Temporal Correlators in the Continuous Time Limit of Strong Coupling Lattice QCD

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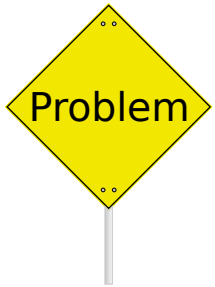
QCD in the Strong Coupling Limit

Study regime where *sign problems* can be made mild:
$\Rightarrow$ limit of infinite gauge coupling

$$g \to \infty, \quad \beta = \frac{2N_c}{g^2} \to 0$$



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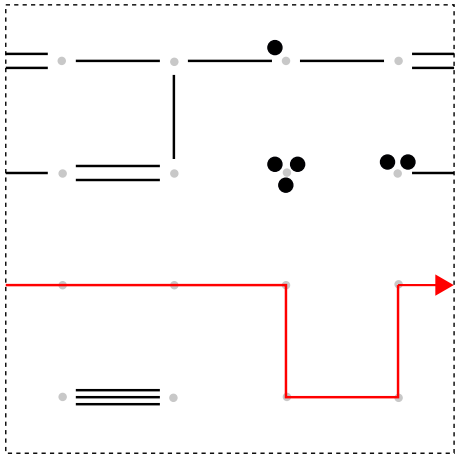

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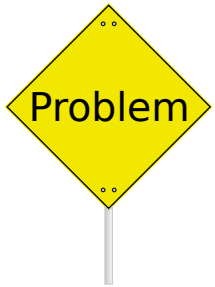

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Fully combinatorial problem, restricted by *Grassmann constraint*:

$$n_x + \sum_{\pm\mu} k_{x\mu} = N_c, \quad \sum_{\pm\mu} l_{x\mu} = 0, \forall x$$

In the following restrict to *chiral limit* where monomer density $\langle n \rangle = 0$

Continuous Time limit within Strong Coupling QCD

First: Introduction of *anisotropy* for continuous temperature variation:

$$aT = \frac{1}{N_\tau} \Rightarrow aT = \frac{\xi(\gamma)}{N_\tau}, \qquad \underbrace{\xi(\gamma) = a/a_\tau}_{\text{anisotropy parameter}}$$

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Continuous Time partition function:

$$\boxed{N_c = 3:}$$

$$Z_{CT}(T) = \sum_{k\in 2\mathbb{N}} \left(\frac{1}{2aT}\right) \sum_{\mathcal{G}'\in\Gamma_k} e^{\mu_B B/T} \hat{\nu}_{\mathcal{T}}^{N_{\mathcal{T}}}$$

$$\text{with } k = \sum_{b=(x,\hat{i})} k_b, \; N_{\mathcal{T}} = \sum_x n_{\mathcal{T}}(x)$$

$$\hat{\nu}_{\mathcal{T}} = \frac{2}{\sqrt{3}} :$$

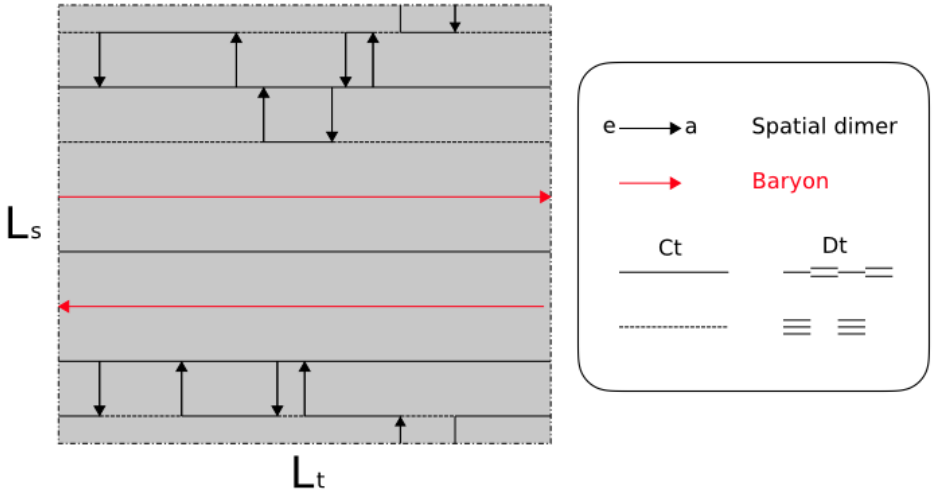
$$\hat{\nu}_L = 1 \quad :$$

Benefits and Comments on Continuous Time Limit

- No *discretization errors* due to finite N_τ
- Only one parameter left (temperature T)
- Baryons become *static* for $N_c \geq 3 \Rightarrow$ no extend in spatial direction

$\Rightarrow$ Sign problem is absent

- Baryons are massive (even though in chiral limit)
- No *multiple spatial dimers* (suppressed by γ)



- Faster algorithm for medium to large temporal extends

Continuous Time Algorithm

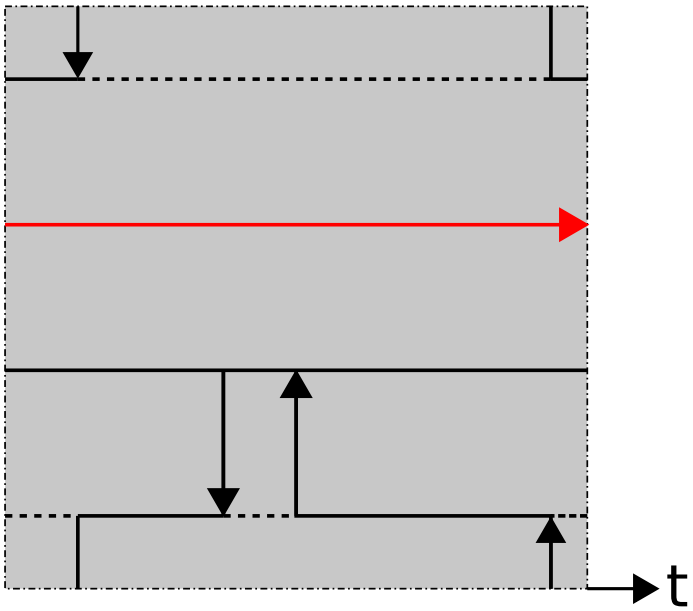
- Worm-type Monte Carlo Algorithm [Adams & Chandrasekharan, 2003]
- *absorption* (even) and *emission* (odd) site decomposition of lattice

Mesonic worm update:

- Place *tail* of mesonic worm on lattice at absorption site

$\Rightarrow$ Violation of Grassmann constraint $\Rightarrow$ propagate *head*

restoration of Grassmann constraint if head at emission site



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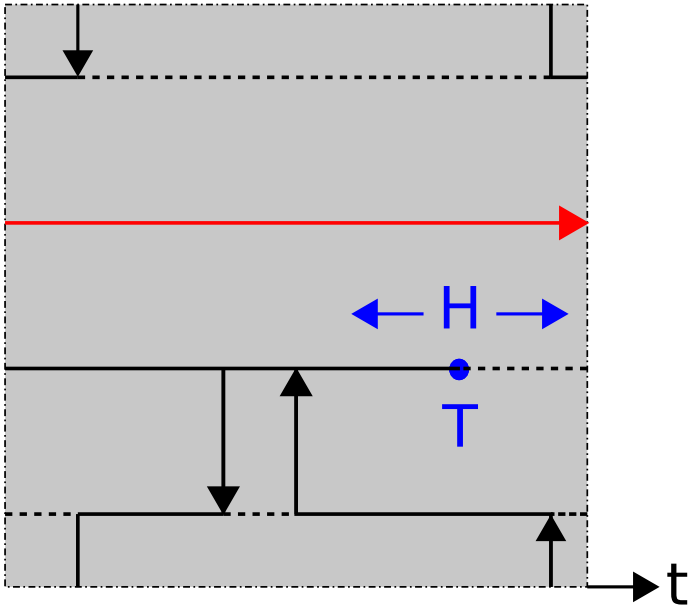

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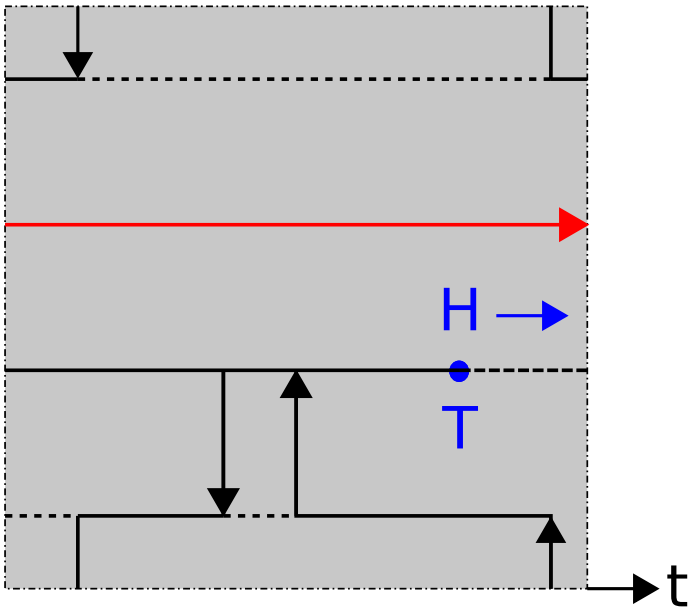

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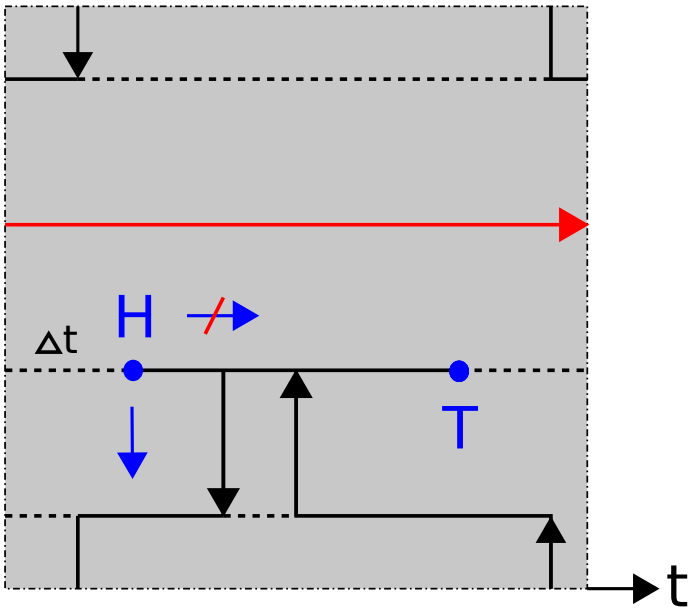

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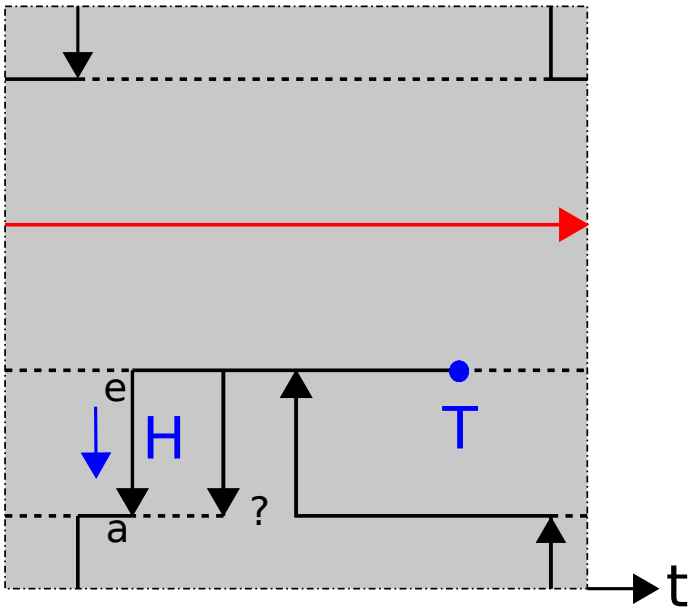

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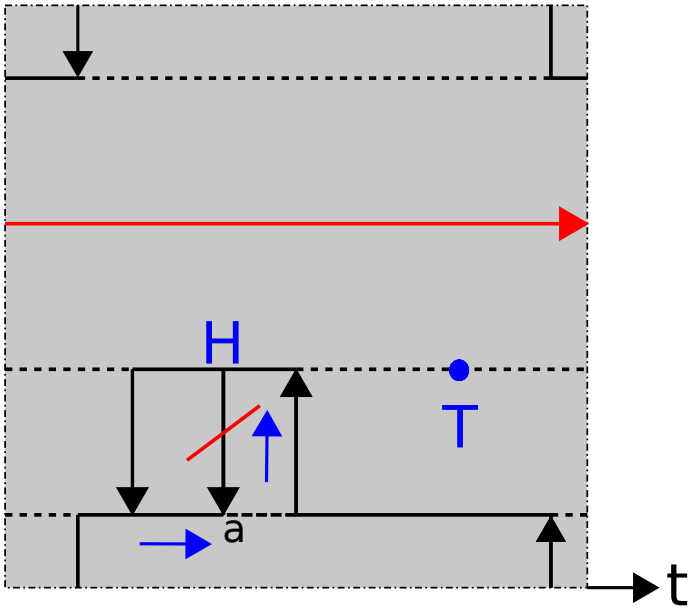

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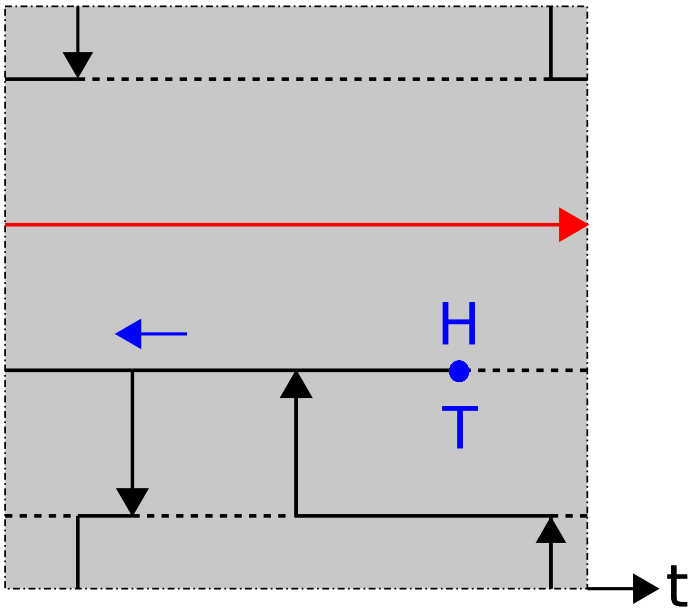

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Weight of configuration ruled by spatial dimer emission/absorption

- *spatial dimer emission* ruled by *Poisson process*
- Vertex weights decide spatial dimer *absorption*

$P(\Delta\beta) \sim \exp(\lambda\Delta\beta), \quad \Delta\beta \in [0, \beta = 1/aT]$
"decay constant" λ for spatial dimer emission:
$\lambda = d_D(x)/4, \quad d_M(x) = 2d - \sum_\mu n_B(x \pm \hat{\mu})$
with $d_M(x)$ the number of mesonic neighbors

Two-point Correlators

- Sample *monomer-monomer two-point correlation functions*

$$C(t_H - t_T, \vec{x}_H - \vec{x}_T) = C(\tau, \vec{x})$$

- accumulate observables during *worm evolution* (tail absorption/source, head emission/sink)

$$C(\tau, \vec{x}) = N_c \frac{O(C(\tau, \vec{x}))}{\#\text{worm updates}}$$

- Measure Chiral Susceptibility χ_σ by summing over worm estimators:

$$\chi_\sigma = \frac{1}{V} \sum_{\vec{x}} C(\tau, \vec{x})$$

Discrete Time: $O(C(\tau, \vec{x})) \to O(C(\tau, \vec{x})) + f(\gamma) \cdot \delta_{x_T, x_1} \delta_{x_H, x_2}, \quad \tau \in [0, \ldots N_\tau]$

Continuous Time: $O(C(\tau, \vec{x})) \to O(C(\tau, \vec{x})) + g(T) \cdot \delta_{x_T, x_1} \delta_{x_H, x_2}, \quad \tau \in [0, \ldots \frac{1}{T}]$

Extracting Meson Masses

- Extract *pole masses* for temporal correlators with *zero spatial momentum*

$$E_0(\vec{p}=0) = m_0, \qquad C(\tau) = \sum_{\vec{x}} \langle \bar{\chi}_0 \chi_0 \bar{\chi}_{\vec{x},t} \chi_{\vec{x},t} \rangle g_x^D$$

- For staggered fermions: Restrict to diagonal of Dirac-taste-kernel ($N_f = 1$)

g_x^D	$\Gamma^D \otimes \Gamma^F$		J^{PC}		Physical states	
	NO	O	NO	O	NO	O
1	$1 \otimes 1$	$\gamma_0\gamma_5 \otimes (\gamma_0\gamma_5)^*$	0^{++}	0^{-+}	σ_S	π_A
$(-1)^{x_i}$	$\gamma_i\gamma_5 \otimes (\gamma_i\gamma_5)^*$	$\gamma_i\gamma_0 \otimes (\gamma_i\gamma_0)^*$	1^{++}	1^{--}	a_A	ρ_T
$(-1)^{x_j+x_k}$	$\gamma_j\gamma_k \otimes (\gamma_j\gamma_k)^*$	$\gamma_i \otimes \gamma_i^*$	1^{+-}	1^{--}	b_T	ρ_V
$(-1)^{x_i+x_j+x_k}$	$\gamma_0 \otimes \gamma_0^*$	$\gamma_5 \otimes (\gamma_5)^*$	0^{+-}	0^{-+}	$-_V$	π_{PS}

channel of primary interest

[Altmeyer et al., 1993]

Temporal Correlators in Continuous Time

- Introduce *binning*
- Evaluate at same spatial site $\Rightarrow$ Zero momentum projection
- Accumulate *histograms* while worm head propagates

Value per bin: $\frac{g(T)}{\#bins}$

- Distinguish histograms for *even* and *odd* interval contributions

Extract *pole masses* from temporal correlators + Study various *channels*

In CT: Masses measured in units of M/T

Discrete and Continuous Time Correlator Fits

Discrete time: *4 parameter* fit

Either combined:

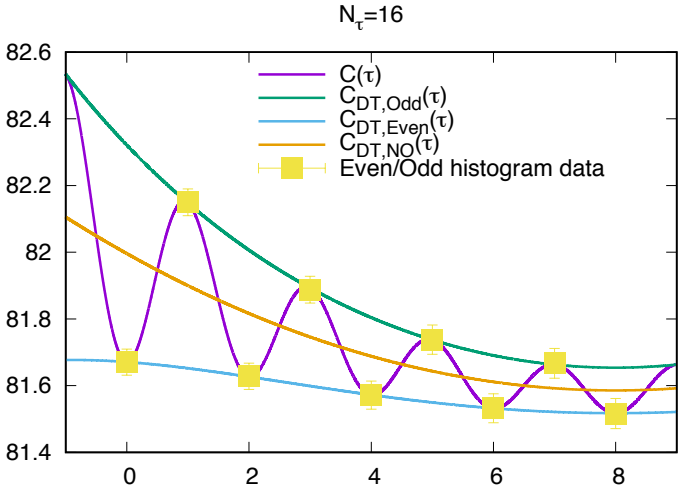
$$C(\tau) = a_{NO}\cosh(m_{NO}(\tau - N_\tau/2)) - a_O\cos(\pi\tau)\cosh(m_O(\tau - N_\tau/2))$$

Or split up for Even and Odd histograms:

$$C_{\mathrm{DT,Even}}(\tau) = a_{NO}\cosh(m_{NO}(\tau - N_\tau/2)) - a_O\cosh(m_O(\tau - N_\tau/2)$$

$$C_{\mathrm{DT,Odd}}(\tau) = \underbrace{a_{NO}\cosh(m_{NO}(\tau - N_\tau/2))}_{\text{Non-oscillating Correlator}} + \underbrace{a_O\cosh(m_O(\tau - N_\tau/2)}_{\text{Oscillating Correlator}}$$

$$\Rightarrow C_{\mathrm{DT,NO}}(\tau) = \frac{1}{2}\left(C_{\mathrm{DT,Even}}(\tau) + C_{\mathrm{DT,Odd}}(\tau)\right), \quad C_{\mathrm{DT,O}}(\tau) = \frac{1}{2}\left(C_{\mathrm{DT,Even}}(\tau) - C_{\mathrm{DT,Odd}}(\tau)\right)$$



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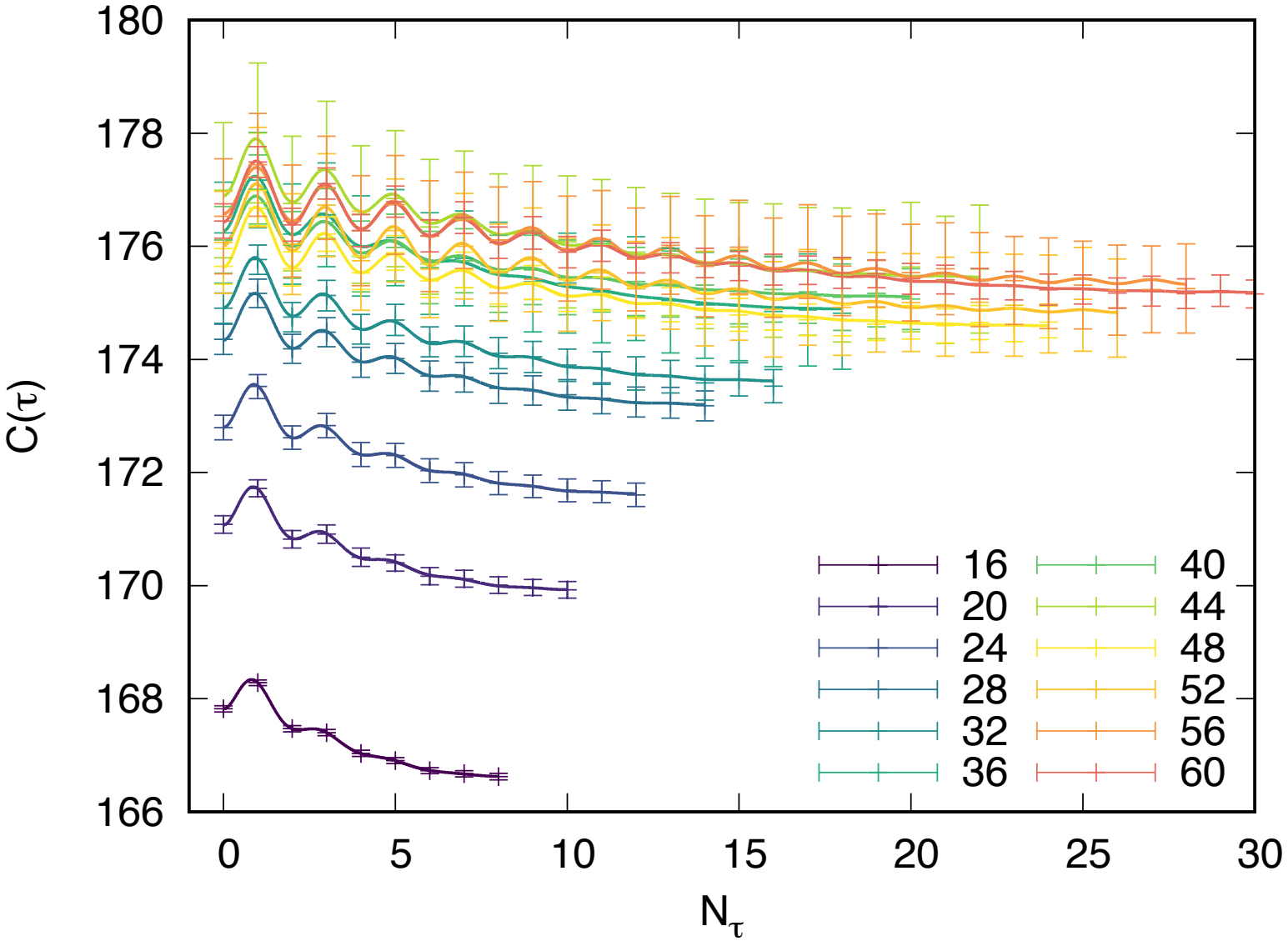
Continuous time: *2/4 parameter* fit of added and subtracted histograms respectively

$$C_{\mathrm{CT,NO}}(\tau) = a_{NO}\cosh(m_{NO}(\tau - 1/2)) = \frac{1}{2}(C_{\mathrm{Odd}}(\tau) + C_{\mathrm{Even}}(\tau))$$

$$C_{\mathrm{CT,O}}(\tau) = a_O\cosh(m_O(\tau - 1/2)) = \frac{1}{2}(C_{\mathrm{Odd}}(\tau) - C_{\mathrm{Even}}(\tau))$$

From discrete Histograms to Correlators and Masses



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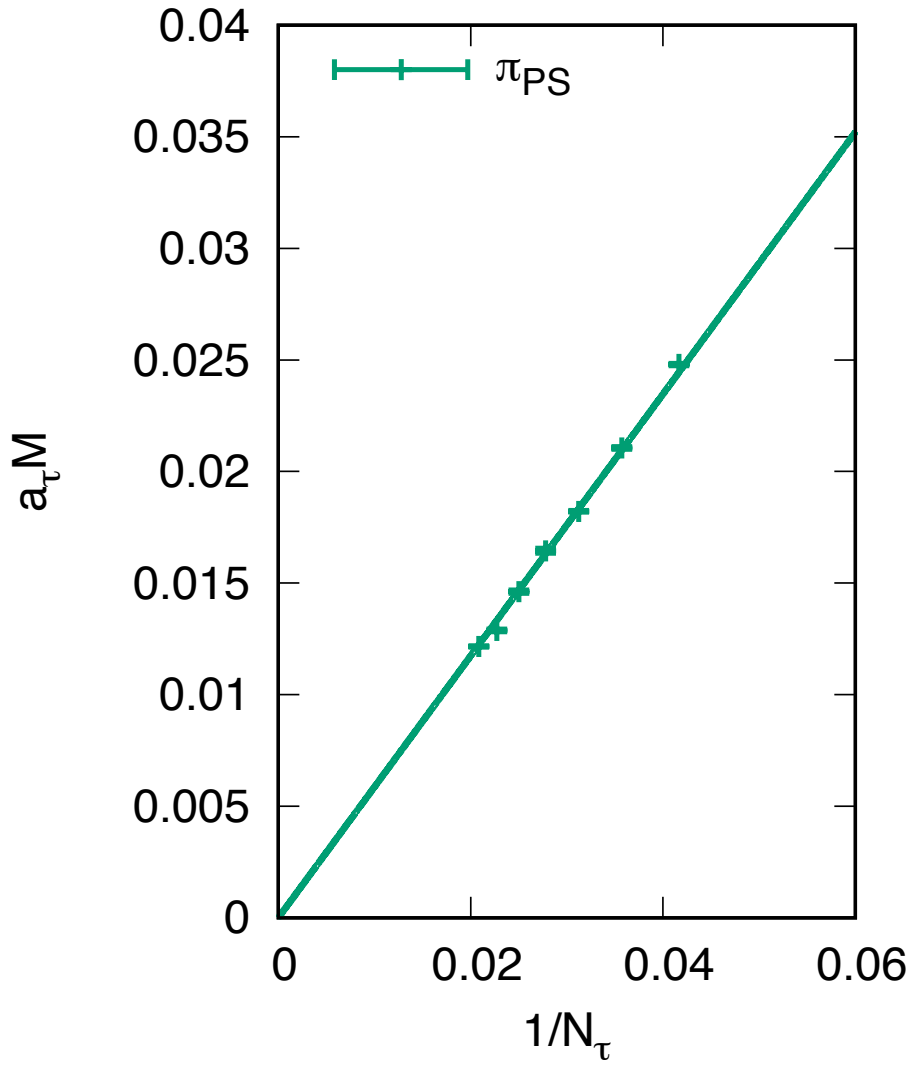

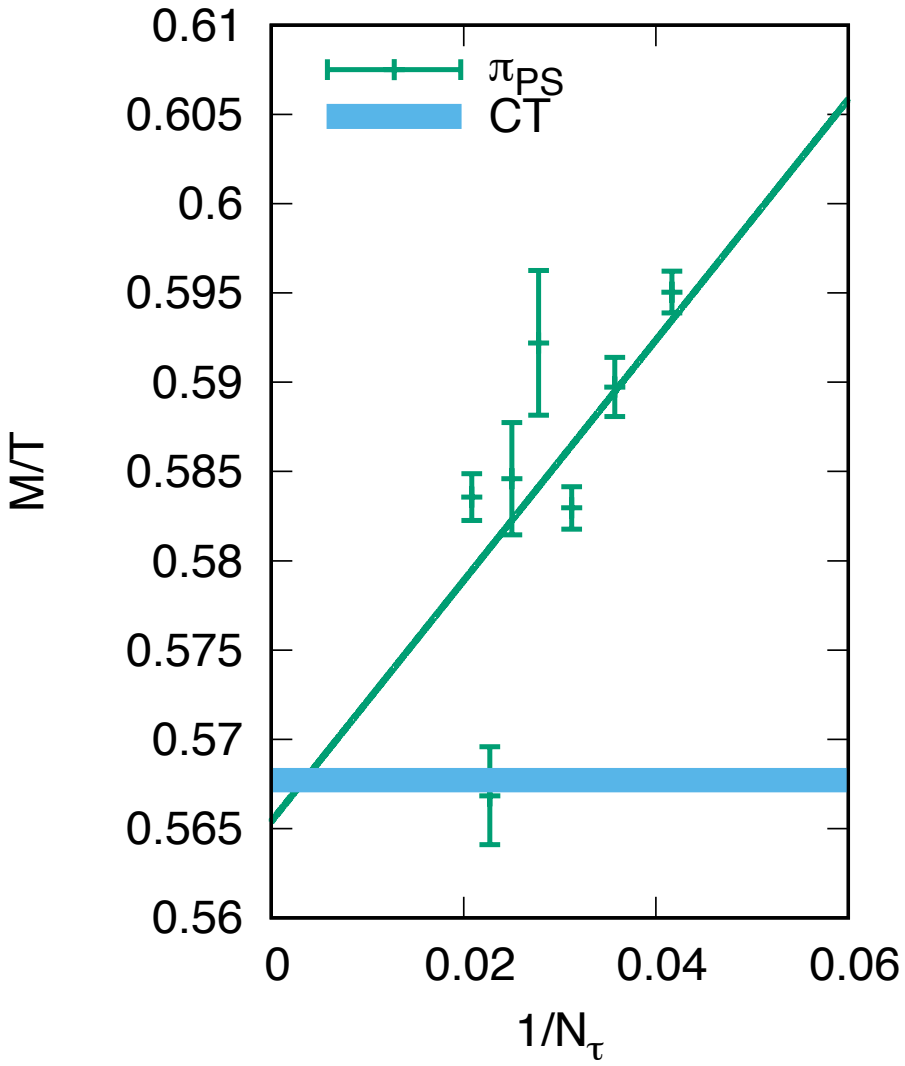

Change $a_\tau M \to M/T$ in order to compare with Continuous Time results

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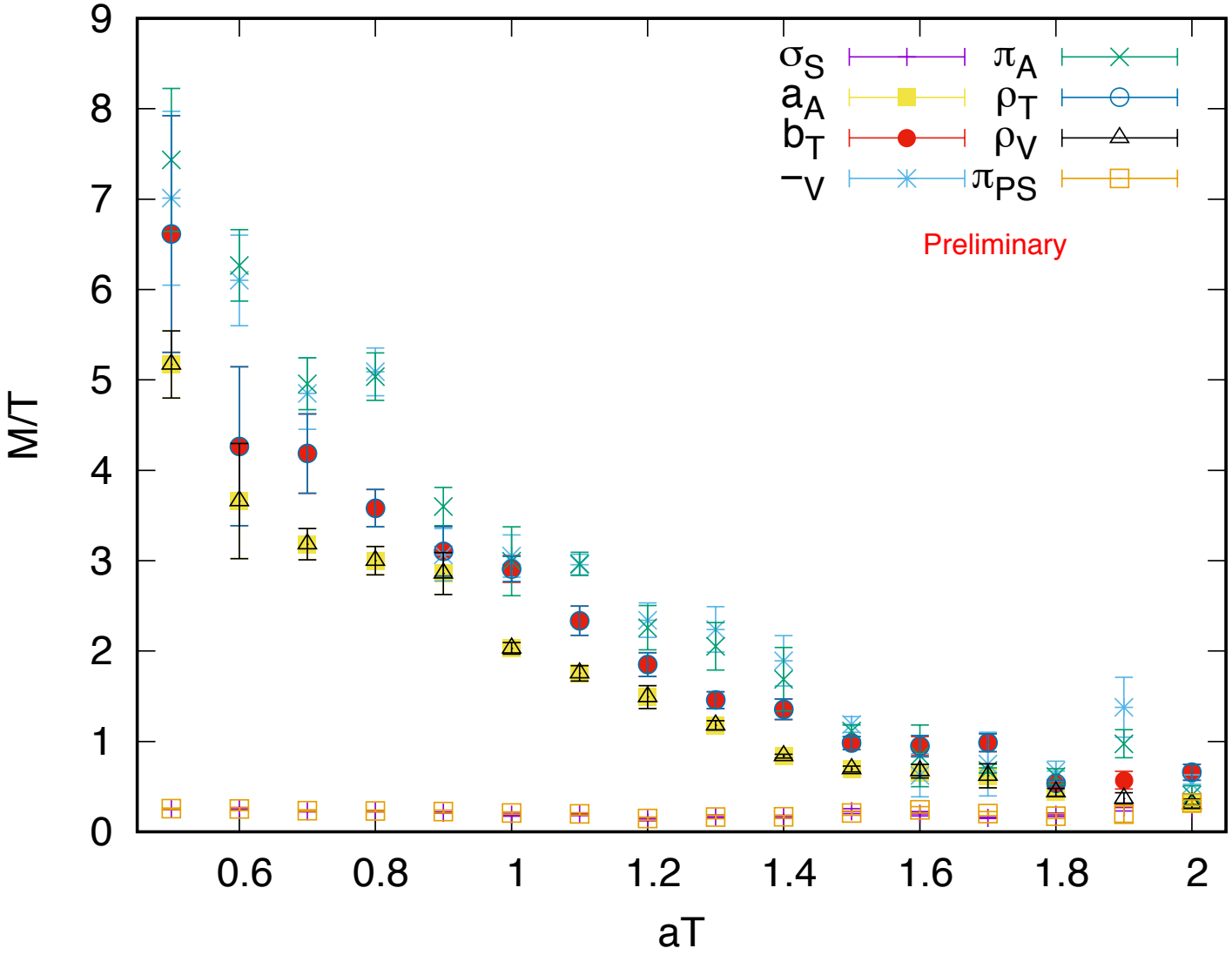

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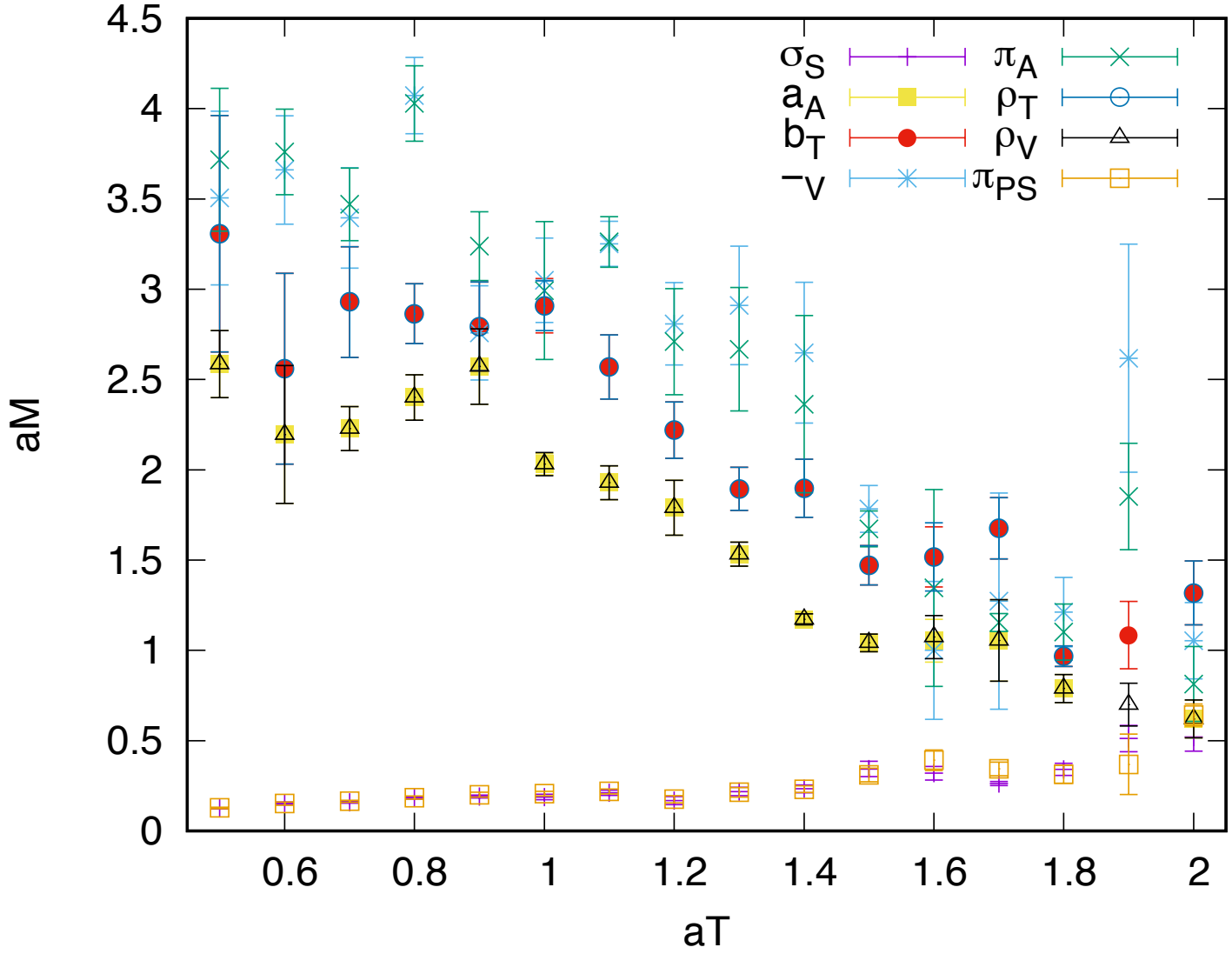
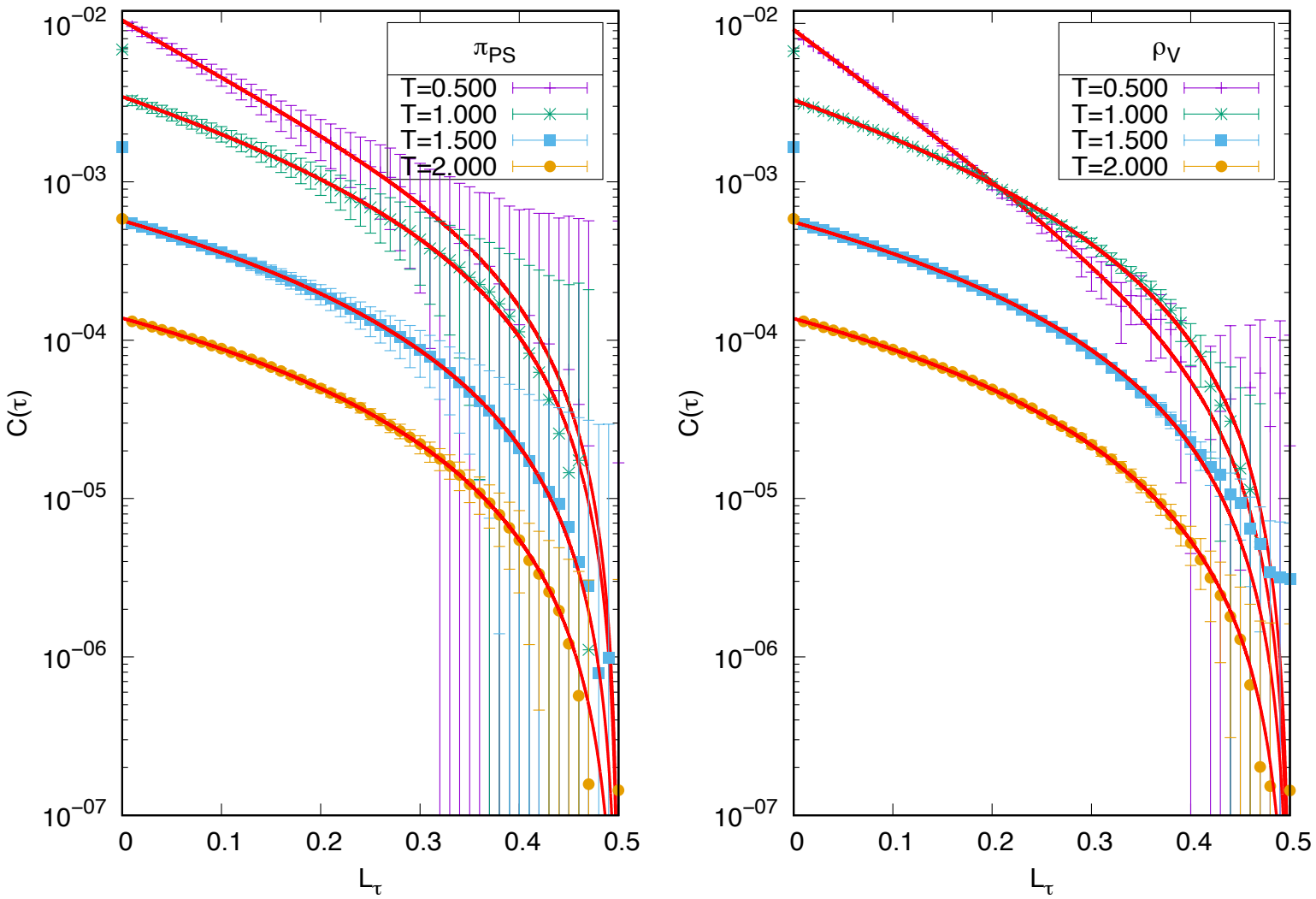
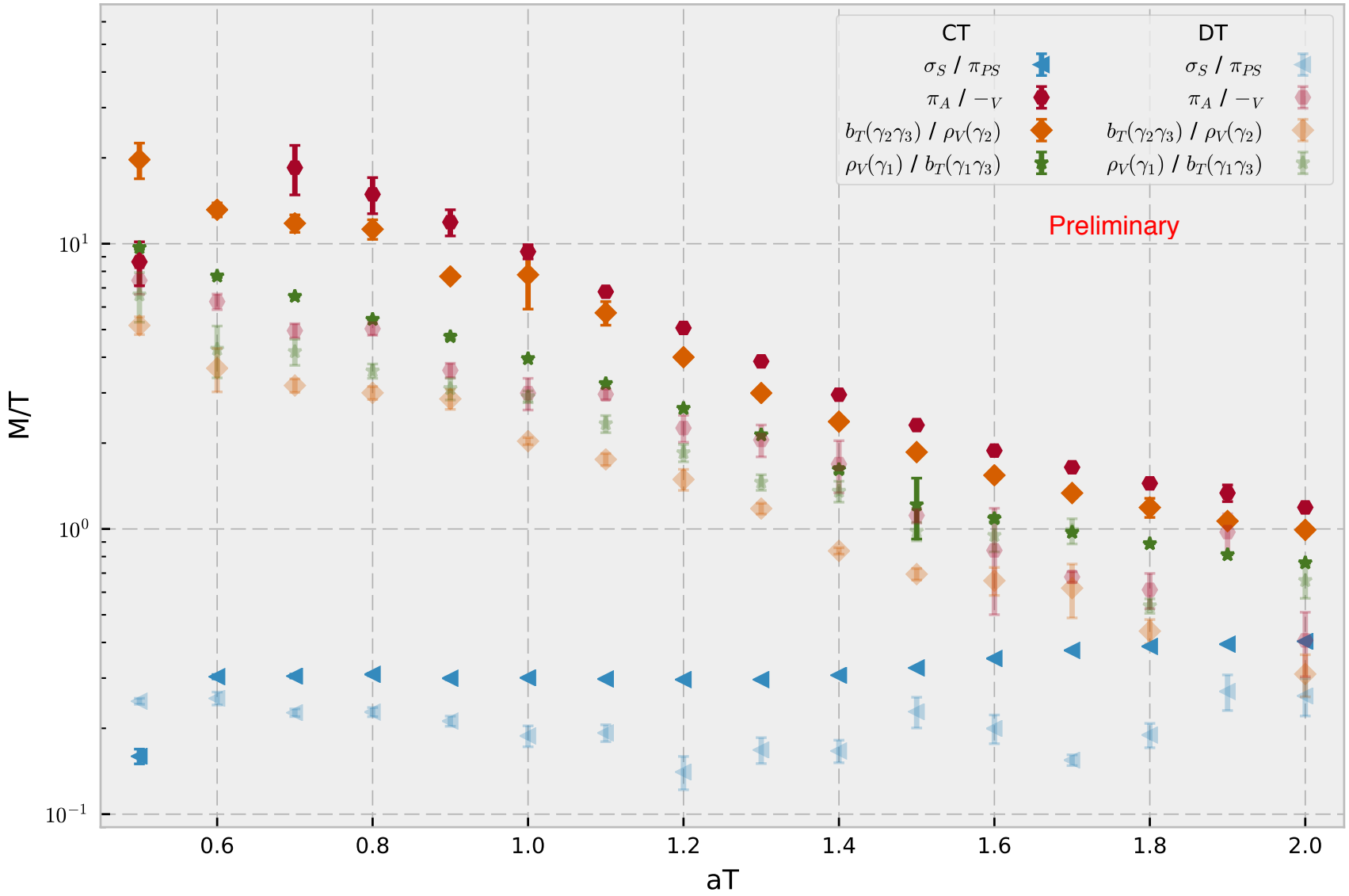


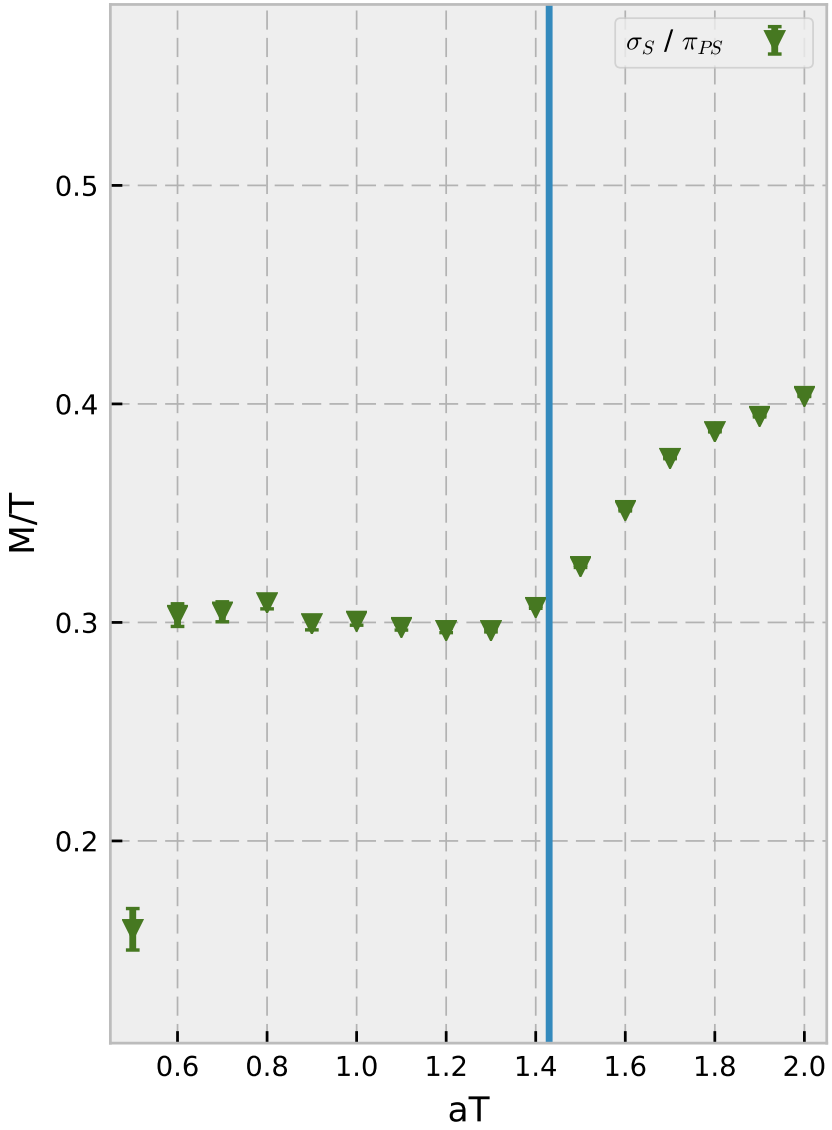
Continuous Time Correlators

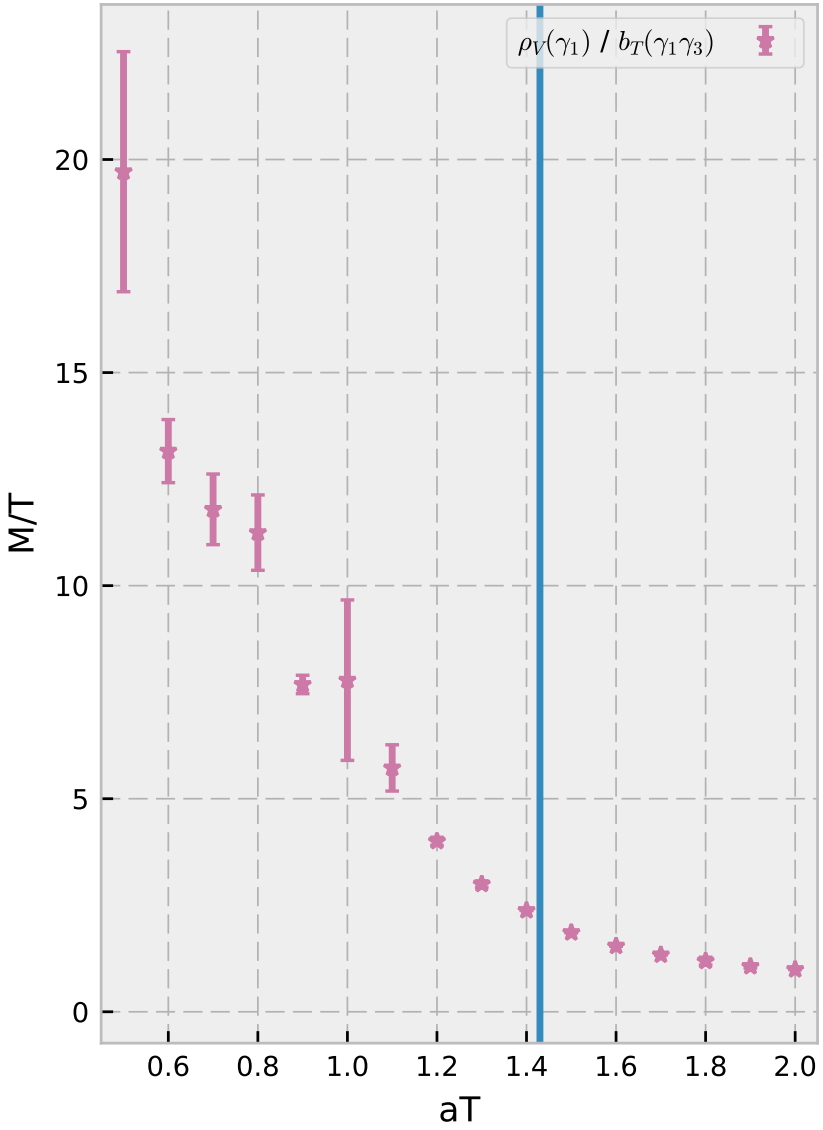


Four Parameter Fit



Behaviour around chiral transition





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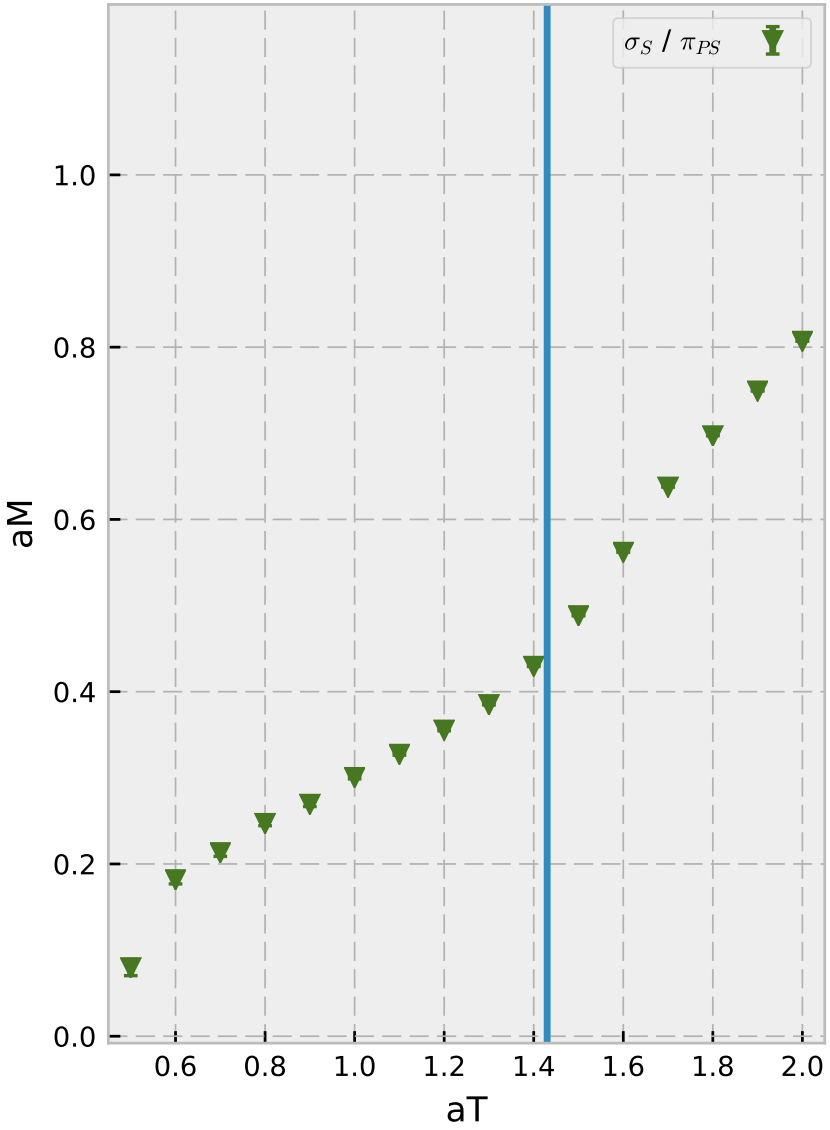

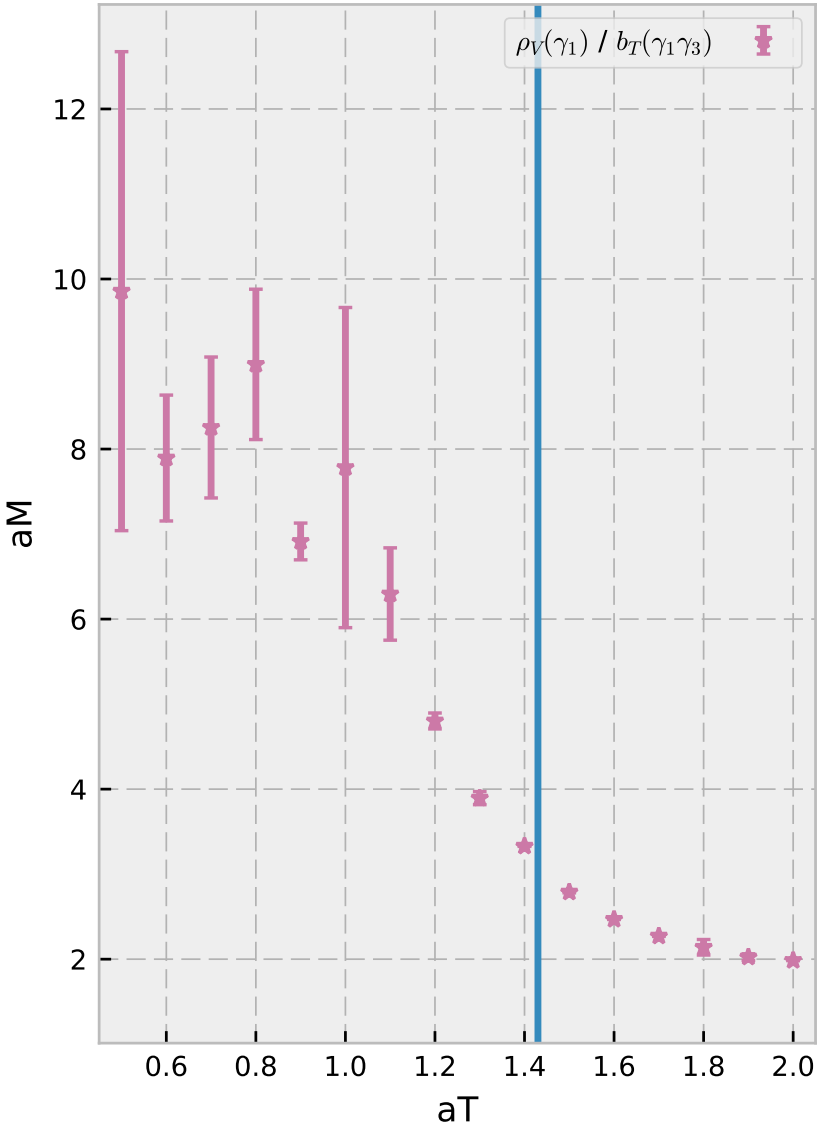

Summary and Outlook

- Measured monomer-monomer two-point functions $\rightarrow$ constructed temporal correlators
- On our way to extract and compare pole masses for discrete and continuous time
- Consider excited states, especially for low temperatures $\Rightarrow$ Mass extraction and analysis not yet fully completed

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- Obtain diffusion constant from zero momentum meson correlators

$\Rightarrow$ Extract spectral function from correlation data

$$C(\tau, T) = \int_0^\infty d\omega K(\omega, \tau)\sigma(\omega, T) = \int_0^\infty d\omega \frac{\cosh(\tau(\omega - \frac{1}{2T}))}{\sinh(\frac{\omega}{2T})}\sigma(\omega, T)$$

Typical bottleneck: #data points in temporal direction $\rightarrow$ advantage of large binning

- Reconstruct spectral function by standard methods like MEM
- $\omega \rightarrow 0$ extrapolation

- Non-zero mass
- $N_f = 2$
- β corrections