Phase Unwrapping and 1D Sign Problems

W Detmold, **G Kanwar**, M L Wagman

MIT

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[1806.01832]

Overview

1. Sign/Phase Problems in Finite Density
2. Phase Unwrapping
3. Toy Model Results: 0+1D Scalar Theory
4. Large Phase Jumps
5. Conclusions and Outlook

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Sign/Phase Problems in Finite Density

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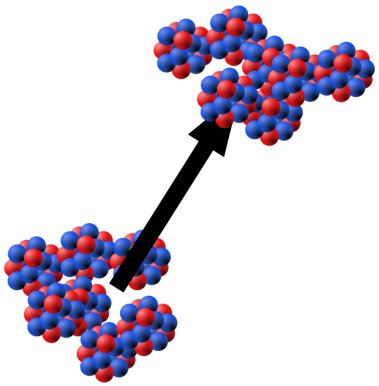
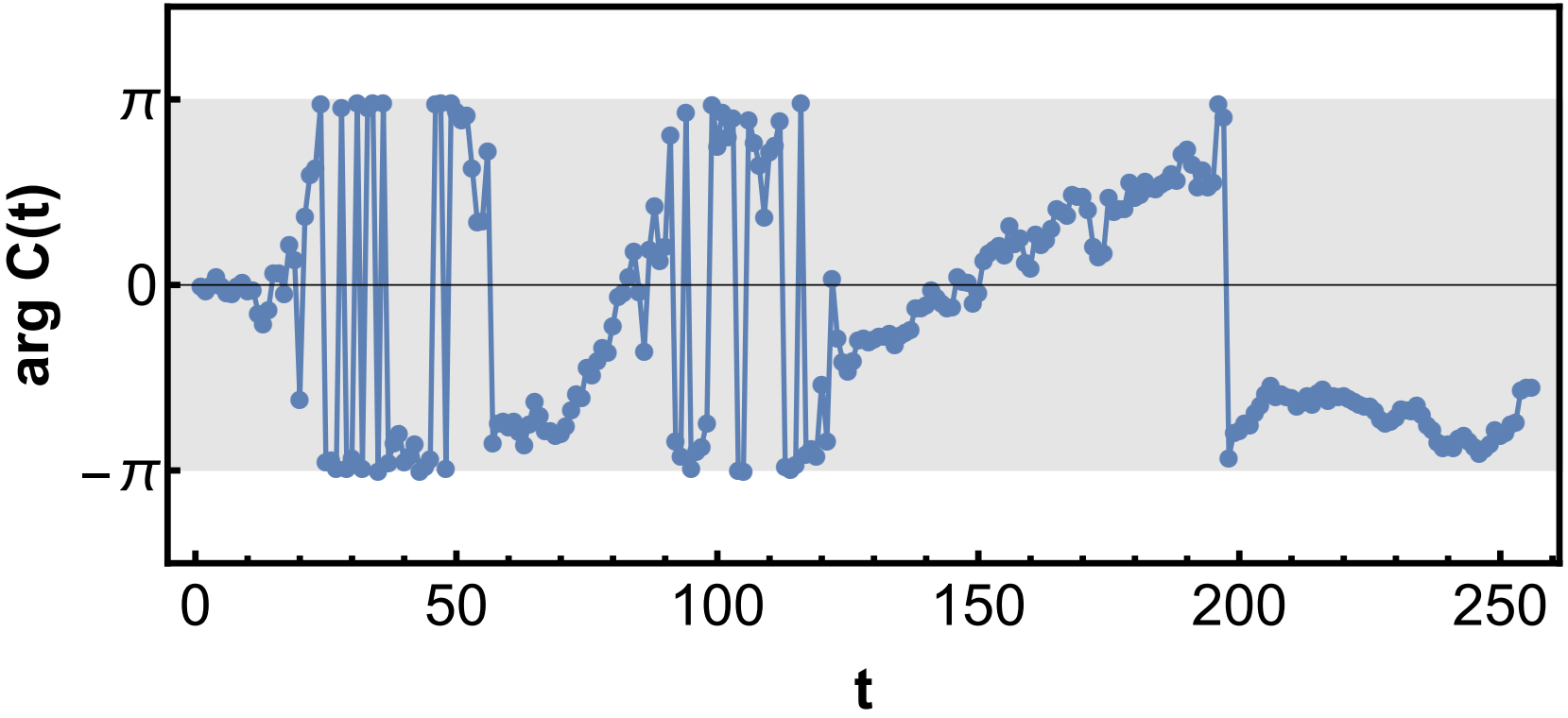
2. Complex observables ($C(t,0)$ for $U(1)_B \neq 0$) **[our focus]**:

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[\phi]\, \mathcal{O}[\phi]\, e^{-S[\phi]}$$



Sign problem = Signal-to-noise problem for correlators

Correlator phases drift relatively smoothly on typical configurations.



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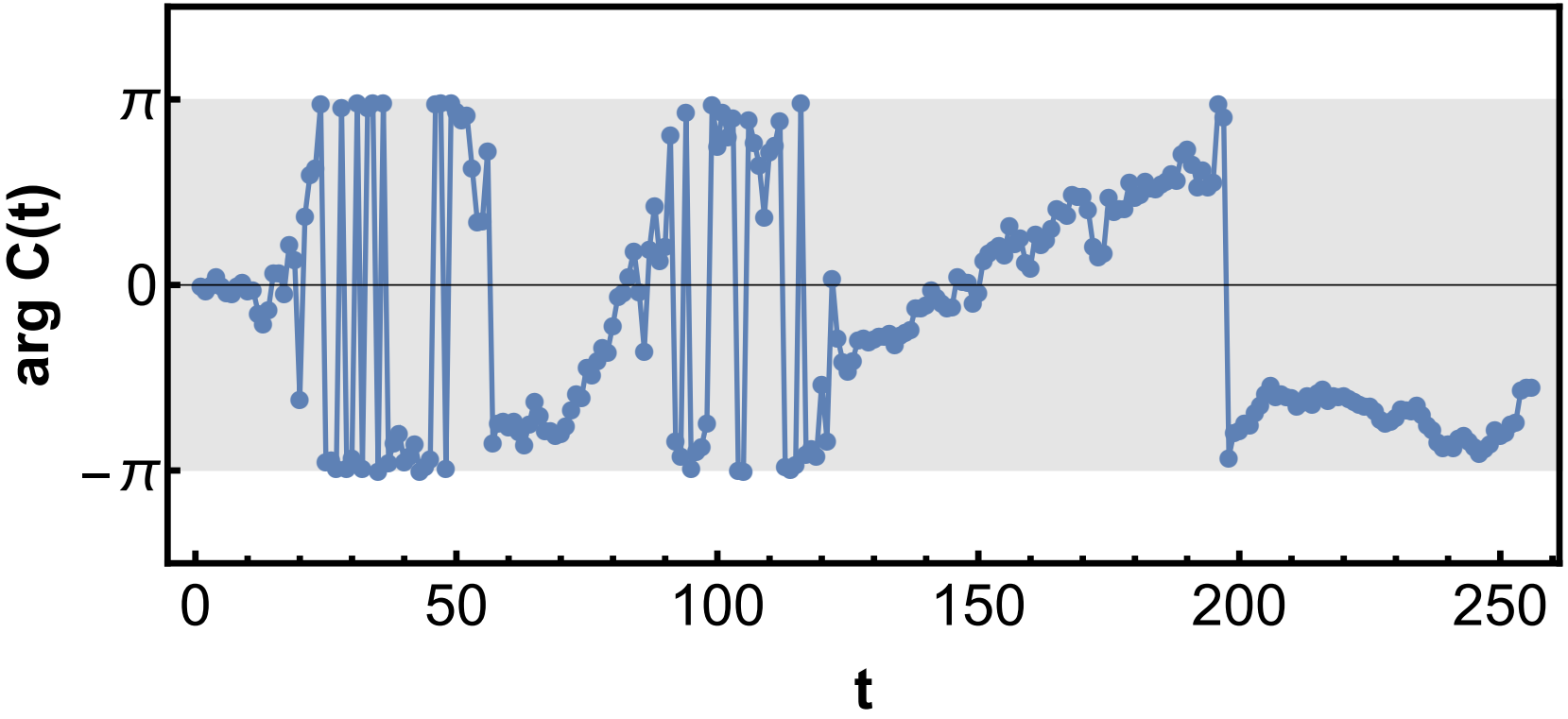

Distributions of $\arg C(t)$ across the ensemble become increasingly uniform at large t.

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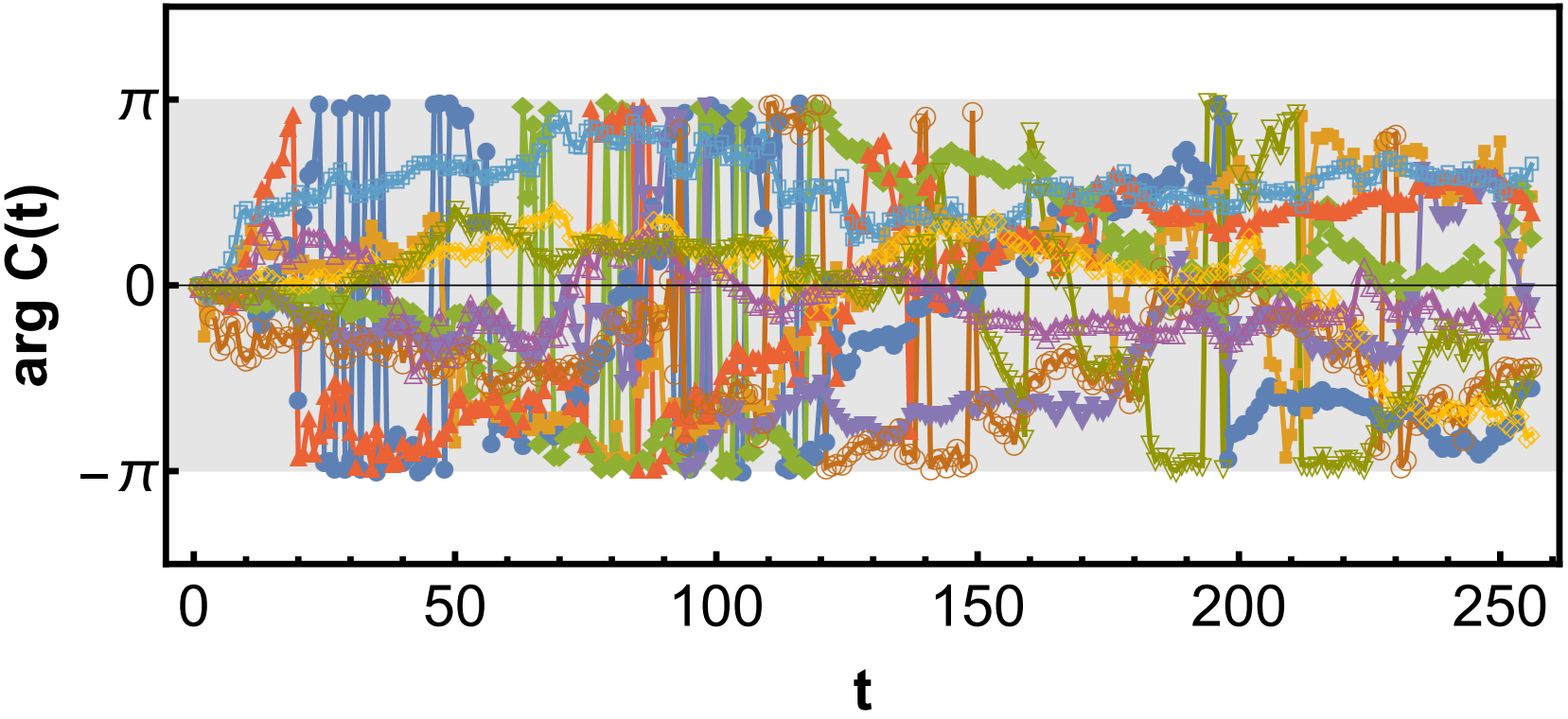

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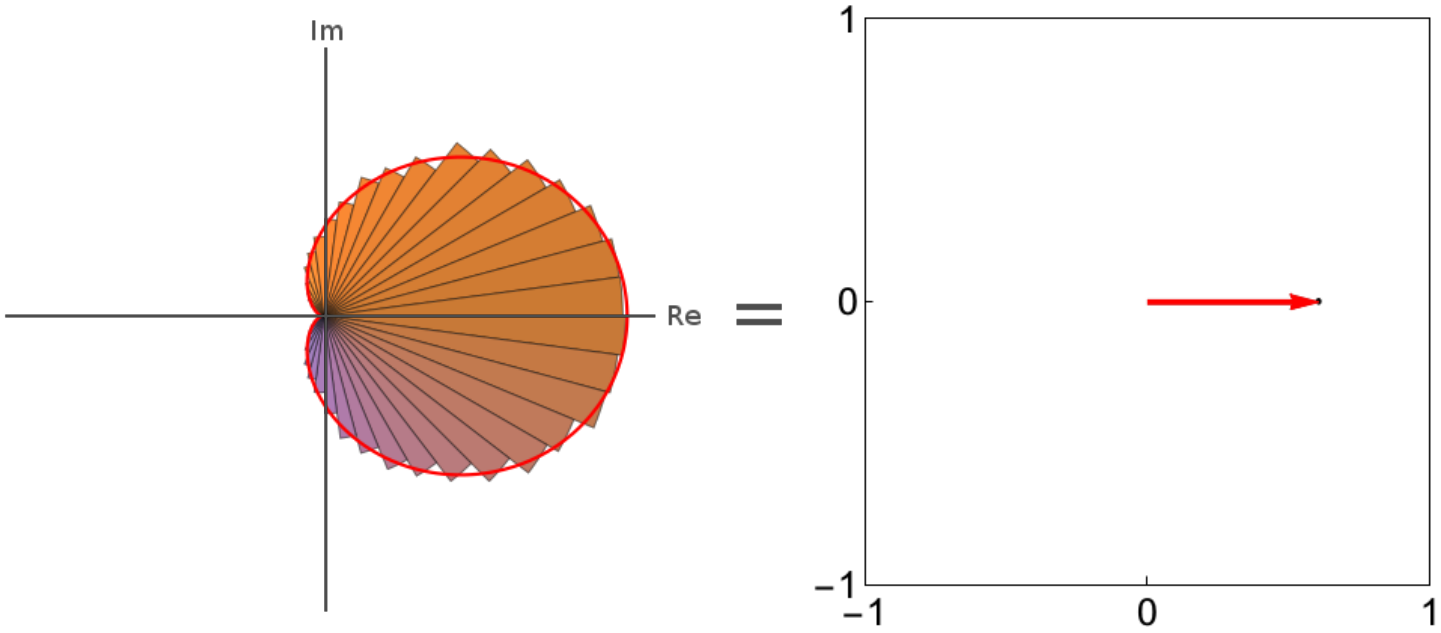
Estimating $\langle C(t) \rangle$ at large t requires **exponentially precise cancellations** of nearly-uniform phase distributions.



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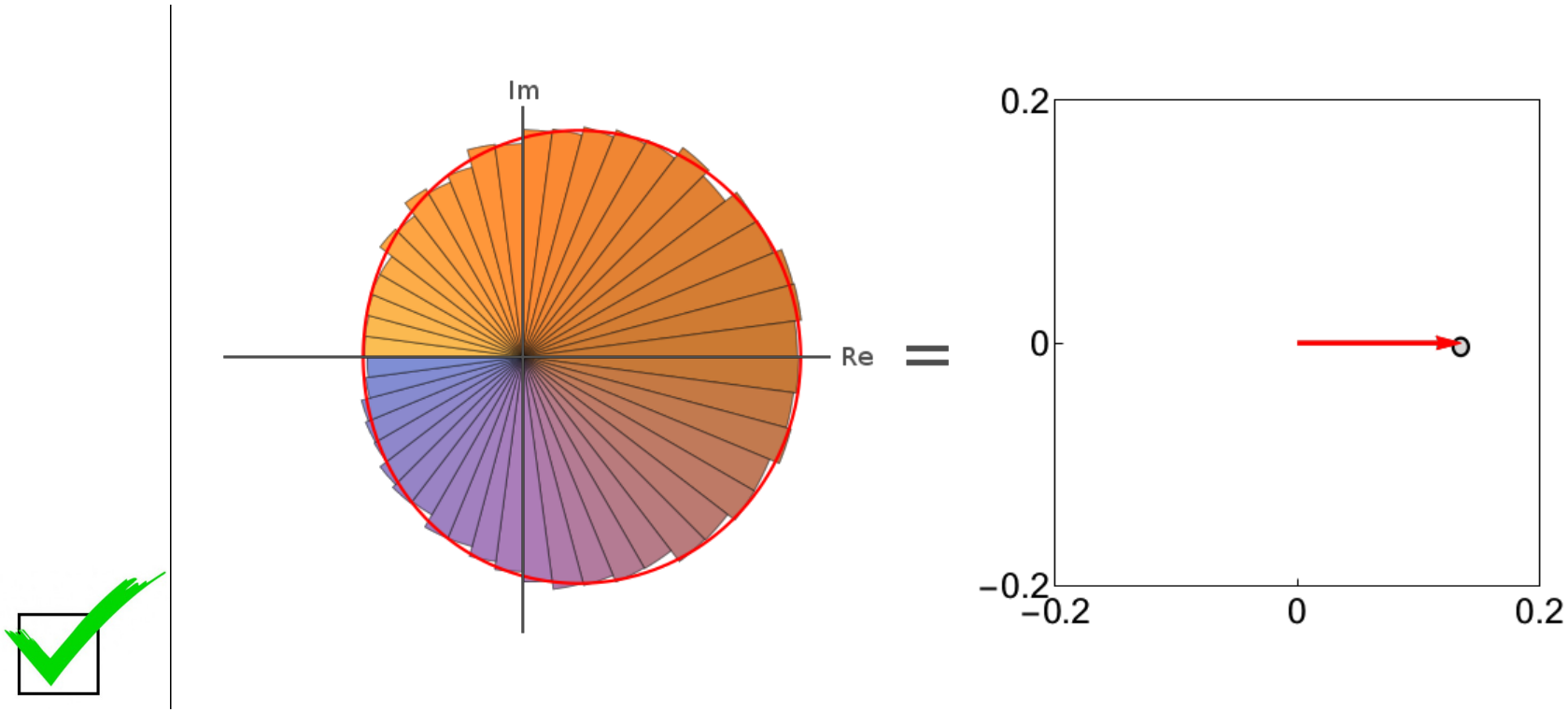

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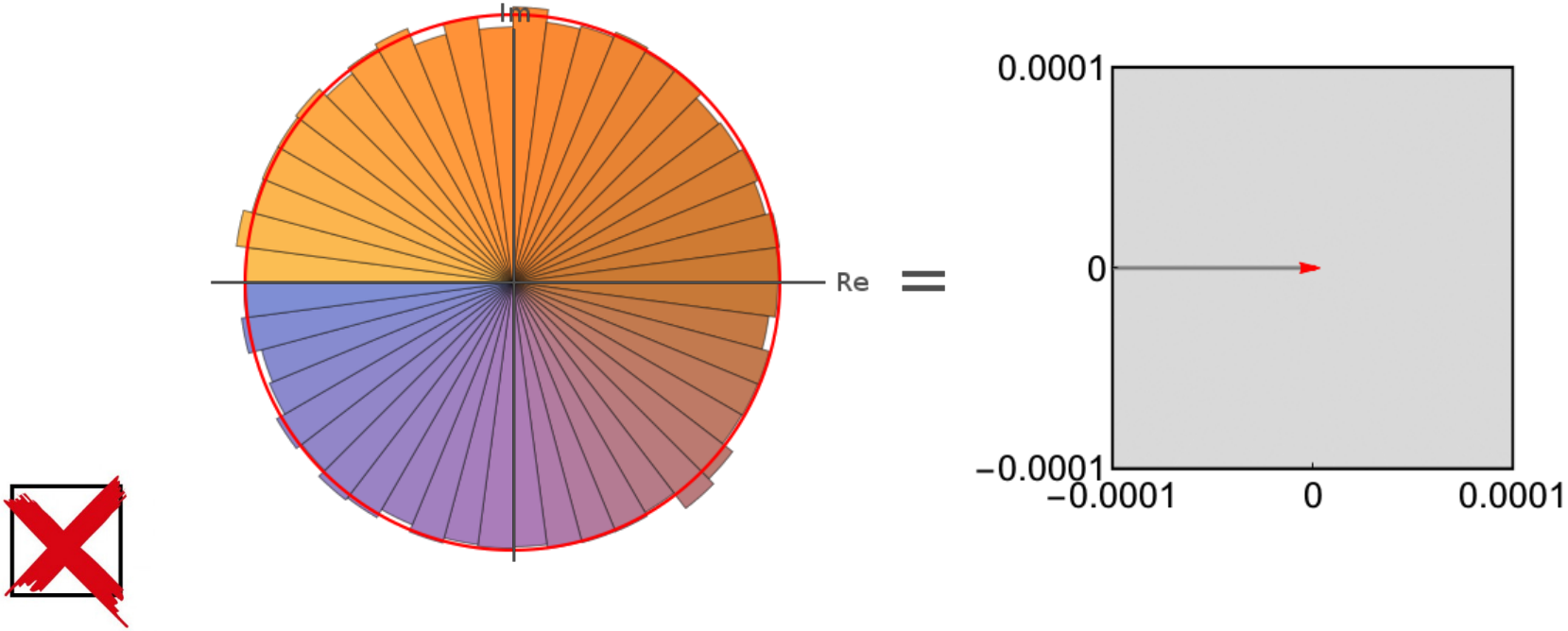

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Phase Unwrapping in 0+1D

1. Decompose $C_i(t)$ on each config i into phase $\Theta_i(t)$, log magnitude $\mathcal{R}_i(t)$.
2. "Unwrap" phase
$$\widetilde{\Theta}_i(t) = \Theta_i(t) + 2\pi\nu_i(t).$$
3. Cumulant expansion allows precise estimate of $\langle C(t)\rangle$ by
$$\langle e^{\mathcal{R}(t)+i\Theta(t)}\rangle = \langle e^{\mathcal{R}(t)+i\widetilde{\Theta}(t)}\rangle = e^{\sum_n \kappa_n/n!}.$$

[M G Endres, et al. PRL 107 (2011) 201601]

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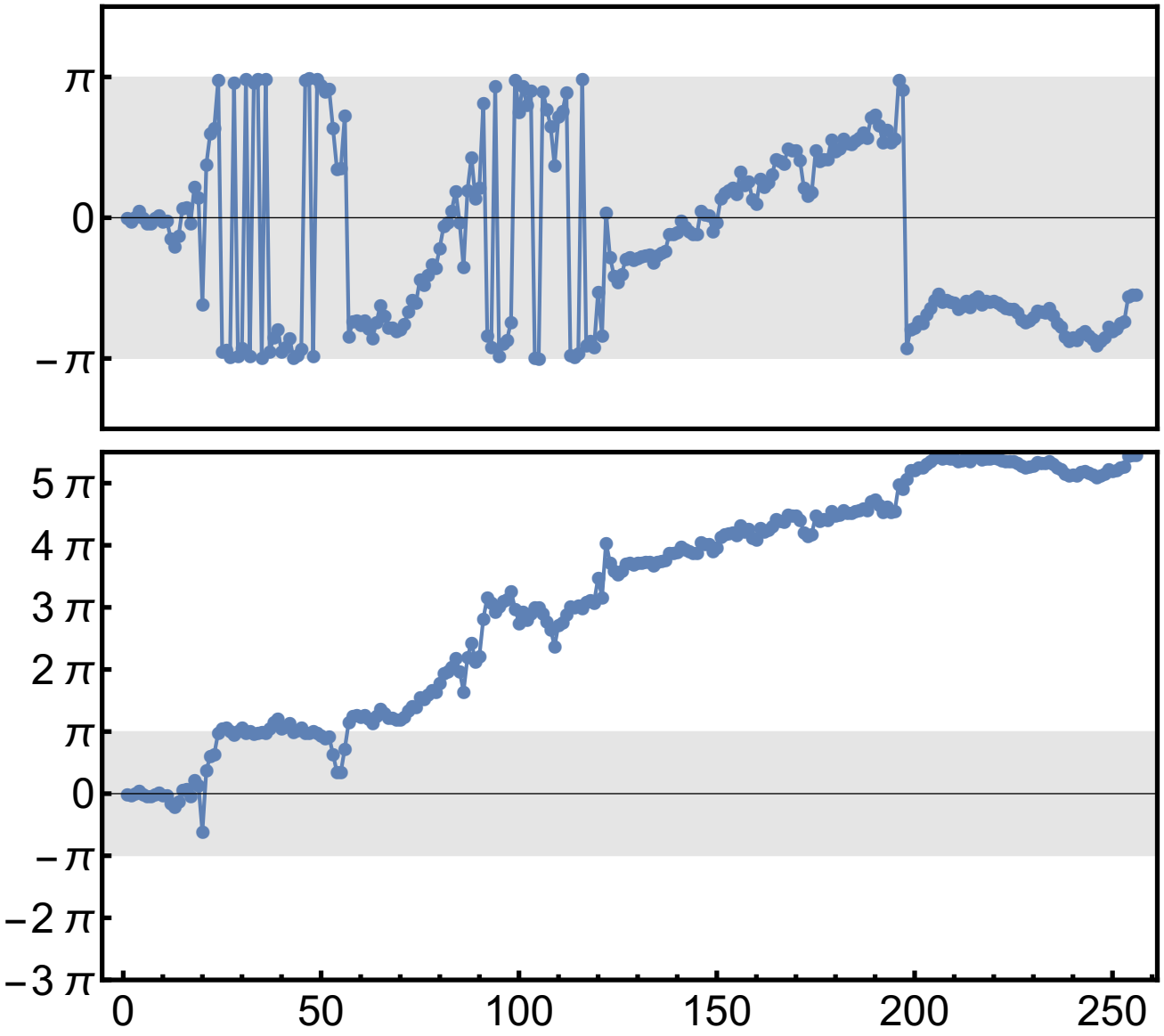
For nearly-Gaussian, κ_2 dominates. Truncate $\sum_n$ at sufficiently large n_{max} to get a good estimate.

Unwrapping Phases

- Add $2\pi\nu$ to each $\Theta_i(t)$ according to a smoothing criterion ("unwrapping scheme").

- Local iterative unwrapping: $\Theta_i(0) = 0$, unwrap forwards/backwards around half the lattice each way.
  - **Nearest-neighbor scheme:** minimize difference between previous unwrapped and current point.
  - **Window-based scheme:** minimize magnitude of differences between average of w previous unwrapping phases and current point.

Nearest-Neighbor Example



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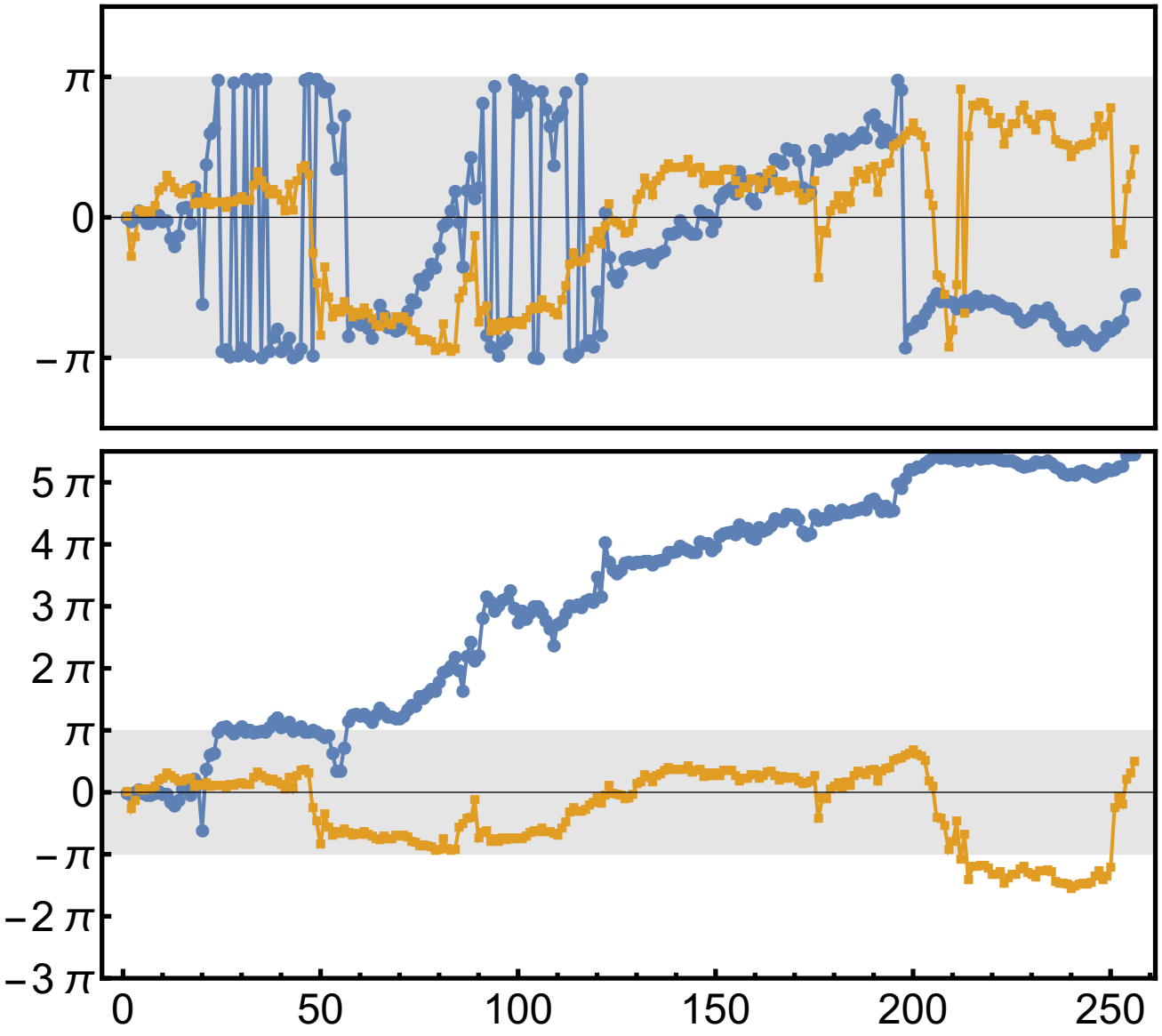

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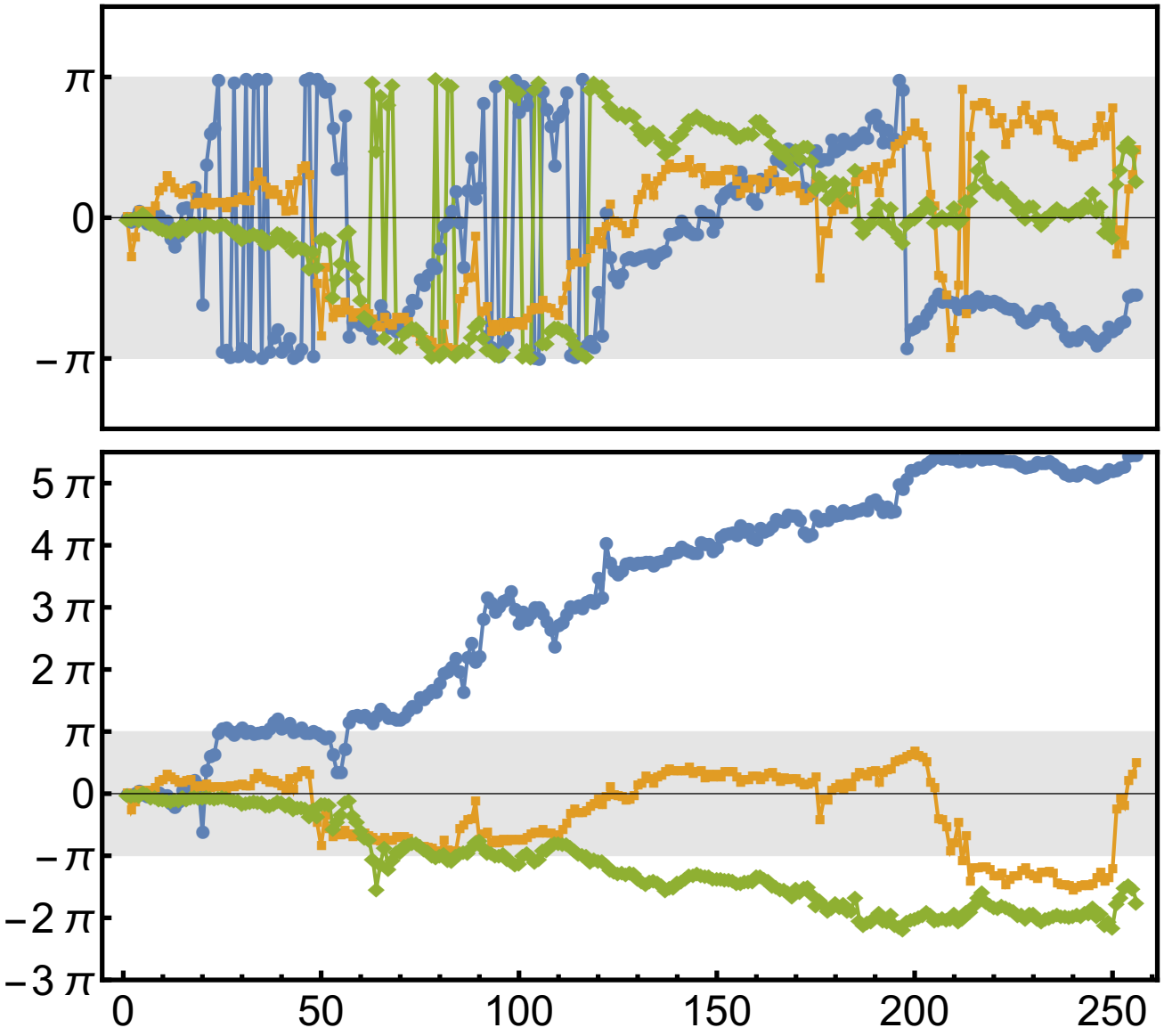

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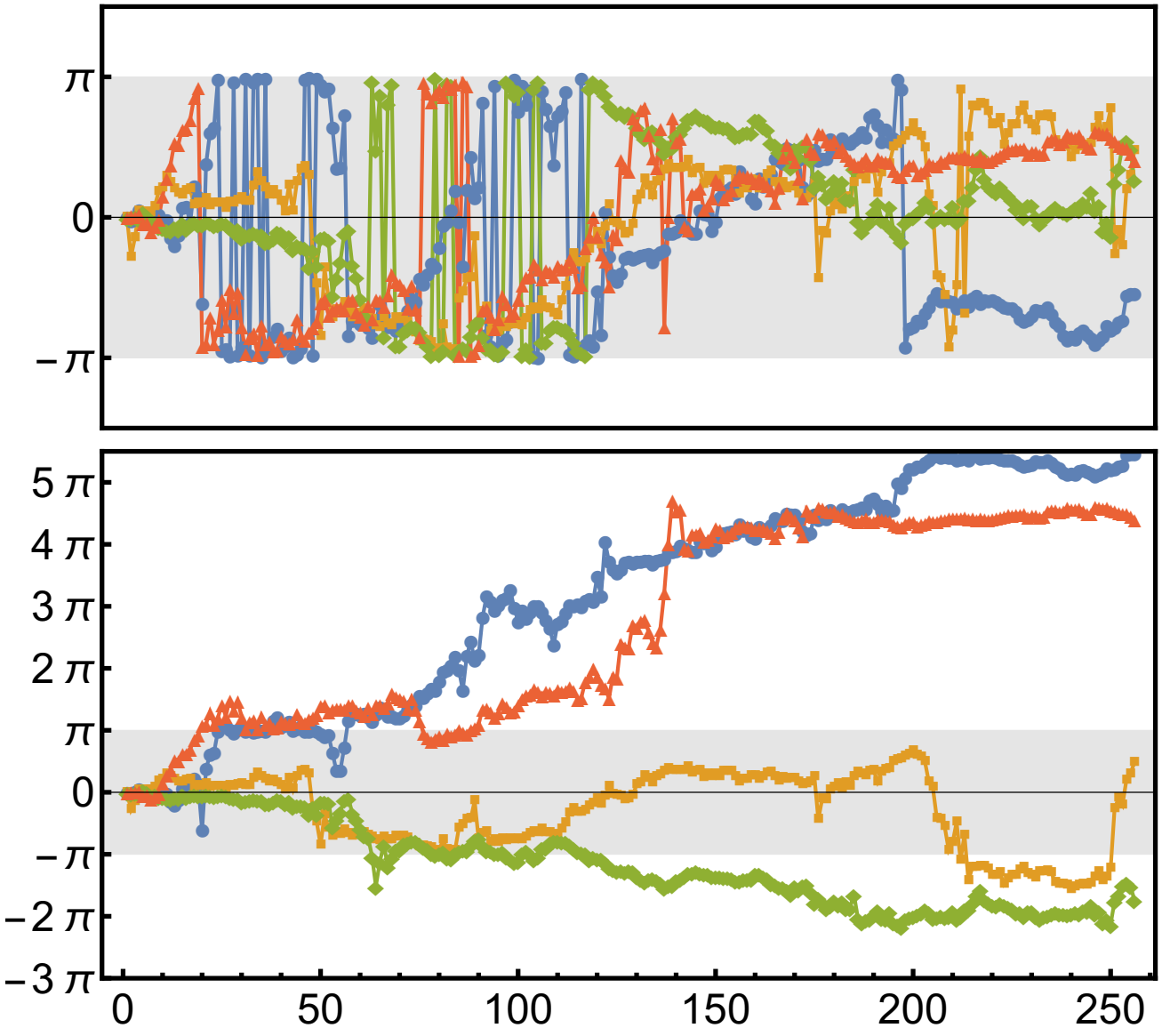

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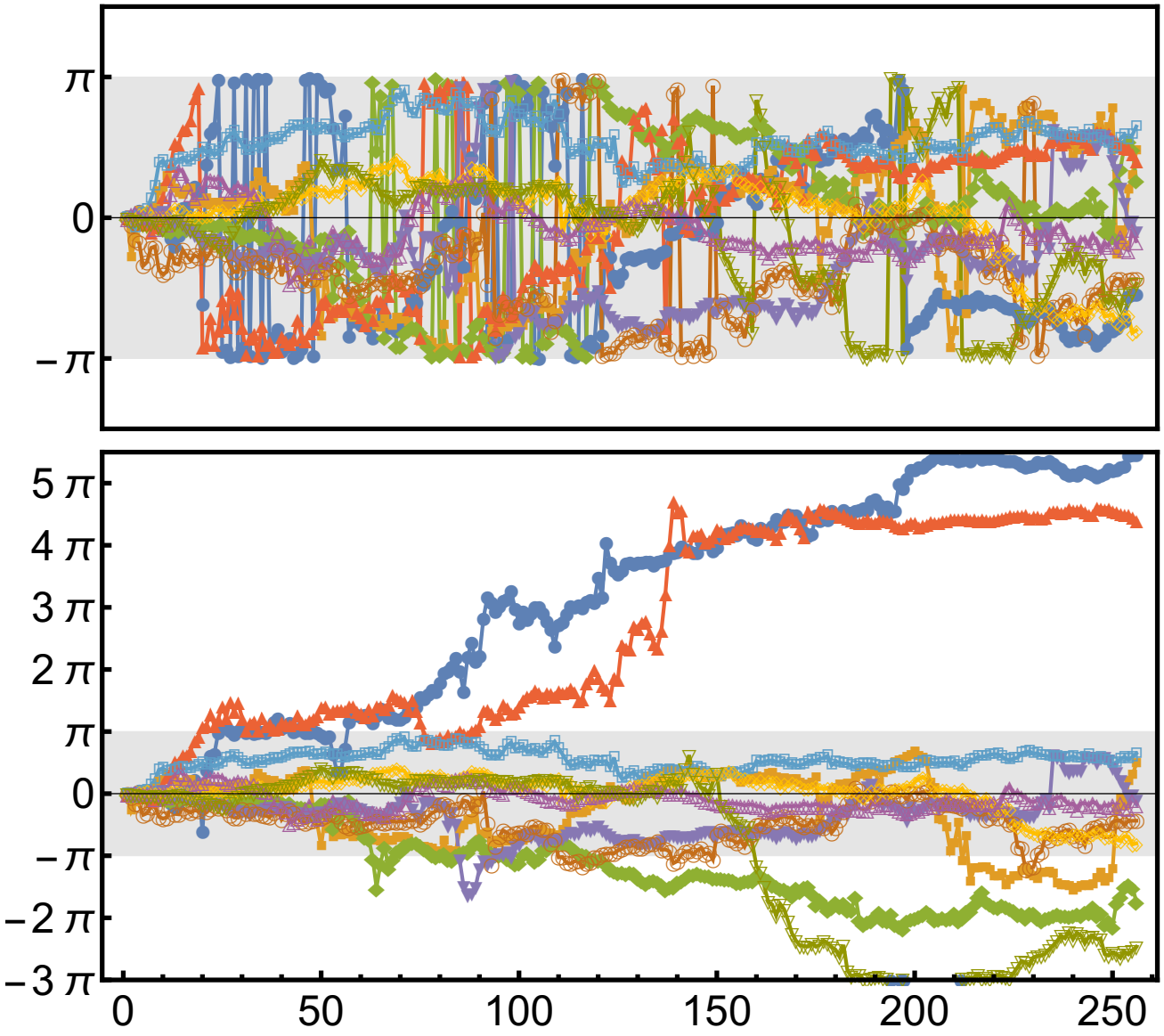

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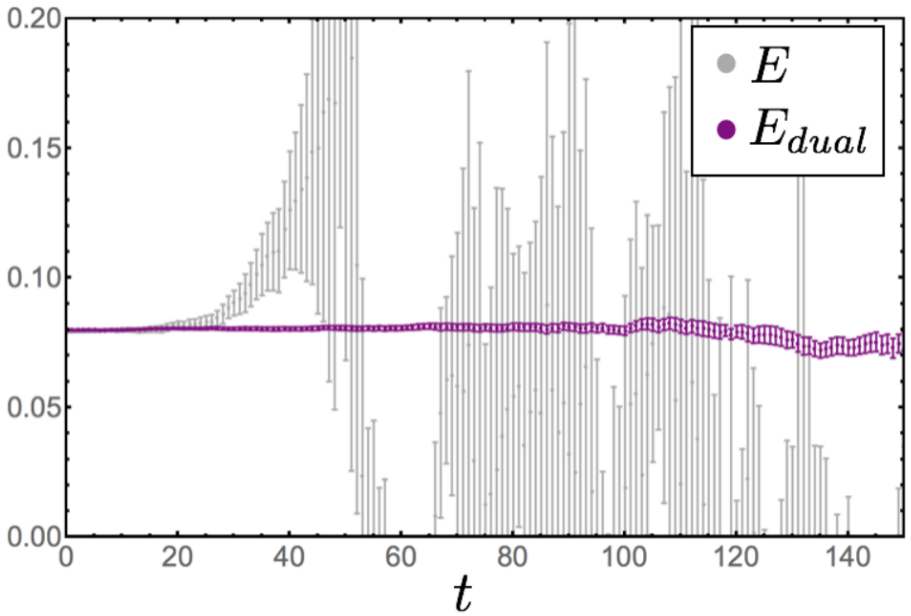
0+1D Scalar Theory

$$S = \sum_{t=0}^{L-1} |\phi(t) - \phi(t-1)|^2 + M^2|\phi(t)|^2 + \lambda|\phi(t)|^4$$

- Global $U(1)$ symmetry $\implies$ spectrum splits into charge sectors.

- Dual variables solution removes sign problem, giving a comparison. [M G Endres, PRD 75 (2007) 065012]



$$Z = 2\sum_{q\in\mathbb{Z}} \int_0^\infty \prod_{t=0}^{L-1} \left[d|\varphi(t)|\ |\varphi(t)|\ e^{-2|\varphi(t)|^2 - V(|\varphi(t)|)}\ 2\ I_{|q|}\left(\kappa(t)\right) \right]$$

0+1D Scalar Theory

Correlators experience maximally poor signal-to-noise:

$$\text{Sig} = \langle \phi^{Q}(t)\phi^{*Q}(0)\rangle \sim e^{-EQt}$$

$$\text{Var} = \langle |\phi|^{2Q}(t)|\phi|^{2Q}(0)\rangle = \mathcal{O}\left(1\right)$$

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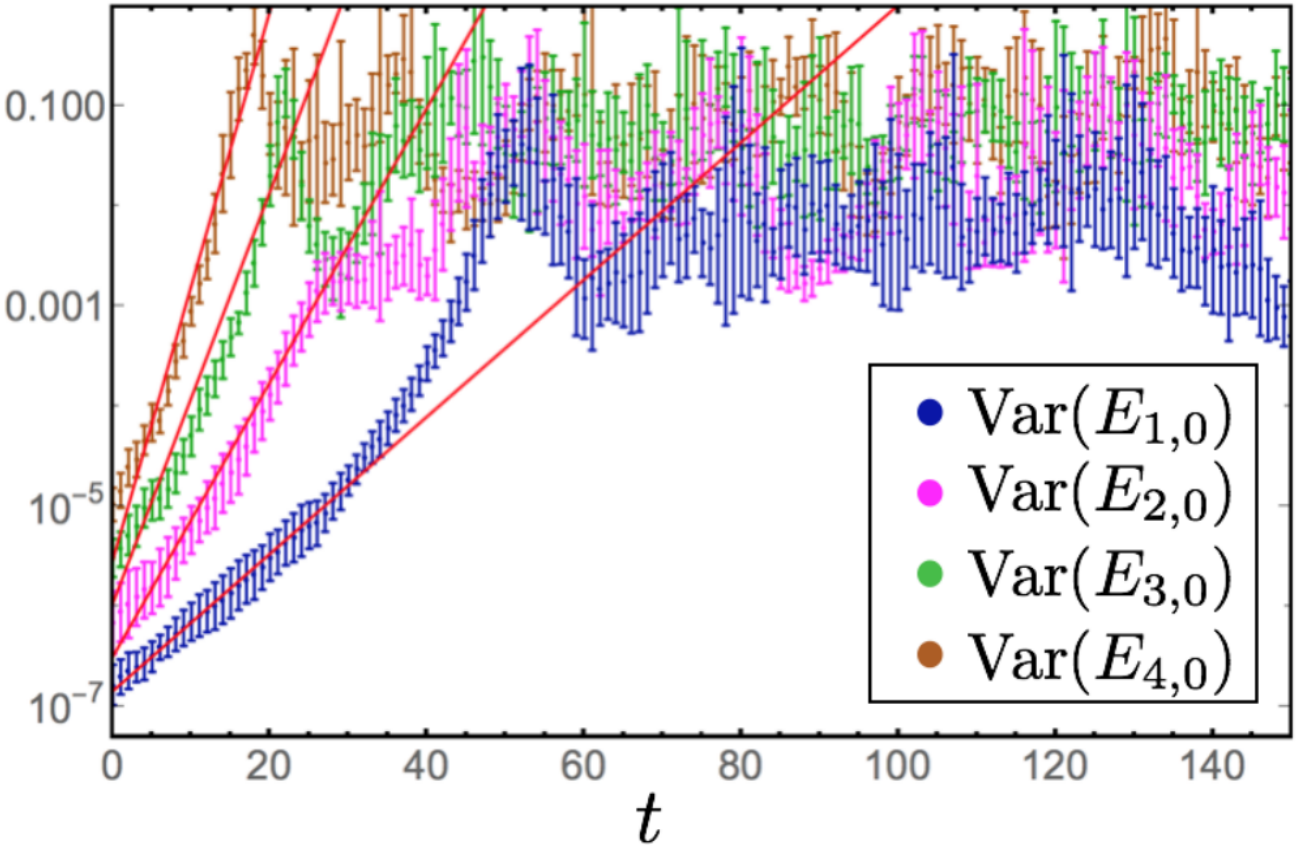
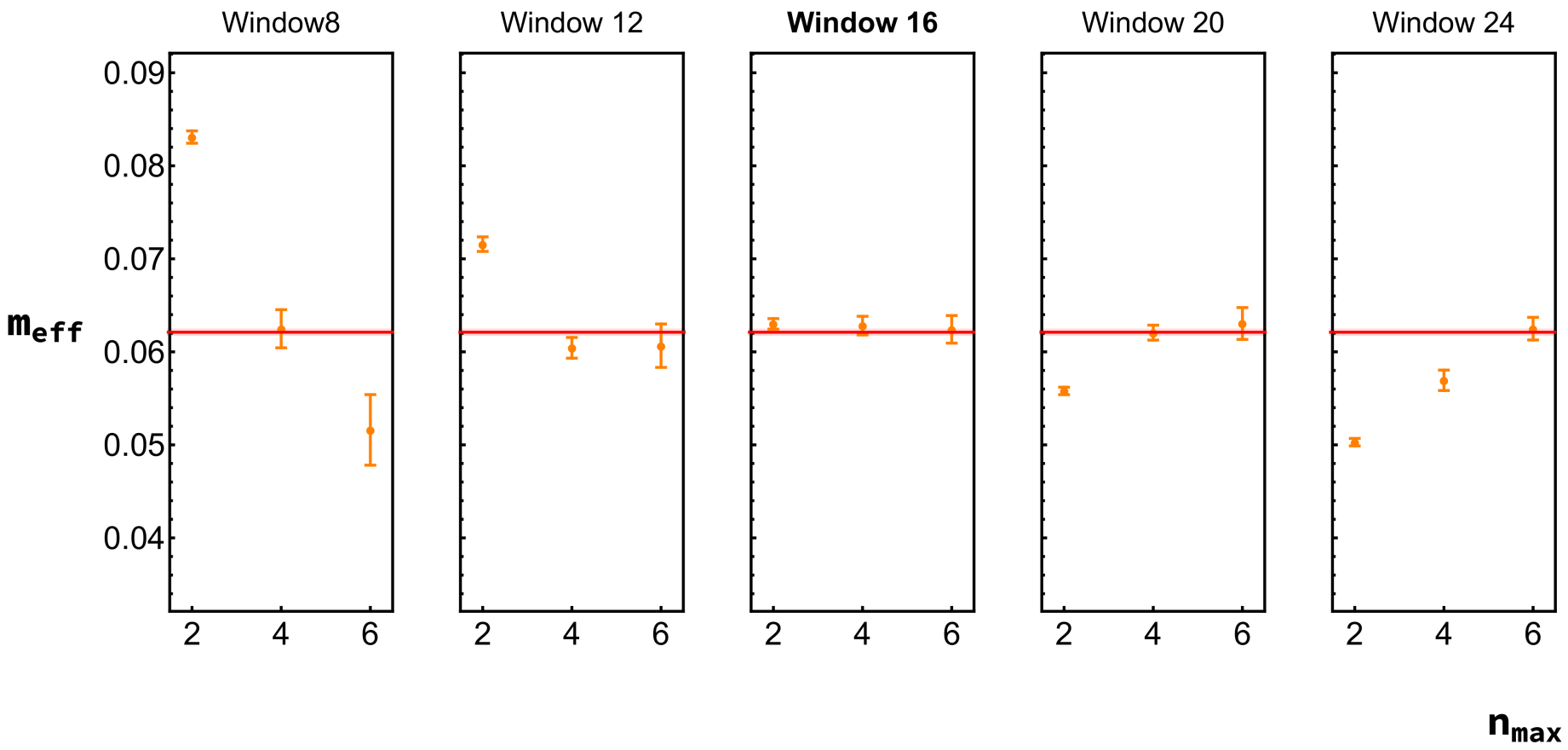


$Q = 1$ Phase Unwrapping

Per ensemble, unwrapping window w tuned to minimize higher-order cumulant corrections.

$$w \sim \xi \approx 16.1$$



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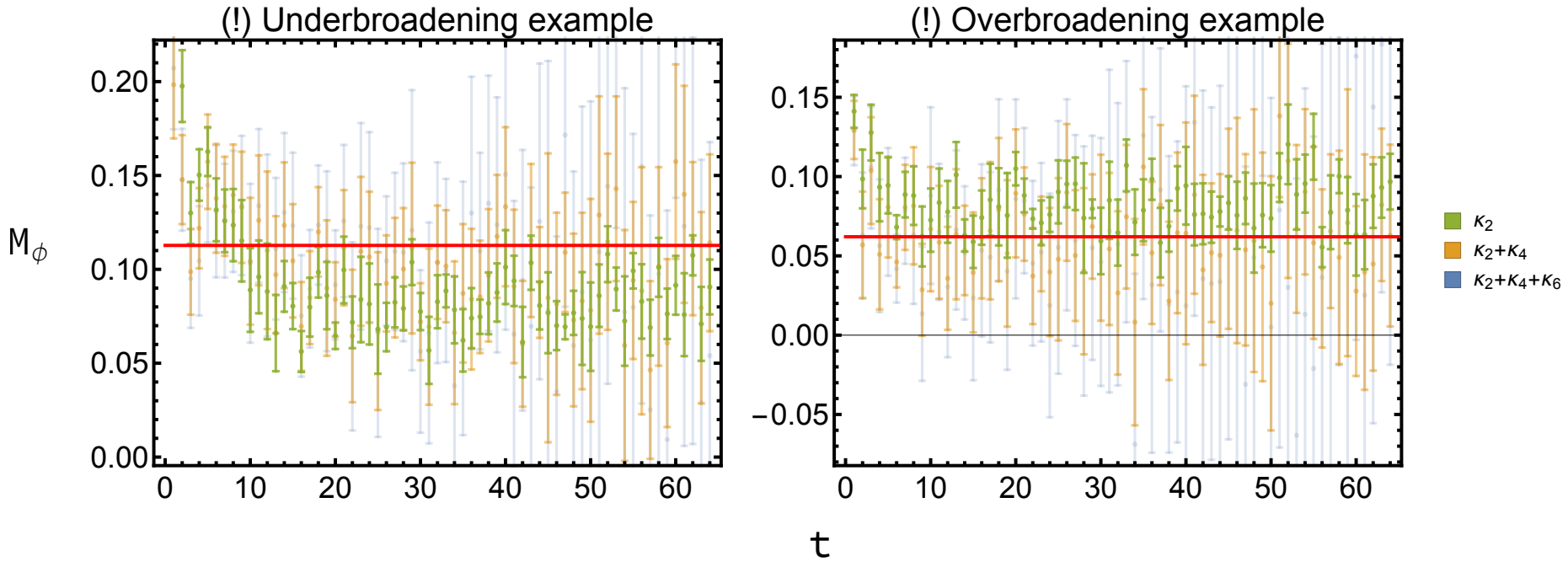

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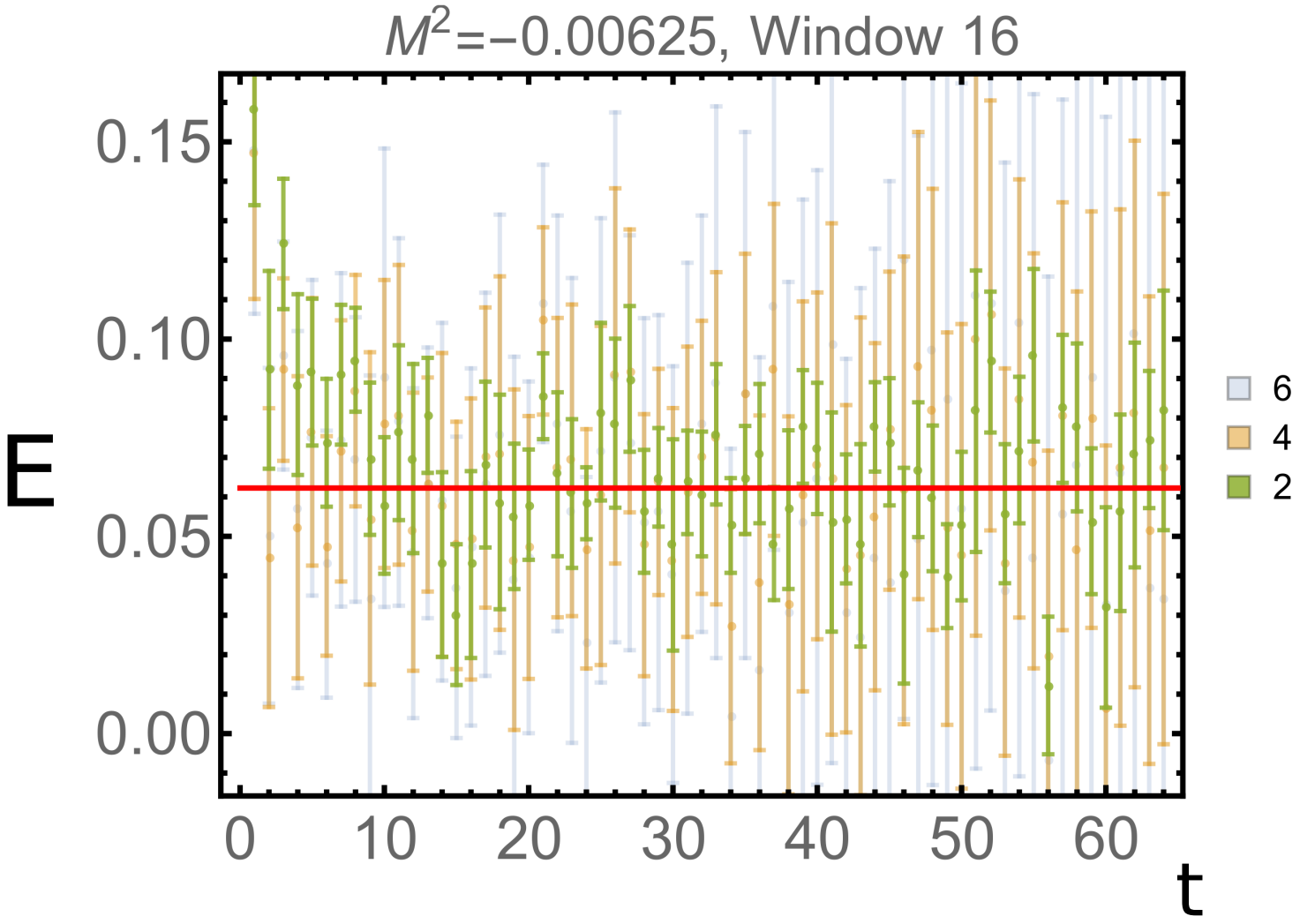
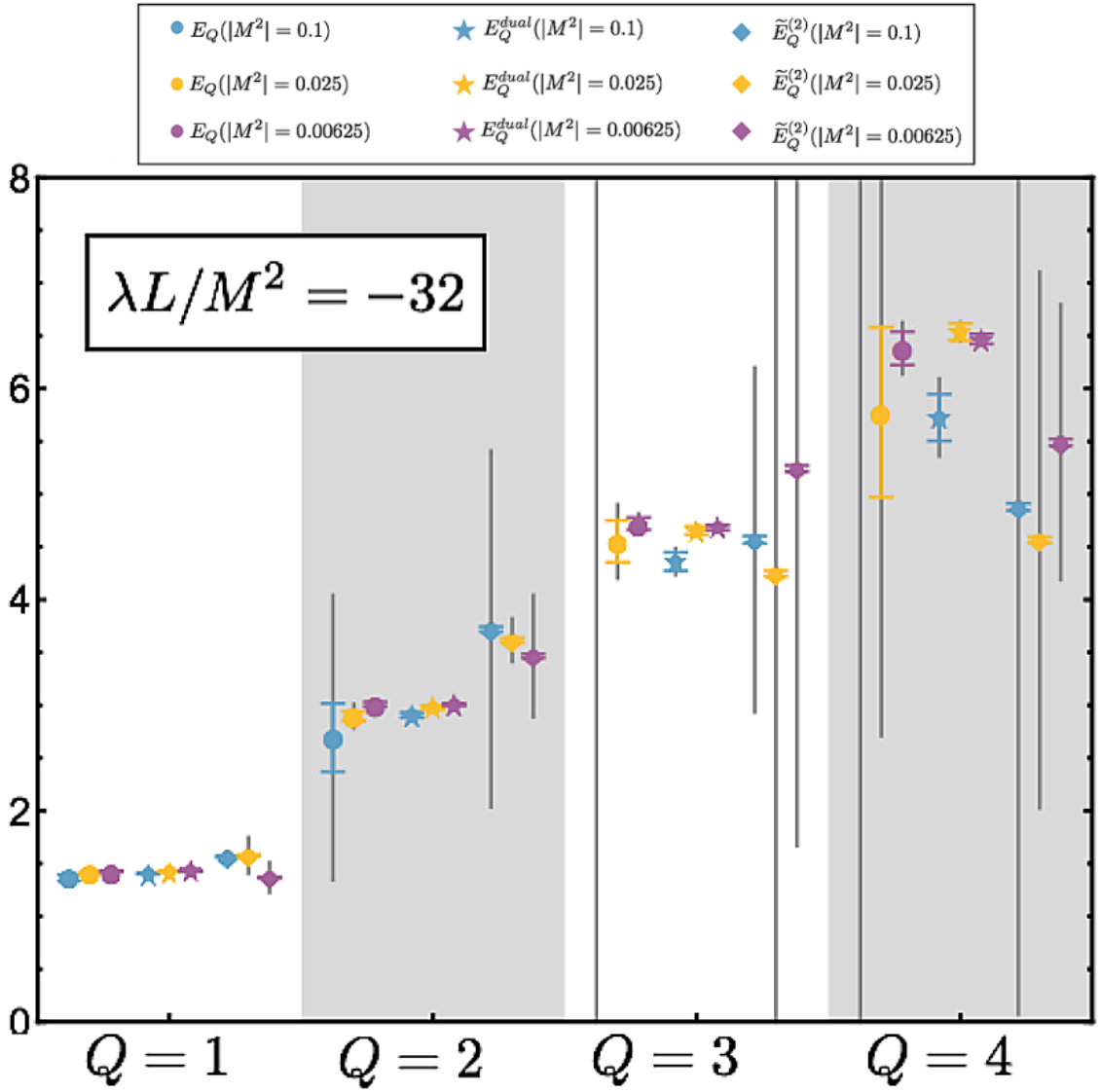


Spectrum Results



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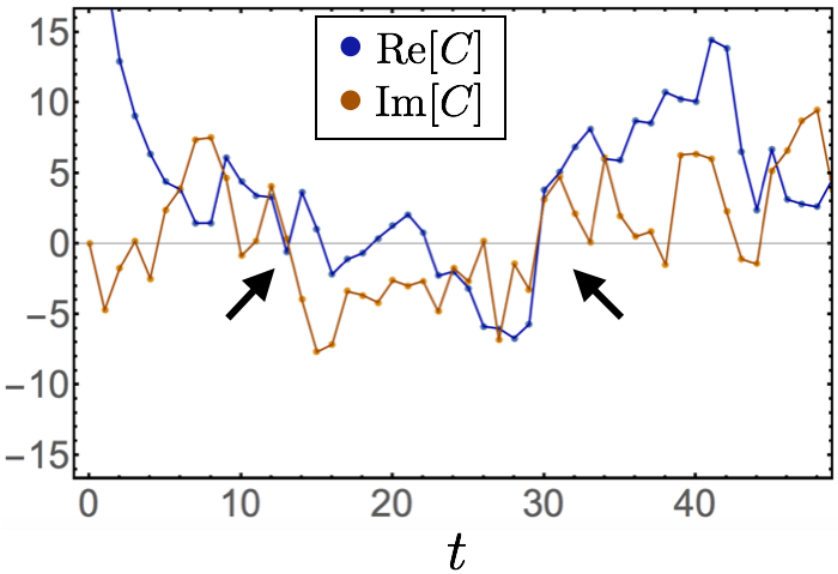
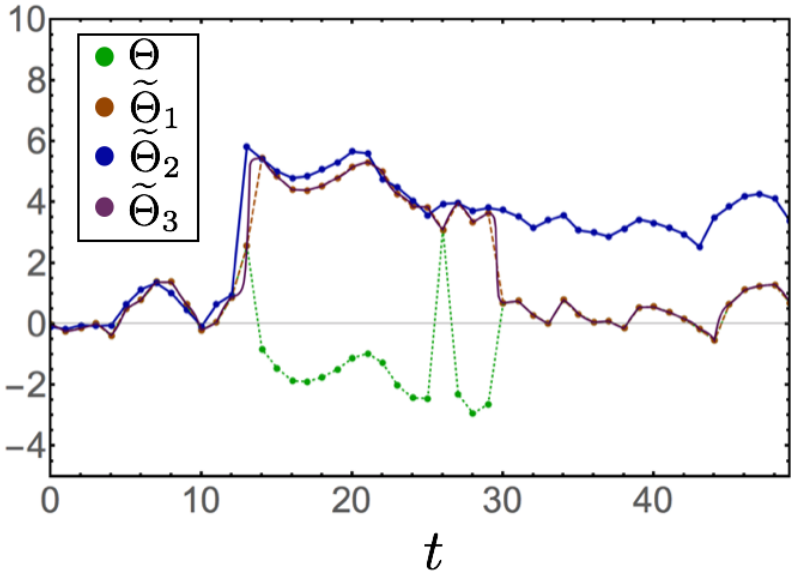
Large phase jumps cause scheme sensitivity

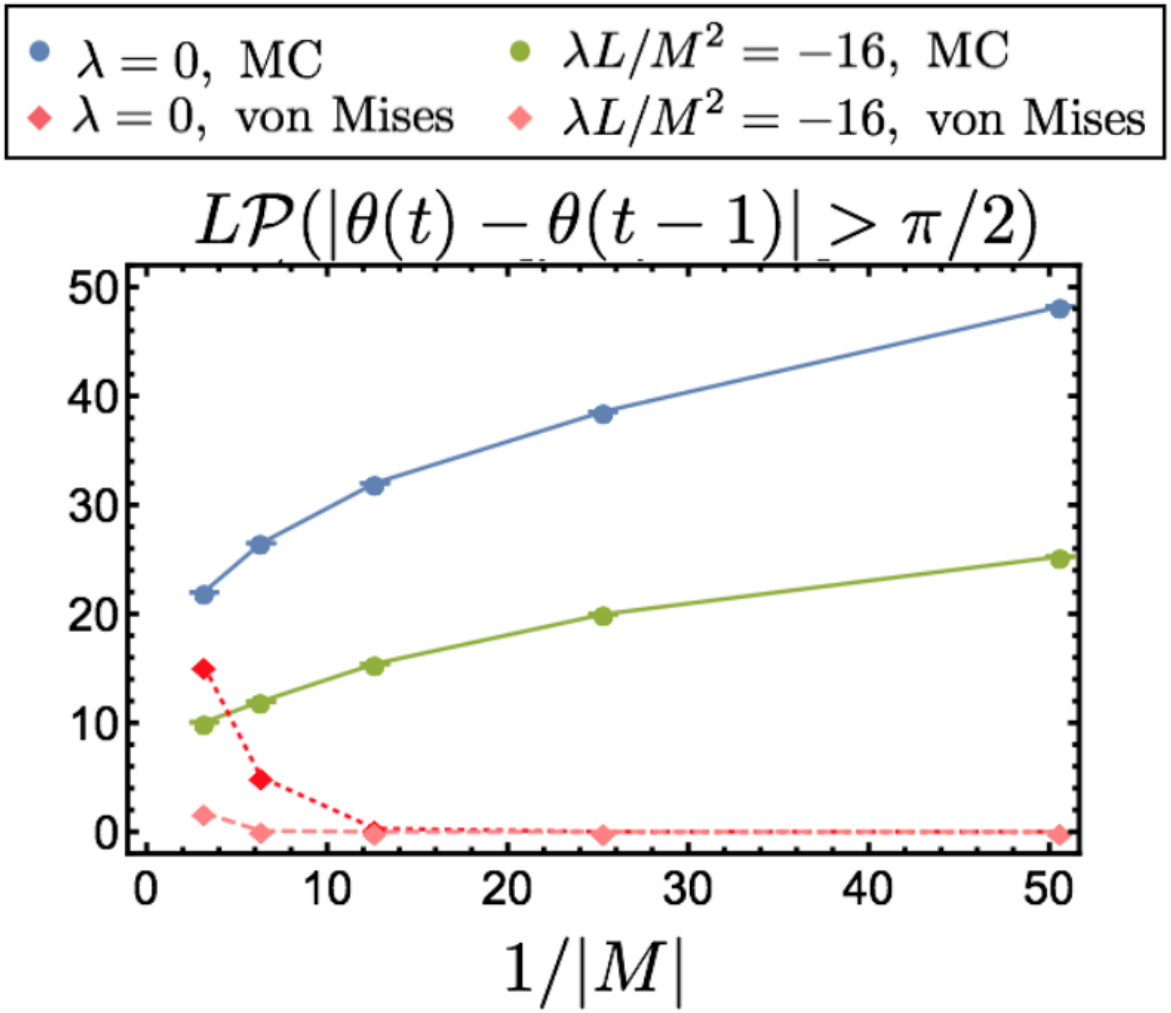
- Slowly drifting phases: insensitive to scheme.
- Quickly drifting phases: 2π differences in all subsequent points.





Large phase jumps cause scheme sensitivity

Frequency of phase jumps increase!



Sensitivity in higher dimensions

- 2D and higher enables unwrapping along multiple paths.
- Scheme dependent fluctuations in regions of unwrapped phase require large phase jumps along entire surface.

Unlike in 0+1D, expect no $O(V)$ fluctuations in the $a \to 0$ limit for higher dimensions. Scheme dependence might be removable as a discretization effect.

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1. Imaginary pieces of correlators contain physics.
2. Unwrapping performs an unbiased transform of the distribution:

$$\langle C(t) \rangle \equiv \langle e^{\mathcal{R}(t)+i\Theta(t)} \rangle = \langle e^{\mathcal{R}(t)+i\widetilde{\Theta}(t)} \rangle$$

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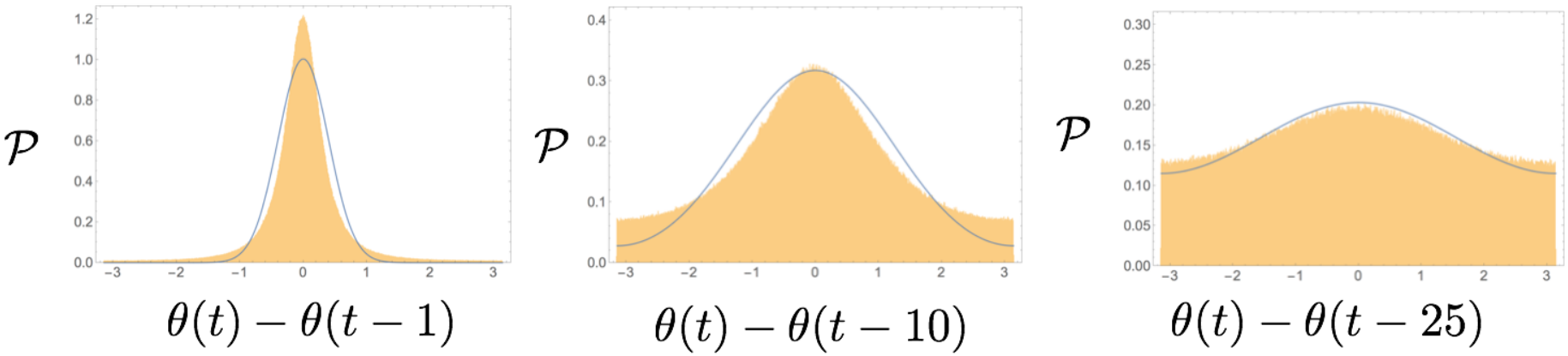
Questions?

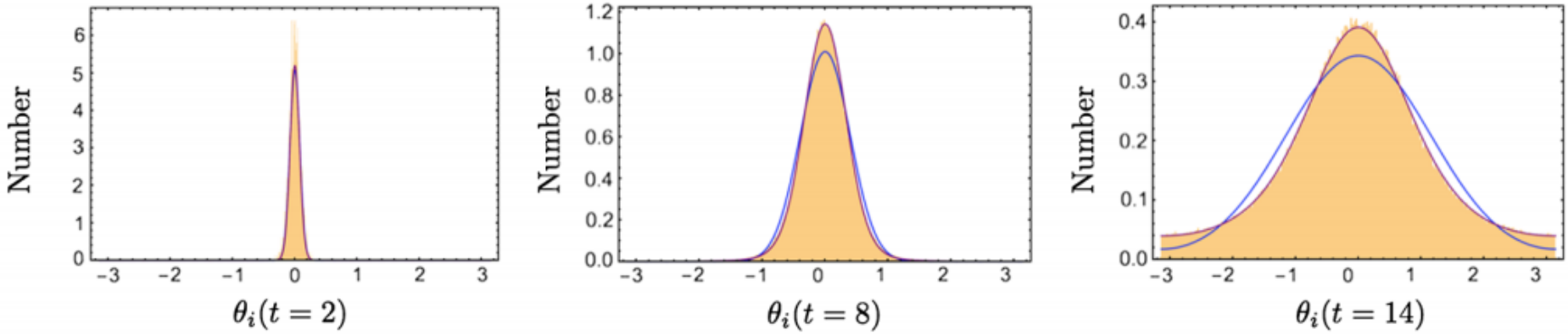
Backup Slides

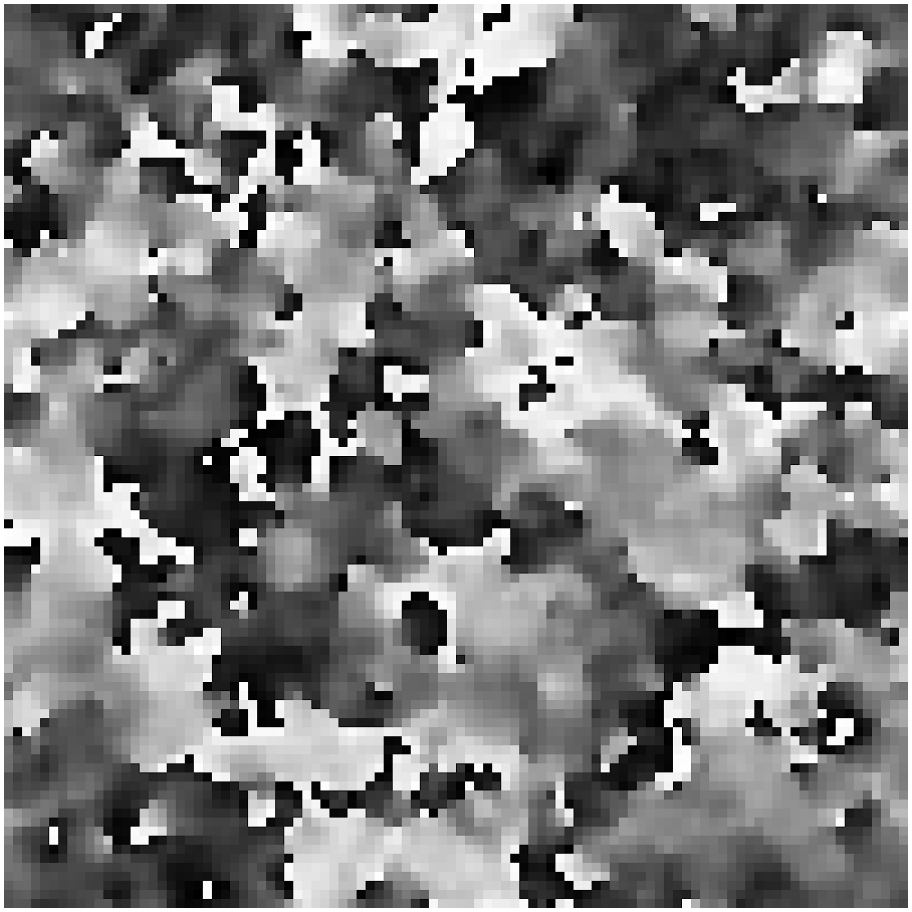
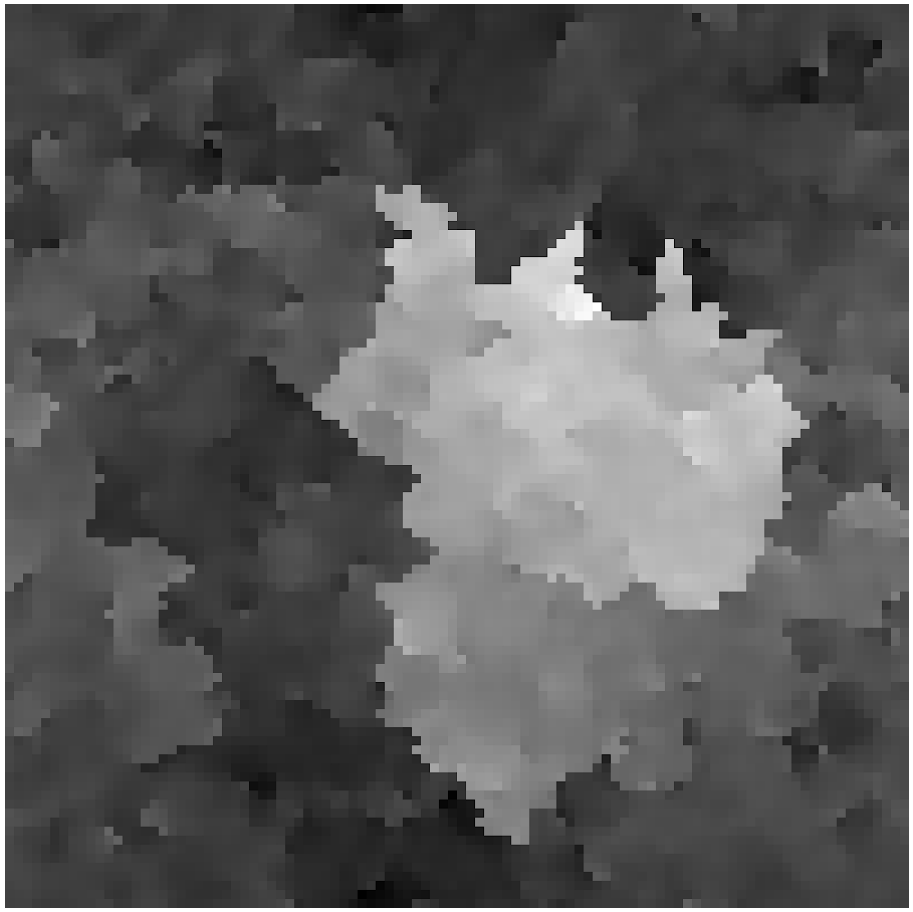
How Gaussian?

0+1D Scalar Theory



LQCD Baryons [Wagman and Savage, PRD 96 (2017) 114508]



Multi-dim Outlook

Topological defects need to be considered. Real physics or simply a technical pathology of compact variables?