Universität Bielefeld

CRC-TR 211
Strong-interaction matter under extreme conditions

Exploring the QCD Phase diagram with imaginary chemical potential with HISQ action

Jishnu Goswami

Collaborators: Frithjof Karsch, Christian Schmidt, and Anirban Lahiri

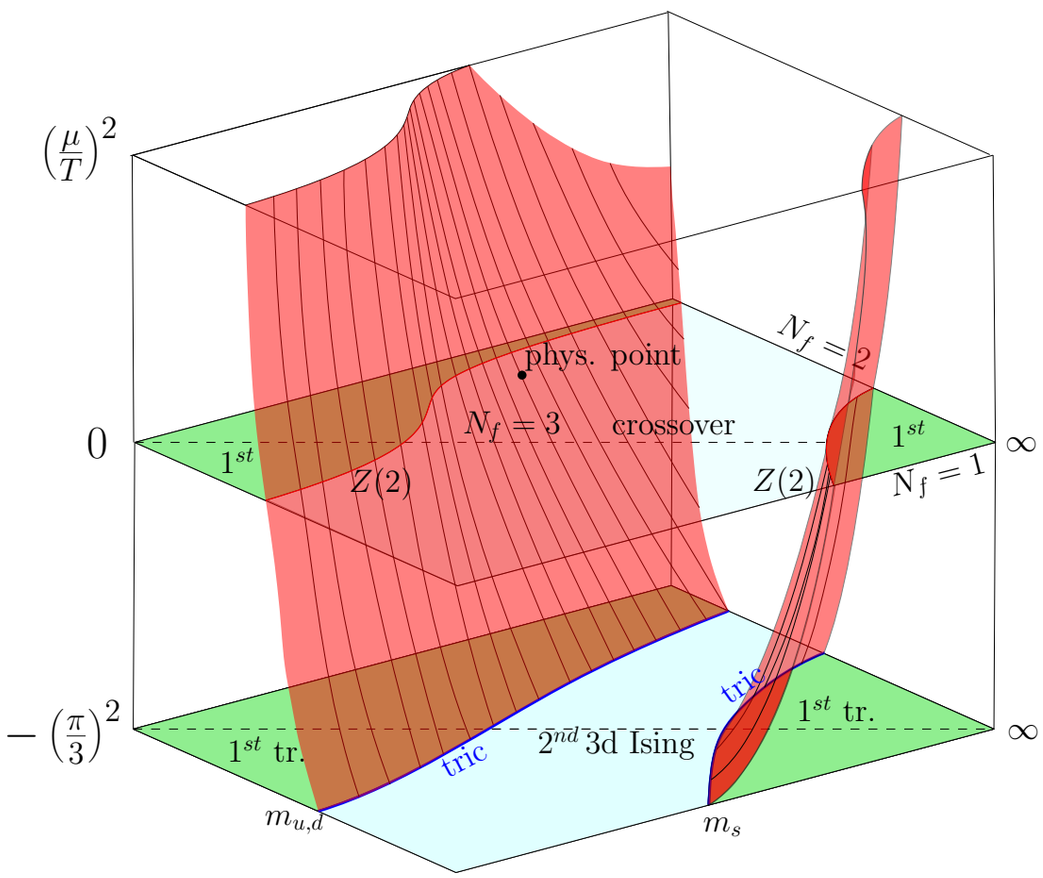
LATTICE 18
East Lansing, MI, USA

Plan of the talk

- Introduction
- Status on chiral phase transition with HISQ at μ =0
- Status on chiral phase transition in the RW plane.
- Results and discussions.

Introduction

Central Question:
Nature of the chiral symmetry restoring transition at μ=0 at the chiral limit??



Possible scenario of extended 3d Columbia plot

Does a 1st order chiral symmetry restoring transition exist at μ=0 below a certain critical quark mass (m_{cri}) ??

The 1st order region is expected to be largest in the RW plane($\mu/T=(2k+1)\pi/3$). Thus critical mass in the RW plane puts a bound on the critical mass at μ=0.

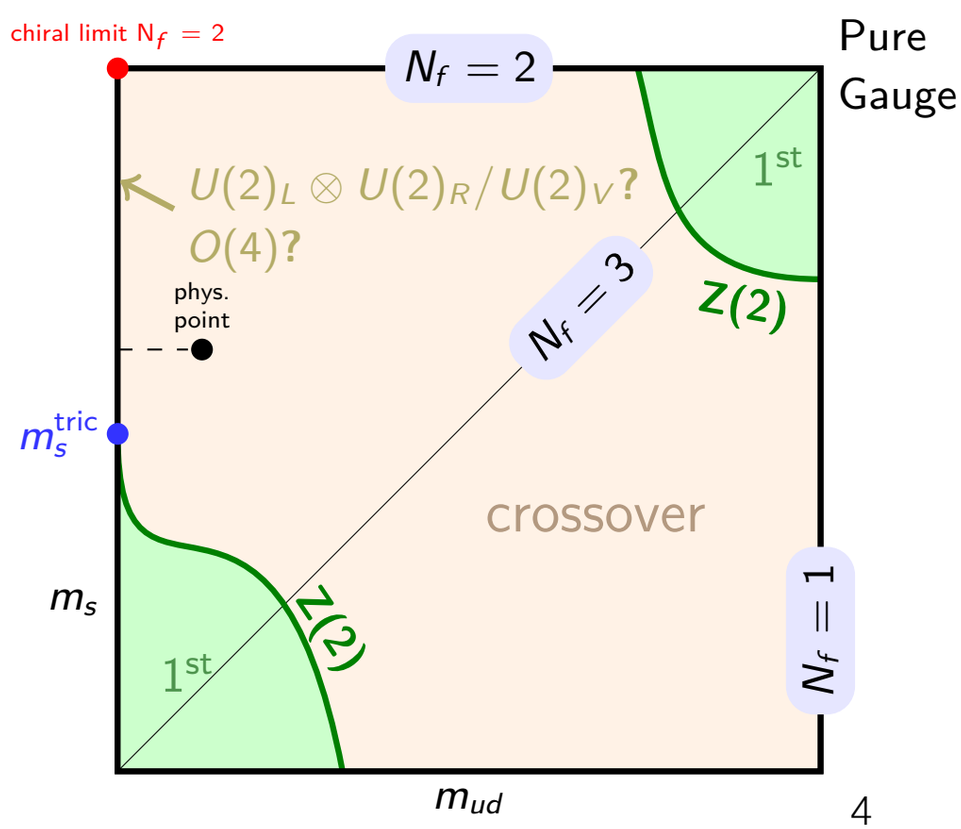
O. Philipsen and C. Pinke. Phys. Rev. D93, 114507, 2016.

Chiral transition for zero chemical potential with HISQ

- HotQCD results on chiral phase transition, [$\mu = 0$]

N_f=2+1 : No hint of 1st order phase transition for m_π >55 MeV. chiral transition is most likely 2nd order O(N) rather than Z(2).

A. Lahiri et. al. , QM 2018, arXiv:1807.05727



N_f=3 : 1st order phase transition ruled out for 230 MeV > m_π >80 MeV. Bound on critical pion mass is given as, $m_\pi^{cr} \lesssim 50$ MeV from the scaling analysis.

Bazavov et. al. PRD 95, 074505 (2017)

Studies in the RW plane

Mostly with unimproved actions

Possible scenario of RW end point
—plenary by S. Mukherjee

$m_\pi > 1$ GeV, for the 'heavy quark mass RW transition'

'small quark mass RW transition' $(N_f = 2)$

Standard staggered action: $m_\pi \sim 400$ MeV (N_τ=4)

Standard Wilson action: $m_\pi \sim 930$ MeV(N_τ=4)

$m_\pi \sim 680$ MeV (N_τ=6)

➡ 1st order end point (of the line of 1st order RW transitions) exist already for $\mu/T=\pi/3$ and $m_{cri} > m_{phy}$.

- ☆ The results are strongly fermion discretization scheme and cut-off(N_τ) dependent.

P. de Forcrand et. al, PRL 105, 152001(2010), Owe Philipsen et. al, PRD 89, 094504(2014), Christopher Czaban et al, PRD 93, 054507 (2016)

Studies in the RW plane

Very recent studies with improved actions,

- Stout improved staggered fermions(N_f=2+1): At the physical quark mass point($m_\pi \sim 135$ MeV) a 2nd order transition in the 3d-Ising universality class happens instead of a 1st order at the RW endpoint.

C. Bonati et. al,PRD 93, 074504 (2016)

No 1st order end point (of the line of 1st order RW transitions) for m_π >50 MeV.

C. Bonati et. al,arXiv:1807.02106 [hep-lat]

- HISQ(N_f=2): Order of the phase transition at physical point is not clear(large cut-off effects) .

L.K.Wu, et al. PRD 97,114514(2018)

Studies with HISQ in the RW plane

Action, $$Z(T,\mu) = \int [\mathscr{D}U] det[M_{ud}(\mu_f)]^{1/2} det[M_s(\mu_f)]^{1/4} \exp[-S_G]$$

$$M_q = D_{HISQ}(\mu_f) + m_q$$

Simulation details,

N_σ	N_τ	$\frac{m_l}{m_s}$	m_π (MeV)
8	4	1/27	135
12	4	1/27	135
16	4	1/27,1/40, 1/60	135, 110, 90
24	4	1/27,1/40	135, 110

$$N_f = 2 + 1, \ \frac{\mu}{T} = \frac{\pi}{3}$$

We vary β in the range [5.850-6.038], corresponds to, $\sim T_c \pm 0.1 T_c$

Generally we generated 20k trajectory per β value away from β_c and 80k trajectory near β_c

We work on the 2nd RW plane

Ising endpoint of a first order line

$$H_{eff}(t,\xi) = t\mathscr{E} + h\mathscr{M}$$

Effective Ising Hamiltonian which defines the universal critical behaviour of the system

temperature like field

energy like operator

magnetic field like

magnetization like operator (order parameter)

under Z(2) transformation,

$$\mathscr{E} \to \mathscr{E}$$
$$\mathscr{M} \to -\mathscr{M}$$

Corresponding critical behaviour of QCD in 2nd RW plane [Z(2) transformation],

$$Im\ L \to -Im\ L$$ order parameter

$$Re\ L \to Re\ L$$ energy like

i.e. at $\mu = \mu_{RW}$

$$\lim_{h\to 0}\lim_{V\to\infty}\langle Im\ L\rangle \equiv \lim_{V\to\infty}\langle |Im\ L|\rangle = \begin{cases} 0, & \text{if } \beta < \beta_c \\ \text{non-zero}, & \text{if } \beta > \beta_c \end{cases}$$

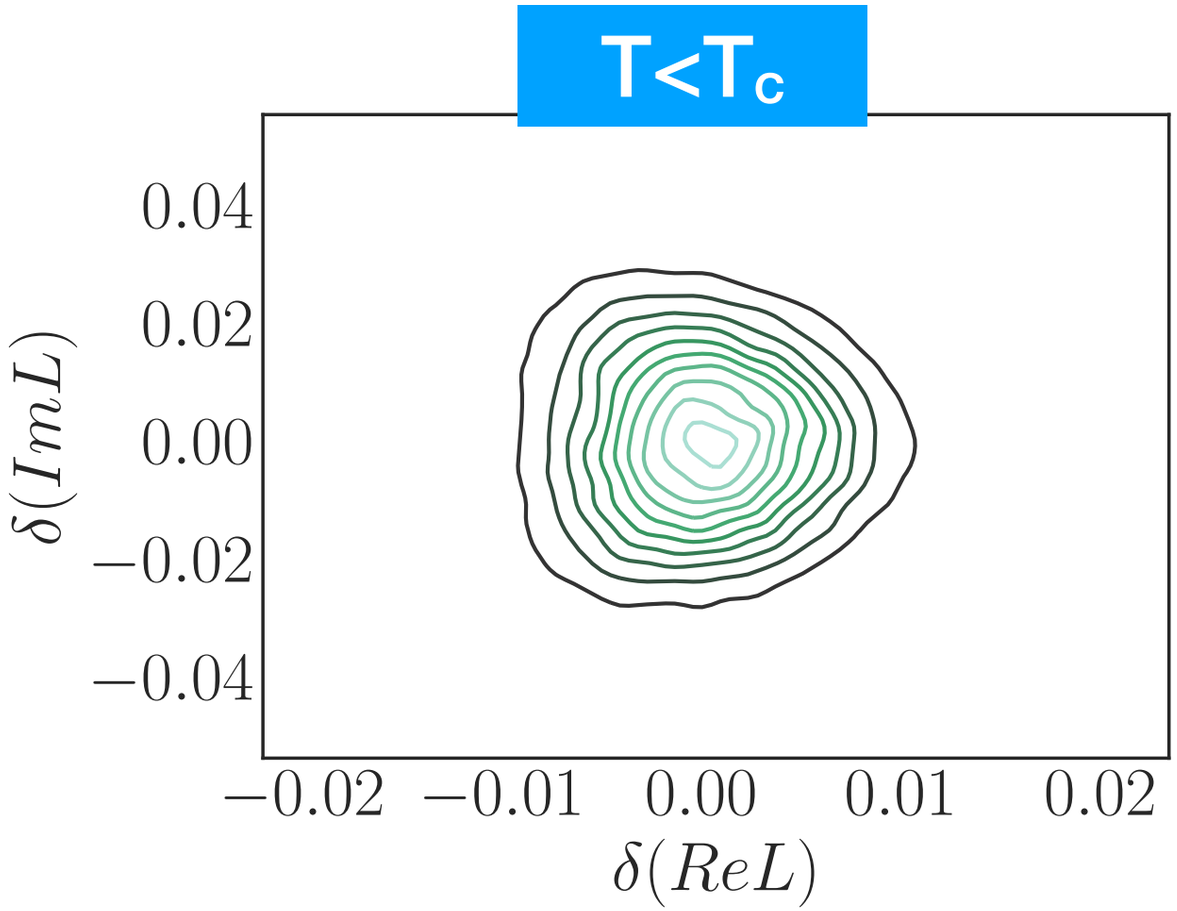
Corresponding critical behaviour of QCD at 2nd RW plane [Z(2) transformation],

$$Im\ L \to -\ Im\ L$$ → order parameter

$$Re\ L \to Re\ L$$ → energy like

i.e. at $\mu = \mu_{RW}$

$$\lim_{h\to 0}\lim_{V\to\infty}\langle Im\ L\rangle \equiv \lim_{V\to\infty}\langle |Im\ L|\rangle = \begin{cases} 0, & \text{if } \beta < \beta_c \\ \text{non-zero}, & \text{if } \beta > \beta_c \end{cases}$$



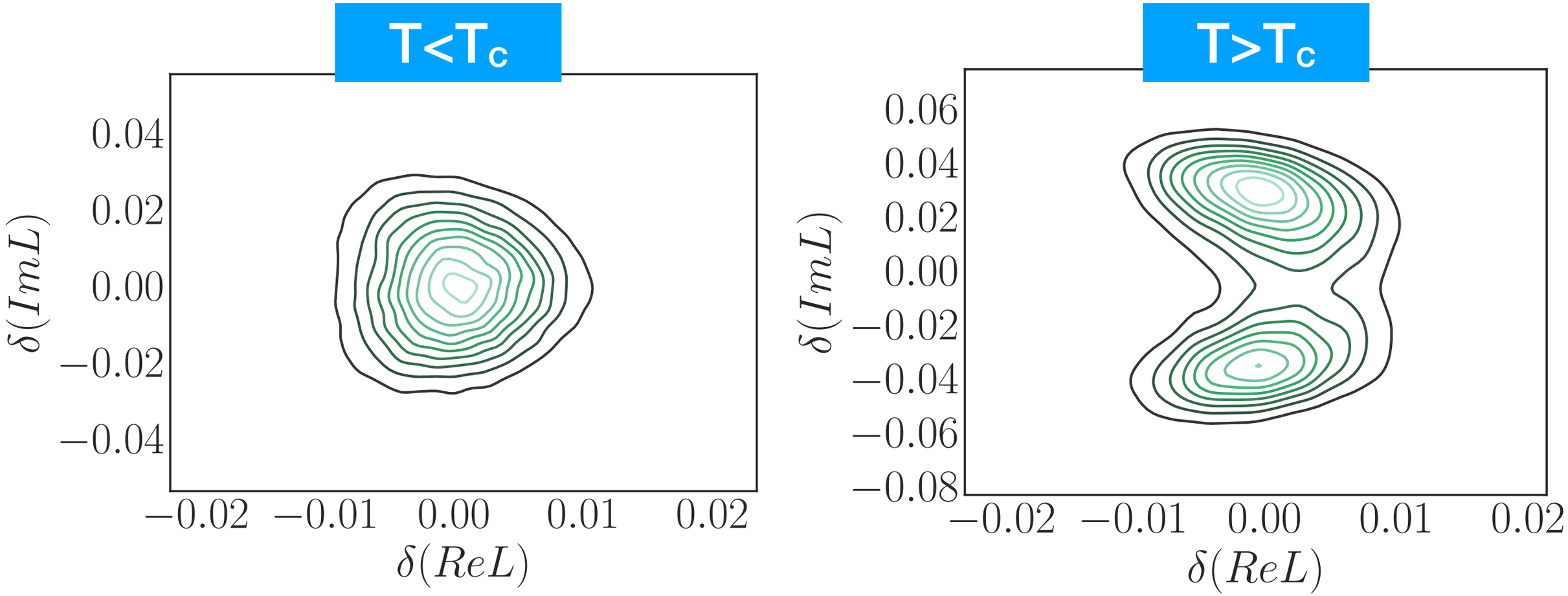
Corresponding critical behaviour of QCD at 2nd RW plane [Z(2) transformation],

$$Im\ L \to -\ Im\ L$$ → order parameter

$$Re\ L \to Re\ L$$ → energy like

i.e. at $\mu = \mu_{RW}$

$$\lim_{h\to 0}\lim_{V\to\infty}\langle Im\ L\rangle \equiv \lim_{V\to\infty}\langle |Im\ L|\rangle = \begin{cases} 0, & \text{if } \beta < \beta_c \\ \text{non-zero}, & \text{if } \beta > \beta_c \end{cases}$$



Corresponding critical behaviour of QCD at 2nd RW plane [Z(2) transformation],

$$Im\ L \to -\,Im\ L$$ → order parameter

$$Re\ L \to Re\ L$$ → energy like

i.e. at $\mu = \mu_{RW}$

$$\lim_{h\to 0}\lim_{V\to\infty}\langle Im\ L\rangle \equiv \lim_{V\to\infty}\langle |Im\ L|\rangle = \begin{cases} 0, & \text{if } \beta < \beta_c \\ \text{non-zero}, & \text{if } \beta > \beta_c \end{cases}$$

Finite size scaling and Z(2) universality class

Free energy,

$$f = f_{ns} + b^{-d} f_s(b^{y_t} u_t, b^{y_h} u_h, b^{-1} N_\sigma)$$

Responsible for the universal critical behaviour

$$t = \frac{T - T_c}{T_c} \sim \beta - \beta_c$$

near, $T \to T_c$ $\quad u_t \sim c_t t, u_t \sim c_h h$

universal functions →

$$< O > = (\ldots.)\frac{\partial}{\partial h} f_s(\ldots)|_{h \to o}$$ order parameter

$$\chi_h = (\ldots.)\frac{\partial^2}{\partial h^2} f_s(\ldots)|_{h \to o}$$ susceptibility of op

$$\chi_t = (\ldots.)\frac{\partial^2}{\partial t^2} f_s(\ldots)|_{h \to o}$$ specific heat

$$\chi_t = z_2 N_\sigma^{\alpha/\nu} f_t(z_0 t N_\sigma^{1/\nu})$$

$$\chi_h = z_1 N_\sigma^{\gamma/\nu} f_h(z_0 t N_\sigma^{1/\nu})$$

Susceptibility of $|Im\ L|$

Susceptibility of $|Re\ L|$

Finite size scaling and Z(2) universality class

Free energy,

$$f = f_{ns} + b^{-d} f_s(b^{y_t} u_t, b^{y_h} u_h, b^{-1} N_\sigma)$$

Responsible for the universal critical behaviour

$$t = \frac{T - T_c}{T_c} \sim \beta - \beta_c$$

near, $T \to T_c$ $\quad u_t \sim c_t t, u_t \sim c_h h$

universal functions →

$$< O > = (\ldots.)\frac{\partial}{\partial h} f_s(\ldots)|_{h \to o}$$ order parameter

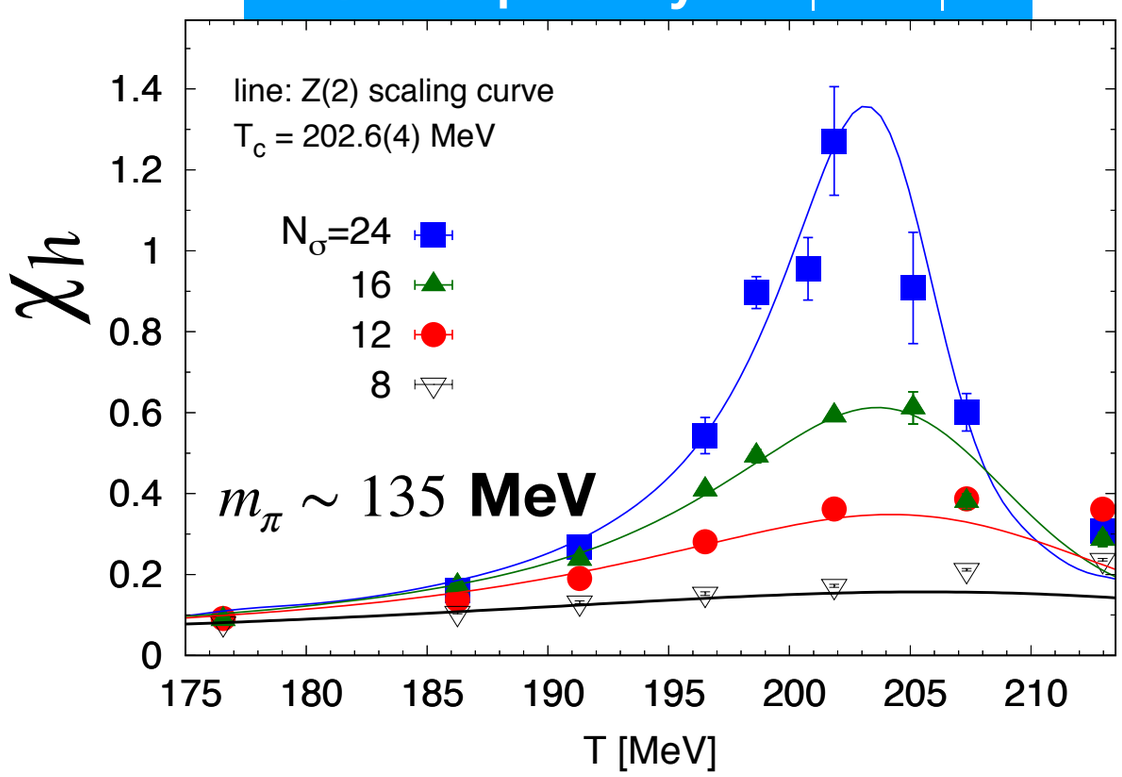
$$\chi_h = (\ldots.)\frac{\partial^2}{\partial h^2} f_s(\ldots)|_{h \to o}$$ susceptibility of op

$$\chi_t = (\ldots.)\frac{\partial^2}{\partial t^2} f_s(\ldots)|_{h \to o}$$ specific heat

$$\chi_t = z_2 N_\sigma^{\alpha/\nu} f_t(z_0 t N_\sigma^{1/\nu})$$

$$\chi_h = z_1 N_\sigma^{\gamma/\nu} f_h(z_0 t N_\sigma^{1/\nu})$$

Susceptibility of $|Im\ L|$



Susceptibility of $|Re\ L|$

Finite size scaling and Z(2) universality class

Free energy,

$$f = f_{ns} + b^{-d} f_s(b^{y_t} u_t, b^{y_h} u_h, b^{-1} N_\sigma)$$

Responsible for the universal critical behaviour

$$t = \frac{T - T_c}{T_c} \sim \beta - \beta_c$$

near, $T \to T_c$ $\quad u_t \sim c_t t, u_t \sim c_h h$

universal functions →

$$< O > = (\ldots.)\frac{\partial}{\partial h} f_s(\ldots)|_{h\to o}$$ order parameter

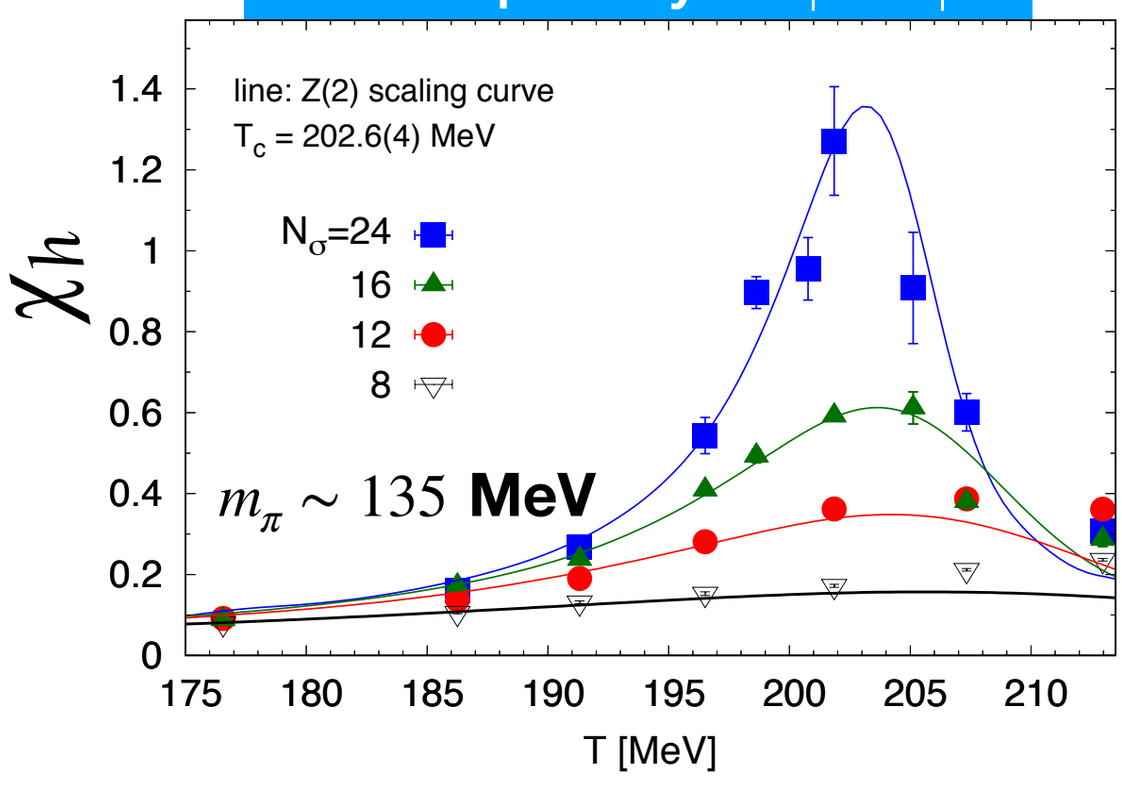
$$\chi_h = (\ldots.)\frac{\partial^2}{\partial h^2} f_s(\ldots)|_{h\to o}$$ susceptibility of op

$$\chi_t = (\ldots.)\frac{\partial^2}{\partial t^2} f_s(\ldots)|_{h\to o}$$ specific heat

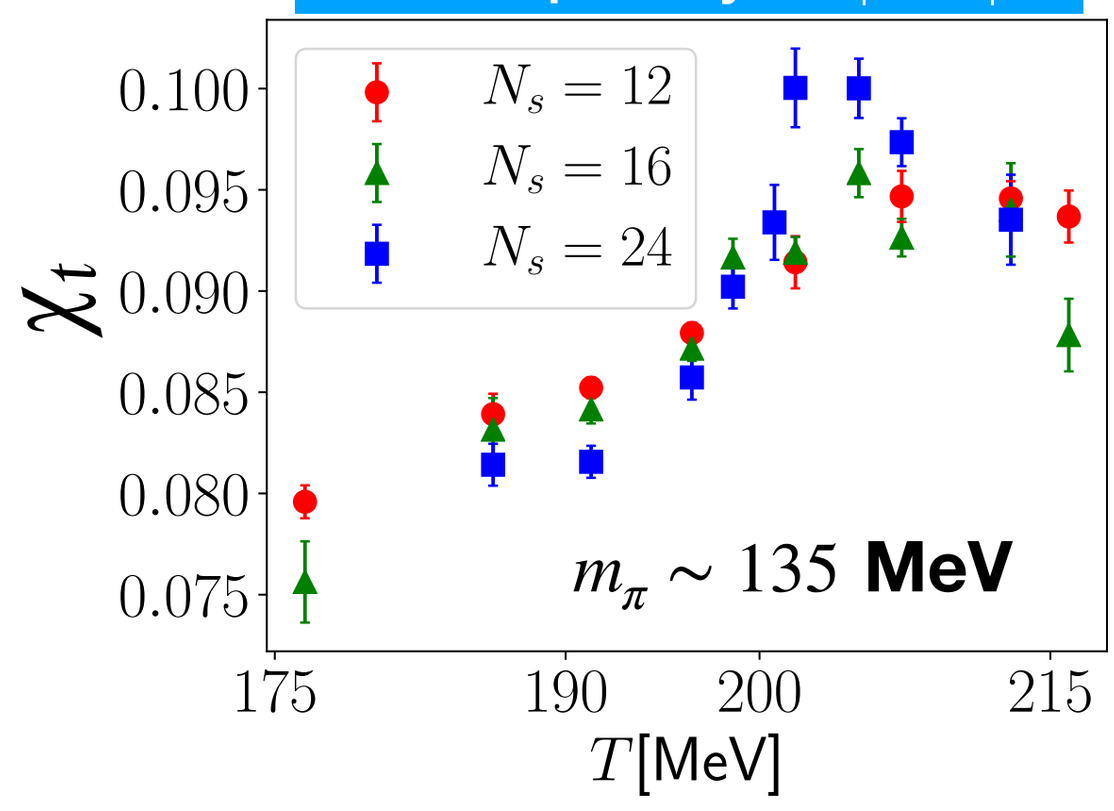
$$\chi_t = z_2 N_\sigma^{\alpha/\nu} f_t(z_0 t N_\sigma^{1/\nu})$$

$$\chi_h = z_1 N_\sigma^{\gamma/\nu} f_h(z_0 t N_\sigma^{1/\nu})$$

Susceptibility of $|Im\ L|$



Susceptibility of $|Re\ L|$



Finite size scaling and Z(2) universality class

Free energy,

$$f = f_{ns} + b^{-d} f_s(b^{y_t} u_t, b^{y_h} u_h, b^{-1} N_\sigma)$$

α, γ, ν are 3d Ising Exponents

Responsible for the universal critical behaviour

$$t = \frac{T - T_c}{T_c} \sim \beta - \beta_c$$

$u_t \sim c_t t, u_t \sim c_h h$ near, $T \to T_c$

"Susceptibility" and "specific heat" scale with corresponding Z(2) finite size universal scaling functions

Finite size scaling and Z(2) universality class

Free energy,

$$f = f_{ns} + b^{-d} f_s(b^{y_t} u_t, b^{y_h} u_h, b^{-1} N_\sigma)$$

α, γ, ν are 3d Ising Exponents

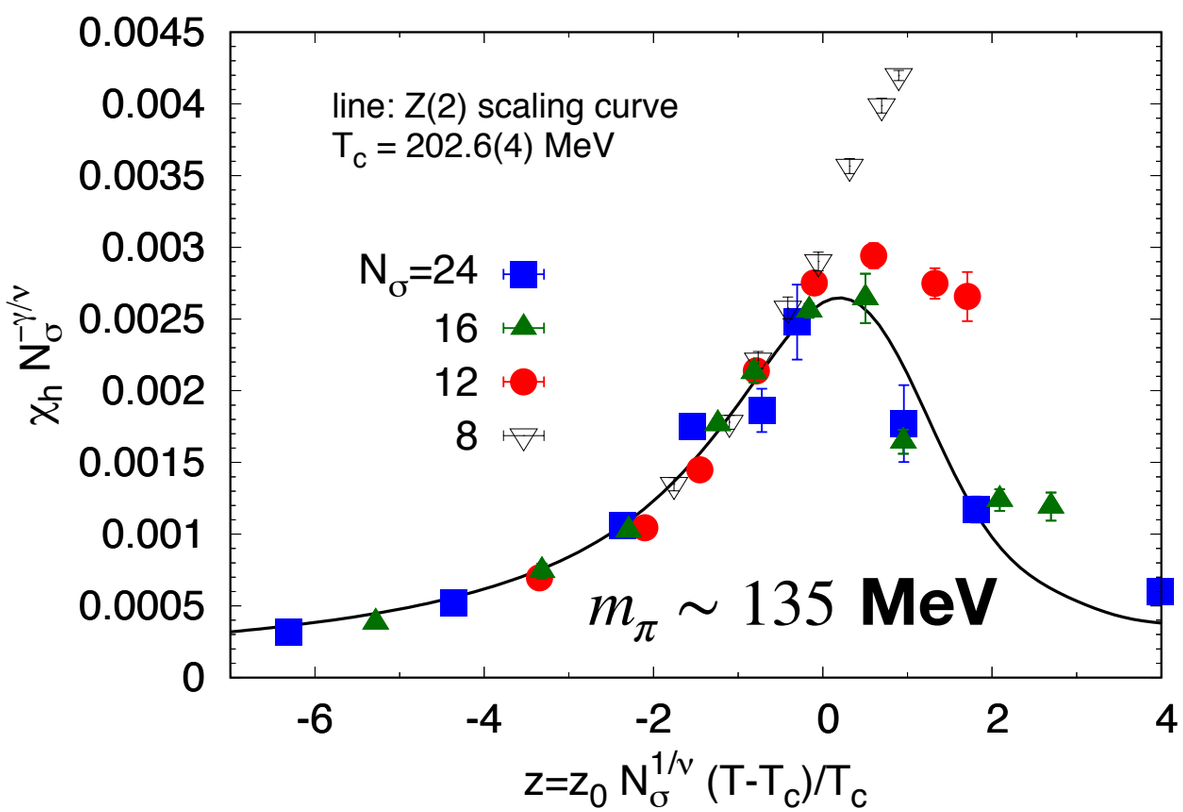
Responsible for the universal critical behaviour

$$t = \frac{T - T_c}{T_c} \sim \beta - \beta_c$$

$u_t \sim c_t t, u_t \sim c_h h$ near, $T \to T_c$

"Susceptibility" and "specific heat" scale with corresponding Z(2) finite size universal scaling functions

$$\chi_h = z_1 N_\sigma^{\gamma/\nu} f_h(z_0 t N_\sigma^{1/\nu})$$



Finite size scaling and Z(2) universality class

Free energy,

$$f = f_{ns} + b^{-d} f_s(b^{y_t} u_t, b^{y_h} u_h, b^{-1} N_\sigma)$$

α, γ, ν are 3d Ising Exponents

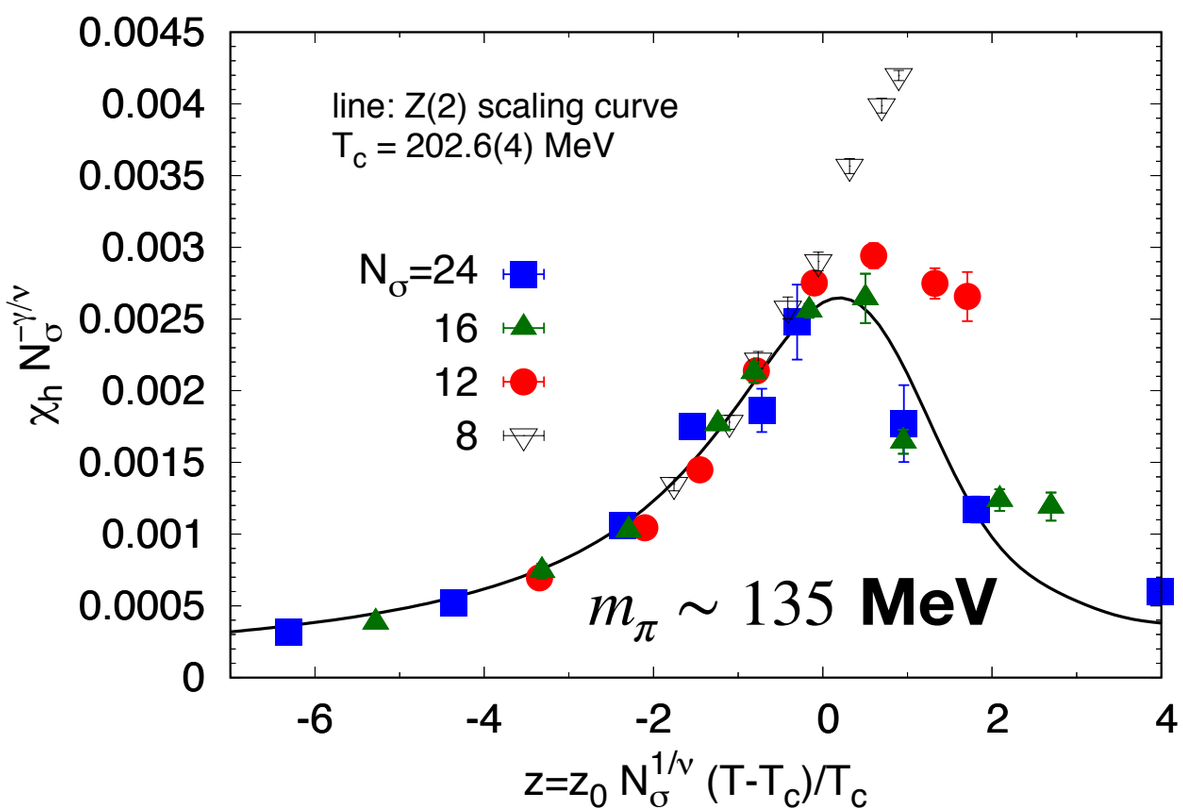
Responsible for the universal critical behaviour

$$t = \frac{T - T_c}{T_c} \sim \beta - \beta_c$$

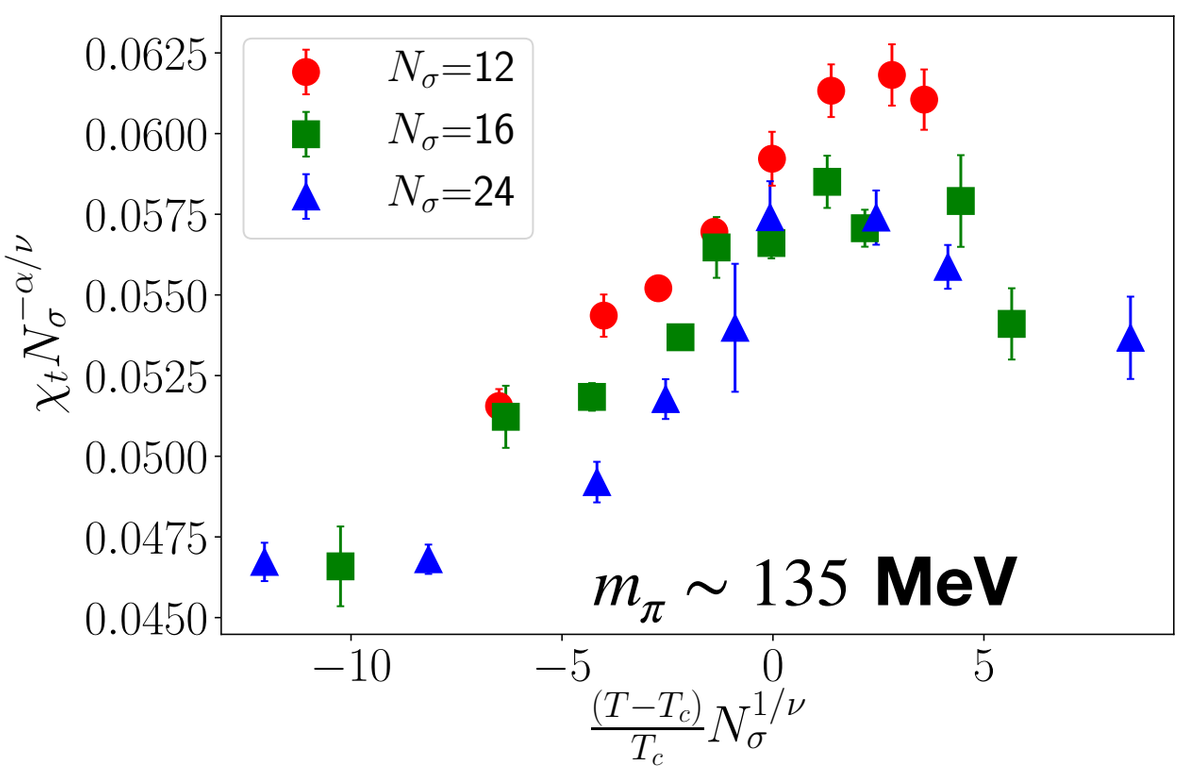
$u_t \sim c_t t, u_t \sim c_h h$ near, $T \to T_c$

"Susceptibility" and "specific heat" scale with corresponding Z(2) finite size universal scaling functions

$$\chi_h = z_1 N_\sigma^{\gamma/\nu} f_h(z_0 t N_\sigma^{1/\nu})$$

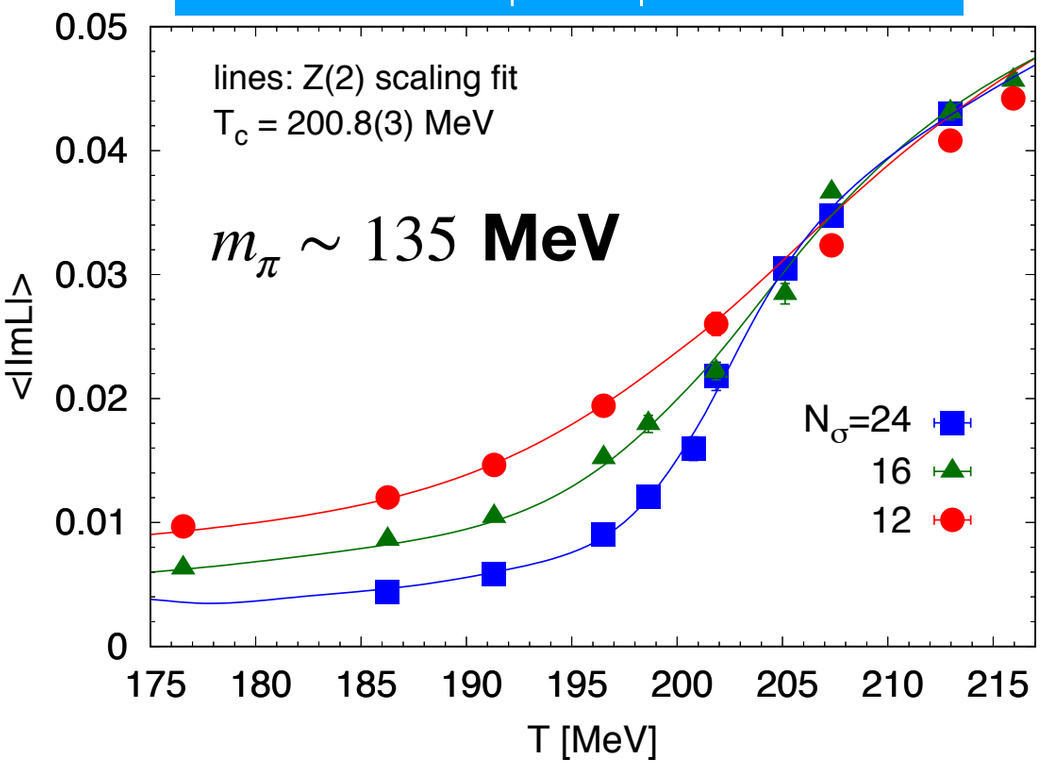


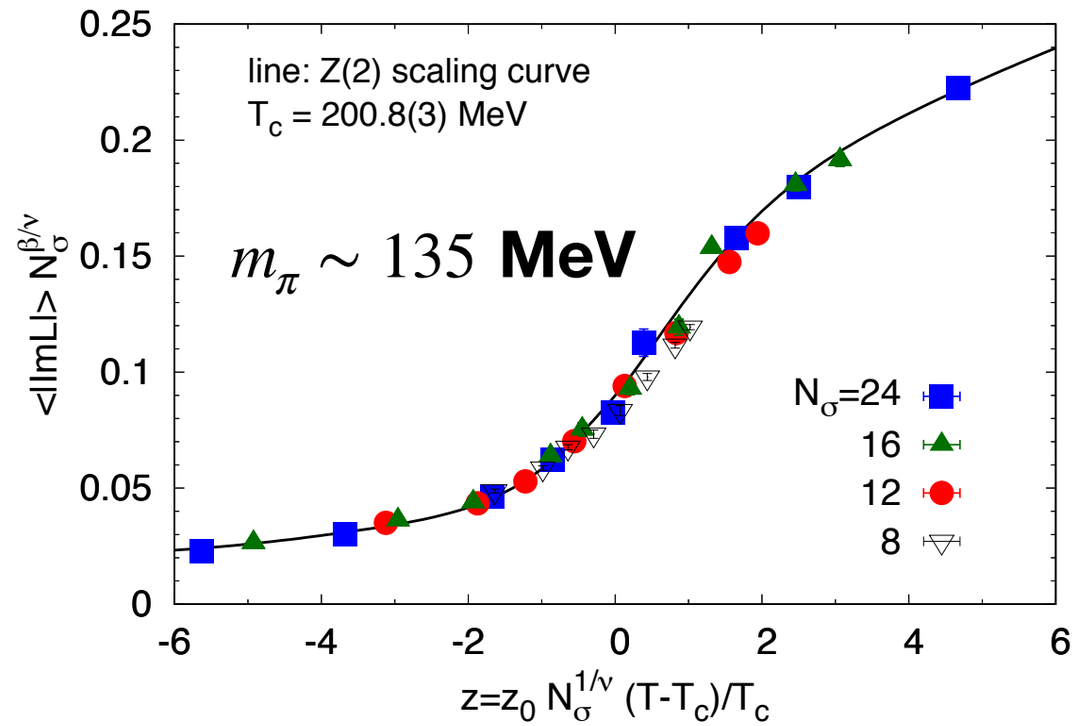
$$\chi_t = z_2 N_\sigma^{\alpha/\nu} f_t(z_0 t N_\sigma^{1/\nu})$$



Binder cumulants of order parameter

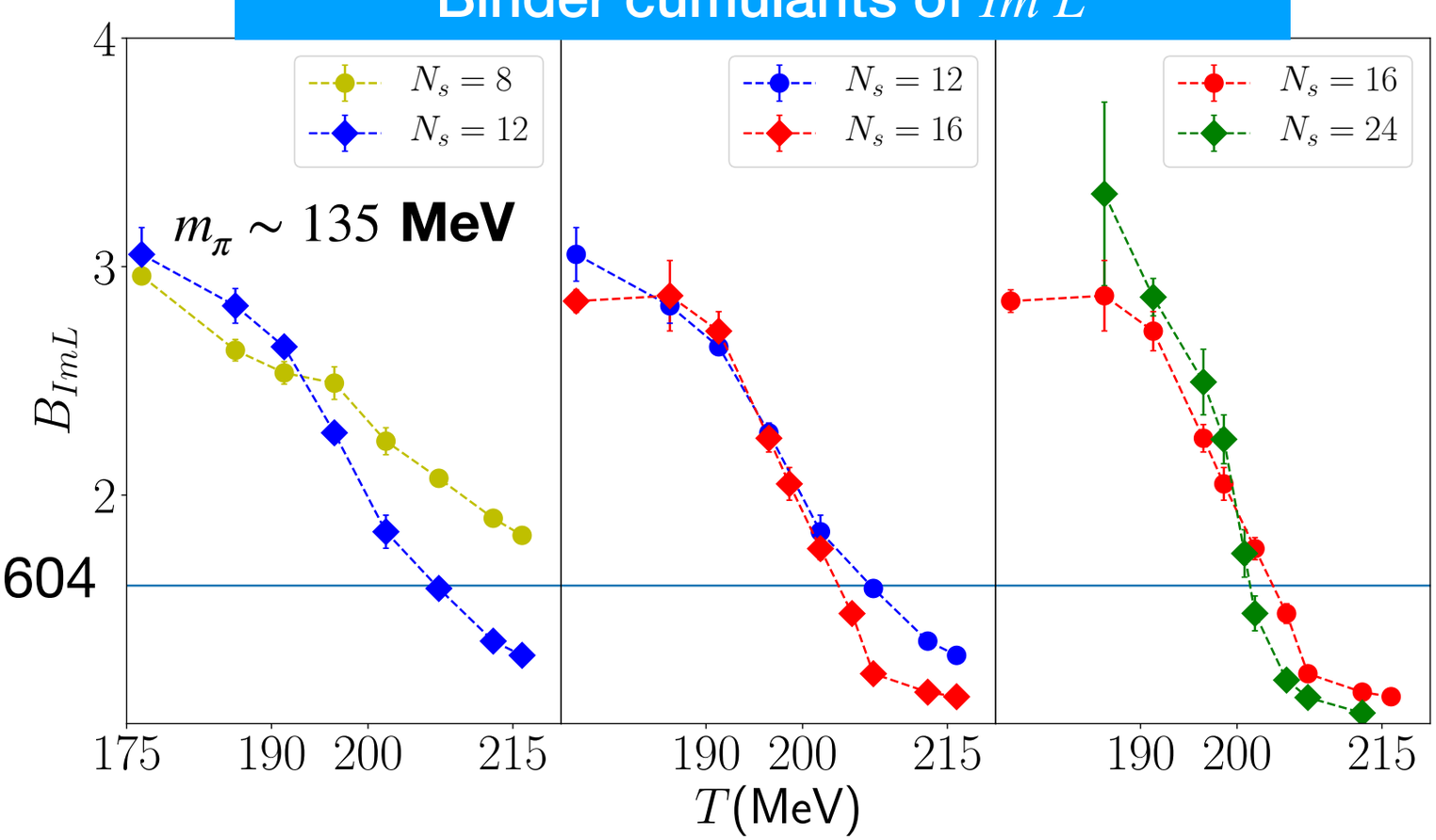
$|Im\ L|$





Universal Z(2) scaling of the order parameter

Binder cumulants of $Im\ L$

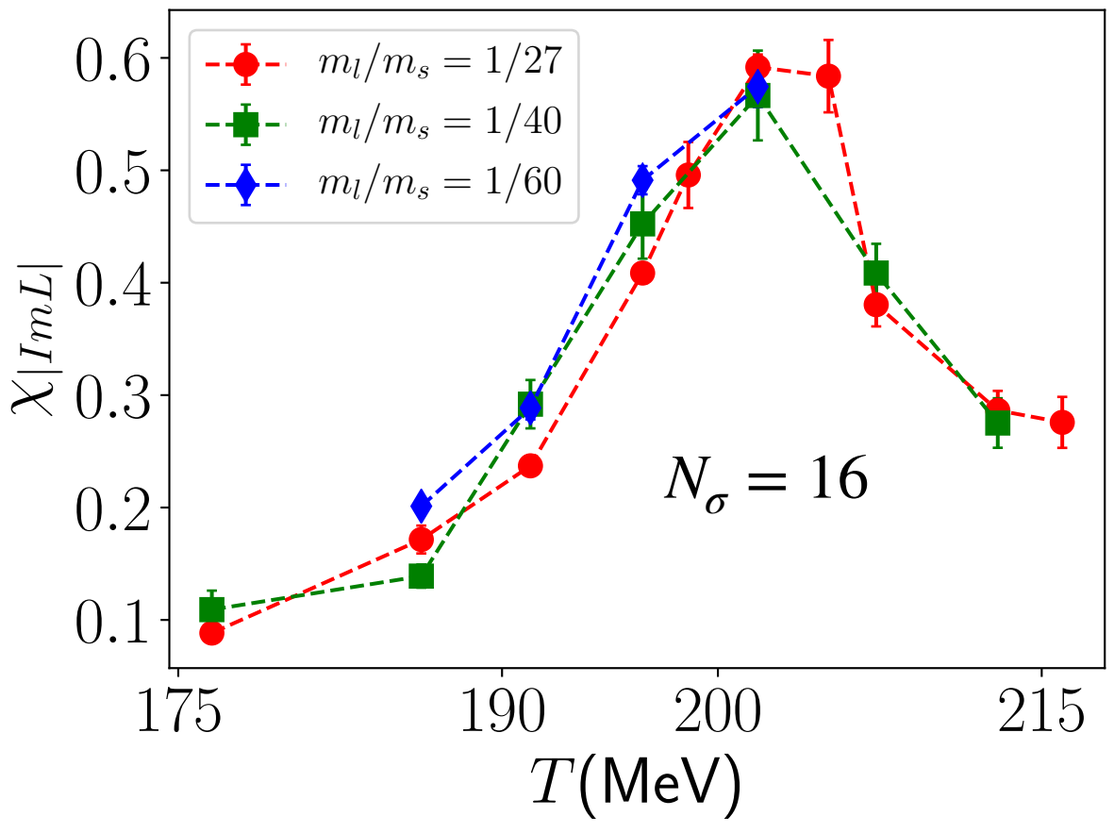


Crossing of the Binder cumulants approaches to the universal value in $V \to \infty$

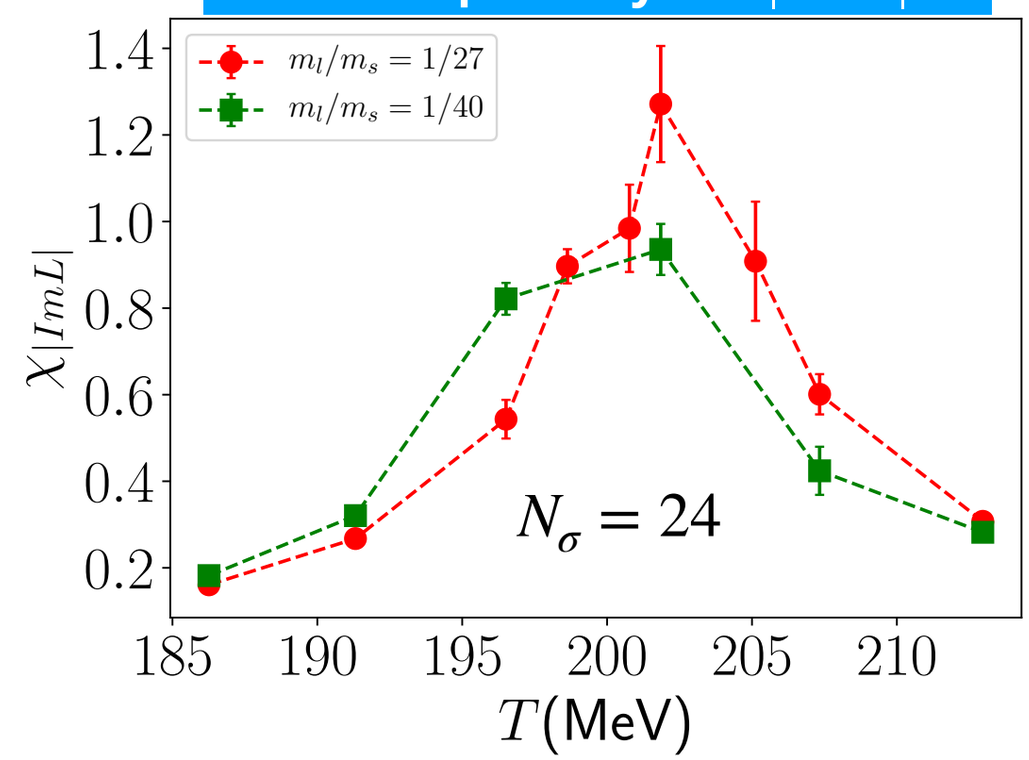
Ising value=1.604

Quark mass dependence of RW transition

Susceptibility of $|Im\ L|$



Susceptibility of $|Im\ L|$

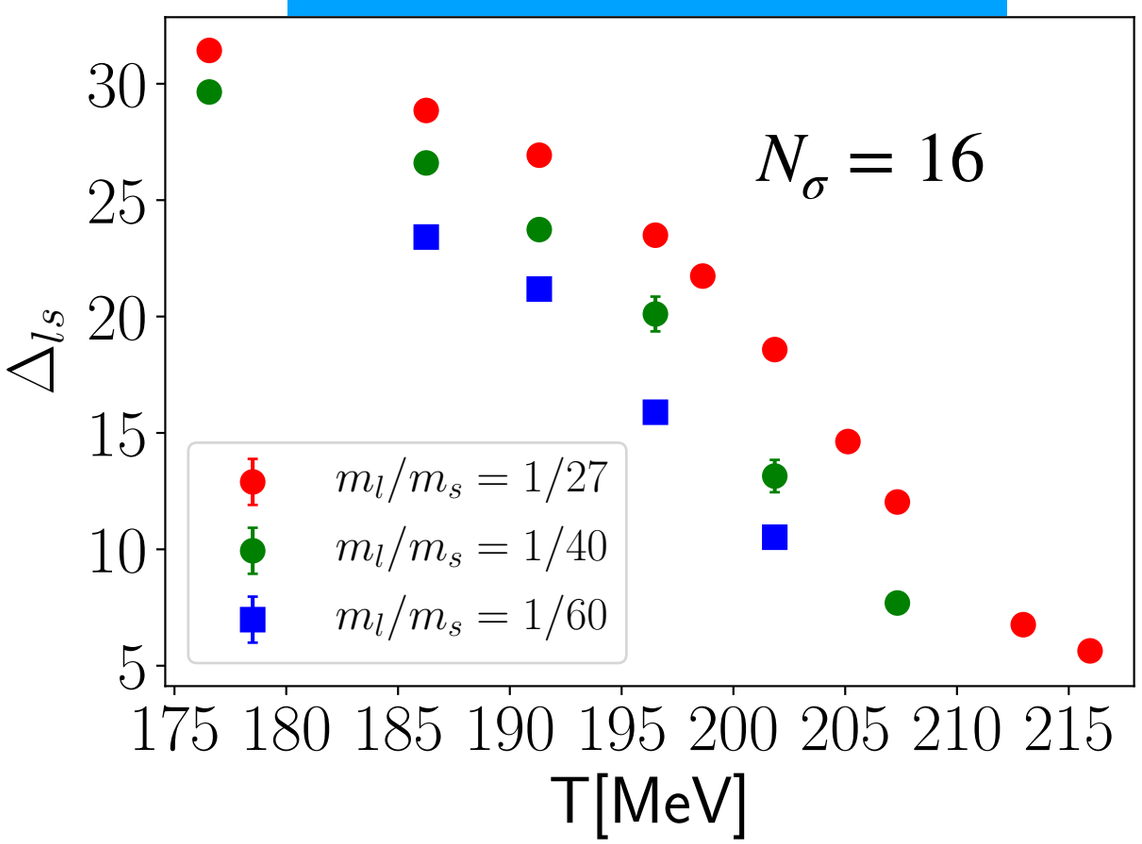


No unusual change in the susceptibility of the order parameter with respect to pion mass upto, $m_\pi \sim 90$ MeV

Order of the transition seems to be unchanged ??

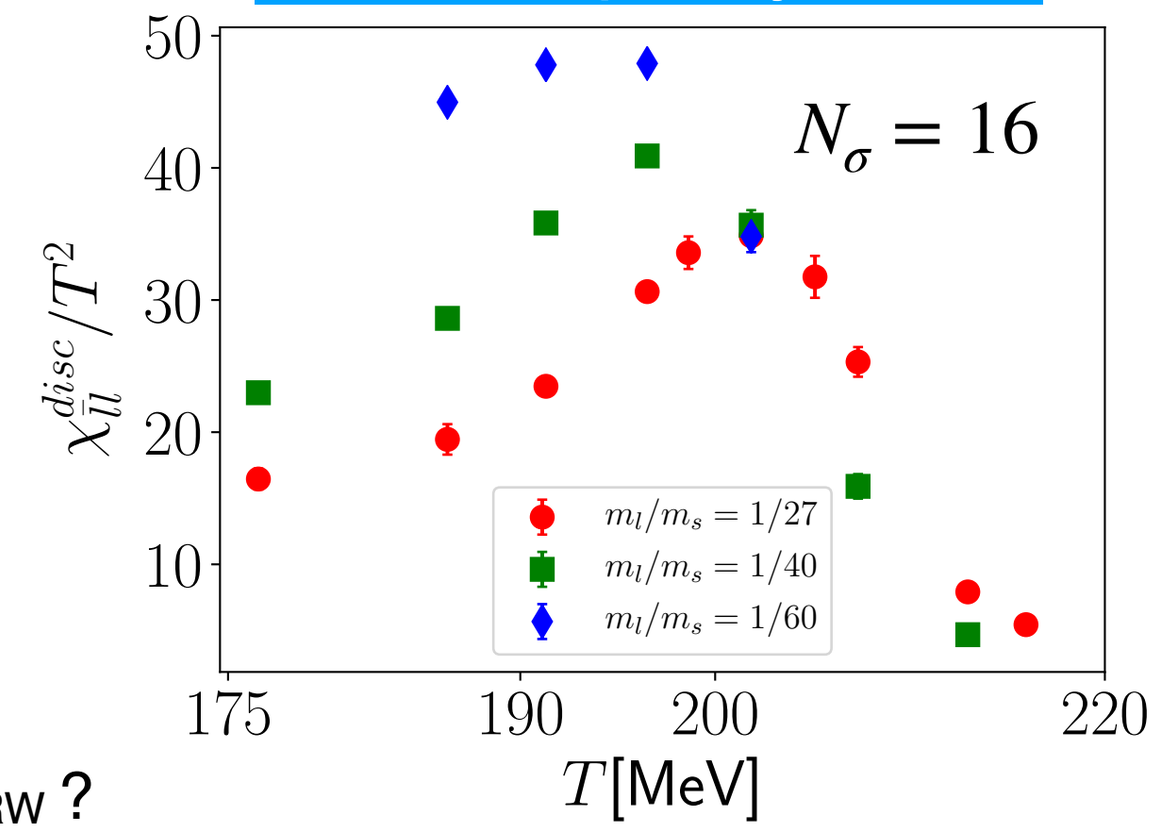
Chiral limit and RW transition

Sub. Chiral condensate



$$\Delta_{ls} = (m_s/f_k^4)\left(\langle \bar{l}l \rangle - (m_l/m_s)\langle \bar{s}s \rangle\right)$$

Susceptibility of $\langle \bar{l}l \rangle$

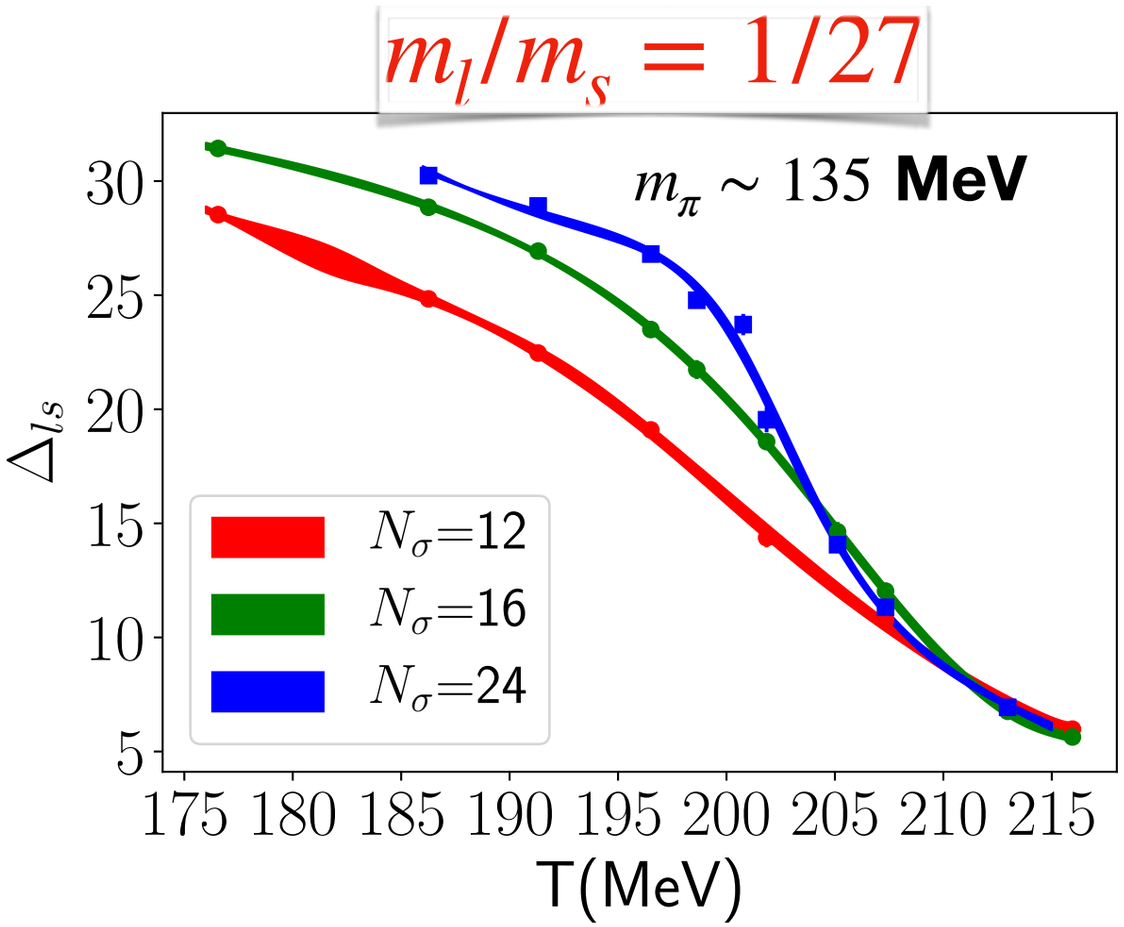


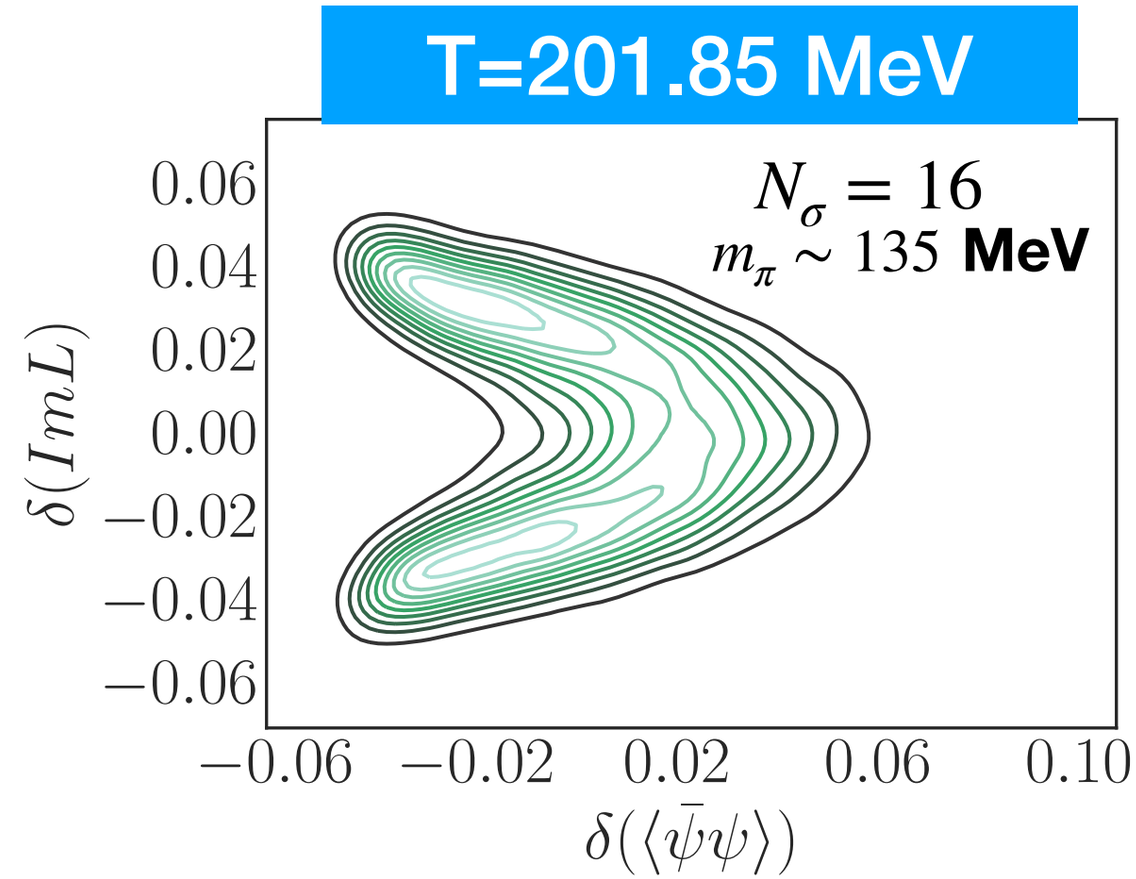
$$\langle \bar{\psi}\psi \rangle \sim const + m^{1/2}$$

$$\chi_{\bar{\psi}\psi}^{disc} \sim m^{-1/2} \quad \text{For, } T < T_c$$

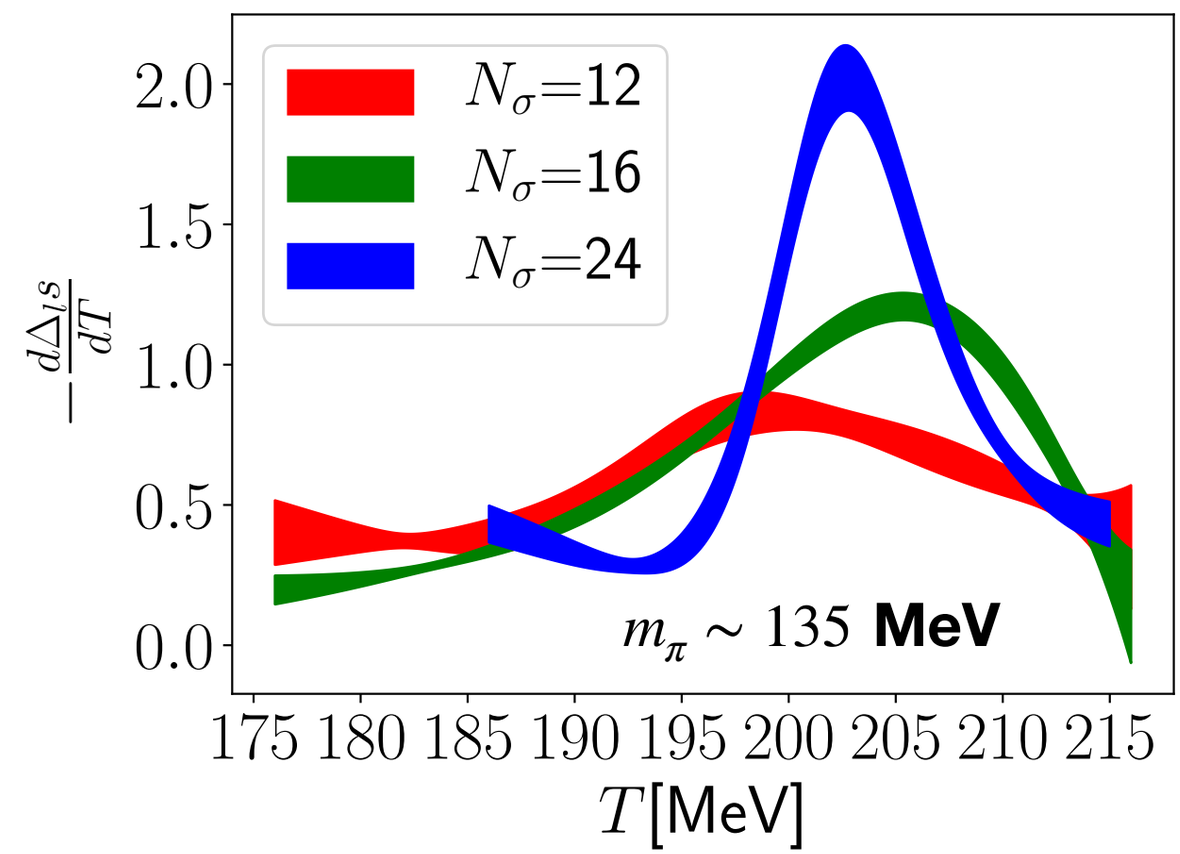
Goldstone effect (square root singularity) in $\chi_{\bar{l}l}^{disc}$ below T_{RW} is evident.

→ chiral symmetry restoration at T_{RW} ?





Strong volume dependence of "subtracted chiral condensate" at fixed m_l/m_s below T_{RW}



mixed chiral susceptibility sensitive to transition at the RW endpoint

Conclusions

- Our preliminary findings from calculations with the HISQ action with physical pion mass suggest that the RW-end point is 2nd order and belongs to the Z(2) universality class. This is consistent with the earlier result found with the stout-improved staggered action.
- preliminary trends of $m_\pi \sim 110$, 90 MeV results are also consistent with a 2nd order phase transition.
- RW transition and chiral phase transition may coincide in the chiral limit.
- Calculations on larger lattices and smaller quark masses are ongoing.

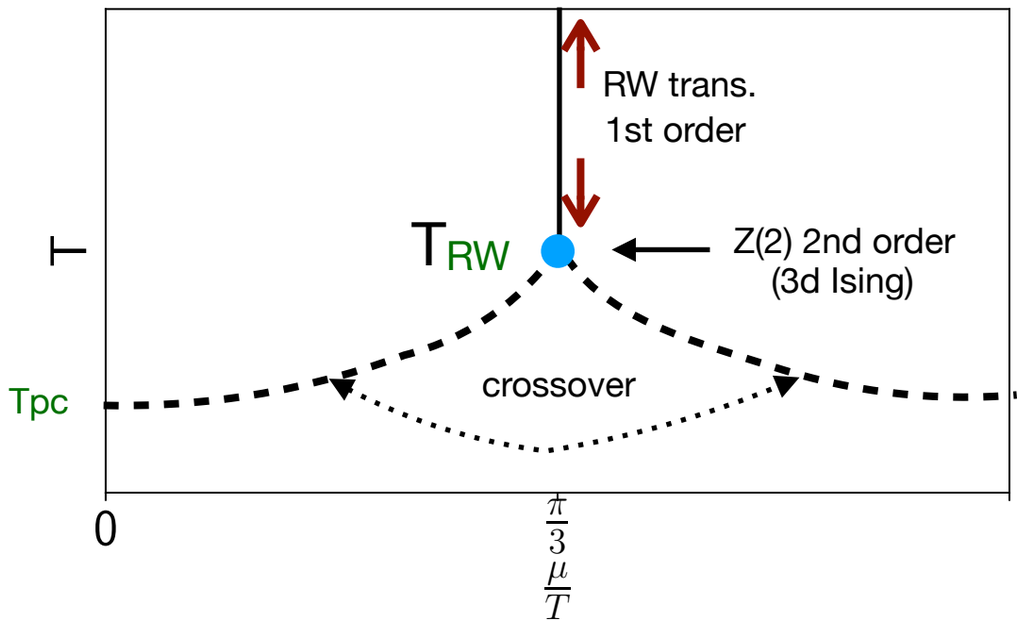
Conclusions

- Our preliminary findings from calculations with the HISQ action with physical pion mass suggest that the RW-end point is 2nd order and belongs to the Z(2) universality class. This is consistent with the earlier result found with the stout-improved staggered action.
- preliminary trends of $m_\pi \sim 110$, 90 MeV results are also consistent with a 2nd order phase transition.
- RW transition and chiral phase transition may coincide in the chiral limit.
- Calculations on larger lattices and smaller quark masses are ongoing.

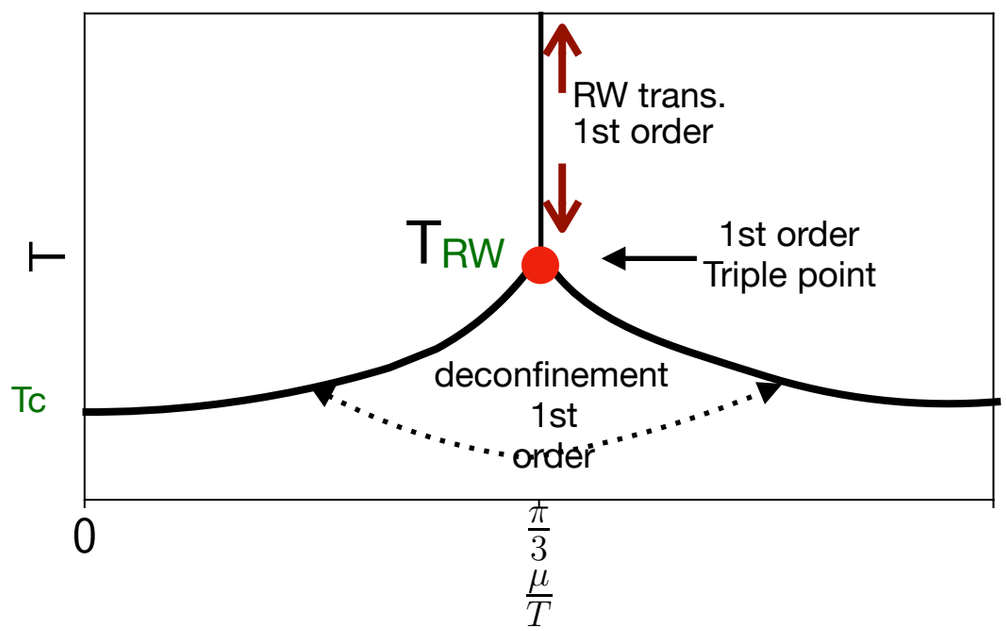
Back up slides

Conjectured phase diagrams in the imaginary chemical potential plane

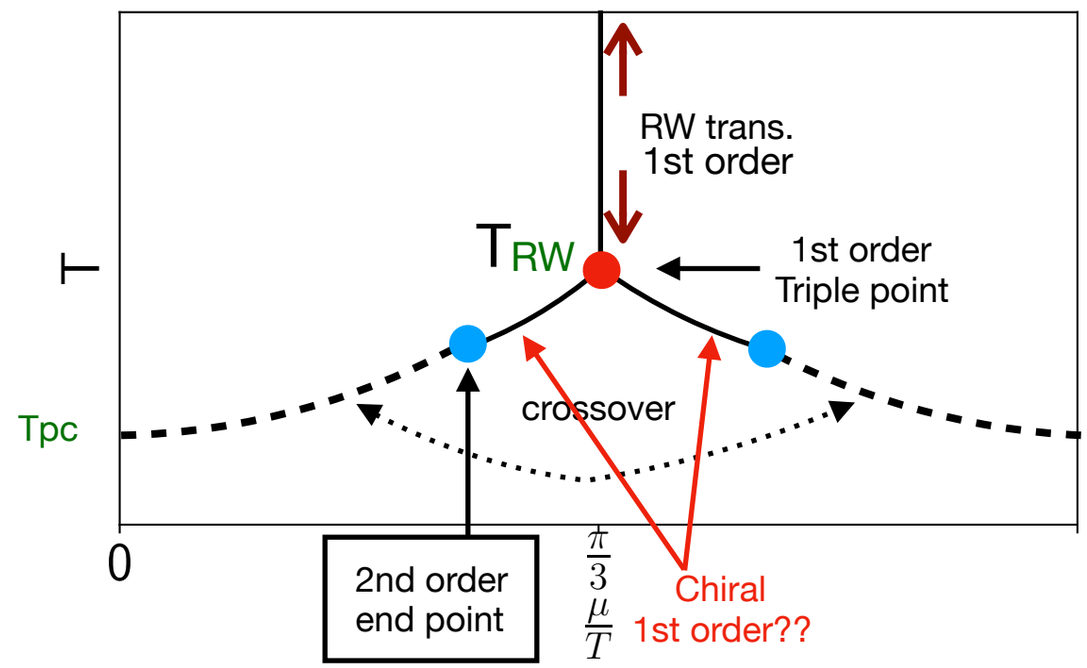
Intermediate quark mass



Heavy quark mass



small quark mass



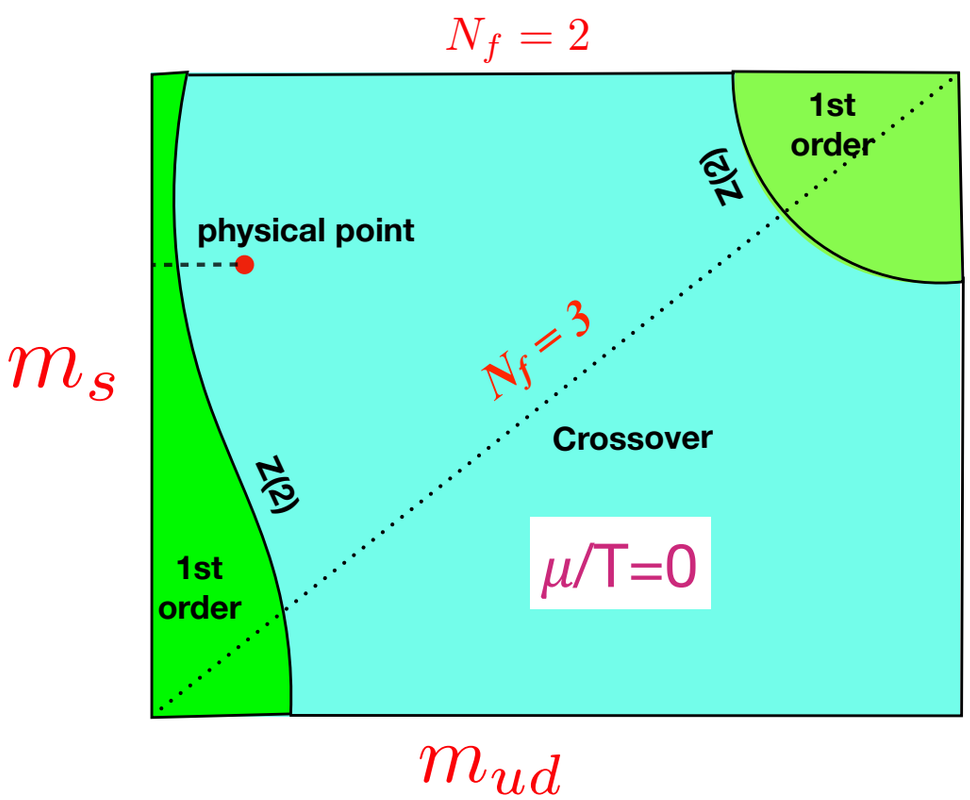
Different scenarios
for different quark masses

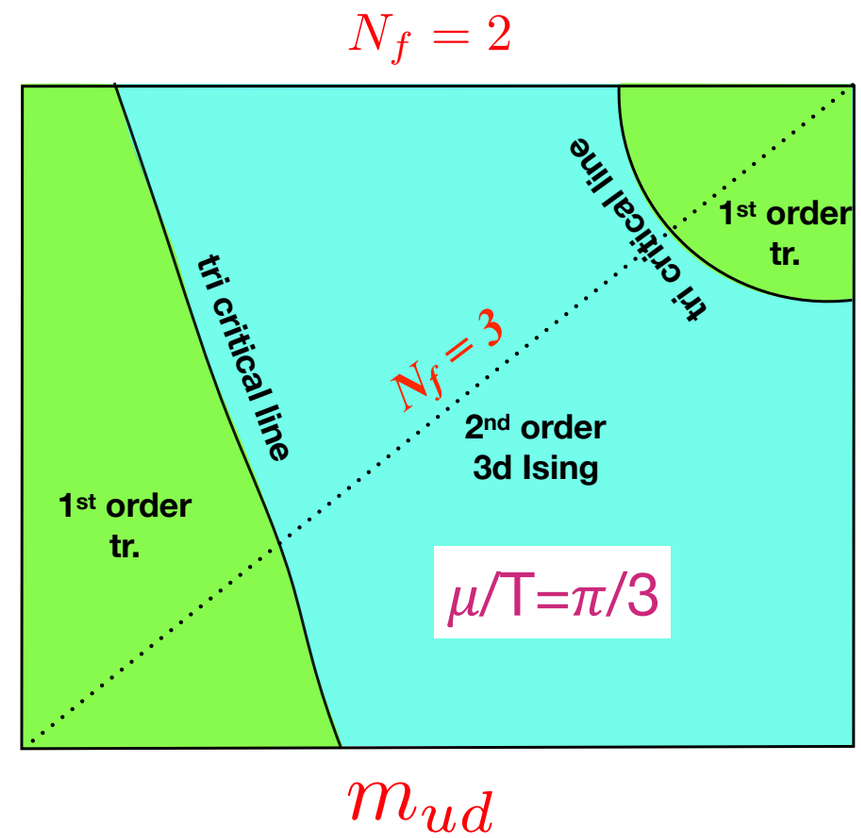
Phases in the RW plane

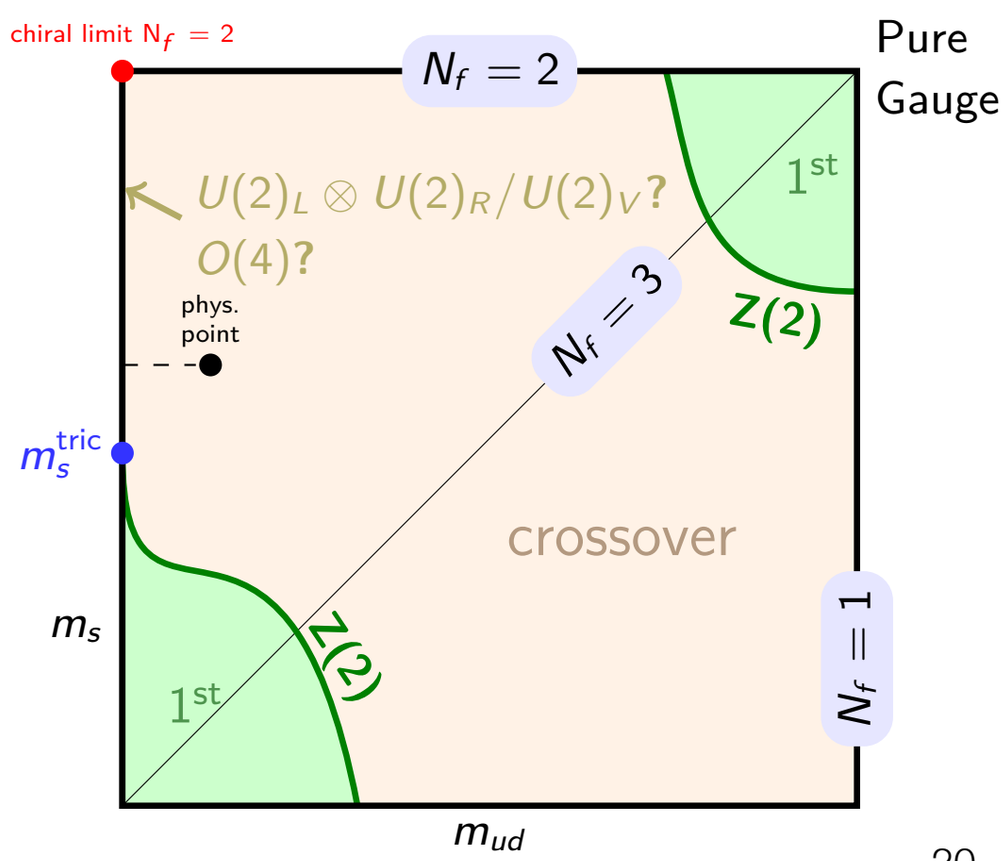
- RW transition happens between two Z(3) sectors of the Polyakov loop. Hence, the order parameter can be the phase or the imaginary part of the Polyakov loop.

- In the RW plane, the 1st order region (for small mass) consists of three 1st order transitions, where high temperature RW transition meets two chiral phase transitions.

- The physical point which is crossover for $\mu=0$ can be 1st or 2nd order in the RW plane. So, our first goal is to confirm this issue and then going to the chiral limit to "search for a 1st order" transition.







Columbia plot in μ=0 and RW plane