Chiral transition using the Banks-Casher relation

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Columbia plot

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- crossover at physical point

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- $m \to 0$ limit controversial
- here: learn about the chiral limit using a novel technique
Outline

- problems of the chiral limit
- new approach
  - Banks-Casher relation
  - determination of the spectral density
  - chiral extrapolations
- results
- conclusions
Towards the chiral limit

- with unimproved actions: critical point with huge lattice artefacts [de Forcrand, D’Elia ’17]

- with improved actions: no critical point only strengthening

[Endrӧdi et al ’07] [Ding et al ’18]
Strategy

- attempt an extrapolation to the chiral limit directly

\[ \langle \bar{\psi} \psi(m) \rangle_m = \frac{1}{Z_m} \int \mathcal{D}U \ e^{-S_g} \ \det[\hat{\mathcal{D}} + m] \ \text{tr}[(\hat{\mathcal{D}} + m)^{-1}] \]

- \( m \to 0 \) using Banks-Casher relation [Banks, Casher '80]
- \( m \to 0 \) using leading-order reweighting
Banks-Casher relation

- In the eigenbasis of $\mathcal{D}$, the condensate $\bar{\psi}\psi \propto \text{tr}(\mathcal{D} + m)^{-1}$

$$\bar{\psi}\psi(m) = \frac{T}{V} \sum_{i} \frac{m}{\lambda_{i}^{2} + m^{2}} \xrightarrow{V \to \infty} \int_{-\infty}^{\infty} d\lambda \rho(\lambda) \frac{m}{\lambda^{2} + m^{2}} \xrightarrow{m \to 0} \pi \rho(0)$$

- The eigenvalues contain much more information than just $\bar{\psi}\psi(m)$, they encode also its dependence on $m$
Leading-order reweighting

- reweight configurations towards \( m = 0 \)

\[
\langle \rho(\lambda) \rangle_0 = \frac{\langle \rho(\lambda) W(m) \rangle_m}{\langle W(m) \rangle_m}
\]

with

\[
W(m) = \frac{\det[\mathcal{D}]}{\det[\mathcal{D} + m]} = \exp \left[ -\frac{V}{T} m \cdot \bar{\psi} \psi(m) + \mathcal{O}(m^4) \right]
\]

- work with the so reweighted spectral density in the following
Spectral density

- find $\rho(0)$ via extrapolation of integrated spectral density

$$N(\lambda) = \int_0^\lambda d\lambda' \rho(\lambda')$$

$$\rho(0) = \lim_{\lambda \to 0} \frac{N(\lambda)}{\lambda}$$

- build histogram of intersects to define mean and systematic error of fit
Extrapolations

- remaining $m_{ud}$-dependence much smaller than in the full condensate $\langle \bar{\psi} \psi(m) \rangle_m$
Chiral transition

- sharpening of the order parameter as $V$ grows
  $\sim$ real phase transition?

- chiral transition temperature at crossing point of two volumes:
  $T_c^{N_f=2+1} \approx 140$ MeV
**Chiral transition**

- Sharpening of the order parameter as $V$ grows leads to a real phase transition?

- Chiral transition temperature at crossing point of two volumes: $T_c^{N_f=2+1} \approx 140$ MeV

- The same signal is hidden in the full condensate
Chiral transition

- Sharpening of the order parameter as $V$ grows $\Rightarrow$ real phase transition?

- Chiral transition temperature at crossing point of two volumes: $T_{c}^{N_f=2+1} \approx 140$ MeV

- The same signal is hidden in the full condensate

- For $\langle \bar{\psi}\psi(m = 0) \rangle$, no additive renormalization necessary
Number of massless flavors

- same analysis along $m_s/m_{ud} = \text{const. line}$

![Graph showing $\langle \bar{\psi} \psi \rangle$ versus $T$ (MeV) for different lattice sizes $16^3 \times 6$ and $24^3 \times 6$. The $m_{ud} \rightarrow 0$ limit is indicated, as well as $m_s = m_s^{ph}$]
Number of massless flavors

- same analysis along $m_s/m_{ud} = \text{const. line}$

![Graphs showing the variation of $\langle \bar{\psi} \psi \rangle$ with temperature for different lattice sizes and mass ratios.](graphs.png)
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- vacuum condensate reduced consistent with $\chi$PT [Moussalam '99, Descotes et al '99]
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- volume-dependence more pronounced $\sim$ stronger transition?
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- volume-dependence more pronounced $\rightsquigarrow$ stronger transition?

- chiral transition is reduced to $T_{c}^{N_f=3} \approx 125 \: \text{MeV}$
Nature of the transition

- fit for slope of order parameter

![Graph showing critical scaling](image)

- critical scaling: $\bar{\psi}\psi'_{T=T_c} \overset{V \to \infty}{\longrightarrow} \infty$
Nature of the transition

- fit for slope of order parameter

- critical scaling: $\bar{\psi}\psi'_{T=T_c} \xrightarrow{V \to \infty} \infty$
Nature of the transition

▶ fit for slope of order parameter

\[ \overline{\psi}\psi \propto T^{-\frac{11}{12}} \]

▶ critical scaling: \( \overline{\psi}\psi \bigg|_{T=T_c} \xrightarrow{V \to \infty} \infty \)
Nature of the transition

- fit for slope of order parameter

- critical scaling: $\bar{\psi}\psi'_{T=T_c} \xrightarrow{V \to \infty} \infty$
Nature of the transition

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\[ \bar{\psi}\psi \rightarrow T = T_c \quad V \rightarrow \infty \quad \infty \]

- critical scaling: \( \bar{\psi}\psi \rightarrow T = T_c \quad V \rightarrow \infty \quad \infty \)
Nature of the transition

- fit for slope of order parameter

\[ \bar{\psi} \psi \rightarrow \sqrt{\nu} \rightarrow \infty \]

- critical scaling: \( \bar{\psi} \psi'_{T=T_c} \rightarrow \infty \)
Summary

▶ extract chiral condensate via Banks-Casher relation
   \( \sim \) flat extrapolation

▶ finite volume analysis of chiral condensate
   (no additive renormalization required)

▶ \( N_F = 2 + 1 \) chiral limit
   consistent with \( O(4) \) scenario