Thermodynamics for SU(2) pure gauge theory using gradient flow

Takehiro Hirakida, Etsuko Itou[1,2], Hiroaki Kouno[3]

Kyushu U., RCNP[1], Kochi U.[2], Saga U.[3]

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arXiv:1805.07106 [hep-lat]

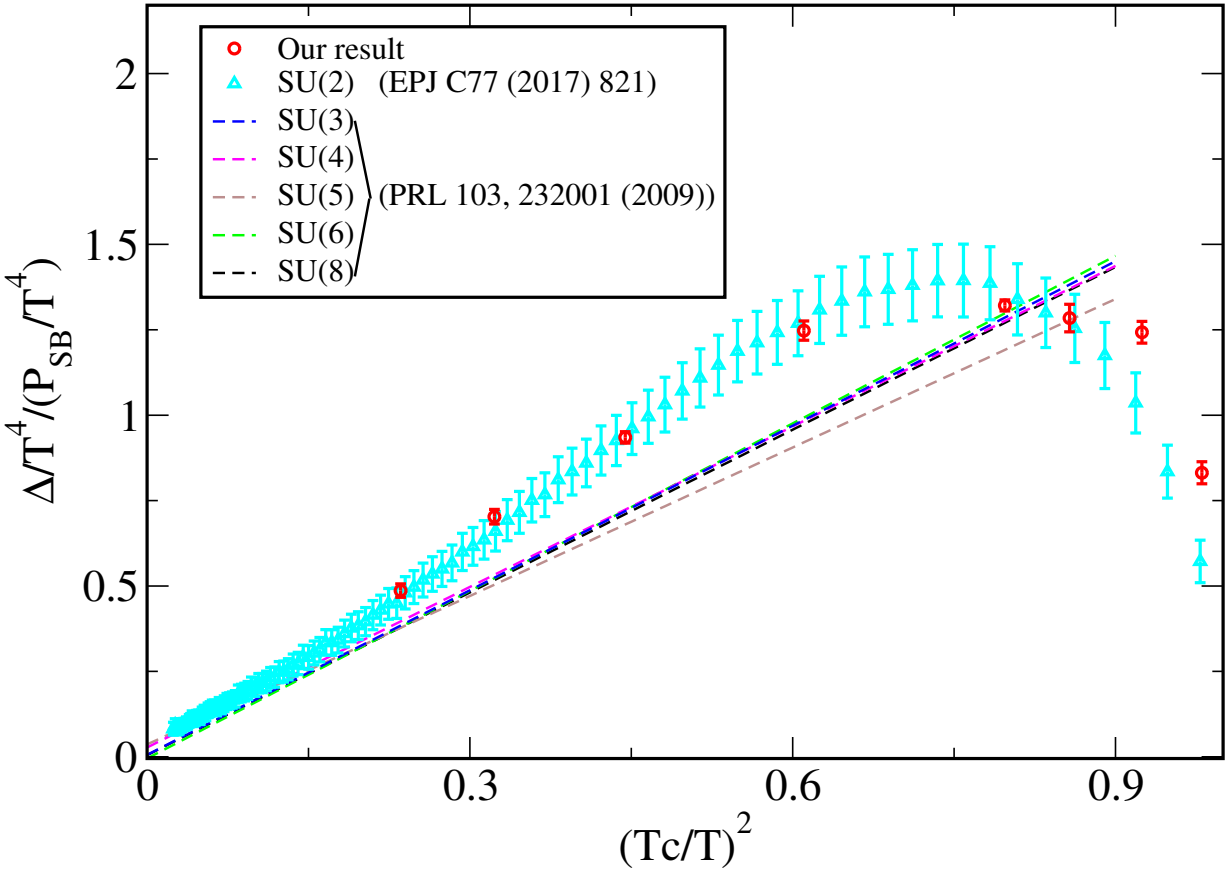
QCD in intermediate temperature

- Experimental data
  $\longrightarrow$ small shear viscosity-to-thermal entropy ratio (η/s)
  $\longrightarrow$ perfect-liquid property rather than gas
- Large-N_c analysis based on AdS/CFT
  $\longrightarrow$ lower bound for shear viscosity
- Lattice calculation
  - Shear viscosity η
    First step: Calculate correlation function of EMT
    - Bad signal-to-noise ratio
      (eg. 6-million Conf. needed in SU(3))
    - Definition of the correctly renormalization of EMT
    - Solving inverse-problem to obtain spectral function
  - Thermal entropy s
    Method: Integral method[1], Gradient flow method[2] etc.

[1] J. Engels et al., Phys. Lett. B **252**, 625 (1990).
[2] H. Suzuki, PTEP 2013, 083B03 (2013)

SU(2) pure gauge theory

- Focus on **SU(2) pure gauge theory**
  - Numerical cost is lower than SU(3) gauge theory
  - Provide larger signal of correction term of $1/N_c$
- Difference from SU($N_c \geq 3$) gauge theory



In this study

Thermodynamics for pure SU(2) using Gradient flow

1. Scale setting using t_0 **reference scale**
   to determine relation between β and lattice spacing
2. Measure thermodynamics quantities
   $$(s/T^3,\ \Delta/T^4,\ \varepsilon/T^4,\ P/T^4)$$

Gradient flow

- Yang-Mills gradient flow equation on lattice[3]

$$\partial_t V_t(x,\mu) - g_0^2 \left\{ \partial_{x,\mu} S_W \right\} V_t(x,\mu)$$

$$V_t(x,\mu)\Big|_{t=0} = U(x,\mu)$$

g_0: bare coupling, t: flow time, $U(\mu,x)$: link variable, S_W: Wilson-plaquette action.

- Renormalized EMT with gradient flow[4]

$$T^R_{\mu\nu} = \lim_{t\to 0}\left[\frac{U_{\mu\nu}}{\alpha_U} + \frac{\delta_{\mu\nu}}{4\alpha_E}\left\{E - \langle E\rangle_0\right\}\right]$$

$$U_{\mu\nu} = G_{\mu\rho}G_{\nu\rho} - \frac{\delta_{\mu\nu}}{4}G_{\rho\sigma}G_{\rho\sigma},\ \ E = \frac{1}{4}G_{\mu\nu}G_{\mu\nu}$$

$G_{\mu\nu}$: field strength consisting of V_t

α_U, α_E: calculated in 1-loop order of running coupling

[3] M. Luscher, JHEP **1008** (2010) 071.

[4] H. Suzuki, PTEP **2013**, 083B03 (2013).

Scale Setting

- Observable: $t^2 \langle E \rangle \propto N_c^2 - 1$
- Reference scale: $t^2 \langle E(t) \rangle \Big|_{t=t_0} = 0.1$

  $\rightarrow$ a natural scaling-down of the SU(3) case[5]
- Configuration generation
  - Wilson-plaquette action, $N_s = N_t = 32$
  - 1 sweep = 1 pseudo-heatbath + 20 over-relaxation
  - 100 sweep separation between measurements

β	2.42	2.50	2.60	2.70	2.80	2.85
# of Conf.	100	300	300	300	300	600

- Gradient flow
  - $t/a^2 \in [0.00, 32.00]$, $\Delta t/a^2 = 0.01$

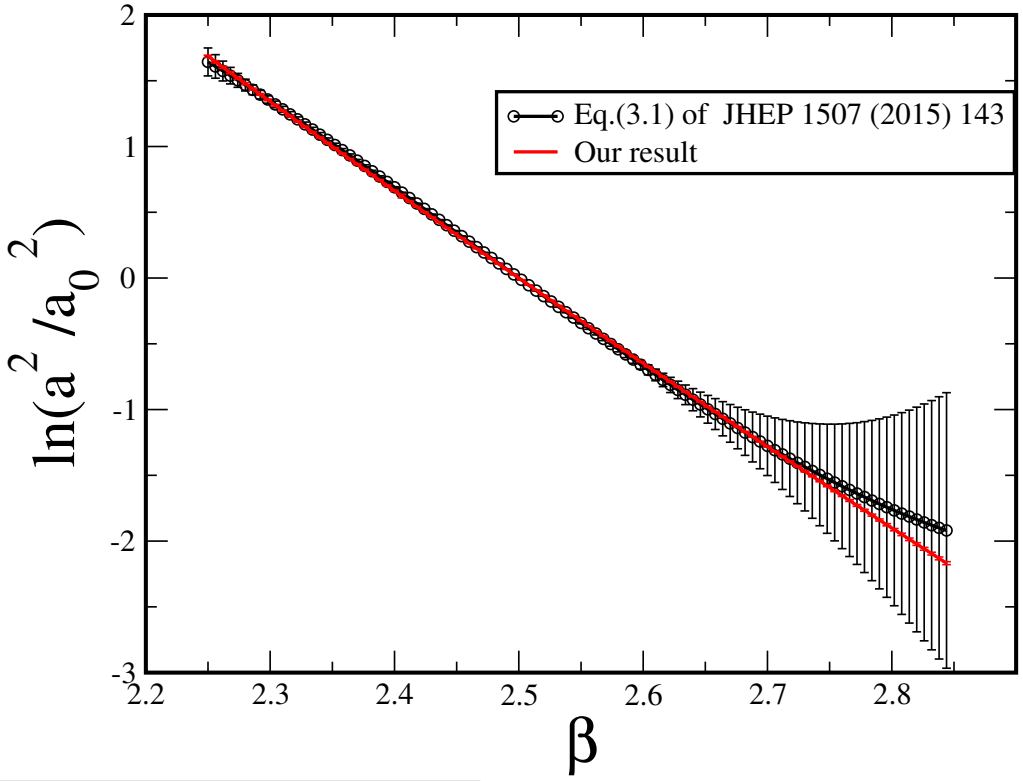
[5] M. Luscher, JHEP **1008** (2010) 071.

Scale Setting

- Best fit function (t_0/a^2 .vs. β): $\beta \in [2.42, 2.85]$

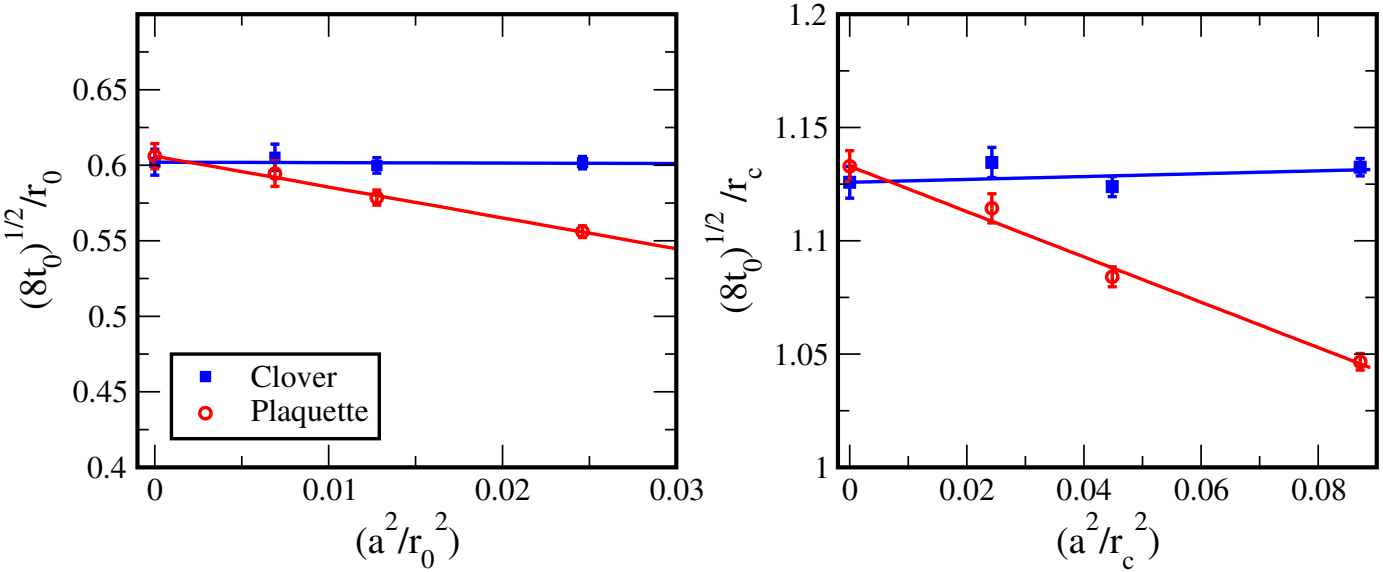
$$\ln(t_0/a^2) = 1.258 + 6.409(\beta - 2.600) - 0.7411(\beta - 2.600)^2$$

- Compare with scale setting using string tension[6]



[6] M. Caselle, A. Nada and M. Panero, JHEP **1507** (2015) 143.

Scale Setting



- Compare with "r_0 scale[7] (r_c scale[8])"

$$\rightarrow \frac{\sqrt{8t_0}}{r_0} = 0.6020(86)(40), \quad \frac{\sqrt{8t_0}}{r_c} = 1.126(7)(7),$$
$$\rightarrow \sqrt{8t_0} = 0.3010(43)(20)[\text{fm}].$$

$$r_0 = 0.5[\text{fm}], \quad r_c = 0.26[\text{fm}].$$

[7] R. Sommer, Nucl. Phys. B 411, 839 (1994).
[8] S. Necco and R. Sommer, Nucl. Phys. B 622, 328 (2002).

Thermodynamics: T/T_c and Observables

- β .vs. T/T_c for each N_t [9]

T/T_c	$N_\tau = 6$	$N_\tau = 8$	$N_\tau = 10$	$N_\tau = 12$
0.95	—	2.50	2.57	2.62
0.98	2.42	2.51	2.58	2.63
1.01	2.43	2.52	2.59	2.64
1.04	2.44	2.53	2.60	2.65
1.08	2.45	2.54	2.61	2.66
1.12	2.46	2.55	2.62	2.67
1.28	2.50	2.59	2.66	2.72
1.50	2.55	2.64	2.71	2.77
1.76	2.60	2.69	2.76	2.82
2.07	2.65	2.74	2.81	—

- Observables: entropy density (s), trace anomaly (Δ), energy density (ε), pressure (P)

$$sT = \varepsilon + P = T^R_{11} - T^R_{44}, \ \Delta = \varepsilon - 3P = -\sum_{\mu=1}^{4} T^R_{\mu\mu}$$

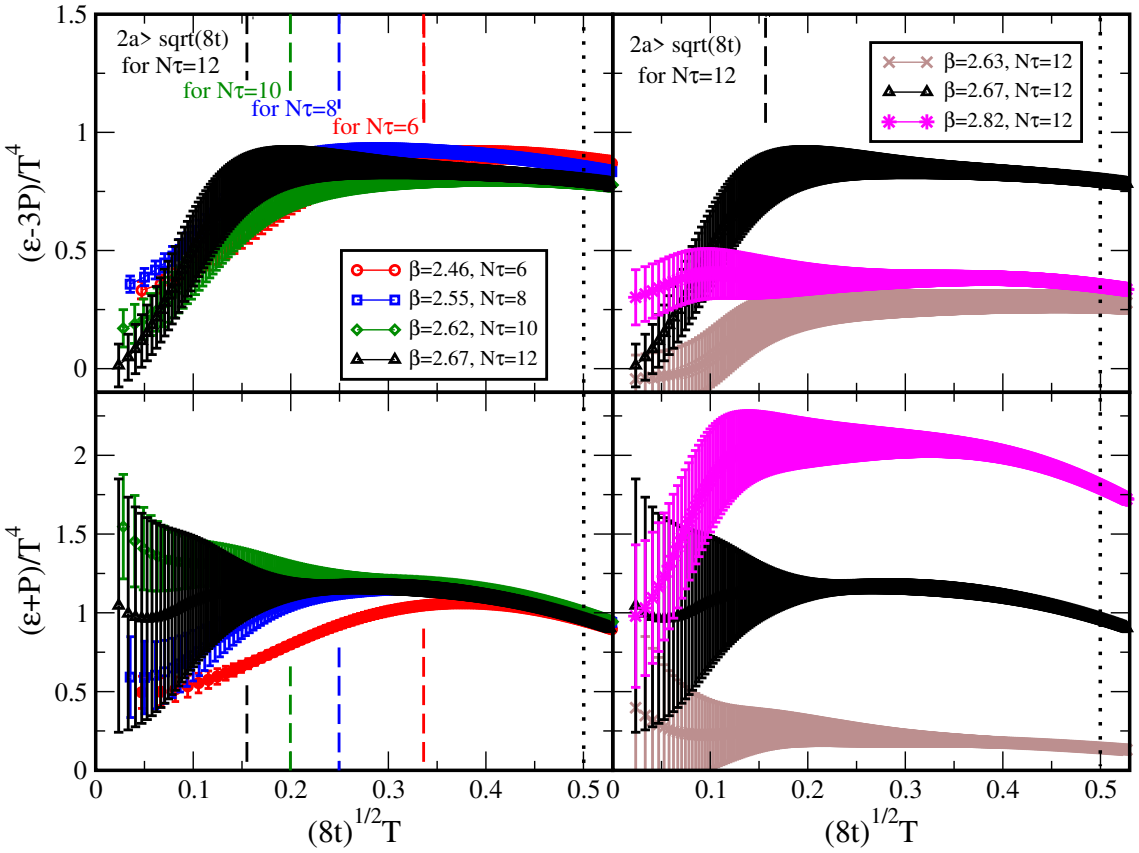
[9] the critical β on $N_t = 6$ from [J. Engels, J. Fingberg and D. E. Miller, Nucl. Phys. B **387** (1992) 501.]

Procedure and Simulation setup

- Steps to calculate renormalized EMT[10]
  1. Generate configuration at $t = 0$ on $N_s^3 \times N_\tau$
  2. Solve gradient flow eq. in $a \ll \sqrt{8t} \ll R$
  3. Construct renormalized EMT at each t
  4. Carry out an extrapolation, first $a \to 0$, next $t \to 0$
- Simulation setup
  - Wilson-plaquette action
  - $N_s/N_\tau = 4,\ N_\tau = 6, 8, 10, 12$
  - # of Conf. for each parameter: 200
  - 1 sweep = 1 pseudo-heatbath+N_t over-relaxation
  - 100 sweep separation between measurements
- Gradient flow
  - $t/a^2 \in [0.00, 5.00],\ \Delta t/a^2 = 0.01$

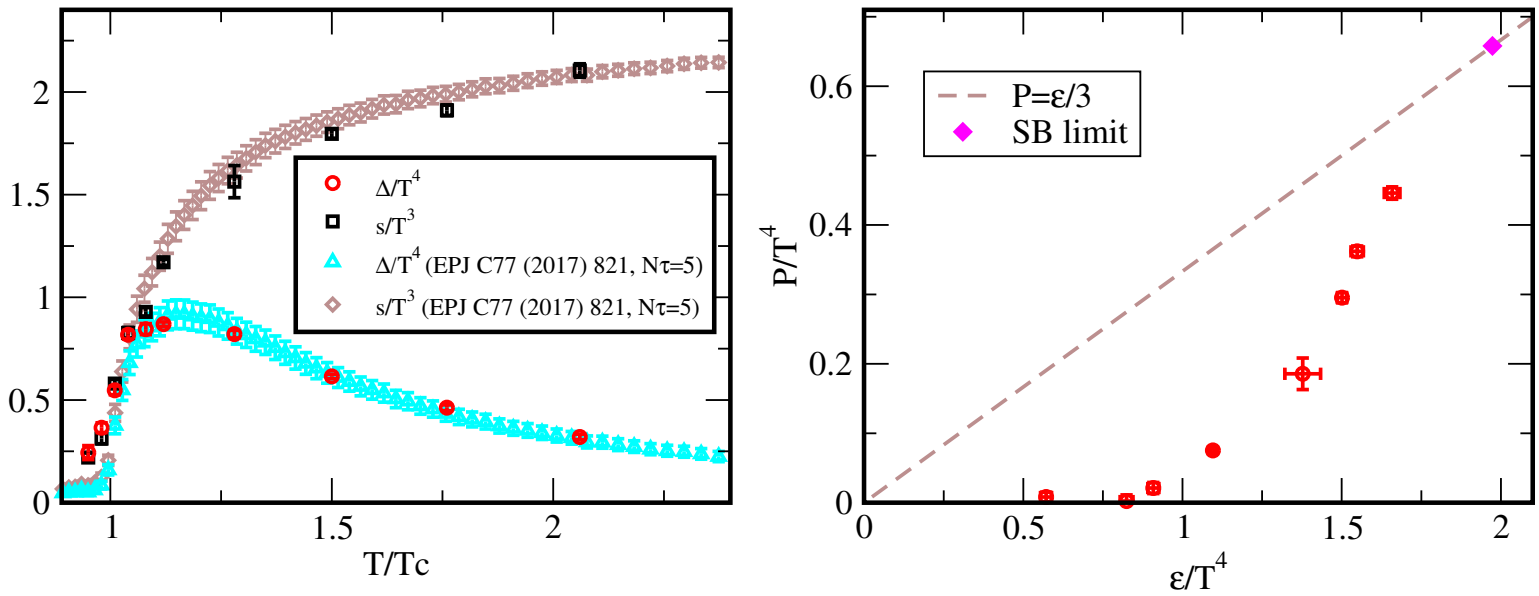
[10] M. Asakawa *et al.*, Phys. Rev. D **90**, no. 1, 011501 (2014)

Flow-time dependence of s/T^3 and Δ/T^4



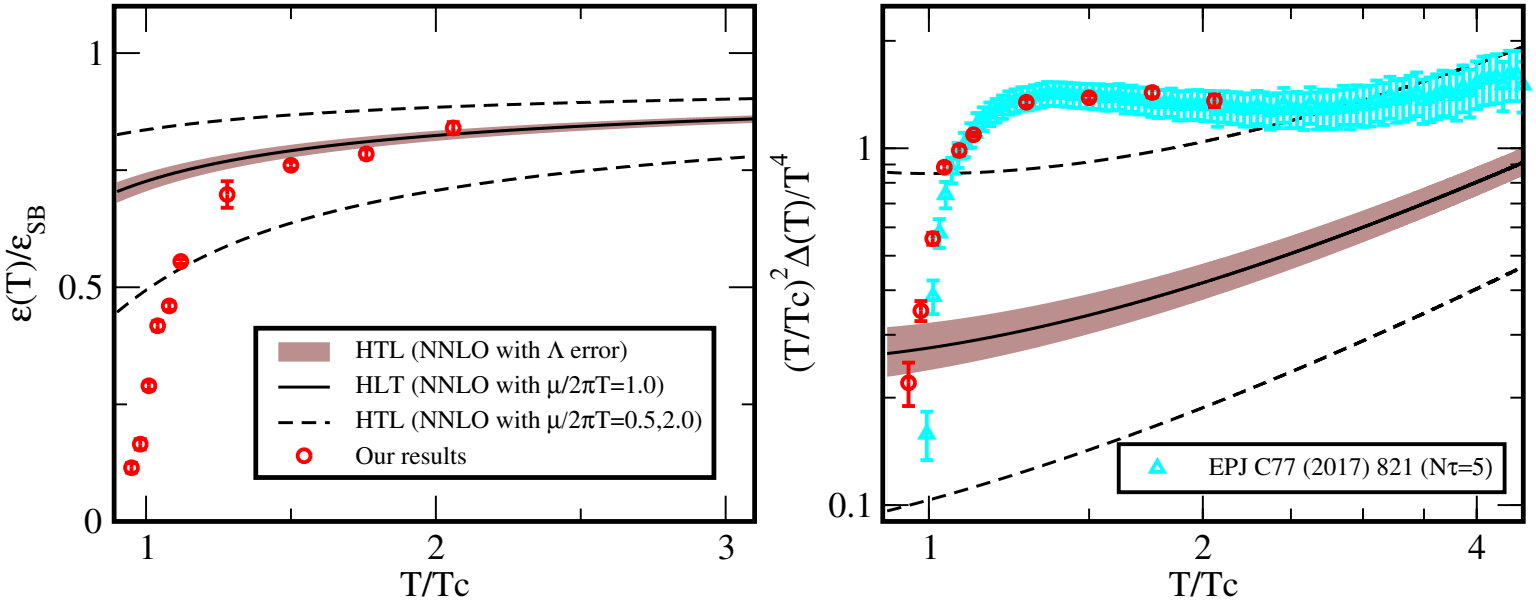
- left: N_t-dep. @$T/T_c = 1.12$, right: T-dep. @$N_\tau = 12$
- Fiducial window: $1/N_t \leq \sqrt{8t}\,T \leq 0.5$
- $(a, t) \to (0, 0)$ limit: constant- & linear-extrapolation

Result



- Left panel: s/T^4 (black symbol), Δ/T^4 (red symbol)
  - $(a,t) \to (0,0)$ sys. error of extrapolation (linear & constant) consistent in $T/T_c \geq 1.12$
- Right panel: ε/T^4 .vs. P/T^4 (EOS) in $T > T_c$
  - Toward to SB limit $(\varepsilon/T^4, P/T^4) = (\pi^2/5, \pi^2/15)$
  - $70 \sim 80\%$ of SB limit for $T \sim 2T_c$
    $\to$ NOT describe two-color QGP around $T \leq 2T_c$

Compare with HTL analysis



- Hard-Thermal-Loop (HTL) analysis[11]
  ... 2-color case in NNLO
- Left panel: $\varepsilon/\varepsilon_{\mathrm{SB}}$
  Our result is consistent with HTL in $T > T_c$
- Right panel: $(T/T_c)^2\Delta/T^4$
  plateau and approaches to HTL result in $1.2T_c \leq T$

[11] J. O. Andersen *et al.*, JHEP **1008** (2010) 113.

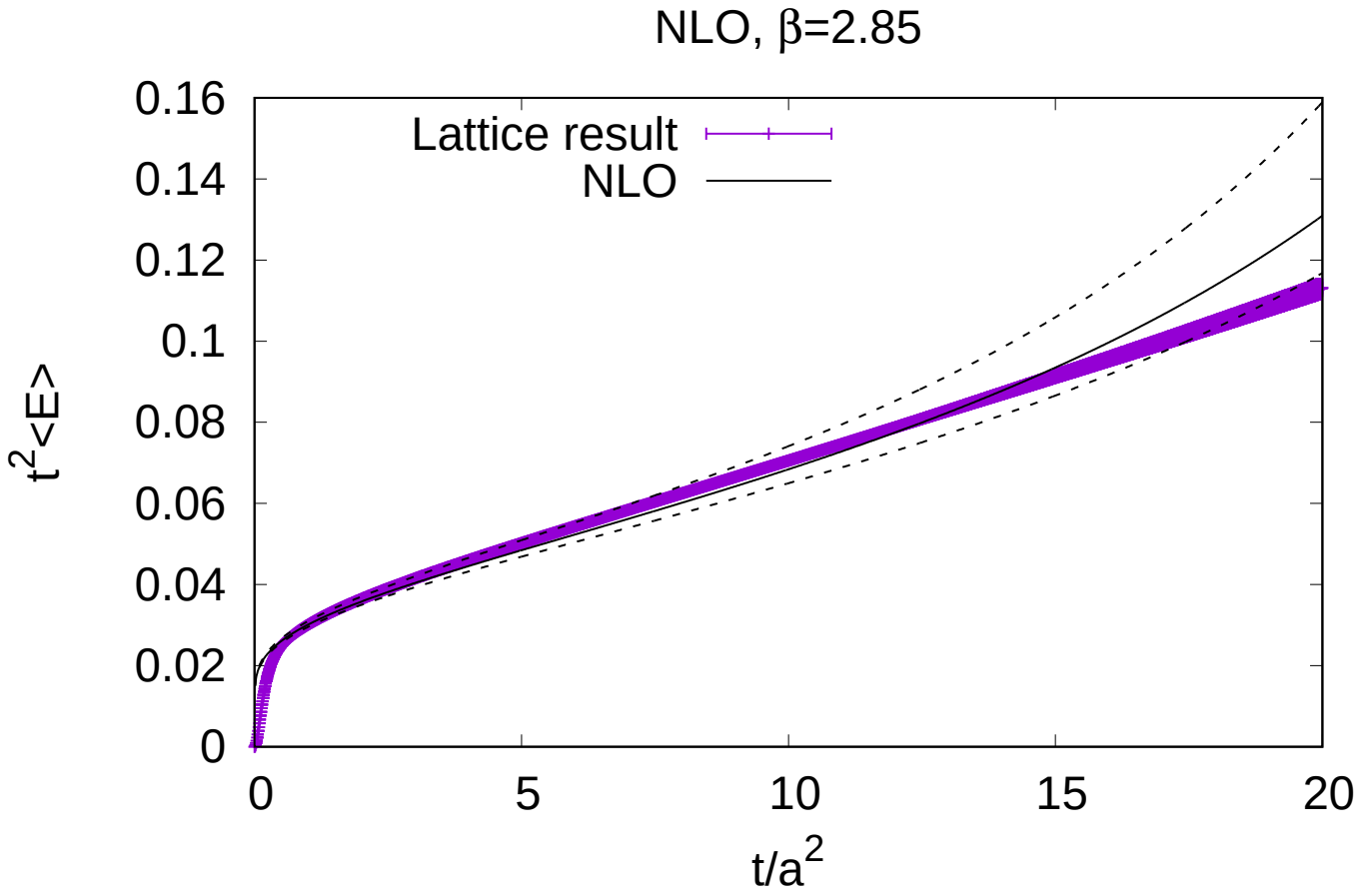
Summary

We investigate the thermodynamics of SU(2) pure gauge theory

1. Scale setting
   - $t^2 \langle E(t) \rangle \Big|_{t=t_0} = 0.1$ for SU(2)
   - our scale-setting function is more precisely and cover wider β region
2. Obtaining s/T^3, Δ/T^4, EOS
   - Confirm that the traceanomaly in the SU(2) pure gauge theory has a different scaling property from the $N_c \geq 3$ cases
   - Our results are more precisely than integral method
   - Consistent with integral method and HTL analysis in high temperature region

Compare $t^2 \langle E \rangle$ with perturbative analysis

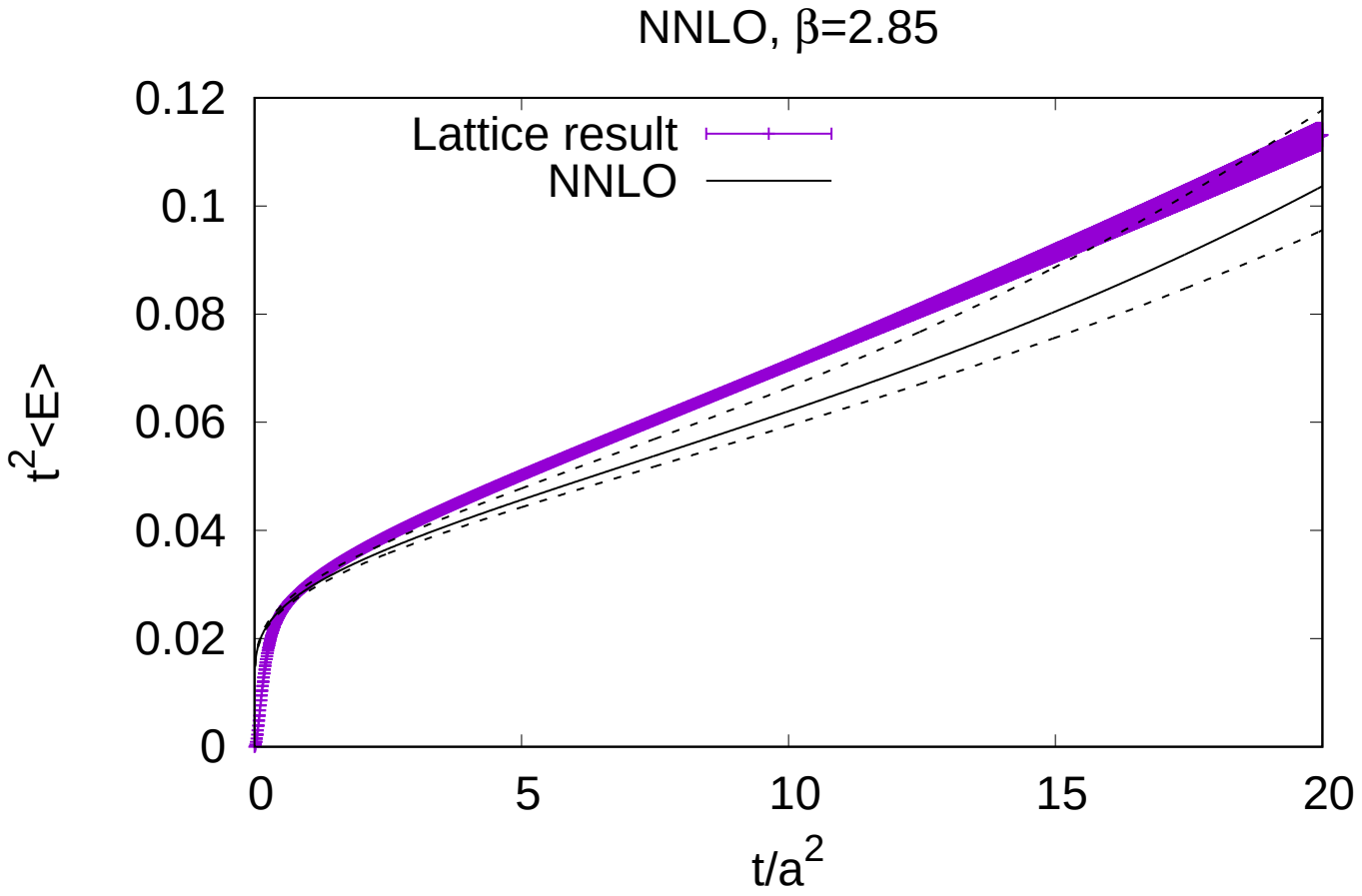
Compare with NLO result[12]



[12] R. V. Harlander and T. Neumann, JHEP **1606** (2016) 161

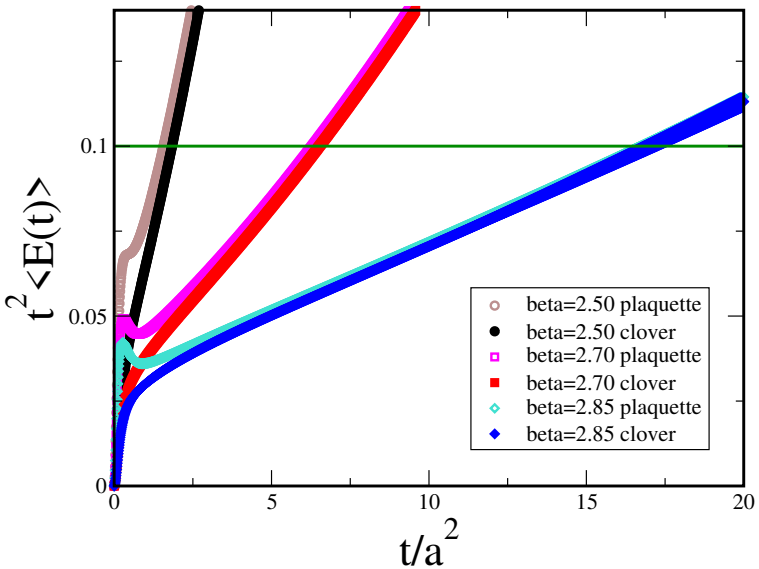
Compare $t^2 \langle E \rangle$ with perturbative analysis

Compare with NNLO result[13]



[13] R. V. Harlander and T. Neumann, JHEP **1606** (2016) 161

Scale Setting

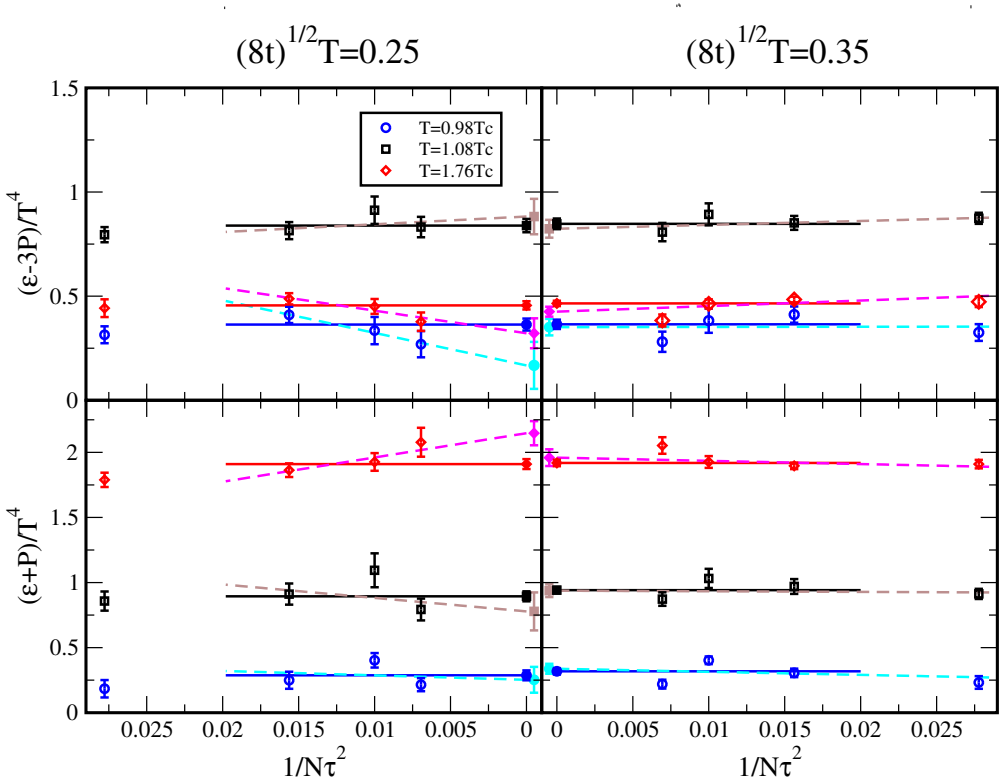


β	2.42	2.50	2.60
t_0/a^2	1.083(2)	1.839(3)	3.522(10)
β	2.70	2.80	2.85
t_0/a^2	6.628(36)	11.96(12)	16.95(17)

- Best fit function (t_0/a^2 .vs. β)

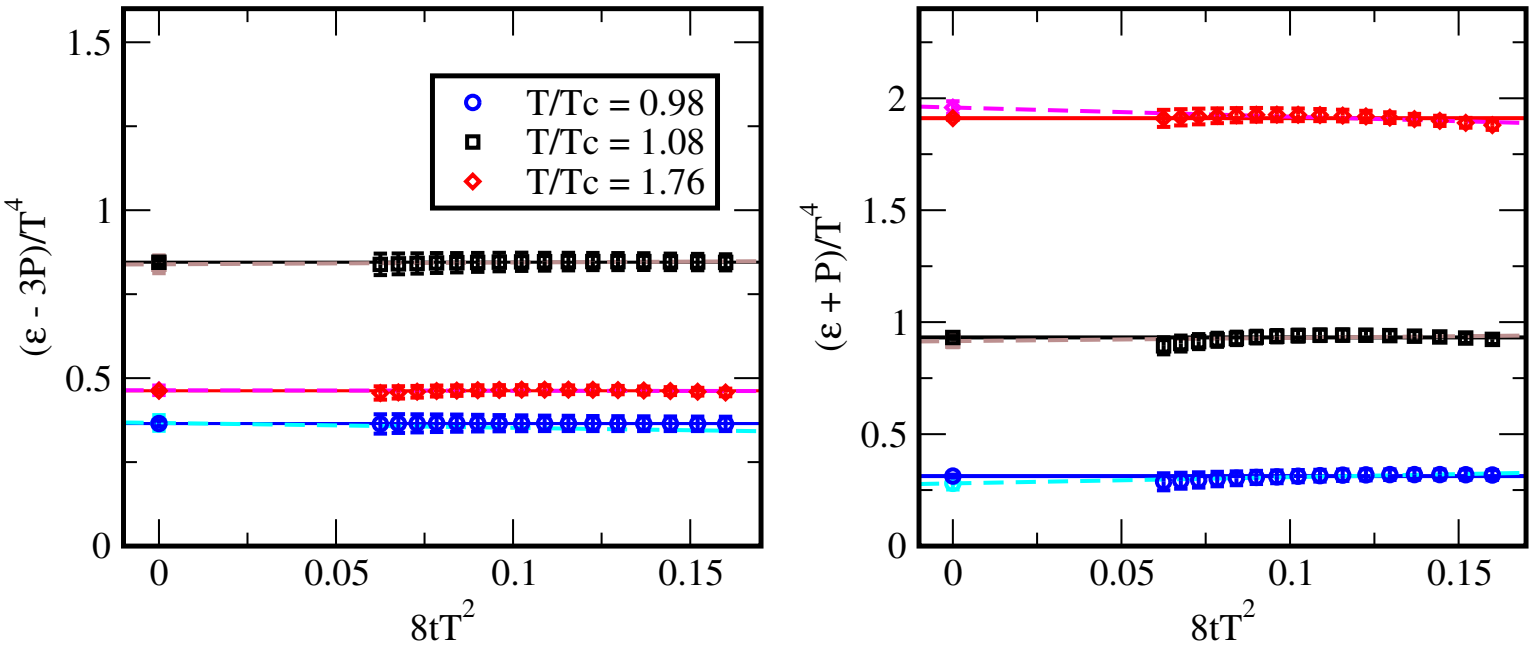
$$\ln(t_0/a^2) = 1.258 + 6.409(\beta - 2.600) - 0.7411(\beta - 2.600)^2.$$

$a \to 0$ limit



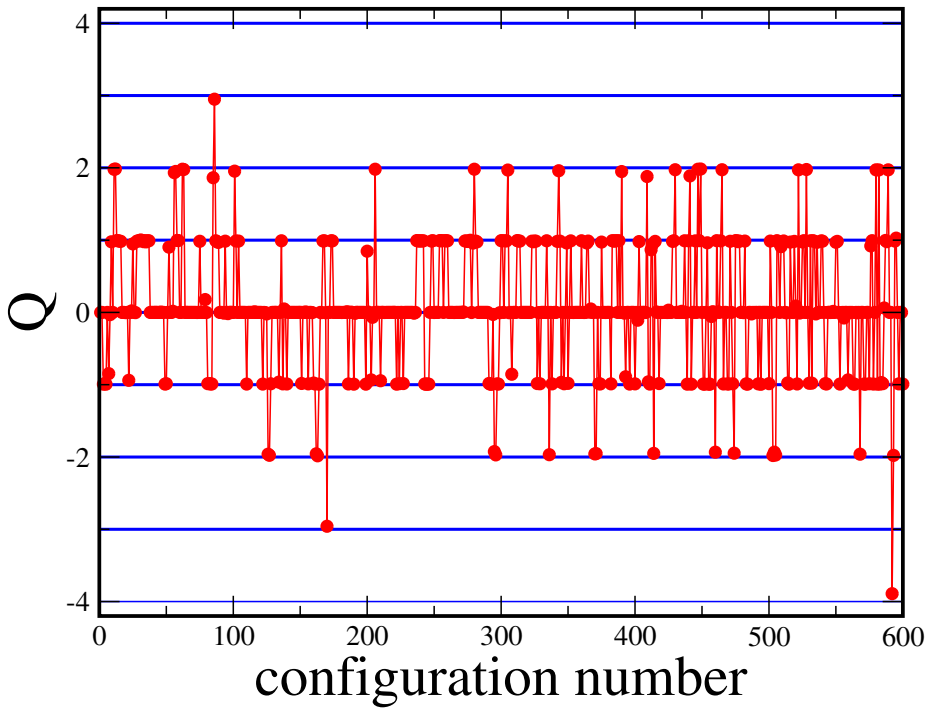
- $\sqrt{8t}\,T \in [0.25, 0.40]$, $\delta(\sqrt{8t}\,T) = 0.01$
- Each data is adopted closest to the fixed $\sqrt{8t}\,T$
- Constant extrapolation: to calculate the central value
- Linear extrapolation: to estimate the systematic error with constant extrapolation
- Sys. error ... at most 3–σ (s/T^3) and 2–σ (Δ/T^4)

$t \longrightarrow 0$ limit



- $\sqrt{8t}\,T \in [0.25, 0.40],\ \delta(\sqrt{8t}\,T) = 0.01$
- Carry out both constant- and linear-extrapolation
- We take the central result which is the better χ^2/d.o.f
- Sys. error ... at most 2–σ (s/T^3) and 1–σ (Δ/T^4)

Back Up: Topological Charge using gradient flow



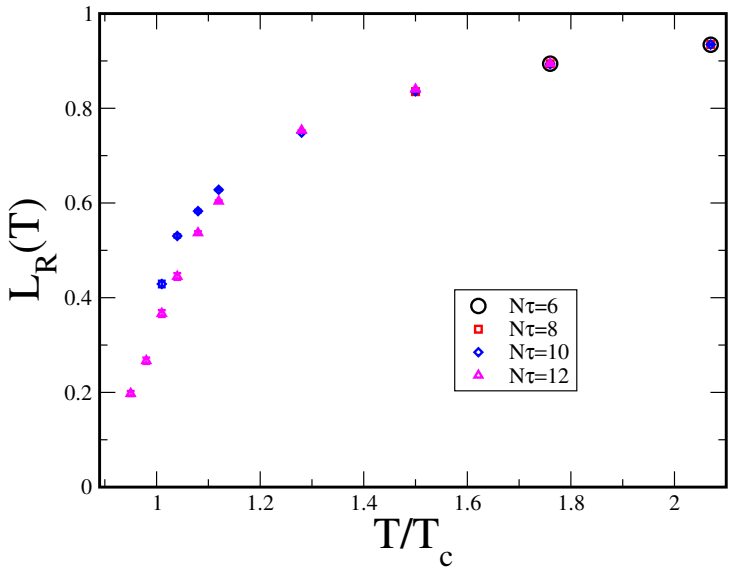
- Figure: Result at $\beta = 2.85, t/a^2 = 32$
- Topological charge Q

$$Q = \frac{1}{32\pi^2} \int d^4x \; \epsilon_{\mu\nu\rho\sigma} \mathrm{Tr} G_{\mu\nu} G_{\rho\sigma}$$

- Q takes an almost integer-value
  $\longrightarrow$ autocorrelation can be negligible in our data sets

Back Up: Renormalized Polyakov loop

- It is believed that universality class of pure SU(2) is same as that of 3-D Ising model
- Renormalization condition[14] $L_R(T = 1.76\,Tc) = 0.894$



- Critical exponent (0.3265(3) in 3-D Ising model)
  - $N_\tau = 10$: 0.159(3)
  - $N_\tau = 12$: 0.242(3)

[14] S. Borsanyi *et al.*, Phys. Lett. B **713** (2012) 342.