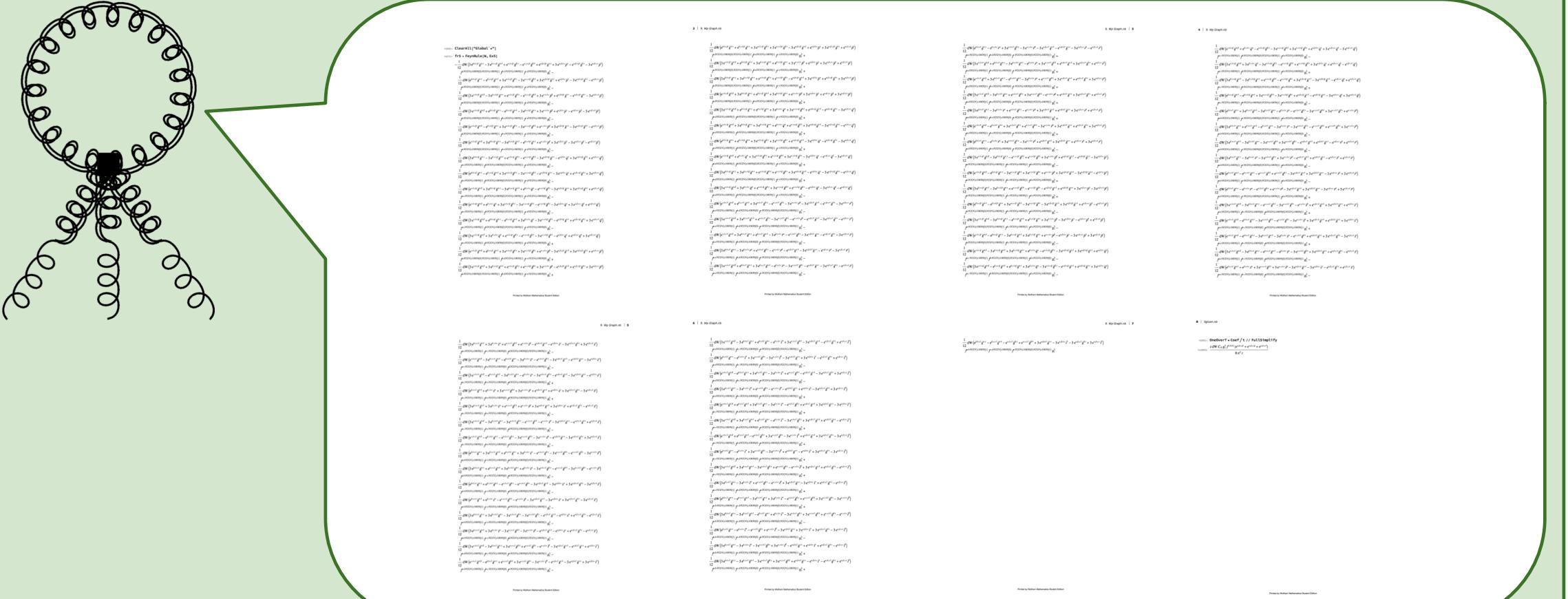
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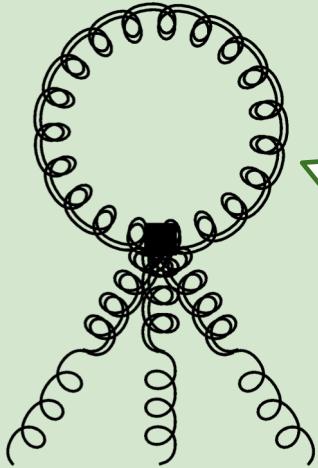
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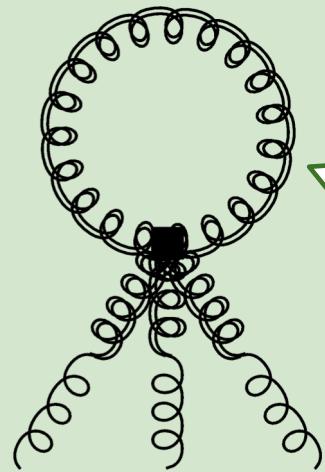


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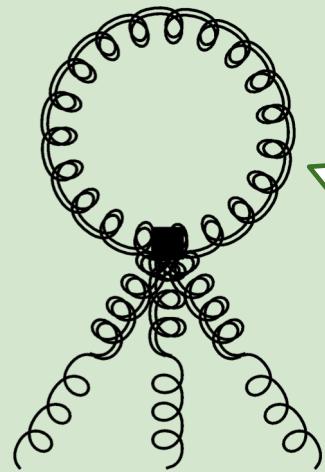
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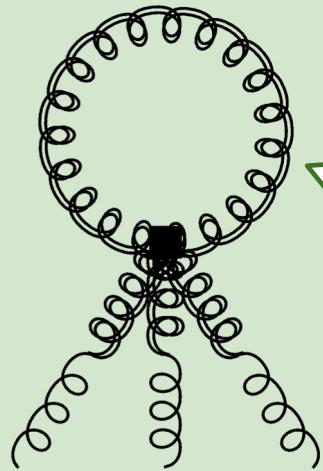
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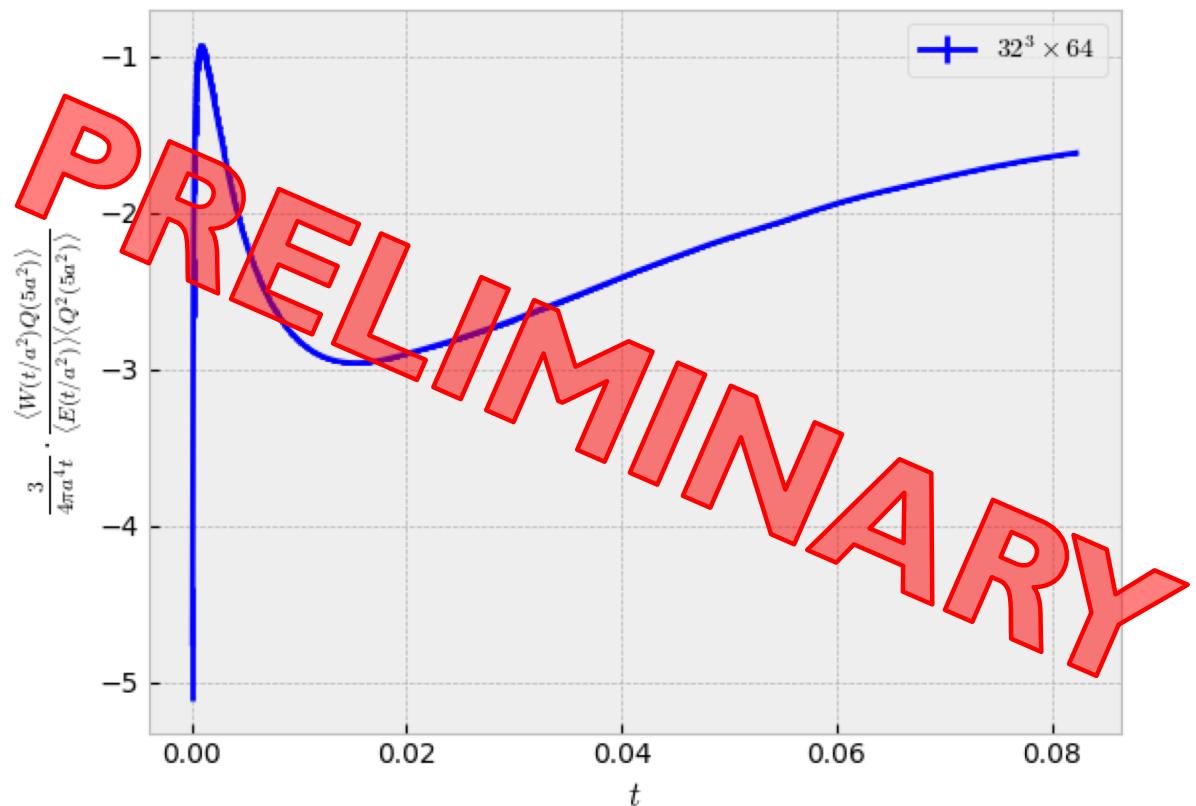
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- Data.

# FUTURE

- Full operator basis:

- $\gtrsim 20$  at  $d = 5$

- [Bhattacharya, Cirigliano, Gupta, Mereghetti, Yoon, 2015]

- Hilbert Series Formalism?

- Background field method?

$$\mathcal{A}_\mu \rightarrow \mathcal{A}_\mu + a_\mu$$

Allows for non-zero vacuum diagrams.

- Data.

*"I like to joke that the lattice people have half the computing power on Earth."*

-S. Brodsky