Large $N_c$ Thermodynamics with Dynamical Fermions

Presented by Daniel Hackett
Thursday, July 25
Lattice 2018

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Overview

What is large $N_c$? What does it have to say about thermo?

Numerical tests of large $N_c$

Previous work, in general: mostly quenched, recently some dynamical
[Review by Lucini, Panero 1210.4997]

Previous thermodynamics work: quenched

This talk: dynamical Wilson fermions [Throughout, $N_f = 2$ a.k.a. “QCD”]

Automation

Early physics results

Phase diagram collapse, fermion independence(?), order of transition(?)

[Disclaimers: Currently in “proof of concept” phase; not intended to ever be a high-precision study]
What is large $N_c$?

Consider “QCD”: SU($N_c$) with some fermion content, vary $N_c$ holding everything else fixed

**Basic assertion:** Power series in $1/N_c$ exists for any observable

$$\langle \hat{O} \rangle = N_c^\alpha O_0 \left[ 1 + \frac{1}{N_c} O_1 + \cdots \right]$$

[leading $N_c$ dependence] [nonperturbative physics]

[subleading corrections $\sim 1/N_c$]

‘t Hooft limit: $N_c \to \infty \Rightarrow \langle \hat{O} \rangle \to N_c^\alpha O_0$

Theory simplifies in limit of infinite number of colors

e.g. Mesons become infinitely narrow, quark model & OZI rule become exact

Holography duals typically apply to this limit

**Lattice:** Test that large $N_c$ works
Thermodynamics at large $N_c$

Fermion loops suppressed by $1/N_c$ vs equivalent diagrams with gluon loops

⇒ Fermions “quenched out” at large $N_c$
⇒ Theories with fermions act like pure gauge theory as $N_c \to \infty$

Quenched large-$N_c$ studies **assume** this works
Test this assumption with dynamical fermions

**Previous work:** $T = 0$ spectroscopy [DeGrand & Liu 1606.01277]
**This study:** Finite $T$ – do large $N_c$ predictions hold?
Numerical details

Variant of MILC for arbitrary $N_c$ [DeGrand]

Unimproved Wilson gauge action

$N_F = 2$ flavors of clover-improved Wilson fermions ($c_{SW} = 1$)

nHYP smeared fat links for fermions

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<th>$N_s$</th>
<th>$N_t$</th>
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This talk: explored $12^3 \times 6$ phase diagrams for $N_c = 3, 4, 5$

Moving forward: Want to vary $N_t$, $N_s/N_t$, ...

$\{N_c\} \times \{N_s\} \times \{N_t\} \times \cdots \rightarrow$ Need to explore many Wilson phase diagrams

Logistically intractable without automation
Automation

**Workflow Manager**
- Manages ongoing HMC runs
- Runs spectroscopy, flow on gauge configurations as they’re generated
- [Minimal, naïve] automatic parameter tuning/failure recovery
  
github.com/dchackett/taxi

**SQL Database & Bulk Analysis**
- DB enforces conventions, structure
- ~ nightly analysis scripts process data into useful observables: $m_q$, phase diagnostics, etc.
  [See DH poster from earlier this week]

**Automated Phase Diagram Explorer (APDE)**
Simple criteria to decide where to explore:
- Are ensembles interesting (cut on $m_q$, phase)?
- Are ensembles explorable (nearby somewhere with equilibrated data)?

APDE specifies new simulations, feeds them to workflow manager

APDE looks at current state of analysis in DB

Automatically load all raw data into a relational (SQL) database
Flowed Polyakov loops

Many options for phase diagnostics, but flowed Polyakov loops are convenient for automation

Apply flow to configs while measuring Polyakov loop $P(t)$ in flow time

**Confined:** $P(t)$ wander randomly
**Deconfined:** $P(t)$ rapidly order to $+N_c$

[Behavior shifts gradually between confined-like and deconfined-like]

Paths of $P(t)$ in complex plane
Each trajectory is one config being flowed
SU(3) $N_F = 2$ on $12^3 \times 6$
$\kappa = 0.128$
Can use flowed Polyakov loops as a diagnostic of confinement

[Ayyar, DH, Jay, Neil 1710.03257]

Flow enhances signal in Polyakov loop

[Datta, Gupta, Lytle 1612.07985]
[Schaich, Hasenfratz, Rinaldi 1506.08791]

At long $t/a^2$, $P$ is (roughly) independent of $(\beta, \kappa)$

Make (arbitrary but intuitive) definitions:

- **Deconfined**: $|\langle P(t) \rangle|/N_c > 0.5$
- **Confined**: $|\langle P(t) \rangle|/N_c < 0.25$

...at $t/a^2 = 2$

At right: $SU(3)$ $N_F = 2$ on $12^3 \times 6$

$\kappa = 0.128$ at various different flow times $t/a^2$
Wilson phase diagrams varying $N_c$

Phases defined using flowed Polyakov loops

**Plotted together:**
Phase-ambiguous regions [colored bands] and ambiguously-phased points
Prediction: phase diagram collapse

’t Hooft limit: LO physics constant at constant $\lambda = g^2 N_c$

$$\beta = \frac{2N_c}{g_0^2} = \frac{2N_c^2}{\lambda_0} \Rightarrow \frac{\beta}{N_c^2} = \frac{2}{\lambda_0} = \text{constant}$$

No LO $N_c$ dependence for $m_q$ [and thus $\kappa$]

$\Rightarrow$ Constant physics at constant $(\beta/N_c^2, \kappa)$ [up to $1/N_c$ corrections]
Result: phase diagram collapse

Plot \((\beta / N_c^2, \kappa)\) instead of \((\beta, \kappa)\)

Steeper \(\rightarrow\) Less sensitive to \(\kappa/\text{fermionic effects}\)

\[1/N_c \text{ corrections}\]
Fermion independence(?)

As $N_c \to \infty$, any observable should converge to its pure-gauge value independent of $m_q$

**Plots:** Uncertainties due to uncertainty in location of transition, wash out statistical errors

$am_q$ from finite-$T$ ensembles; empirically, small error vs properly calculating with $T = 0$ data

[Pure gauge data from Lucini, Teper, Wenger [hep-lat/0307017]; Lucini, Rago, Rinaldi [1202.6684]]
Disappearance of pure-gauge transition (?)

Deconfinement/chiral transition
\[ m_q = \infty: \text{First-order for } N_c > 2 \]
\[ m_q \text{ finite, easily simulated: Crossover} \Rightarrow \exists \text{ some } m_q^{PG} (N_c) \text{ where transition changes order} \]

Fermionic effects suppressed as \( N_c \) increases
\[ \Rightarrow \text{Expect } m_q^{PG} \rightarrow 0 \text{ as } N_c \rightarrow \infty \]

“Result”: At present, no obvious first-orderness in data
- All observables continuous at transition
- No observed metastability near transition
- Polyakov loop doesn’t become binary under flow

\[ \Rightarrow a m_q^{PG} \gtrapprox 0.5 \]

[Image from de Forcrand 2017]
Conclusions & Future Directions

Proof-of-concept works: fully automated phase diagram exploration, ~ ready for production
Initial physics results look promising

Explore $N_t > 6$ [$N_t = 8$ in progress]
→ Get control over $a$ dependence
→ Get away from bulk transitions

More ensembles, statistics near transitions
→ Find $\beta_c$, lines of constant $m_q$ more precisely via interpolation, reweighting?

Explore $N_s/N_t > 2$
→ Get control over finite volume artifacts
→ Volume scaling analysis to determine order of transition

Matching $T = 0$ data
→ Scales to get e.g. $T_c$, $m_q$ in physical units

Bulk transitions are an issue, block access to small $m_q$ for $N_c > 3$
→ Try improved actions?