The 36th Annual International Symposium on Lattice Field Theory

Axial U(1) symmetry and Dirac spectra in high-temperature phase of N_f=2 lattice QCD

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from JLQCD Collaboration:
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Guido Cossu (Edinburgh), Hidenori Fukaya (Osaka U.),
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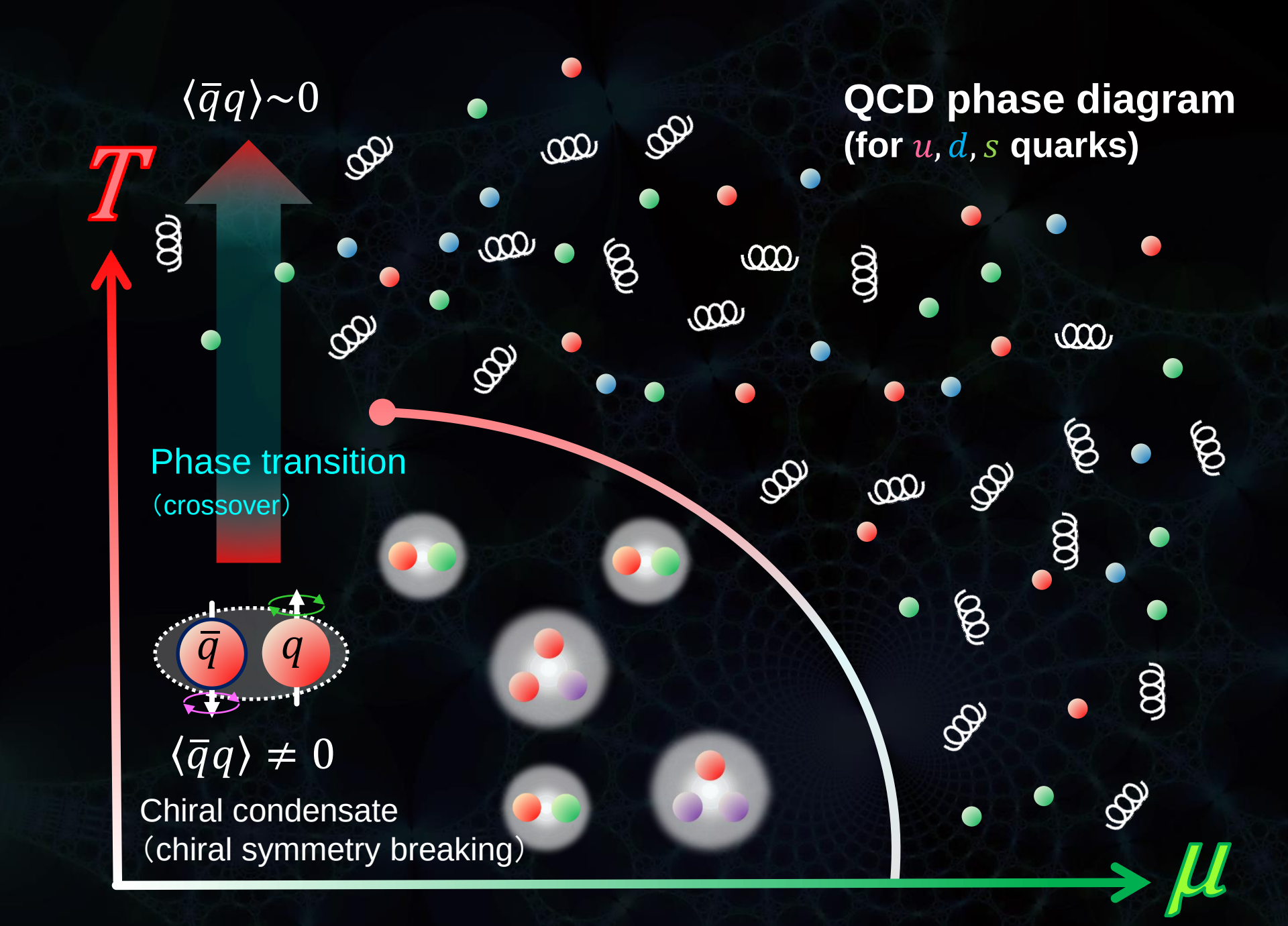
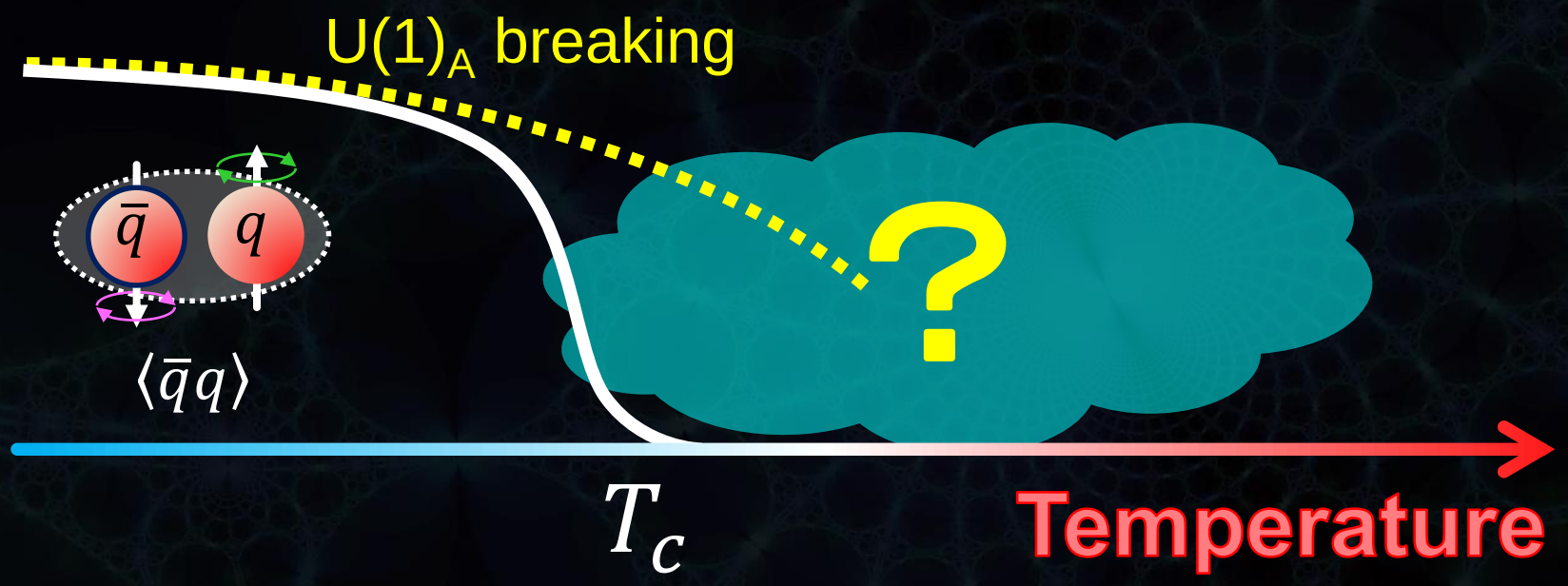
24/Jul/2018


QCD phase diagram
(for u, d, s quarks)
$\langle \bar{q}q \rangle \sim 0$
T
Phase transition
(crossover)
$\bar{q}$
q
$\langle \bar{q}q \rangle \neq 0$
Chiral condensate
(chiral symmetry breaking)
μ

$U(1)_A$ symmetry (in vacuum, broken by anomaly) is restored above Tc？

- Above Tc, chiral symmetry breaking by $\langle \bar{q}q \rangle$ is restored
  ⇒How about <u>$U(1)_A$ symmetry?</u>

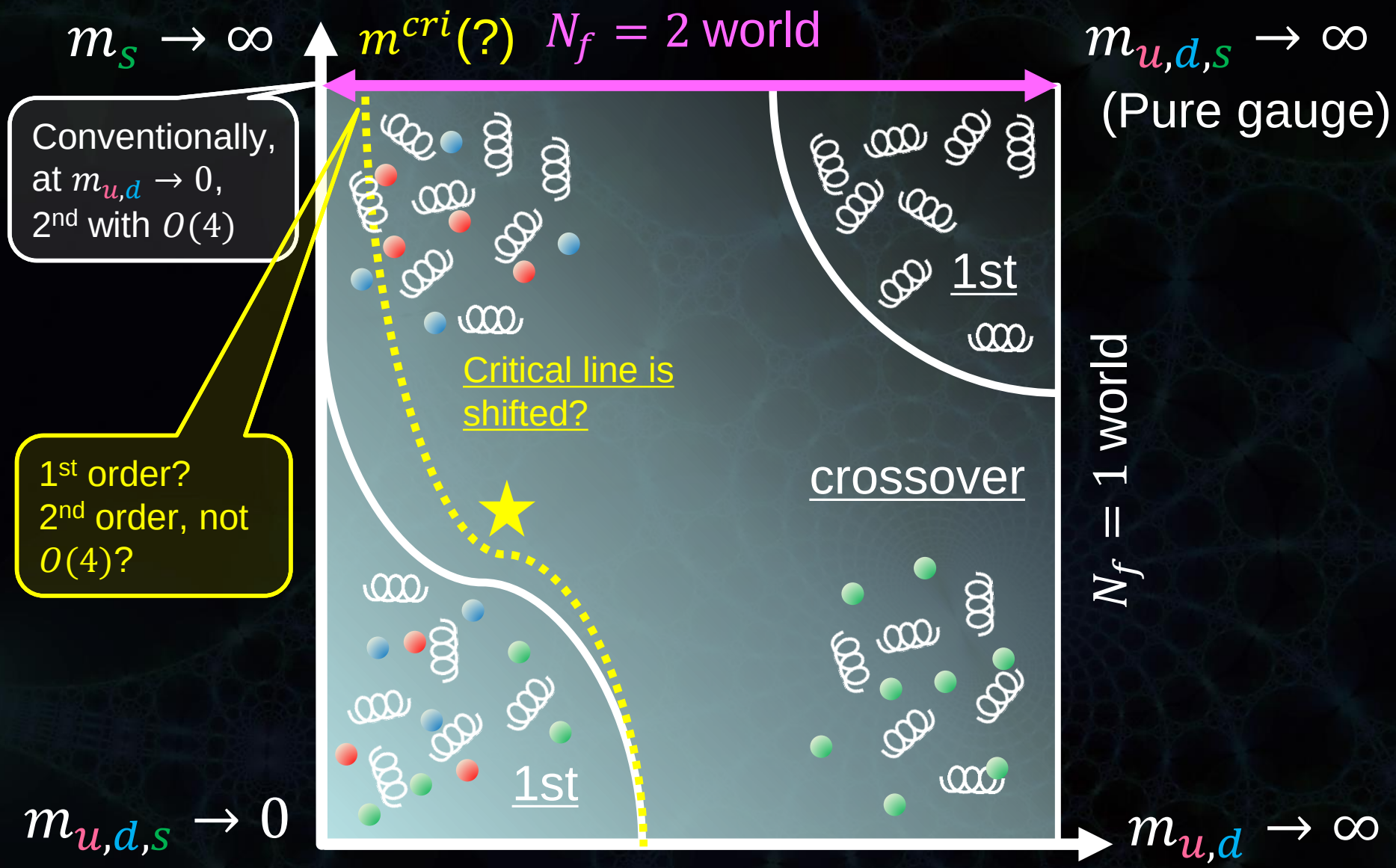
$$\Delta_{\pi-\delta} = \int_0^\infty d^4x \, [\pi^a(x)\pi^a(x) - \delta^a(x)\delta^a(x)]$$



Cf.) S. Aoki, H. Fukaya, and Y. Taniguchi, PRD86

If $U(1)_A$ is restored…

Colombia plot is modified?



$U(1)_A$ symmetry above Tc ⇒Long-standing problem in QCD

- Gross-Pisarski-Yaffe (Dilute instanton gas model, 1981) restored at enough high T
- Cohen (1996) w/o zero mode (or instanton)⇒restored
- Aoki-Fukaya-Taniguchi (2012) zero mode suppressed, restored in chiral limit at $N_f = 2$
- HotQCD (DW, 2012) broken
- JLQCD (topology fixed overlap, 2013) restored
- TWQCD (optimal DW, 2013) restored
- LLNL/RBC (DW, 2014) broken (restored at higher T?)
- Dick et al. (overlap on HISQ, 2015) broken
- Sharma et al. (overlap on DW, 2015,2016,2018) broken
- Brandt et al. (Wilson, 2016) restored
- Ishikawa et al. (Wilson, 2017) restored
- JLQCD (reweighted overlap on DW, 2017) restored
- Rohrhofer et al. (DW, 2017) restored

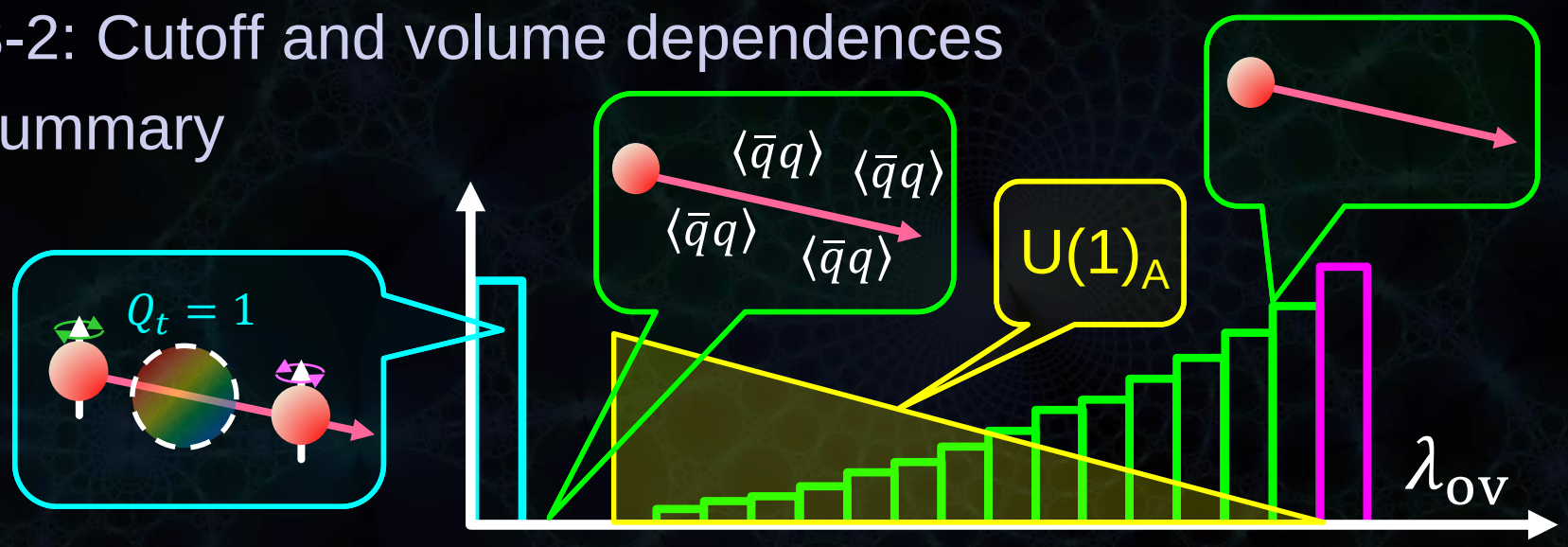
⇒ Many suggestions from lattice QCD (and models)…

$U(1)_A$ symmetry restoration by JLQCD Collaboration
⇒overlap fermion (exact chiral symmetry on the lattice)

	valence/sea quark	Setup
G. Cossu et al. PRD87 (2013)	OV on OV (Topology fixed sector)	
A. Tomiya et al. PRD96 (2017)	DW on DW OV on DW OV on (reweighted) OV	1/a=1.7GeV (a=0.11fm)
In progress	OV on DW OV on (reweighted) OV	1/a=2.6GeV (a=0.076fm) (Finer lattice)

Outline

1. Introduction
2. $U(1)_A$ susceptibility from Dirac spectra
   （zero mode and ultraviolet divergence）
3. Results
   3-1: $U(1)_A$ susceptibility at finite T
   3-2: Cutoff and volume dependences
4. Summary

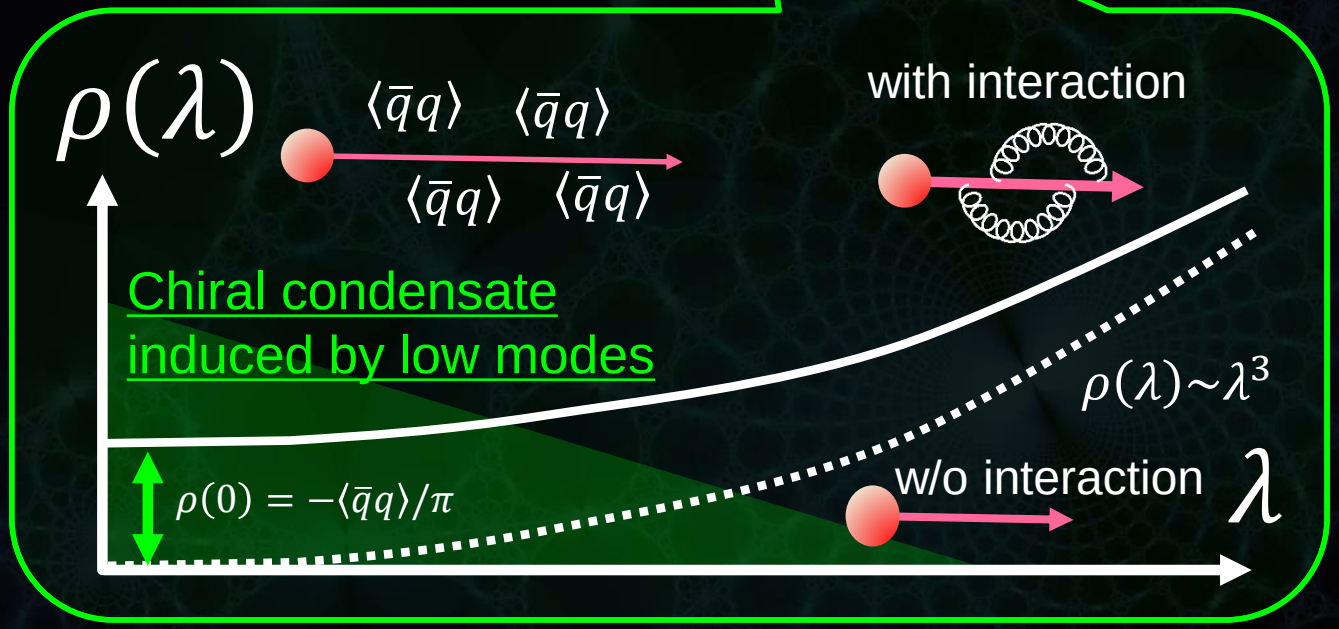


Chiral condensate and Dirac spectra

Banks-Casher relation:

$$\langle \bar{q}q \rangle = \lim_{m \to 0} \int_0^\infty d\lambda\, \rho(\lambda)\, \frac{2m}{\lambda^2 + m^2}$$

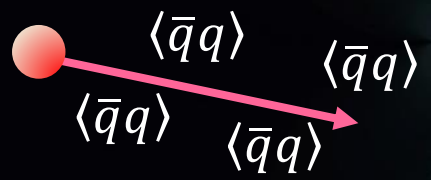
$$\rho(\lambda) \equiv \lim_{V \to \infty} \frac{1}{V} \Sigma_{\lambda'} < \delta(\lambda - \lambda') >$$



G. Cossu et al. (JLQCD) PRD87 (2013), 114514

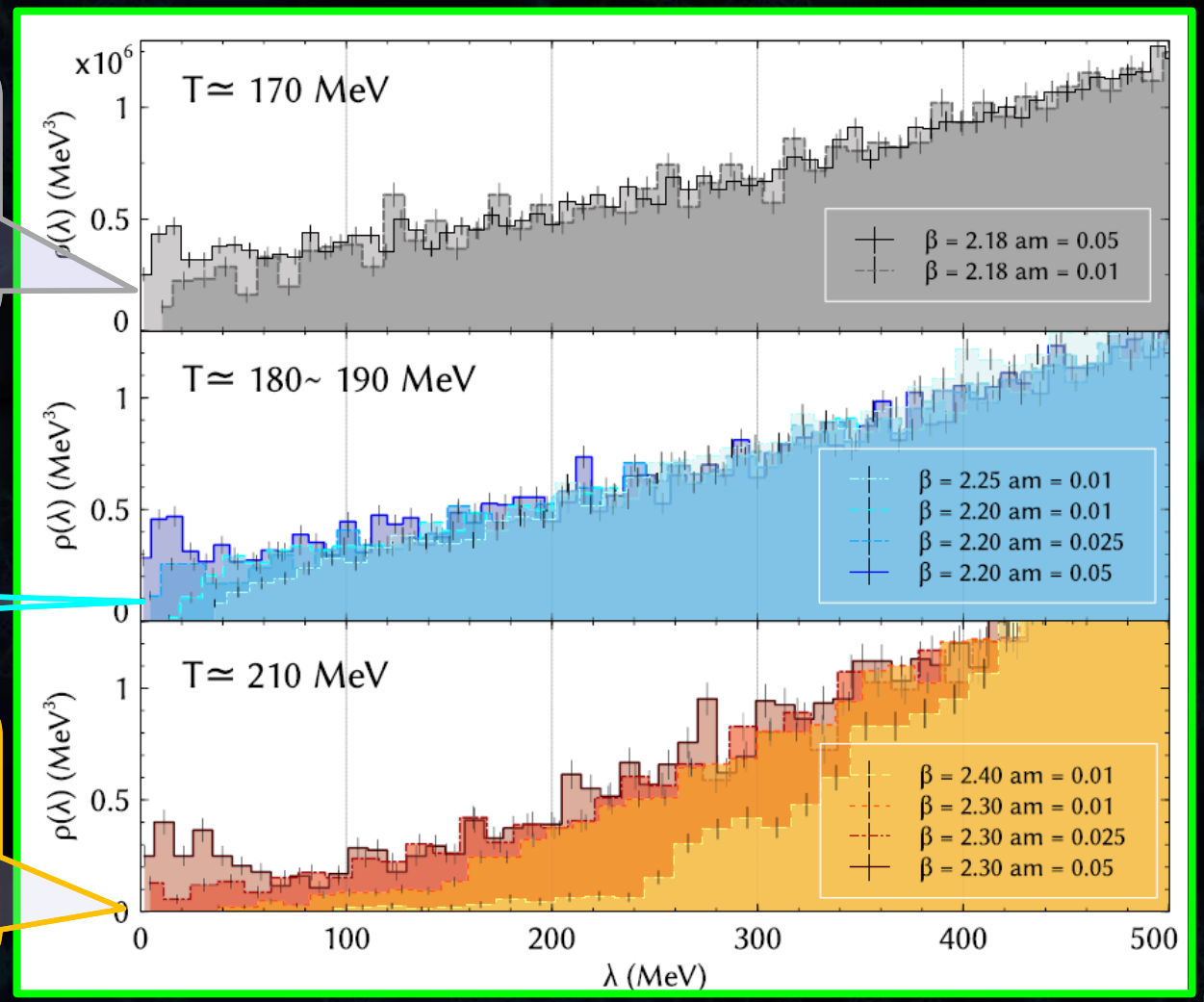
T-dependence of Dirac spectra

Low T:
ρ(0)≠0
⇒Spontaneous chiral symmetry breaking



Critical Temp.

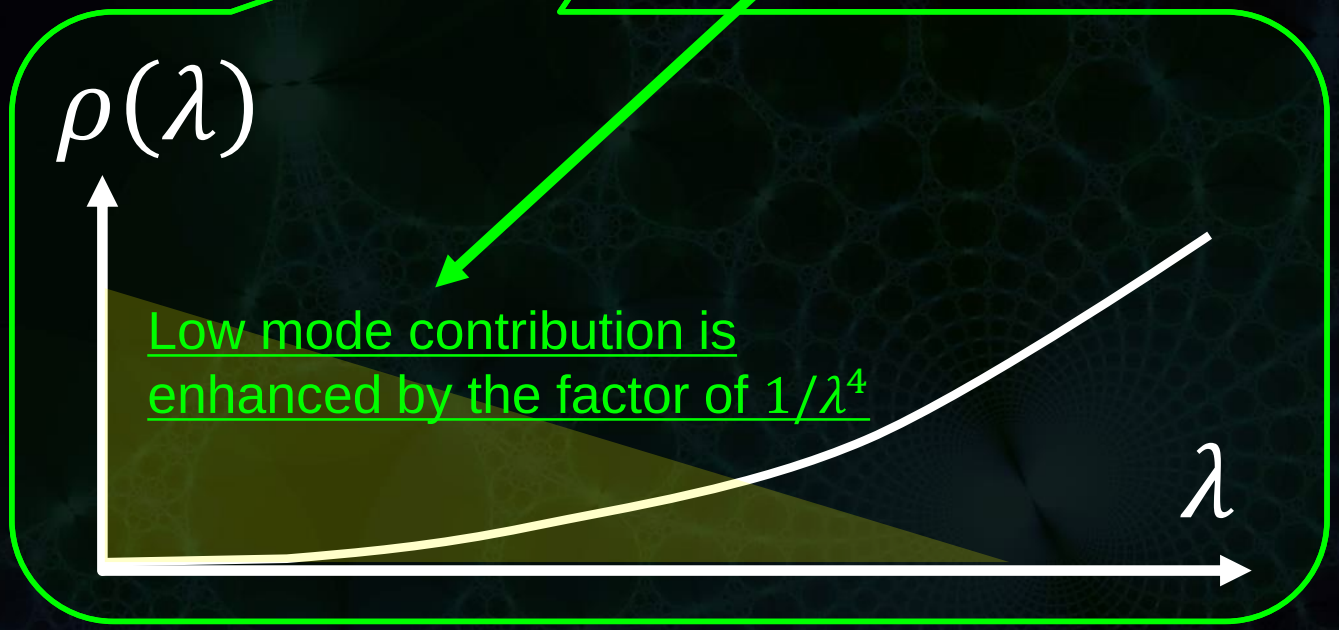
High T:
ρ(0)=0
⇒Chiral symmetry restoration



Low energy → High energy

$U(1)_A$ susceptibility and low modes of Dirac spectra

$$\Delta_{\pi-\delta} = \int_0^\infty d\lambda\, \rho(\lambda)\, \frac{2m^2}{(\lambda^2 + m^2)^2}$$



Cf.) Banks-Casher relation: $\langle \bar{q}q \rangle = \lim_{m\to 0} \int_0^\infty d\lambda\, \rho(\lambda)\, \frac{2m}{\lambda^2 + m^2}$

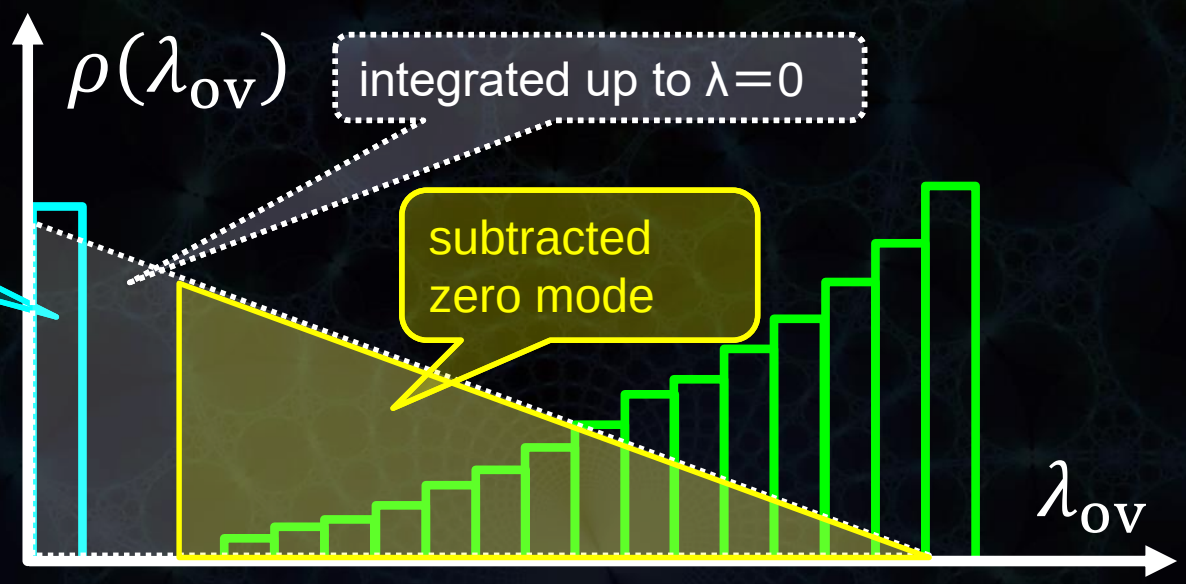
S. Aoki, H. Fukaya, and Y. Taniguchi PRD86 (2012), 114512
A. Tomiya et al. (JLQCD) PRD96 (2017), 034509

Note 1 :

$U(1)_A$ susc. = Low modes + ~~Zero~~ mode ?

$$\Delta_{\pi-\delta} = \int_0^\infty d\lambda\, \rho(\lambda)\, \frac{2m^2}{(\lambda^2+m^2)^2} \Rightarrow \Delta^{\mathrm{ov}}_{\pi-\delta} \equiv \frac{1}{V(1-m^2)^2} \sum_i \frac{2m^2(1-\lambda^{(i)2}_{\mathrm{ov}})^2}{\lambda^{(i)4}_{\mathrm{ov}}}$$

The factor of $1/\lambda^4$ enhances zero-mode contribution?



In $V \to \infty$ limit, we know zero-mode contribution is suppressed:

$$\Delta^{\mathrm{ov}}_{0-mode} = \frac{2N_0}{Vm^2} (\propto 1/\sqrt{V})$$

New order parameter: we subtract zero mode

$$\bar{\Delta}^{\mathrm{ov}}_{\pi-\delta} \equiv \Delta^{\mathrm{ov}}_{\pi-\delta} - \frac{2N_0}{Vm^2}$$

Note 2:

$U(1)_A$ susc. = Physics + ~~Ultraviolet divergence~~?

$$\Delta_{\pi-\delta} = \int_0^\infty d\lambda\, \rho(\lambda) \frac{2m^2}{(\lambda^2+m^2)^2}$$

$\rho(\lambda) \sim \lambda^3$ $\sim 1/\lambda^4$

$$\Delta_{\pi-\delta}^{\mathrm{ov}} \propto m^2 \ln\Lambda + \cdots$$

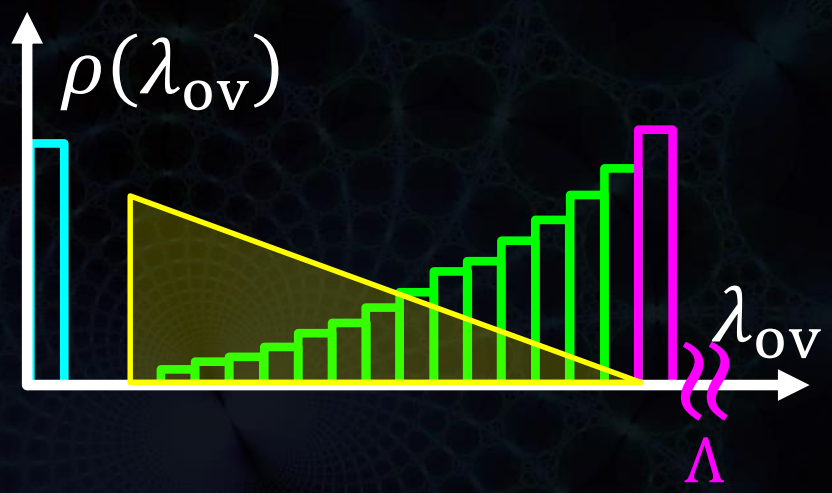
The term depends on cutoff Λ and valence quark mass m

We assume valence quark mass dependence of $\Delta_{\pi-\delta}$ (for small m):

$$\Delta_{\pi-\delta}(m) = \frac{a}{m^2} + b + cm^2 + O(m^4)$$

Zero-mode (disappears in $V \to \infty$)

$m^2 \ln\Lambda$ (disappears in $m \to 0$)

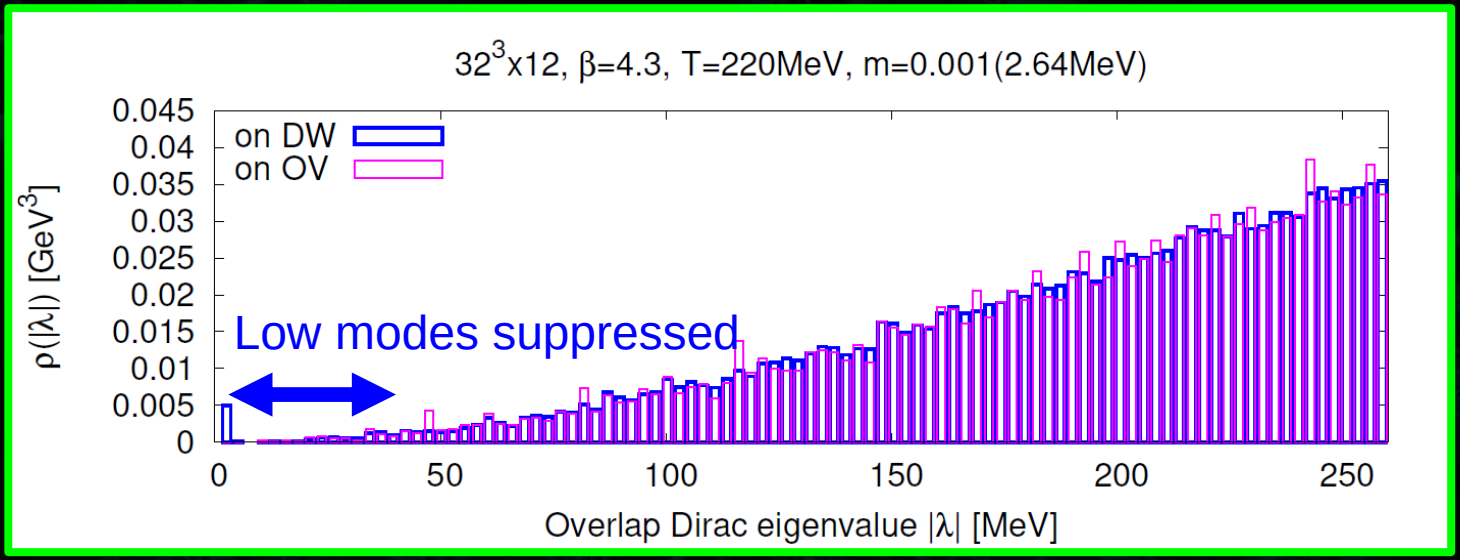


⇒ From 3 eqs. for $\Delta_{\pi-\delta}(m_1)$, $\Delta_{\pi-\delta}(m_2)$, $\Delta_{\pi-\delta}(m_3)$, a and c are eliminated

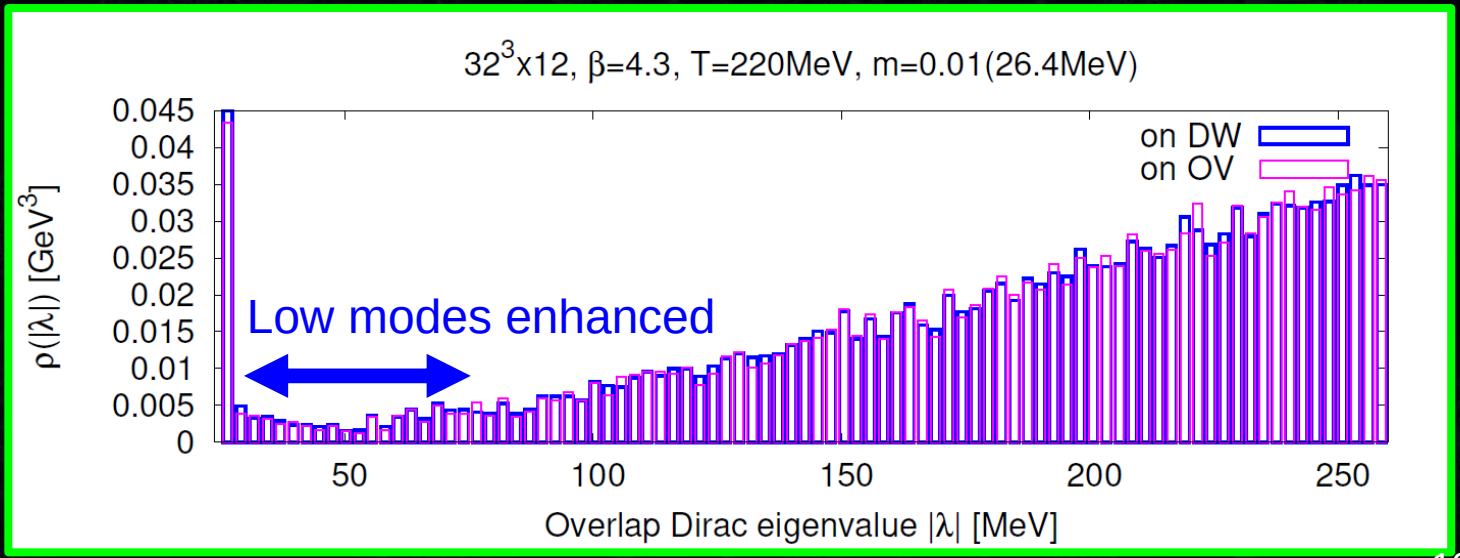
⇒ $\Delta_{\pi-\delta} \sim b + O(m^4)$ (, that depends on sea quark mass)

Overlap Dirac spectra at T = 220MeV

m_q=2.6MeV

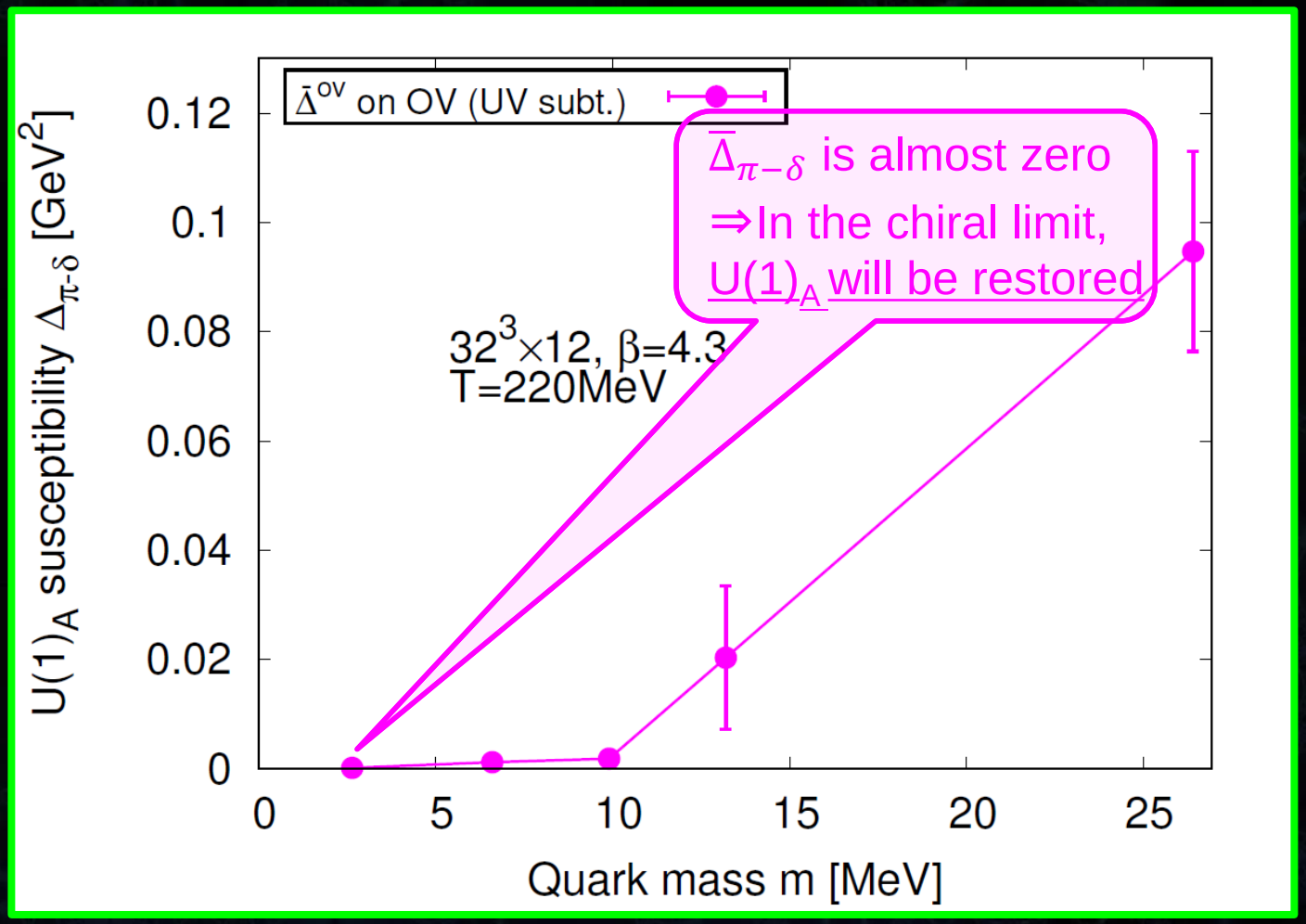


m_q=26MeV

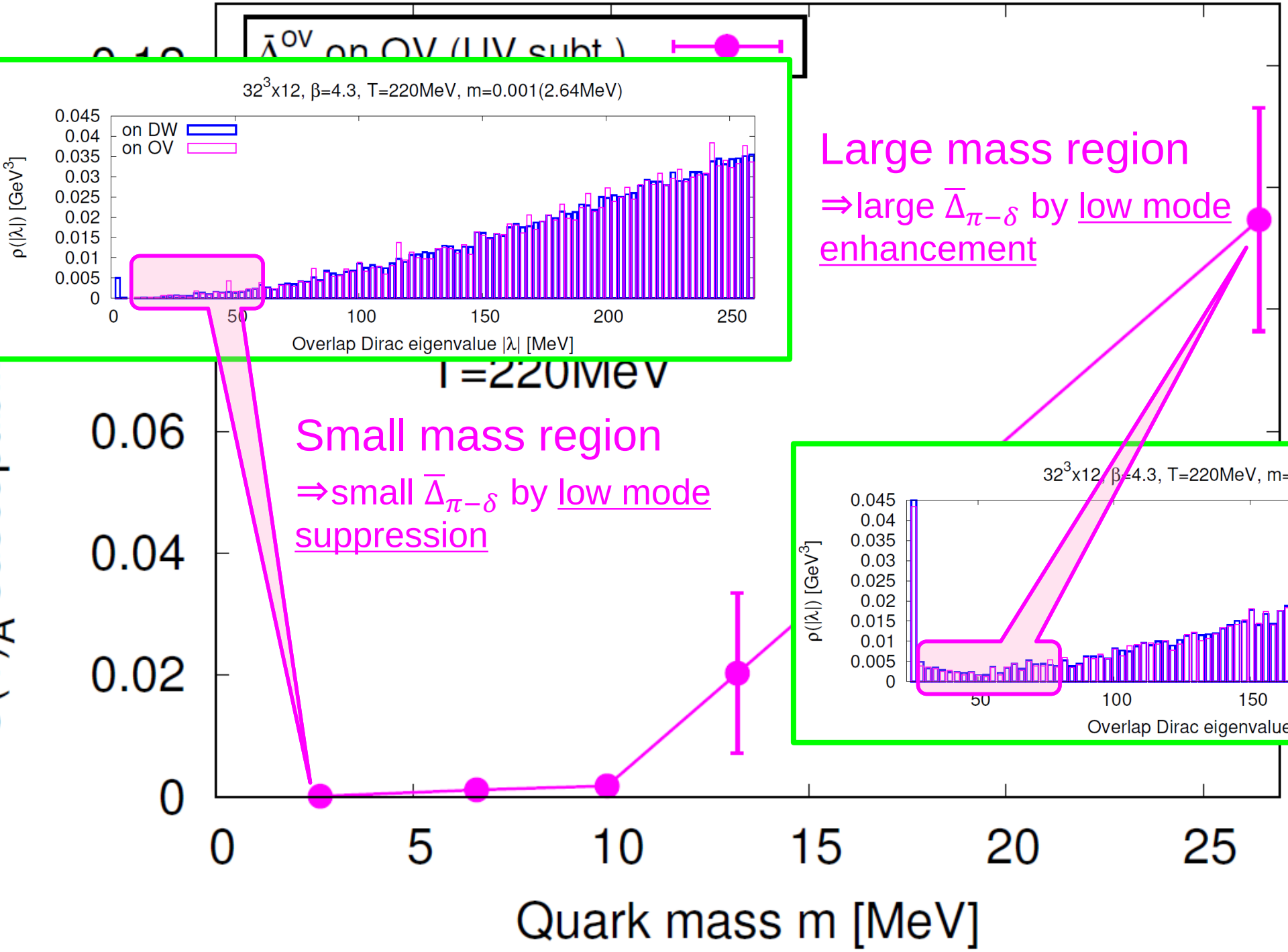


JLQCD, preliminary (2018)

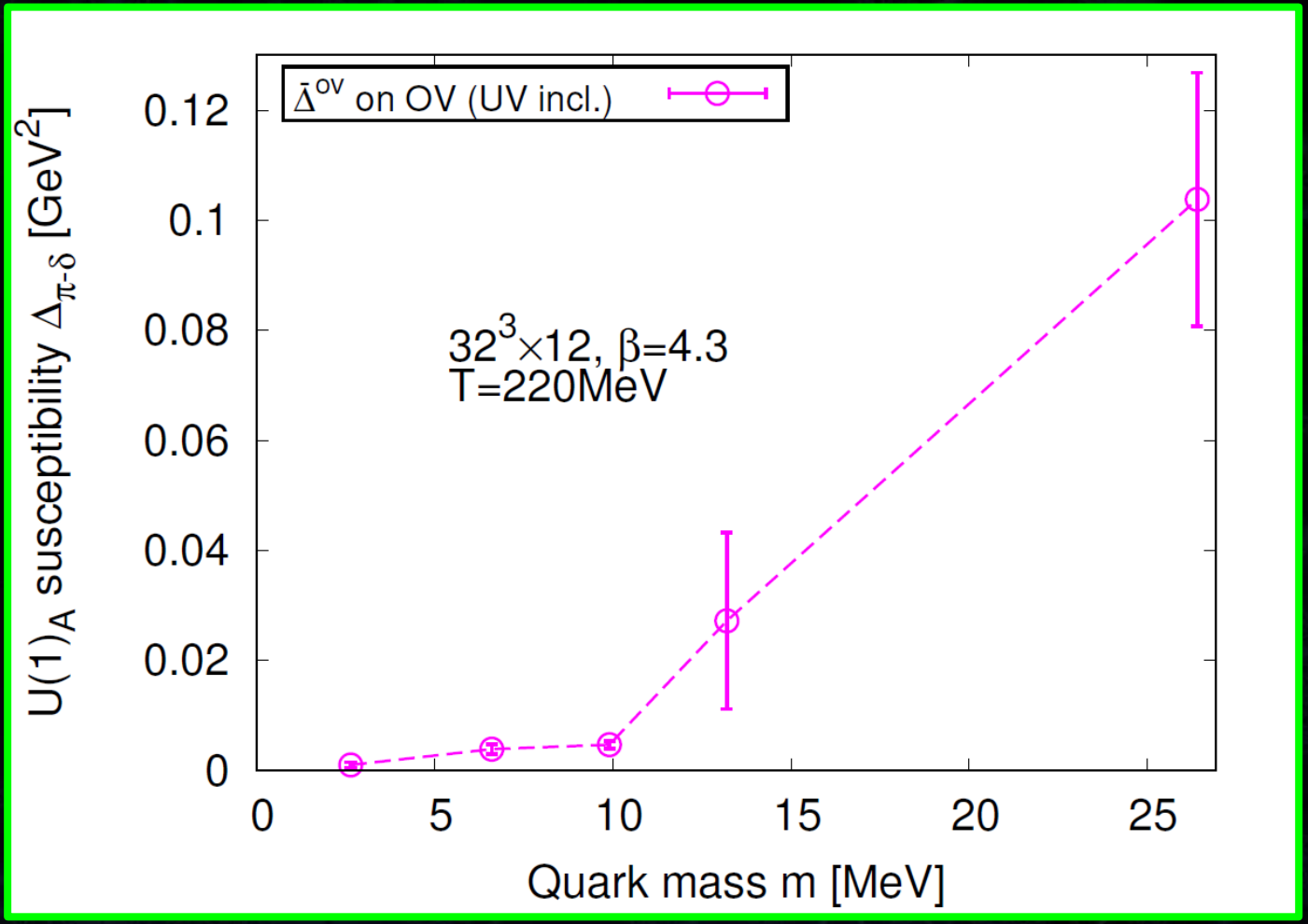
$U(1)_A$ susceptibility at T = 220MeV



⇒At m=2.6MeV, we found suppression of 10^{-4}GeV^2

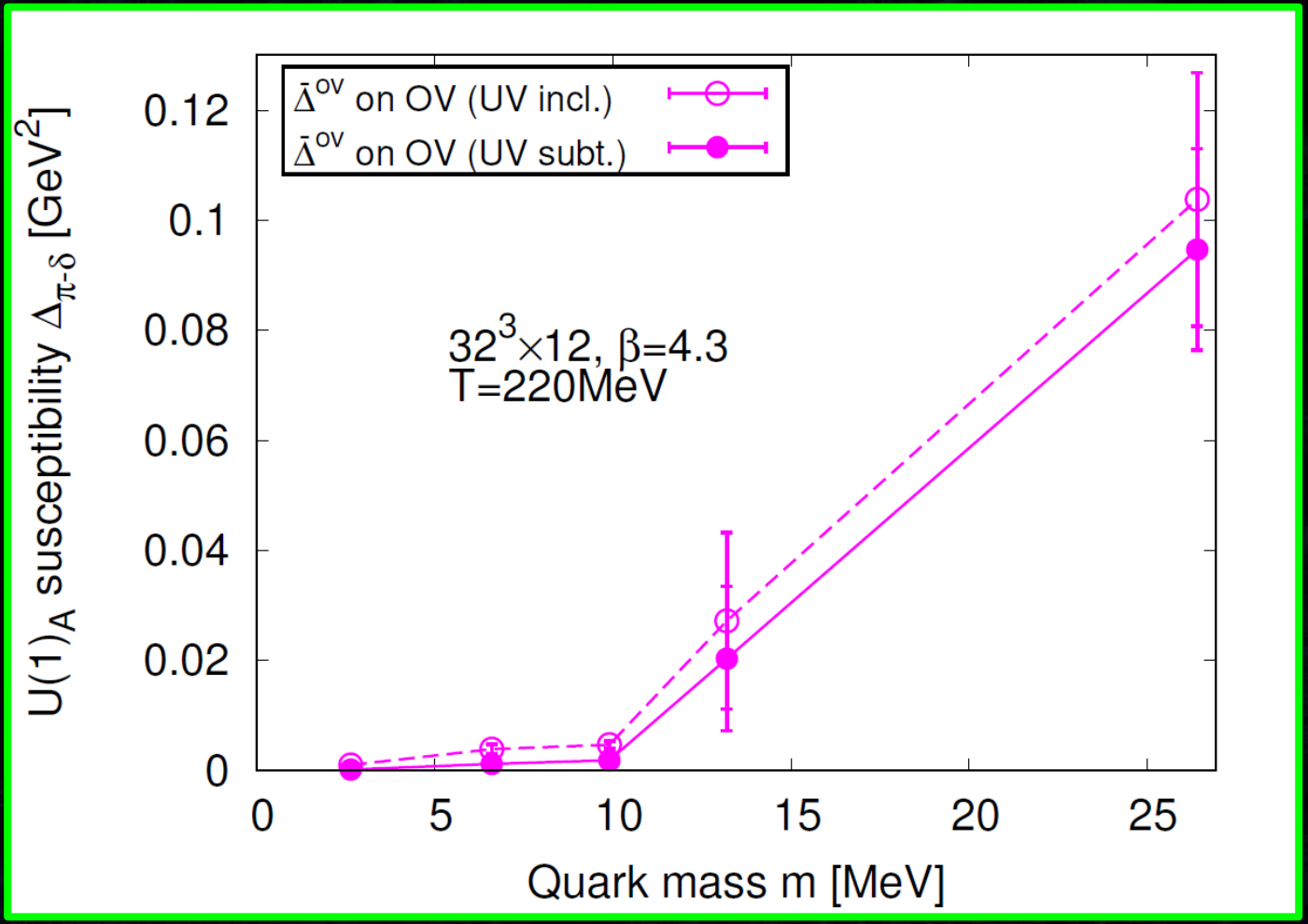

32^3x12, β=4.3, T=220MeV, m=0.001(2.64MeV)
on DW
on OV
ρ(|λ|) [GeV^3]
Overlap Dirac eigenvalue |λ| [MeV]
Large mass region
⇒large $\bar{\Delta}_{\pi-\delta}$ by low mode enhancement
Small mass region
⇒small $\bar{\Delta}_{\pi-\delta}$ by low mode suppression
T=220MeV
0.06
0.04
0.02
0
0
5
10
15
20
25
Quark mass m [MeV]

$U(1)_A$ susceptibility (UV-subt. before/after)



⇒Ultraviolet divergence（$\sim m^2 \ln \Lambda$）is subtracted from $\bar{\Delta}_{\pi-\delta}$

$U(1)_A$ susceptibility (UV-subt. before/after)

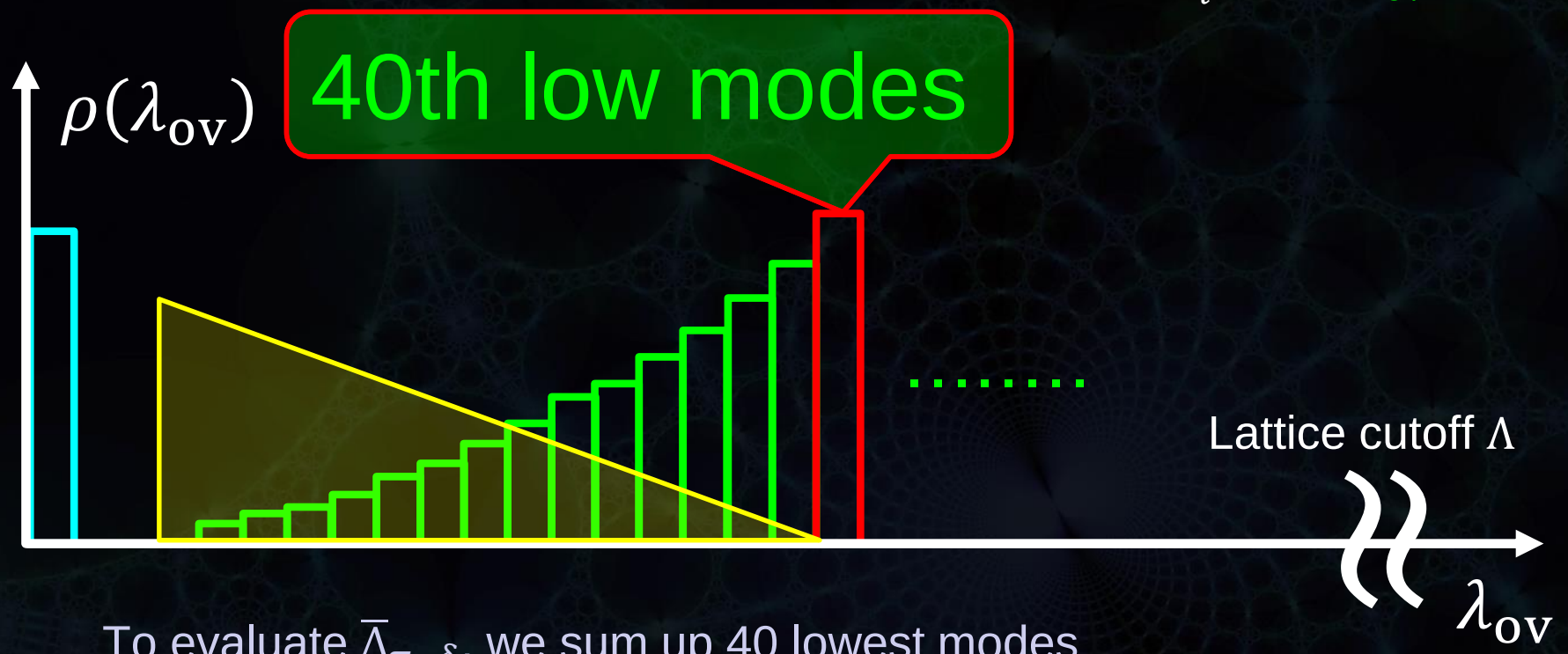


⇒Ultraviolet divergence（$\sim m^2 \ln \Lambda$）is subtracted from $\bar{\Delta}_{\pi-\delta}$

Did we really remove the ultraviolet contribution?

Check of cutoff dependence

$$\Delta_{\pi-\delta} = \int_0^\infty d\lambda\, \rho(\lambda)\, \frac{2m^2}{(\lambda^2+m^2)^2} \Rightarrow \Delta_{\pi-\delta}^{\mathrm{ov}} \equiv \frac{1}{V(1-m^2)^2} \sum_i \frac{2m^2(1-\lambda_{\mathrm{ov}}^{(i)2})^2}{\lambda_{\mathrm{ov}}^{(i)4}}$$

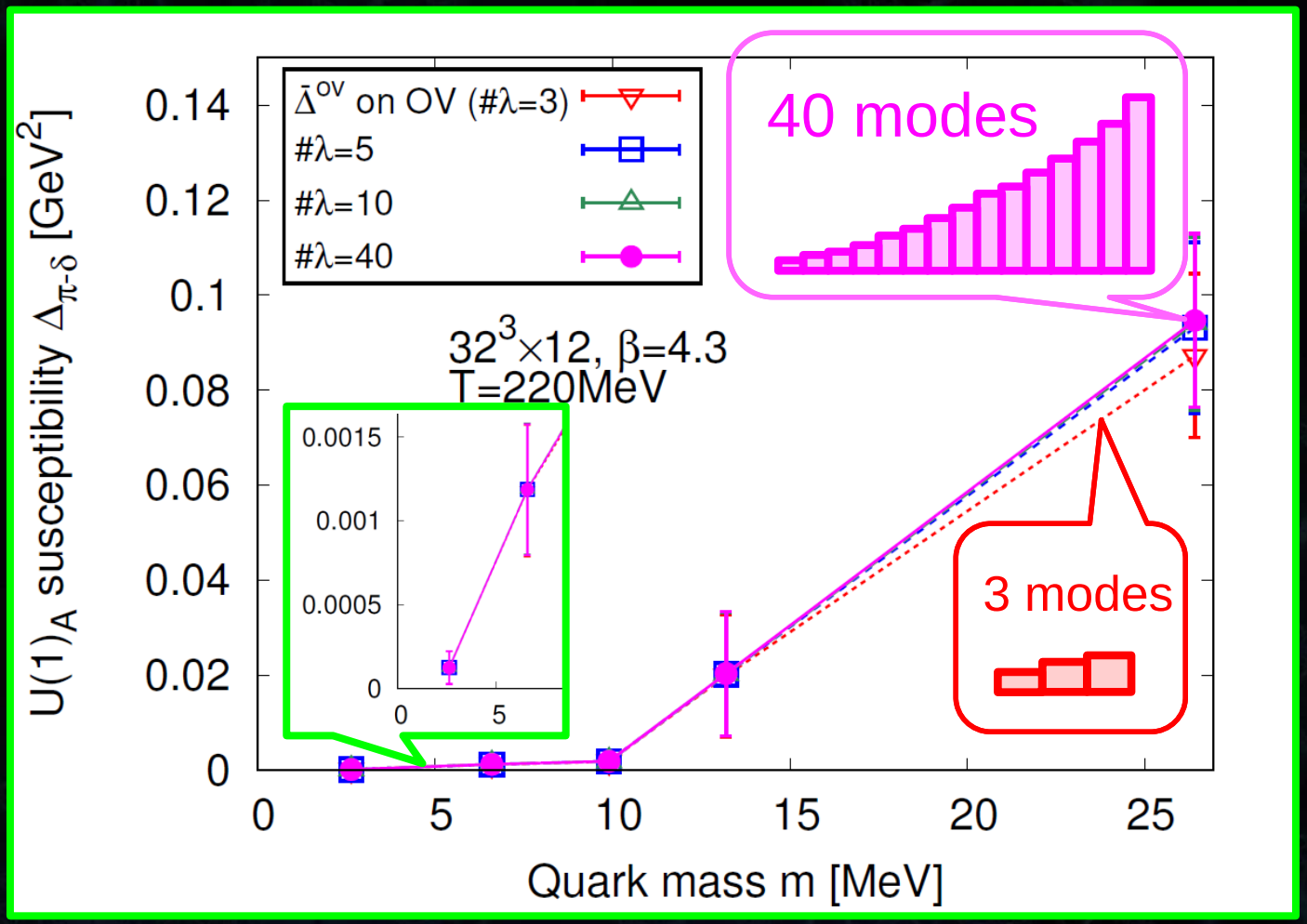


To evaluate $\overline{\Delta}_{\pi-\delta}$, we sum up 40 lowest modes

⇒Cutoff dependence by the number of low modes

JLQCD, preliminary (2018)

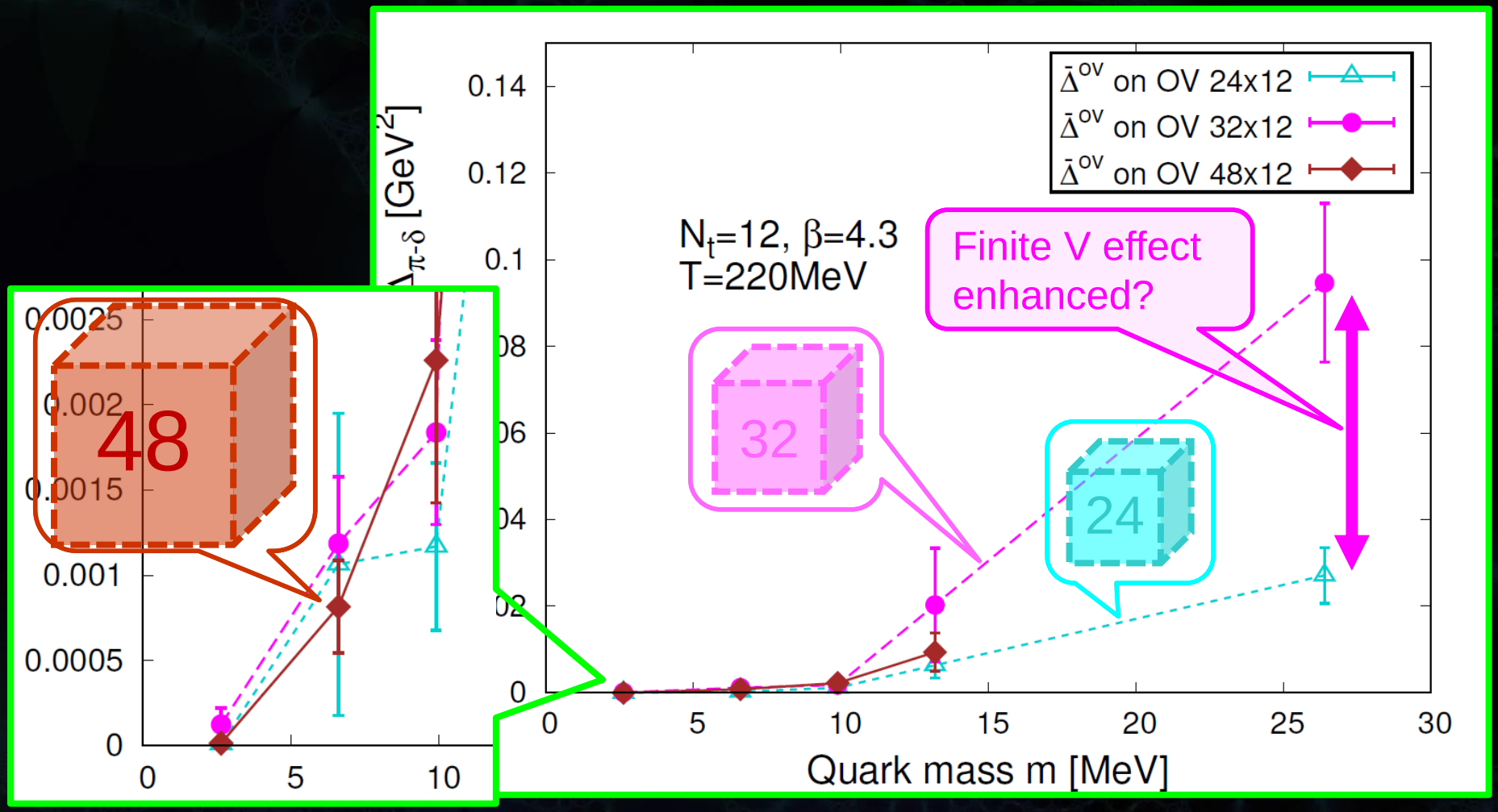
$U(1)_A$ susceptibility (cutoff dependence)



⇒No cutoff dependence（saturated by a few low modes）

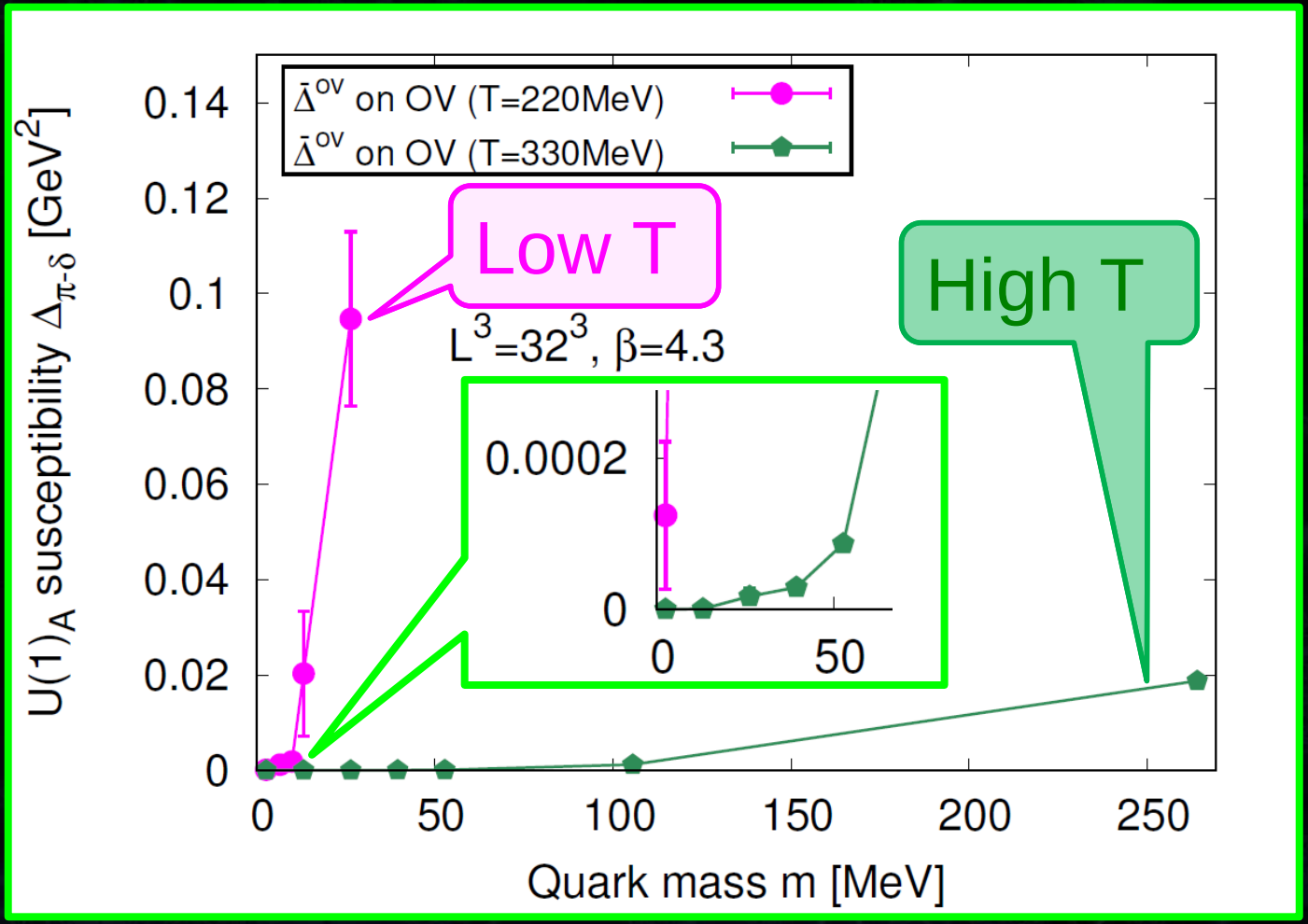
JLQCD, preliminary (2018)

$U(1)_A$ susceptibility (volume effect)



⇒For small m, V-dependence seems to be small

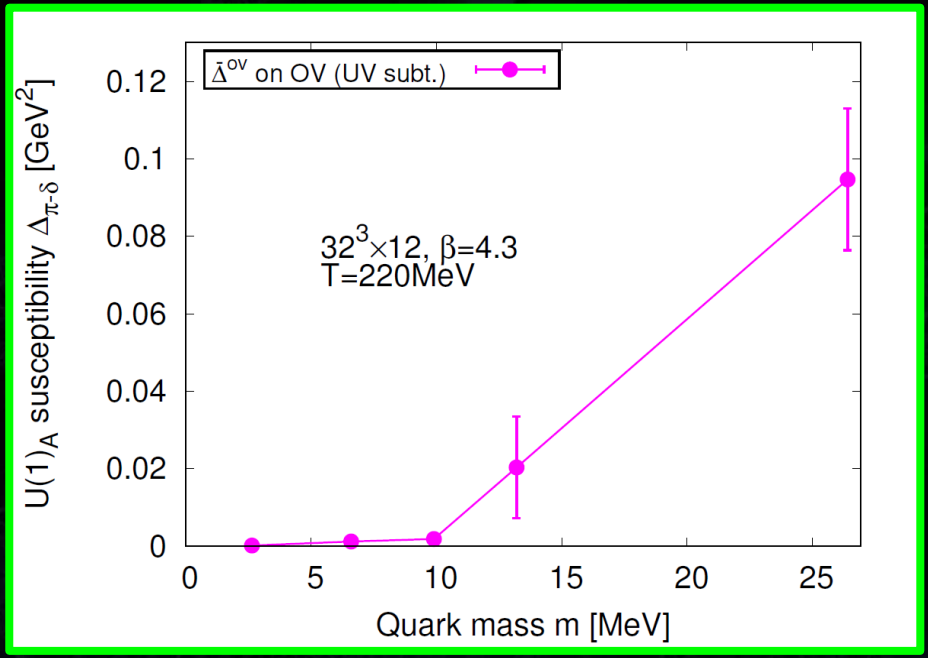
$U(1)_A$ susceptibility (T=220, 330MeV)



⇒ With increasing T, $U(1)_A$ is more resotored

Summary and outlook

- In high-temperature phase ($T > T_c$) at $N_f = 2$, we studied $U(1)_A$ susceptibility
- Strong suppression in the chiral limit (for T=220-330MeV)
- Checked volume and cutoff dependences



- Topological susceptibility ⇒ talk by Y. Aoki
- Parametrization as function of m_q (larger than m_q^2?)
- Near T_c ($N_t = 14$?, chiral transition?)
- $N_f = 2 + 1$ sector

Backup

S. Aoki, H. Fukaya, and Y. Taniguchi PRD86 (2012), 114512
A. Tomiya et al. (JLQCD) PRD96 (2017), 034509

Note 1 :

$U(1)_A$ susc. = Low modes + ~~Zero~~ mode ?

$$\Delta_{\pi-\delta} \equiv \int_0^\infty d\lambda\, \rho(\lambda)\, \frac{2m^2}{(\lambda^2+m^2)^2}$$

$$\rho_{0-mode}(\lambda) = \frac{1}{V}\sum_{0-mode} \delta(\lambda)$$

$$\Delta_{\text{zero}} = \int_0^\infty d\lambda\, \frac{1}{V}\sum_{0-mode} \delta(\lambda)\frac{2m^2}{(\lambda^2+m^2)^2}$$

$$= \frac{1}{V}\sum_{0-mode}\frac{2m^2}{m^4}$$

$$= \frac{1}{V}\sum_{0-mode}\frac{2}{m^2} = \frac{2N_0}{Vm^2}$$

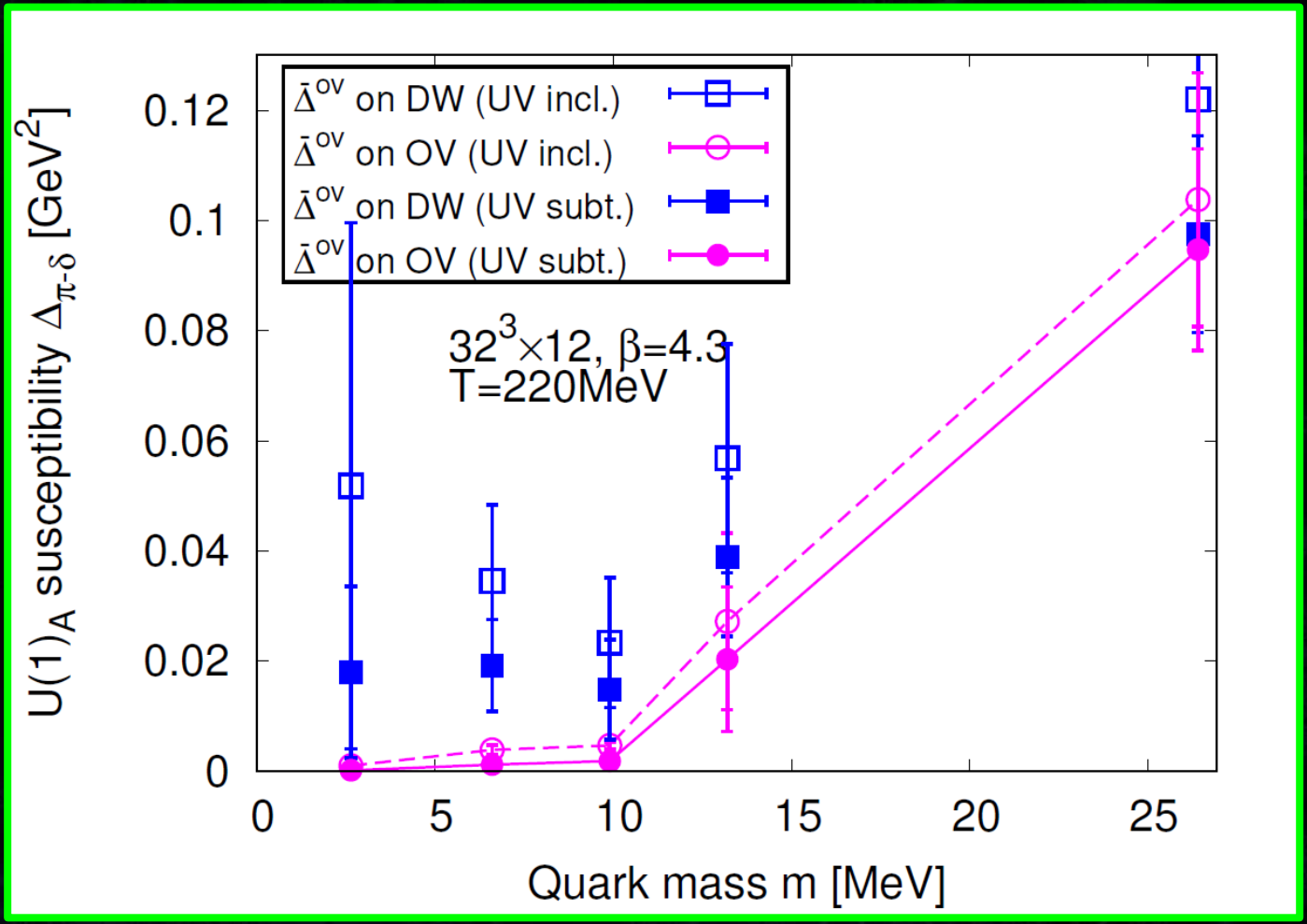
$$\langle N_{L+R}^2\rangle = \mathcal{O}(V)$$
$$\langle N_{L+R}\rangle = \mathcal{O}(\sqrt{V})$$

$$\lim_{V\to\infty}\Delta_{\text{zero}} = 0$$

Zero mode contributions in $\Delta_{\pi-\delta}$ will be suppressed in $V \to \infty$ limit

JLQCD, preliminary (2018)

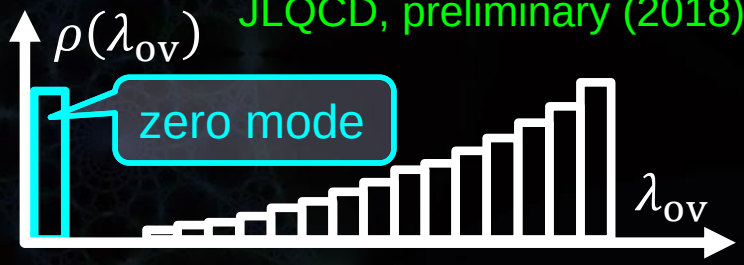
$U(1)_A$ susceptibility (DW/OV reweighting)



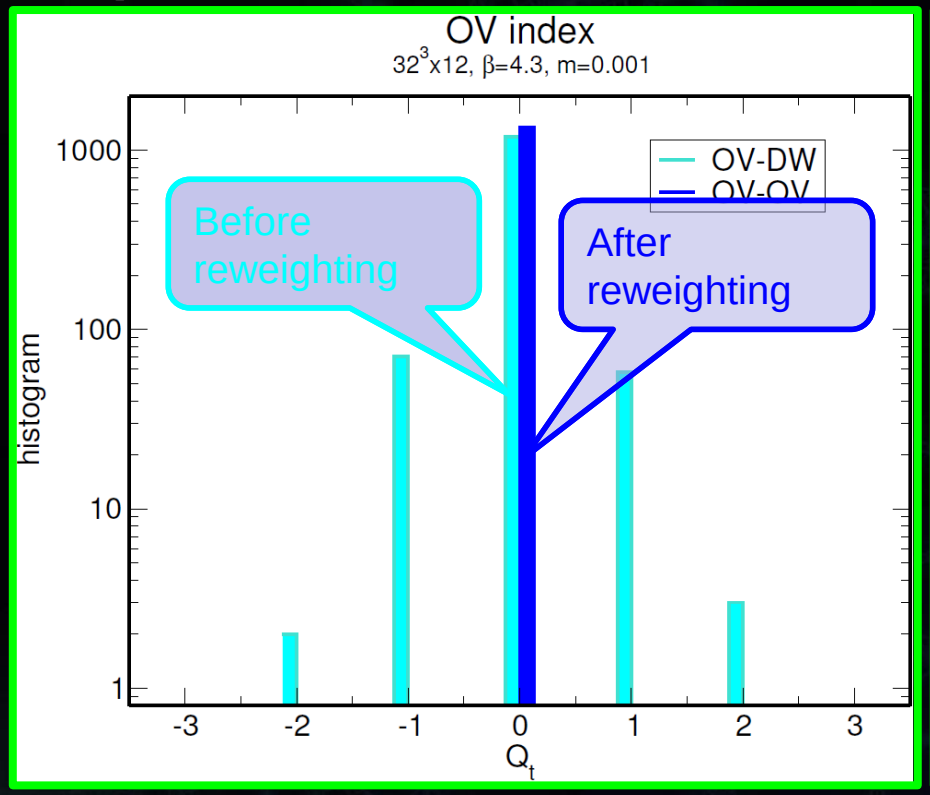
⇒DW/OV reweighting is crucial in small m region

Histogram of topological charge at T = 220MeV

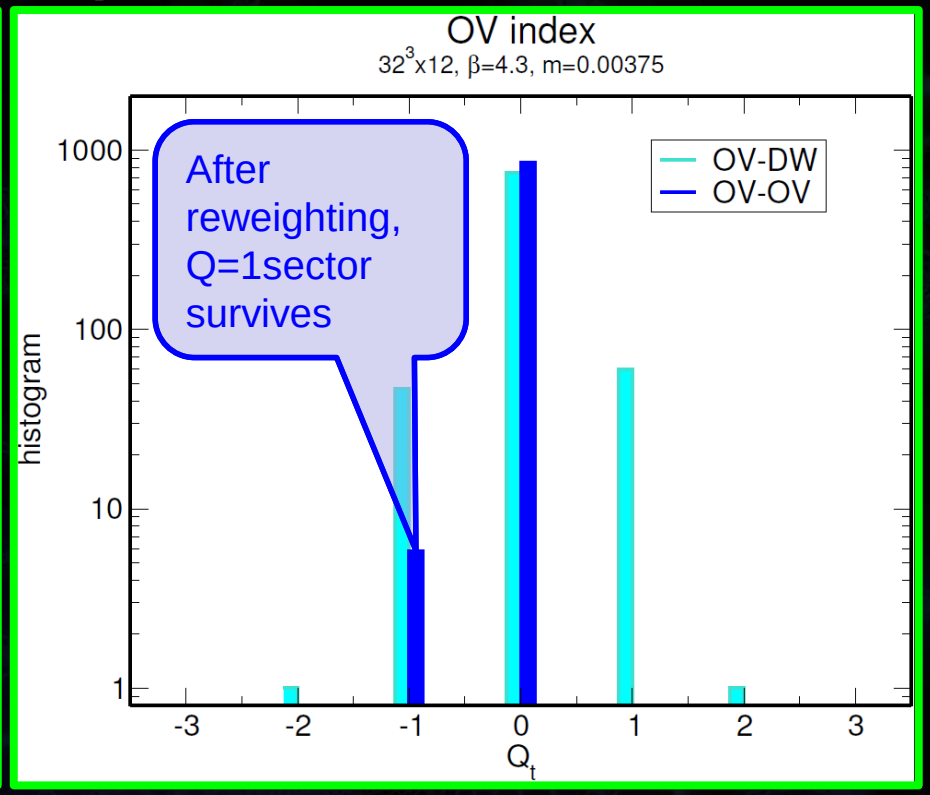
JLQCD, preliminary (2018)



m_q=2.6MeV



m_q =10MeV



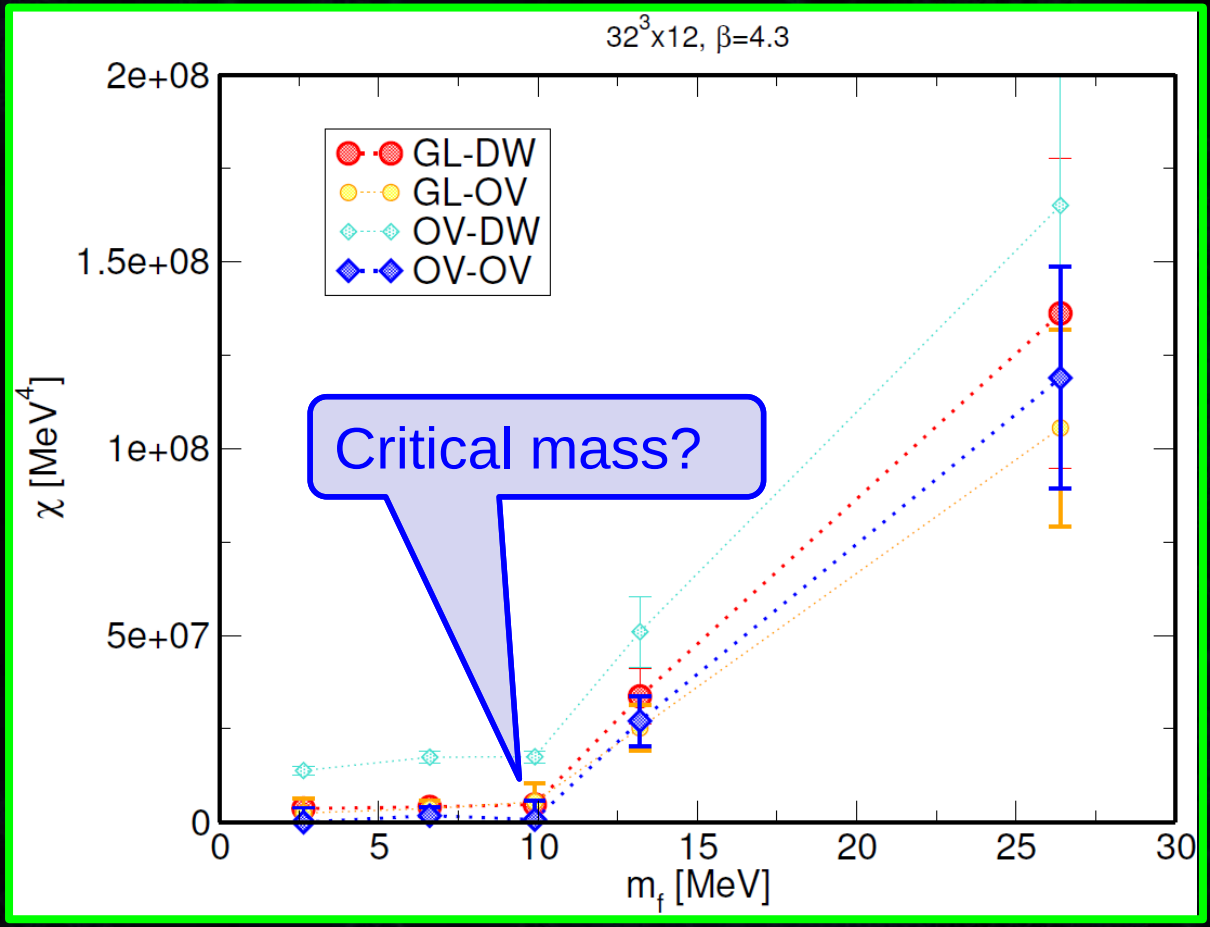
Small m_q: all conf. are Q＝0 sector

Large m_q: Q≠0 sectors appear

Using $\chi_t \equiv \frac{\langle Q_t^2 \rangle}{V}$, we plot χ_t

JLQCD, preliminary (2018)

Topological susceptibility at T = 220MeV

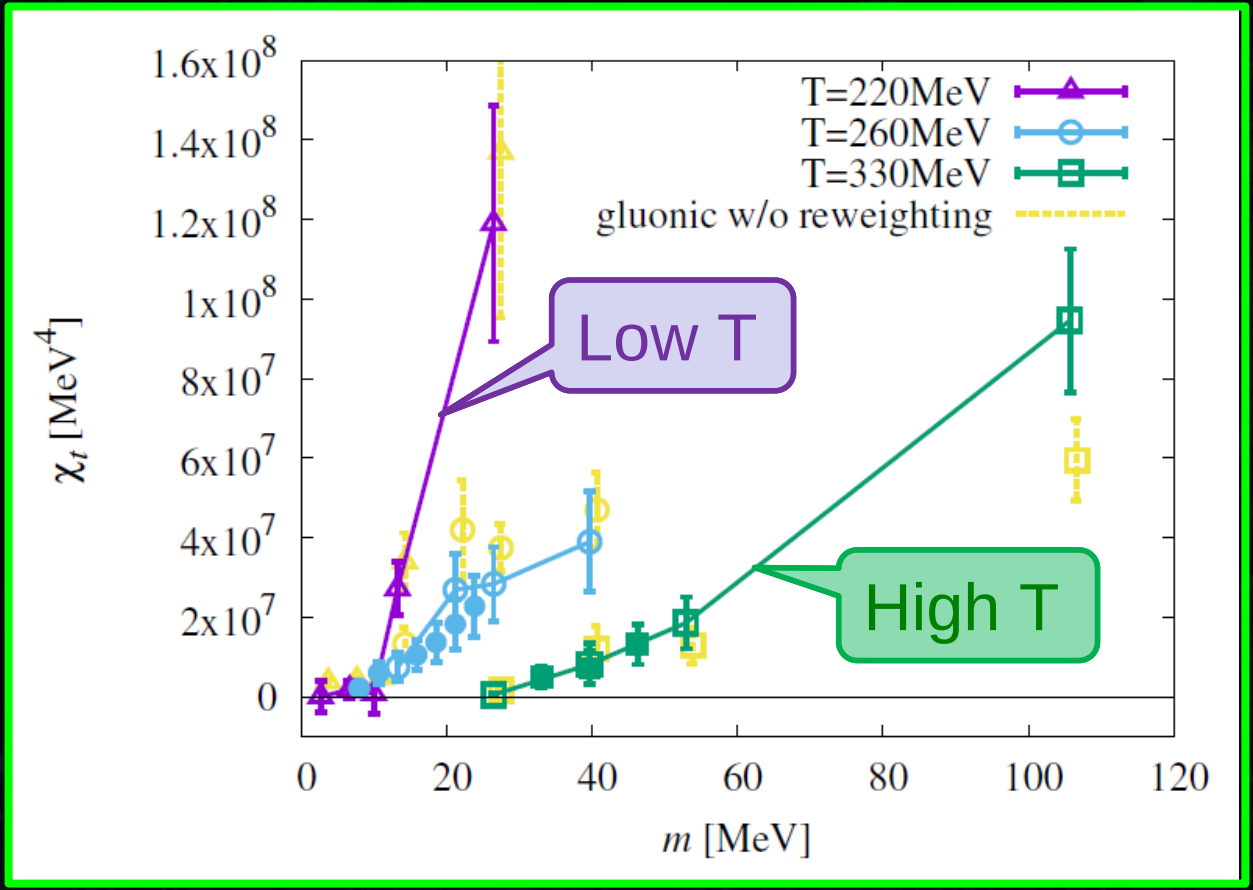


⇒In small m_q region, χ_t=0?

⇒Around m_q~10MeV, we found a jump (critical mass?)

JLQCD, preliminary (2018)

Topological susceptibility
(Temperature dependence)



⇒With increasing T, χ_t becomes small

JLQCD, preliminary (2018)

Topological susceptibility
(Volume dependence)