Exploring the QCD Phase diagram with imaginary chemical potential with HISQ action

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Plan of the talk

• Introduction

• Status on chiral phase transition with HISQ at $\mu = 0$

• Status on chiral phase transition in the RW plane.

• Results and discussions.
Central Question:
Nature of the chiral symmetry restoring transition at $\mu=0$ at the chiral limit?

Does a 1st order chiral symmetry restoring transition exist at $\mu=0$ below a certain critical quark mass ($m_{\text{cri}}$)?

The 1st order region is expected to be largest in the RW plane ($\mu/T=(2k+1)\pi/3$). Thus critical mass in the RW plane puts a bound on the critical mass at $\mu=0$.

Possible scenario of extended 3d Columbia plot

Chiral transition for zero chemical potential with HISQ

- HotQCD results on chiral phase transition, $[\mu = 0]$

$N_f=2+1$ : No hint of 1st order phase transition for $m_\pi > 55$ MeV.
chiral transition is most likely 2nd order $O(N)$ rather than $Z(2)$.

$N_f=3$ : 1st order phase transition ruled out for $230$ MeV $> m_\pi > 80$ MeV.
Bound on critical pion mass is given as, $m_{\pi}^{cr} \approx 50$ MeV from the scaling analysis.

A. Lahiri et. al., QM 2018, arXiv:1807.05727

Bazavov et. al. PRD 95, 074505 (2017)
Studies in the RW plane

m_π > 1 GeV, for the ‘heavy quark mass RW transition’

‘small quark mass RW transition’ \((N_f = 2)\)

Standard staggered action: \(m_π \sim 400\) MeV \((N_τ = 4)\)

Standard Wilson action: \(m_π \sim 930\) MeV \((N_τ = 4)\)

\(m_π \sim 680\) MeV \((N_τ = 6)\)

1st order end point (of the line of 1st order RW transitions) exist already for \(\mu/T = \pi/3\) and \(m_{cri} > m_{phy}\).

The results are strongly fermion discretization scheme and cut-off \((N_τ)\) dependent.

P. de Forcrand et. al, PRL 105, 152001(2010), Owe Philipsen et. al, PRD 89, 094504(2014), Christopher Czaban et al, PRD 93, 054507 (2016)
Very recent studies with improved actions,

- **Stout improved staggered fermions** ($N_f=2+1$): At the physical quark mass point ($m_\pi \sim 135$ MeV) a 2nd order transition in the 3d-Ising universality class happens instead of a 1st order at the RW endpoint.

  C. Bonati et. al, PRD 93, 074504 (2016)

  No 1st order end point (of the line of 1st order RW transitions) for $m_\pi > 50$ MeV.

  C. Bonati et. al, arXiv:1807.02106 [hep-lat]

- **HISQ** ($N_f=2$): Order of the phase transition at physical point is not clear (large cut-off effects).

Studies with HISQ in the RW plane

Action,

\[ Z(T, \mu) = \int [\mathcal{D}U] \text{det}[M_{ud}(\mu_f)]^{1/2} \text{det}[M_s(\mu_f)]^{1/4} \exp[-S_G] \]

\[ M_q = D_{\text{HISQ}}(\mu_f) + m_q \]

Simulation details,

<table>
<thead>
<tr>
<th>$N_\sigma$</th>
<th>$N_\tau$</th>
<th>$\frac{m_l}{m_s}$</th>
<th>$M_\pi$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>4</td>
<td>$1/27$</td>
<td>135</td>
</tr>
<tr>
<td>12</td>
<td>4</td>
<td>$1/27$</td>
<td>135</td>
</tr>
<tr>
<td>16</td>
<td>4</td>
<td>$1/27, 1/40, 1/60$</td>
<td>135, 110, 90</td>
</tr>
<tr>
<td>24</td>
<td>4</td>
<td>$1/27, 1/40$</td>
<td>135, 110</td>
</tr>
</tbody>
</table>

$N_f = 2 + 1, \quad \frac{\mu}{T} = \frac{\pi}{3}$

We vary $\beta$ in the range [5.850-6.038], corresponds to, $\sim T_c \pm 0.1 T_c$

Generally we generated 20k trajectory per $\beta$ value away from $\beta_c$ and 80k trajectory near $\beta_c$

We work on the 2nd RW plane
**Ising endpoint of a first order line**

\[ H_{\text{eff}}(t, \xi) = t \mathcal{E} + h \mathcal{M} \]

**Effective Ising Hamiltonian which defines the universal critical behaviour of the system**

- Temperature like field
- Energy like operator
- Magnetic field like
- Magnetization like operator (order parameter)

Under Z(2) transformation,

\[ \mathcal{E} \rightarrow \mathcal{E} \]
\[ \mathcal{M} \rightarrow -\mathcal{M} \]

**Corresponding critical behaviour of QCD in 2nd RW plane [Z(2) transformation],**

\[ \text{Im } L \rightarrow -\text{Im } L \]
\[ \text{Re } L \rightarrow \text{Re } L \]

i.e. at \( \mu = \mu_{\text{RW}} \)

\[
\lim_{h \rightarrow 0} \lim_{V \rightarrow \infty} \langle \text{Im } L \rangle \equiv \lim_{V \rightarrow \infty} \langle |\text{Im } L| \rangle = \begin{cases} 
0, & \text{if } \beta < \beta_c \\
\text{non-zero}, & \text{if } \beta > \beta_c 
\end{cases}
\]
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\text{non-zero,} & \text{if } \beta > \beta_c
\end{cases}
\]
Finite size scaling and Z(2) universality class

Free energy,\[ f = f_{ns} + b^{-d} f_s(b_t u_t, b_h u_h, b^{-1} N_\sigma) \]

Responsible for the universal critical behaviour

\[ t = \frac{T - T_c}{T_c} \sim \beta - \beta_c \]

near, \( T \to T_c \) \( u_t \sim c_t t, u_t \sim c_h h \)

\[ \chi_t = (\ldots) \frac{\partial^2}{\partial t^2} f_s(\ldots) \bigg|_{h \to o} \]

\[ \chi_h = (\ldots) \frac{\partial^2}{\partial h^2} f_s(\ldots) \bigg|_{h \to o} \]

\[ \chi_t = z_2 N_\sigma^{\alpha/\nu} f_t(z_0 t N_\sigma^{1/\nu}) \]

\[ \chi_h = z_1 N_\sigma^{\gamma/\nu} f_h(z_0 t N_\sigma^{1/\nu}) \]

Susceptibility of \( |Im L| \)

Susceptibility of \( |Re L| \)

Order parameter

Specific heat

Universal functions

\[ <O> = (\ldots) \frac{\partial}{\partial h} f_s(\ldots) \bigg|_{h \to o} \]
Finite size scaling and $\mathbb{Z}(2)$ universality class

Free energy,

$$f = f_{ns} + b^{-d} f_s (b^y u_t, b^y v_u, b^{-1} N_\sigma)$$

Responsible for the universal critical behaviour

$$t = \frac{T - T_c}{T_c} \sim \beta - \beta_c$$

near, $T \to T_c$ 

$$u_t \sim c_t t, u_t \sim c_h h$$

$$<O> = (\ldots) \frac{\partial}{\partial h} f_s (\ldots) \big|_{h \to 0}$$

order parameter

$$\chi_h = (\ldots) \frac{\partial^2}{\partial h^2} f_s (\ldots) \big|_{h \to 0}$$

susceptibility of op

$$\chi_t = (\ldots) \frac{\partial^2}{\partial t^2} f_s (\ldots) \big|_{h \to 0}$$

specific heat

$$\chi_h = z_2 N_\sigma^{\alpha/\nu} f_t (z_0 t N_\sigma^{1/\nu})$$

susceptibility of op

$$\chi_t = z_1 N_\sigma^{\gamma/\nu} f_t (z_0 t N_\sigma^{1/\nu})$$

universal functions

Susceptibility of $|\text{Im } L|$ (top) and $|\text{Re } L|$ (bottom)

Line: $\mathbb{Z}(2)$ scaling curve 
$T_c = 202.6(4) \text{ MeV}$

$N_\sigma = 24$ ( ), $16$ ( ), $12$ ( ), $8$ ( )

$m_\pi \sim 135 \text{ MeV}$
Finite size scaling and Z(2) universality class

Free energy,
\[ f = f_{nS} + b^{-d} f_s(b^u_t u_t, b^u_h u_h, b^{-1} N_\sigma) \]

 Responsible for the universal critical behaviour
\[ t = \frac{T - T_c}{T_c} \sim \beta - \beta_c \]
near, \( T \to T_c \) \( u_t \sim c_t t, u_t \sim c_h h \)

\[ < O > = (\ldots) \frac{\partial}{\partial h} f_s(\ldots)|_{h \to 0} \]

\[ \chi_t = (\ldots) \frac{\partial^2}{\partial t^2} f_s(\ldots)|_{h \to 0} \]

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susceptibility of op

\[ \chi_t = z_2 N_\sigma^{\alpha/\nu} f_t(z_0 t N_\sigma^{1/\nu}) \]

specific heat

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susceptibility of op

\[ m_\pi \sim 135 \text{ MeV} \]

\[ N_s = 12 \]
\[ N_s = 16 \]
\[ N_s = 24 \]
Finite size scaling and Z(2) universality class

Free energy,
\[ f = f_{ns} + b^{-d} f_s(b^{y_t} u_t, b^{y_h} u_h, b^{-1} N_\sigma) \]

\[ \alpha, \gamma, \nu \] are 3d Ising Exponents

“Susceptibility” and “specific heat” scale with corresponding Z(2) finite size universal scaling functions

Responsible for the universal critical behaviour
\[ t = \frac{T - T_c}{T_c} \sim \beta - \beta_c \]
\[ u_t \sim c_t t, u_t \sim c_h h \quad \text{near, } T \to T_c \]
Finite size scaling and Z(2) universality class

Free energy,
\[ f = f_{ns} + b^{-d} f_s(b^{y_1} u_t, b^{y_h} u_h, b^{-1} N_\sigma) \]

\( \alpha, \gamma, \nu \) are 3d Ising Exponents

Responsible for the universal critical behaviour
\[ t = \frac{T - T_c}{T_c} \sim \beta - \beta_c \]
\[ u_t \sim c_t t, u_h \sim c_h h \quad \text{near, } T \to T_c \]
\[ \chi_h = z_1 N_\sigma^{\gamma/\nu} f_h(z_0 t N_\sigma^{1/\nu}) \]

"Susceptibility" and "specific heat" scale with corresponding Z(2) finite size universal scaling functions

\[
\begin{align*}
    m_\pi &\sim 135 \text{ MeV} \\
    T_c &= 202.6(4) \text{ MeV} \\
    N_\sigma &= 24, 16, 12, 8
\end{align*}
\]
Finite size scaling and Z(2) universality class

Free energy,

\[ f = f_{ns} + b^{-d} f_s(b^y u_t, b^y u_h, b^{-1} N_\sigma) \]

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Responsible for the universal critical behaviour

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\( u_t \sim c_t t, u_t \sim c_h h \) near, \( T \to T_c \)

\[ \chi_h = z_1 N_\sigma^{\gamma/\nu} f_h(z_0 t N_\sigma^{1/\nu}) \]

“Susceptibility” and “specific heat” scale with corresponding Z(2) finite size universal scaling functions

\[ \chi_t = z_2 N_\sigma^{\alpha/\nu} f_t(z_0 t N_\sigma^{1/\nu}) \]

\( \chi \) and \( \chi_t \) scale with

\[ m_\pi \sim 135 \text{ MeV} \]
Binder cumulants of order parameter

Universal Z(2) scaling of the order parameter

Crossing of the Binder cumulants approaches to the universal value in \( V \to \infty \)

Ising value = 1.604

\( m_\pi \sim 135 \) MeV

\( m_\pi \sim 135 \) MeV
No unusual change in the susceptibility of the order parameter with respect to pion mass up to $m_\pi \sim 90$ MeV.

Order of the transition seems to be unchanged??
Chiral limit and RW transition

\[ \langle \bar{\psi} \psi \rangle \sim const + m^{1/2} \]

\[ \chi_{\bar{\psi}\psi}^{disc} \sim m^{-1/2} \text{ For, } T < T_c \]

Goldstone effect (square root singularity) in \( \chi_{\bar{\psi}\psi}^{disc} \) below T_{RW} is evident.

\[ \Delta_{ls} = (m_s/f_k^4) \left( \langle \bar{l}l \rangle - (m_l/m_s) \langle \bar{s}s \rangle \right) \]

Susceptibility of \( \langle \bar{l}l \rangle \)

Sub. Chiral condensate

\[ \Delta_{ls} = (m_s/f_k^4) \left( \langle \bar{l}l \rangle - (m_l/m_s) \langle \bar{s}s \rangle \right) \]

\[ \langle \bar{l}l \rangle \]

\[ N_\sigma = 16 \]

\[ m_l/m_s = 1/27 \]
\[ m_l/m_s = 1/40 \]
\[ m_l/m_s = 1/60 \]

\[ m_l/m_s = 1/27 \]
\[ m_l/m_s = 1/40 \]
\[ m_l/m_s = 1/60 \]
$m_l/m_s = 1/27$

$m_\pi \sim 135 \text{ MeV}$

$N_\sigma = 16$

$m_\pi \sim 135 \text{ MeV}$

Strong volume dependence of “subtracted chiral condensate” at fixed $m_l/m_s$ below $T_{RW}$

Mixed chiral susceptibility sensitive to transition at the RW endpoint
Conclusions

• Our preliminary findings from calculations with the HISQ action with physical pion mass suggest that the RW-end point is 2nd order and belongs to the Z(2) universality class. This is consistent with the earlier result found with the stout-improved staggered action.

• Preliminary trends of \( m_\pi \sim 110 \), 90 MeV results are also consistent with a 2nd order phase transition.

• RW transition and chiral phase transition may coincide in the chiral limit.

Calculations on larger lattices and smaller quark masses are ongoing.
Conclusions

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Thank you for your time and attention.
Back up slides
Conjectured phase diagrams in the imaginary chemical potential plane.

Different scenarios for different quark masses.
Phases in the RW plane

- RW transition happens between two $Z(3)$ sectors of the Polyakov loop. Hence, the order parameter can be the phase or the imaginary part of the Polyakov loop.

- In the RW plane, the 1st order region (for small mass) consists of three 1st order transitions, where high temperature RW transition meets two chiral phase transitions.

- The physical point which is crossover for $\mu=0$ can be 1st or 2nd order in the RW plane. So, our first goal is to confirm this issue and then going to the chiral limit to “search for a 1st order” transition.
$N_f = 2$

$m_s$

$N_f = 2$

$m_{ud}$

$m_s$

$N_f = 3$

$N_f = 3$

$m_{ud}$

Crossover

$N_f = 1$

$m_{ud}$

$Z(2)$

$Z(2)$

$Z(2)$

Crossover

$Z(2)$

$Z(2)$

Crossover

Columbia plot in $\mu=0$ and RW plane

Pure Gauge