Linear confinement and stress–energy tensor around static quark and anti–quark pair
– Lattice simulation with Yang–Mills gradient flow –

( RY et al., arXiv : 1803.05656 [ hep-lat ]. )

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QED vs. QCD

**QED**
- Electric field spreads all over the space
- Coulomb potential

**QCD**
- Flux tube, squeezed one-dimensionally
- Confinement potential
QED vs. QCD

QED

- Electric field spreads all over the space
- Coulomb potential

Maxwell stress

- Perpendicular plane: pulling
- Parallel plane: pushing

Local interaction

QCD

- Flux tube, squeezed one-dimensionally
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Lattice 2018 @ MSU (2018/07/23)
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Energy Momentum Tensor (EMT)

Physics around $Q\bar{Q}$ in terms of energy and stress
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$T_{\mu\nu}$
Energy Momentum Tensor (EMT)

Physics around $Q\bar{Q}$ in terms of energy and stress

Energy density

Momentum density

Pressure

Stress tensor

$T_{\mu\nu} = T_{00} \begin{pmatrix} T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{pmatrix}$

Stress is force per unit area

$f_i = \sigma_{ij} n_j$ ; $\sigma_{ij} = -T_{ij}$

Landau and Lifshitz
Energy Momentum Tensor (EMT)

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Landau and Lifshitz

rubber
Energy Momentum Tensor (EMT)

Physics around $Q\bar{Q}$ in terms of energy and stress

\[ T_{\mu\nu} = \begin{pmatrix}
    T_{00} & T_{01} & T_{02} & T_{03} \\
    T_{10} & T_{11} & T_{12} & T_{13} \\
    T_{20} & T_{21} & T_{22} & T_{23} \\
    T_{30} & T_{31} & T_{32} & T_{33}
\end{pmatrix} \]

- Energy density
- Momentum density
- Pressure

- **Goal**: Determine absolute values of all components
- **Gauge invariant**!
A lot of Previous Studies

- Cardoso et al., PRD 86 (2013) 054501.

Color electric field
Cea et al., PRD 88 (2012) 054504.

Action density
Cardoso et al., PRD 86 (2013) 054501.
A lot of Previous Studies


Color electric field

More direct physical quantity : Stress tensor !!
Measurement of the stress on the Lattice

To Do

① prepare $Q\bar{Q}$ on the lattice and ② measure EMT around $Q\bar{Q}$
Measurement of the stress on the Lattice

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① prepare $Q\bar{Q}$ on the lattice and ② measure EMT around $Q\bar{Q}$

Wilson Loop

Imaginary time

space

$\langle W(R,T) \rangle$

= $C_0 \exp[-V(R)T] + C_1 \exp[-V_1(R)T] + \cdots$

$V(R) = -\lim_{T \to \infty} \frac{1}{T} \log \langle W(R,T) \rangle$

Ground state potential

Confinement potential

☑ quenched SU(3) Yang-Mills
☑ $\beta = 6.600$ ($a = 0.038$ fm)
EMT defined via gradient flow

\[ T = \mu \nu, \quad x = 1 \]

\[ \alpha \quad \mathcal{U} \quad t \quad \mathcal{U} \quad \mu \nu \quad t, \quad x \quad + \quad \delta \quad \mu \nu \quad 4 \quad \alpha \quad E \quad (t, \quad x) \quad [E(t, \quad x) - \langle E(t, \quad x) \rangle] + O(t) \]

Gradient flow

\[ \frac{\partial B_\mu(t, x)}{\partial t} = -g_0^2 \frac{\delta S[B]}{\delta B_\mu(t, x)} \]

\[ B_\mu : \text{smeared field} \]

\[ B_\mu(t = 0, x) = A_\mu(x) \]

Entrophy density vs. temperature

FlowQCD (2016)

\[ s/T^3 \]

Lattice 2018 @ MSU (2018/07/23)

To Do

① prepare \( Q \bar{Q} \) on the lattice and ② measure EMT around \( Q \bar{Q} \)
Setup

- Quenched SU(3) Yang-Mills gauge theory
- Wilson gauge action
- Clover operator
- Continuum limit
- APE smearing for spatial links
- Multihit improvement in temporal links
- Simulation using BlueGene/Q @ KEK

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>Lattice spacing</th>
<th>Lattice size</th>
<th># of statistics</th>
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<td>0.057 fm</td>
<td>$48^4$</td>
<td>140</td>
</tr>
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<td>6.600</td>
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<tr>
<td>6.819</td>
<td>0.029 fm</td>
<td>$64^4$</td>
<td>1000</td>
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</table>
Stress distribution in Maxwell theory

\[ T_{ij} = \varepsilon_0 \left( E_i E_j - \frac{\delta_{ij}}{2} E^2 \right) + \frac{1}{\mu_0} \left( B_i B_j - \frac{\delta_{ij}}{2} B^2 \right) \]

- **Stress tensor**
  \[ T_{ij} n_j^{(k)} = \lambda_k n_i^{(k)} \]
  \( (i, j = 1, 2, 3 ; k = 1, 2, 3) \)

- Perpendicular plane: \( \lambda_k < 0 \)
- Parallel plane: \( \lambda_k > 0 \)

\[ f_i = \sigma_{ij} n_j \quad ; \quad \sigma_{ij} = -T_{ij} \]

Note that the length of the arrow is equal to \( \sqrt{\lambda_k} \)

Landau and Lifshitz
Stress distribution in SU(3) YM theory

\[ a = 0.029 \text{ fm (no continuum limit)} \]
\[ t/\alpha^2 = 2.0 \text{ (no } t \to 0 \text{ limit)} \]
\[ R = 0.69 \text{ fm} \]

Note that the length of the arrow is equal to \( \sqrt{\lambda_k} \)
SU(3) YM theory vs. Maxwell theory

SU(3) YM theory

Maxwell theory

- Gauge invariant
- In terms of local interaction
- Propagation of force: squeezed vs. spreading
We focus on the mid-plane: double extrapolation
Double extrapolation @ mid point \( (R = 0.46 \text{ fm}) \)

Double extrapolation

\[
\langle \mathcal{O}_{\text{lat}}(t, 0) \rangle_{Q\bar{Q}} \text{[GeV/fm}^3]\]

\[
\frac{-\langle \mathcal{T}_{zz}^{\text{lat}}(t, 0) \rangle}{a^2} = \mathcal{O}_{\text{cont}} + A_0 t + A_1(t) a^2 + \ldots
\]

Lattice 2018 @ MSU (2018/07/23)
Double extrapolation @ mid point ($R = 0.46$ fm)

FlowQCD (2016)

$$O_{\text{lat}} = O_{\text{cont}} + A_0 t + A_1(t)a^2 + \cdots$$

$Lattice\ 2018\ @\ MSU\ (2018/07/23)$
Double extrapolation @ mid point \( (R = 0.46 \text{ fm}) \)

FlowQCD (2016)

\[
\langle T_{zz}^{(t,0)} \rangle_{Q\bar{Q}} \left[ \text{GeV/fm}^3 \right] = \phi_{a = 0.029 \text{ fm}} + A_0 t + A_1(t) a^2 + \cdots
\]

Double extrapolation

\( O_{\text{lat}} = O_{\text{cont}} + A_0 t + A_1(t) a^2 + \cdots \)
Double extrapolation @ mid point ($R = 0.46$ fm)

FlowQCD (2016)

$$-\langle T_{zz}^{\text{lat}}(t, 0) \rangle_{Q\bar{Q}} \rightleftharpoons \text{GeV/fm}^3$$

\begin{align*}
\Phi & \quad a = 0.029 \text{ fm} \\
\diamond & \quad a = 0.038 \text{ fm} \\
\triangle & \quad a = 0.046 \text{ fm} \\
\bigtriangleup & \quad a = 0.058 \text{ fm}
\end{align*}

continuum

Range-1

Range-2

Range-3

Double extrapolation

Range1

Range2

Range3

Strong discretization effect

\[ O_{\text{lat}} = O_{\text{cont}} + A_0 t + A_1(t) a^2 + \ldots \]

Lattice 2018 @ MSU (2018/07/23)
Cylindrical coordinate

\[ T_{\mu\nu} = \begin{pmatrix} T_{44} & T_{zz} & 0 \\ T_{zz} & T_{rr} & 0 \\ 0 & 0 & T_{\theta\theta} \end{pmatrix} \]

Diagonalization of EMT

(Cylindrical symmetry and parity symmetry)
Cylindrical coordinate

\[ T_{\mu\nu} = \begin{pmatrix} T_{44} & 0 & 0 & 0 \\ 0 & T_{zz} & 0 & 0 \\ 0 & 0 & T_{rr} & 0 \\ 0 & 0 & 0 & T_{\theta\theta} \end{pmatrix} \]

Diagonalization of EMT

(Cylindrical symmetry and parity symmetry)

Degenerate in Maxwell theory!

\[ -T_{44} = -T_{zz} = T_{rr} = T_{\theta\theta} \]
Distribution of $\langle T_{cc} \rangle_{Q\bar{Q}}$ ($c = 4, z, r, \theta$) in mid-plane

$R = 0.46$ fm

$0.92$ fm

$0.69$ fm

$0.46$ fm

$V(R)$ [GeV] vs $R$ [fm]

Cornell type vs \textit{data}
Distribution of $\langle T_{cc} \rangle_{Q \bar{Q}}$ ($c = 4, z, r, \theta$) in mid-plane

(a) $R = 0.46$ fm
(b) $R = 0.69$ fm
(c) $R = 0.92$ fm

- $\langle T_{44}^R(r) \rangle_{Q \bar{Q}}$ [GeV/fm$^3$]
- $\langle T_{zz}^R(r) \rangle_{Q \bar{Q}}$ [GeV/fm$^3$]
- $\langle T_{rr}^L(r) \rangle_{Q \bar{Q}}$ [GeV/fm$^3$]
- $\langle T_{66}^R(r) \rangle_{Q \bar{Q}}$ [GeV/fm$^3$]
Distribution of $\langle T_{cc} \rangle_{Q\bar{Q}}$ ($c = 4, z, r, \theta$) in mid-plane

Properties in non-Abelian theory

- $T_{44} \approx T_{zz}, T_{rr} \approx T_{\theta\theta}$ (degeneracy)
- $T_{44} \neq T_{rr}$ (separation)
- $\sum_{\mu} T_{\mu\mu} \neq 0$

- $T_{44} \approx T_{zz}$, $T_{rr} \approx T_{\theta\theta}$ (degeneracy)
- $T_{44} \neq T_{rr}$ (separation)
- $\sum_{\mu} T_{\mu\mu} \neq 0$

![Graphs showing properties in non-Abelian theory](a) $R = 0.46$ fm  (b) $R = 0.69$ fm  (c) $R = 0.92$ fm

- $\langle T_{44}(r) \rangle_{Q\bar{Q}}$ [GeV/fm$^3$]
- $\langle T_{zz}(r) \rangle_{Q\bar{Q}}$ [GeV/fm$^3$]
- $\langle T_{rr}(r) \rangle_{Q\bar{Q}}$ [GeV/fm$^3$]
- $\langle T_{\theta\theta}(r) \rangle_{Q\bar{Q}}$ [GeV/fm$^3$]
From EMT

\[ F_{\text{stress}} := \int_{\text{mid}} \langle T_{zz} \rangle_{Q\bar{Q}} \, d^2x \]

\[ V(R) = a + bR + c/R \]

\[ F_{\text{pot}} := -\frac{dV(R)}{dR} \]
EMT and confinement potential

From EMT

\[ V(R) = a + bR + c/R \]

\[ F_{\text{stress}} := \int_{\text{mid}} \langle T_{zz} \rangle_{Q\bar{Q}} \, d^2x \]

\[ F_{\text{pot}} := -\frac{dV(R)}{dR} \]
Summary

First measurement of stress distribution on the lattice!!

Outlook

✓ We need to explain the stress distribution using, for example, Abelian-Higgs model (RY+ in progress)
✓ Application: two flux tubes, finite temperature, full QCD...
Ground saturation

\[ -\langle T_{zz}(t, 0)\rangle_{\text{lat}}^{Q\bar{Q}} \text{ [GeV/fm}^3\rangle \]

\[ T/\bar{a} \]

- \( t/\bar{a}^2 = 1.0 \)
- \( t/\bar{a}^2 = 1.7 \)
- \( t/\bar{a}^2 = 2.0 \)
- \( t/\bar{a}^2 = 3.0 \)
- \( t/\bar{a}^2 = 4.0 \)
- \( t/\bar{a}^2 = 6.0 \)