Temporal Correlators in the Continuous Time Limit of Strong Coupling Lattice QCD

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Study regime where *sign problems* can be made mild:
⇒ limit of infinite gauge coupling

\[ g \to \infty, \quad \beta = \frac{2N_c}{g^2} \to 0 \]
QCD in the Strong Coupling Limit

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A change of integration order results in SC-partition function for *staggered fermions*:

\[
Z_{SC} = \sum_{\{n,k,l\}} \prod_x \frac{N_c!}{n_x!} (2am_q)^{n_x} \prod_{b=(x,\mu)} \frac{(N_c - k_b)!}{N_c! k_b!} \gamma^{2k_b \delta_{\mu 0}} \prod_l w(l, \mu) 
\]

- **Monomers**
- **Mesonic hoppings/dimers**
- **Baryonic hoppings**

[Wolff & Rossi, 1984]
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- monomers
- mesonic hoppings/dimers
- baryonic hoppings

Fully combinatorial problem, restricted by *Grassmann constraint*:

\[
n_x + \sum_{\pm \mu} k_{x\mu} = N_c, \quad \sum_{\pm \mu} l_{x\mu} = 0, \forall x
\]

In the following restrict to *chiral limit* where monomer density \( \langle n \rangle = 0 \)

[Wolff & Rossi, 1984]
Continuous Time limit within Strong Coupling QCD

First: Introduction of *anisotropy* for continuous temperature variation:

\[
aT = \frac{1}{N_\tau} \Rightarrow aT = \frac{\xi(\gamma)}{N_\tau}, \quad \xi(\gamma) = \frac{a}{a_\tau}
\]

\( \xi(\gamma) \) is the anisotropy parameter.

Second: Gamma dependence of \( \xi(\gamma) \) non trivial:

\[
\xi(\gamma) \approx \kappa \gamma^2 + \gamma^4 + \lambda \gamma^4, \quad \kappa = 0.781 \text{ for SU(3)}
\]
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Definition of the *Continuous Time Limit* as:

\[ N_\tau \rightarrow \infty, \gamma \rightarrow \infty \text{ with } \frac{\xi(\gamma)}{N_\tau} = \frac{\kappa \gamma^2}{N_\tau} = aT \text{ fixed} \]
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Continuous Time partition function:

\[
Z_{CT}(T) = \sum_{k \in 2\mathbb{N}} \left(\frac{1}{2aT}\right) \sum_{g' \in \Gamma_k} e^{\mu_B B/T} \hat{\nu}_T^{N_T}
\]

\[\hat{\nu}_T = \frac{2}{\sqrt{3}} : \begin{array}{c}
\end{array}\]

\[\hat{\nu}_L = 1 : \begin{array}{c}
\end{array}\]

\(N_c = 3:\) with \(k = \sum_{b=(x,i)} k_b, \ N_T = \sum_x n_T(x)\)
Benefits and Comments on Continuous Time Limit

- No discretization errors due to finite $N_\tau$
- Only one parameter left (temperature $T$)
- Baryons become static for $N_c \geq 3 \Rightarrow$ no extend in spatial direction
  $\Rightarrow$ Sign problem is absent
- Baryons are massive (even though in chiral limit)
- No multiple spatial dimers (suppressed by $\gamma$)

- Faster algorithm for medium to large temporal extends
Continuous Time Algorithm

- Worm-type Monte Carlo Algorithm
- *absorption* (even) and *emission* (odd) site decomposition of lattice

**Mesonic worm update:**
- Place *tail* of mesonic worm on lattice at absorption site
  
  $\Rightarrow$ Violation of Grassmann constraint $\Rightarrow$ propagate *head*

  restoration of Grassmann constraint if head at emission site
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\begin{itemize}
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\end{itemize}

\[ \text{T} \]

\[ \text{H} \]

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[Adams & Chandrasekharan, 2003]
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Weight of configuration ruled by spatial dimer emission/absorption
- \textit{spatial dimer emission} ruled by \textit{Poisson process}
- Vertex weights decide spatial dimer \textit{absorption}

\[ P(\Delta \beta) \sim \exp(\lambda \Delta \beta), \quad \Delta \beta \in [0, \beta = 1/aT] \]
"decay constant" \( \lambda \) for spatial dimer emission:
\[ \lambda = d_D(x)/4, \quad d_M(x) = 2d - \sum \mu \ n_B(x \pm \hat{\mu}) \]
with \( d_M(x) \) the number of mesonic neighbors
Two-point Correlators

- Sample *monomer-monomer two-point correlation functions*

\[ C(t_H - t_T, \vec{x}_H - \vec{x}_T) = C(\tau, \vec{x}) \]

- accumulate observables during **worm evolution** (tail absorption/source, head emission/sink)

\[ C(\tau, \vec{x}) = N_c \frac{O(C(\tau, \vec{x}))}{\text{#worm updates}} \]

- Measure Chiral Susceptibility \( \chi_\sigma \) by summing over worm estimators:

\[ \chi_\sigma = \frac{1}{V} \sum_{\vec{x}} C(\tau, \vec{x}) \]

**Discrete Time:**

\[ O(C(\tau, \vec{x})) \rightarrow O(C(\tau, \vec{x})) + f(\gamma) \cdot \delta_{x_T, x_1} \delta_{x_H, x_2}, \quad \tau \in [0, \ldots N_\tau] \]

**Continuous Time:**

\[ O(C(\tau, \vec{x})) \rightarrow O(C(\tau, \vec{x})) + g(T) \cdot \delta_{x_T, x_1} \delta_{x_H, x_2}, \quad \tau \in [0, \ldots \frac{1}{T}] \]
Extracting Meson Masses

- Extract *pole masses* for temporal correlators with *zero spatial momentum*

\[ E_0(\vec{p} = 0) = m_0, \quad C(\tau) = \sum_{\vec{x}} \langle \bar{\chi}_{0} \chi_{0} \bar{\chi}_{\vec{x}}, t \chi_{\vec{x}}, t \rangle g_{\vec{x}}^{D} \]

- For staggered fermions: Restrict to diagonal of Dirac-taste-kernel \((N_f = 1)\)

<table>
<thead>
<tr>
<th>(g_{\vec{x}}^{D})</th>
<th>(\Gamma^{D} \otimes \Gamma^{F})</th>
<th>(J^{PC})</th>
<th>Physical states</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(1 \otimes 1)</td>
<td>(\gamma_0 \gamma_5 \otimes (\gamma_0 \gamma_5)^*)</td>
<td>(0^{++})</td>
</tr>
<tr>
<td>((-1)^{x_i})</td>
<td>(\gamma_i \gamma_5 \otimes (\gamma_i \gamma_5)^*)</td>
<td>(\gamma_i \gamma_0 \otimes (\gamma_i \gamma_0)^*)</td>
<td>(1^{++})</td>
</tr>
<tr>
<td>((-1)^{x_j+x_k})</td>
<td>(\gamma_j \gamma_k \otimes (\gamma_j \gamma_k)^*)</td>
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</tbody>
</table>

\(0^{+-}\) | \(b_T\) |
\(0^{-+}\) | \(\rho V\) |
\(-\nu\) | \(\pi_{PS}\)

channel of primary interest

[Altmeyer et al., 1993]
Temporal Correlators in Continuous Time

- Introduce *binning*
- Evaluate at same spatial site ⇒ Zero momentum projection
- Accumulate *histograms* while worm head propagates

\[ \text{Value per bin: } \frac{g(T)}{\# \text{bins}} \]

- Distinguish histograms for *even* and *odd* interval contributions

Extract *pole masses* from temporal correlators + Study various *channels*

In CT: Masses measured in units of \( \frac{M}{T} \)
Discrete and Continuous Time Correlator Fits

Discrete time: \textit{4 parameter} fit

Either combined:

\[ C(\tau) = a_{NO} \cosh(m_{NO}(\tau - N_{\tau}/2)) - a_{O} \cos(\pi\tau) \cosh(m_{O}(\tau - N_{\tau}/2)) \]

Or split up for Even and Odd histograms:

\[ C_{DT,\text{Even}}(\tau) = a_{NO} \cosh(m_{NO}(\tau - N_{\tau}/2)) - a_{O} \cosh(m_{O}(\tau - N_{\tau}/2)) \]
\[ C_{DT,\text{Odd}}(\tau) = a_{NO} \cosh(m_{NO}(\tau - N_{\tau}/2)) + a_{O} \cosh(m_{O}(\tau - N_{\tau}/2)) \]

\[ \Rightarrow C_{DT,NO}(\tau) = \frac{1}{2} \left( C_{DT,\text{Even}}(\tau) + C_{DT,\text{Odd}}(\tau) \right), \quad C_{DT,O}(\tau) = \frac{1}{2} \left( C_{DT,\text{Even}}(\tau) - C_{DT,\text{Odd}}(\tau) \right) \]
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Non-oscillating Correlator  Oscillating Correlator

\[ \Rightarrow C_{DT,NO}(\tau) = \frac{1}{2} (C_{DT,Even}(\tau) + C_{DT,Odd}(\tau)), \quad C_{DT,O}(\tau) = \frac{1}{2} (C_{DT,Even}(\tau) - C_{DT,Odd}(\tau)) \]

Continuous time: 2/4 parameter fit of added and subtracted histograms respectively

\[ C_{CT,NO}(\tau) = a_{NO} \cosh(m_{NO}(\tau - 1/2)) = \frac{1}{2} (C_{Odd}(\tau) + C_{Even}(\tau)) \]
\[ C_{CT,O}(\tau) = a_O \cosh(m_O(\tau - 1/2)) = \frac{1}{2} (C_{Odd}(\tau) - C_{Even}(\tau)) \]
From discrete Histograms to Correlators and Masses

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Change \( a_\tau M \rightarrow M/T \) in order to compare with Continuous Time results.
From discrete Histograms to Correlators and Masses
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Continuous Time Correlators

\begin{align*}
\pi_{PS} & \quad T=0.500 \\
& \quad T=1.000 \\
& \quad T=1.500 \\
& \quad T=2.000 \\
\rho_{V} & \quad T=0.500 \\
& \quad T=1.000 \\
& \quad T=1.500 \\
& \quad T=2.000 \\
\end{align*}
Four Parameter Fit

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Temporal Correlators in the Continuous Time Limit of Strong Coupling Lattice QCD
Behaviour around chiral transition

\[ \frac{\sigma_s}{\pi_{PS}} \]

\[ \frac{\rho_{V(\gamma_1)}}{b_{T(\gamma_1 \gamma_3)}} \]

\[ \text{Marc Klegrewa} \]

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Behaviour around chiral transition

\[ \frac{\sigma_S}{\pi_{PS}} \]

\[ \frac{\rho_V(\gamma_1)}{b_T(\gamma_1 \gamma_3)} \]
Summary and Outlook

- Measured monomer-monomer two-point functions $\rightarrow$ constructed temporal correlators
- On our way to extract and compare pole masses for discrete and continuous time
- Consider excited states, especially for low temperatures $\Rightarrow$ Mass extraction and analysis not yet fully completed

- Obtain diffusion constant from zero momentum meson correlators
$\Rightarrow$ Extract spectral function from correlation data

$$C(\tau, T) = \int_{0}^{\infty} d\omega K(\omega, \tau)\sigma(\omega, T) = \int_{0}^{\infty} d\omega \frac{\cosh(\tau(\omega - \frac{1}{2T}))}{\sinh(\frac{\omega}{2T})}\sigma(\omega, T)$$

Typical bottleneck: #data points in temporal direction $\rightarrow$ advantage of large binning
- Reconstruct spectral function by standard methods like MEM
- $\omega \rightarrow 0$ extrapolation

- Non-zero mass
- $N_f = 2$
- $\beta$ corrections