Computing $\hat{q}$ on a quenched SU(3) lattice

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Outline

- ❑ Quark and gluon plasma (QGP) produced in heavy-ion collision

1) Importance of transport coefficient $\hat{q}$
2) Need for an *ab-initio* calculation of $\hat{q}$

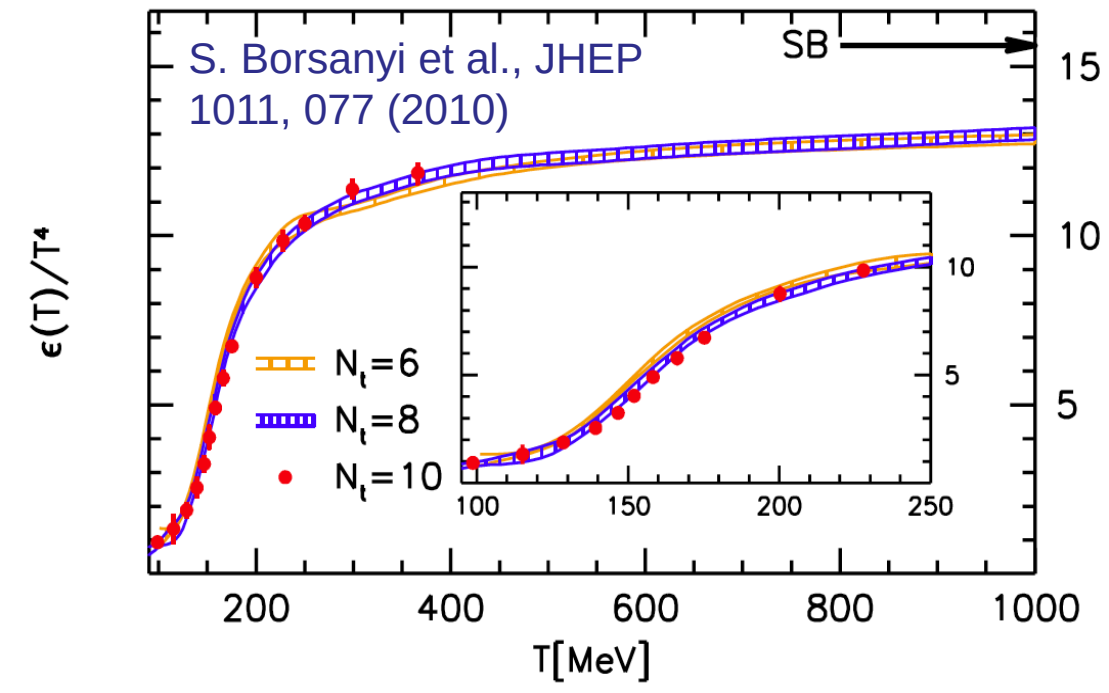
- ❑ Formulating $\hat{q}$ on the lattice using Lattice gauge theory

1) Previous study done on a quenched SU(2) lattice
2) Extending calculations to a quenched SU(3) lattice

- ❑ Estimates of $\hat{q}$ on a quenched QGP plasma

Nuclear matter under extreme condition

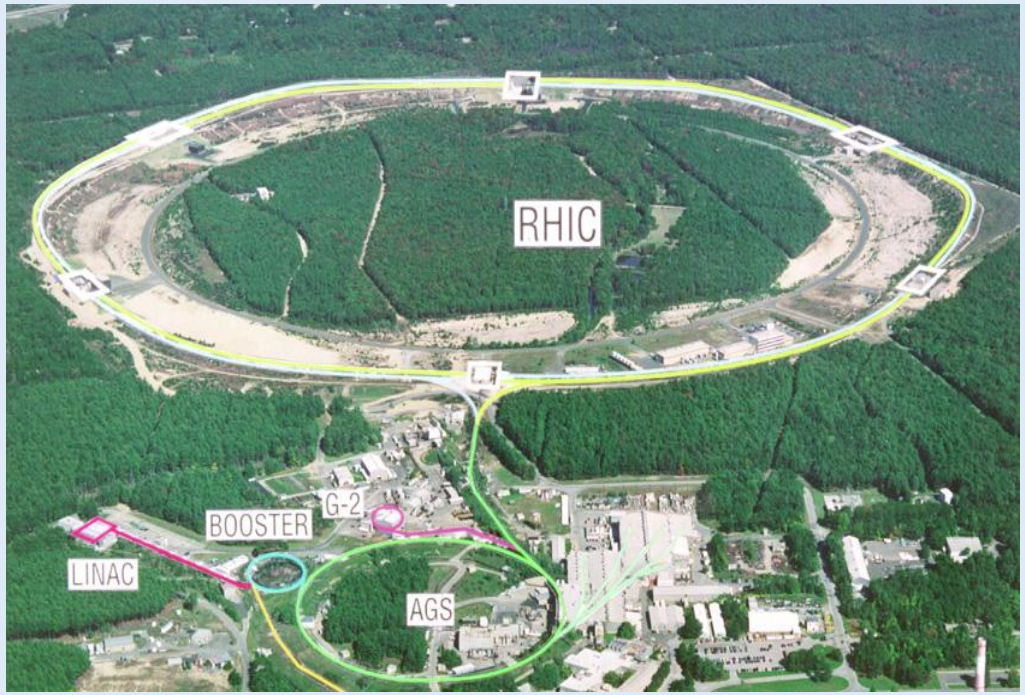
The Lattice QCD prediction of EOS



- Sudden increase in ∈ near the transition region.
- Due to increase in the number of DOF
- Corresponds to a deconfined state of quarks and gluons

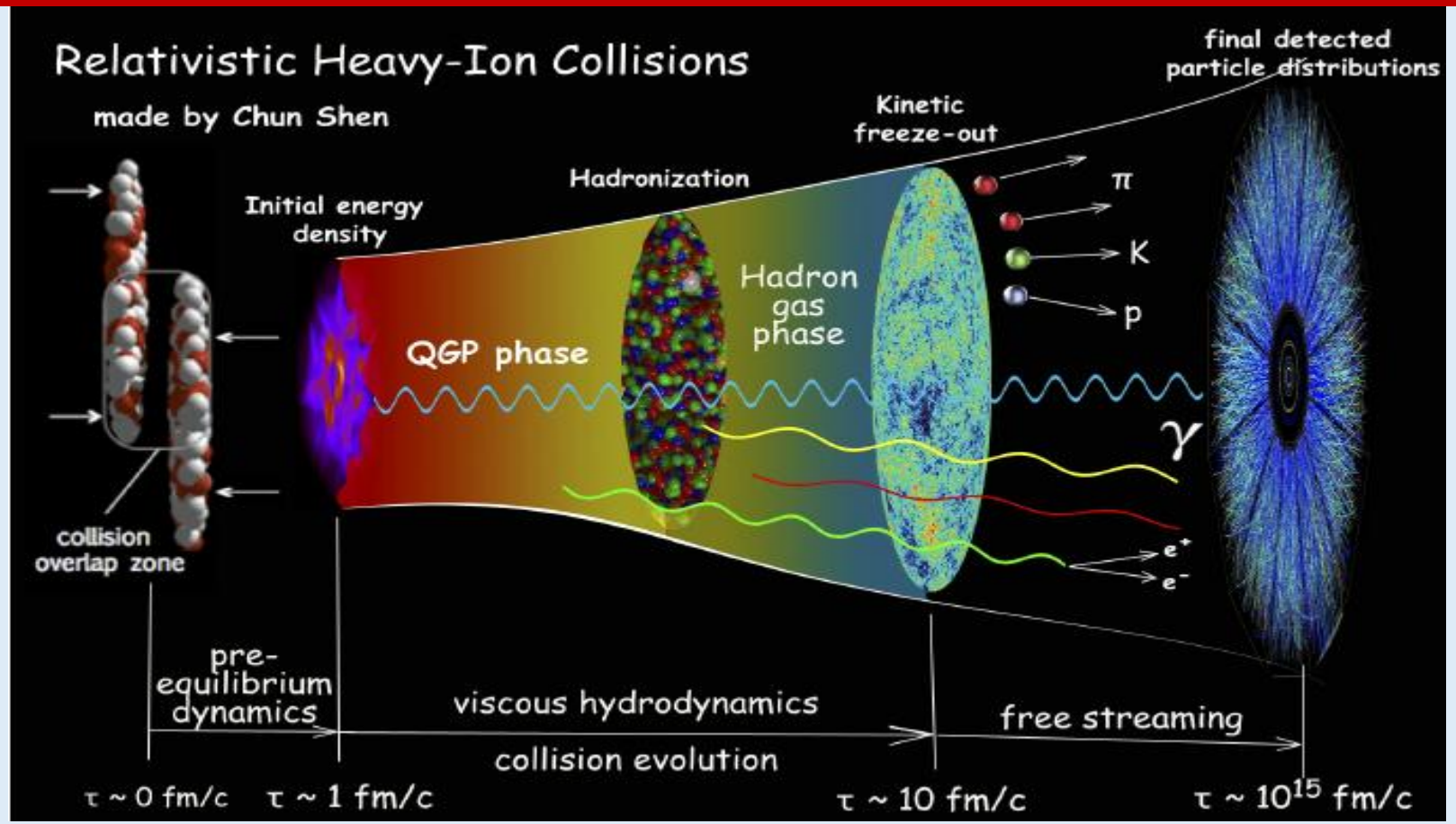
Creation of the Quark-Gluon Plasma (QGP)

Relativistic Heavy Ion Collider (RHIC) at BNL: Collide two Au/Cu nuclei @ 20-200GeV per nucleon pair

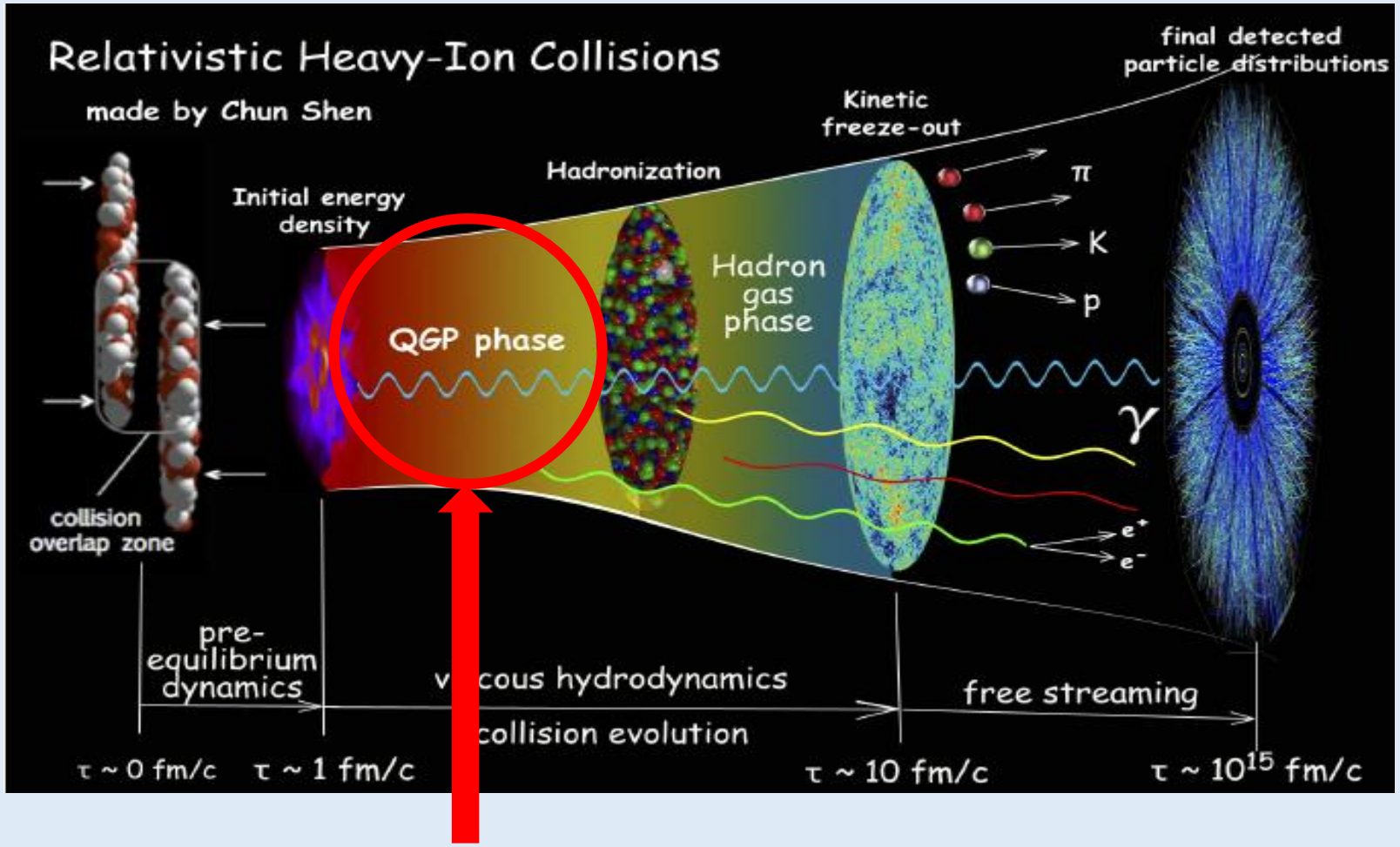


Large Hadron Collider (LHC) at CERN: Collide two Pb nuclei @ 2.76TeV/5.5TeV per nucleon pair

Creation of the Quark-Gluon Plasma (QGP)

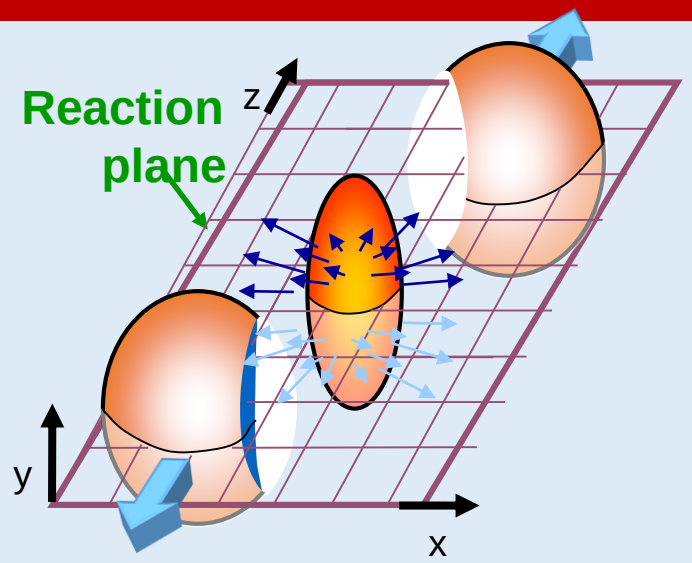


- Initial state: two Lorentz-contracted nuclei approach each other
- Pre-equilibrium state: undergo hard collisions produce hard probes and drive the system to thermalization in the form of QGP matter
- QGP phase: the QGP expands hydrodynamically
- Hadronization: the QGP cools down and new hadrons are formed
- Freeze-out: hadron gas is so dilute that the interactions cease

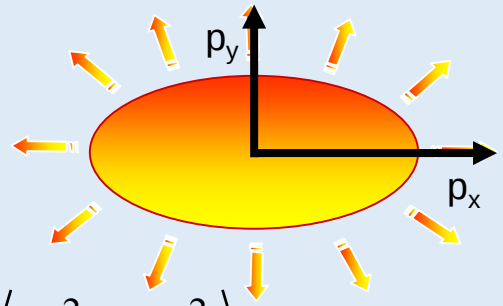


Thermalized, non-perturbative, and strongly interacting plasma

Flow measurement: Evidence for strongly interacting QGP

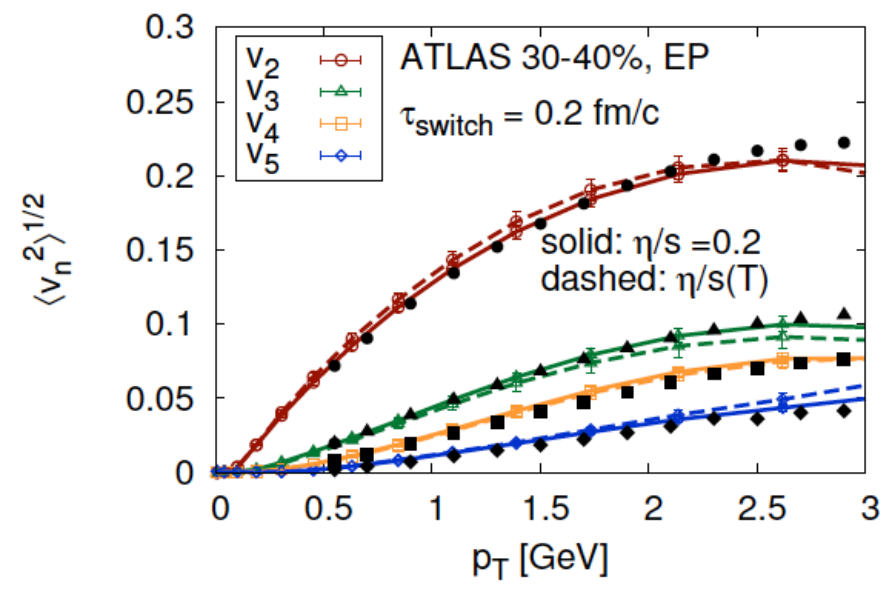


To momentum space



Elliptic flow: $v_2 \equiv \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$; $\frac{dN}{d\varphi} \propto N(1 + \sum_n 2v_n \cos(n\varphi))$

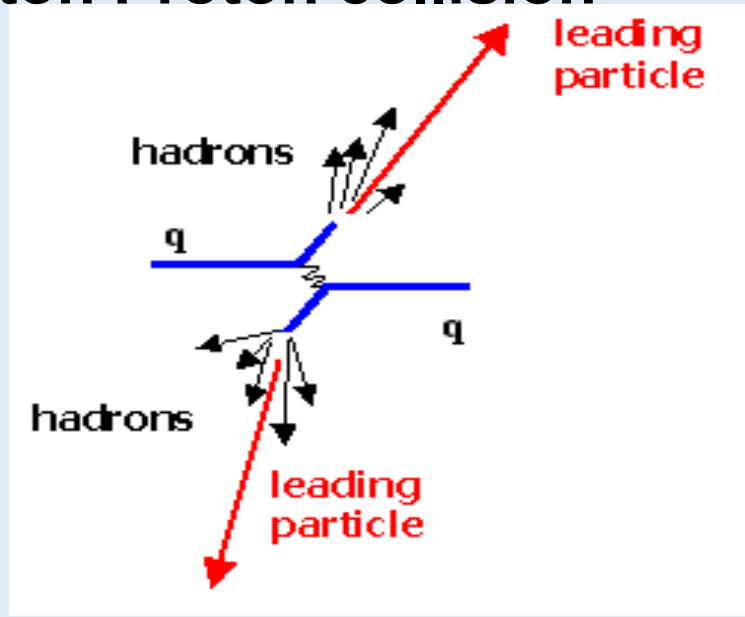
Schenke et al., PRL 108 252301 (2012)



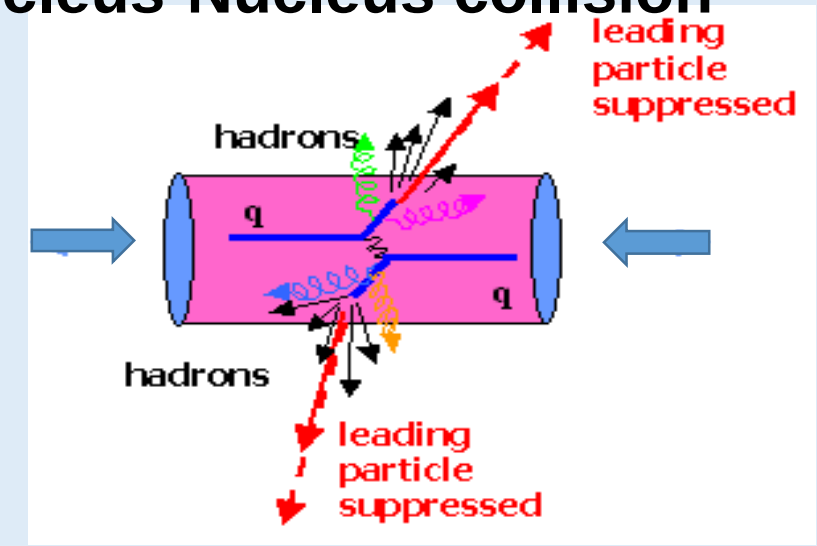
- Data indicate that v_2 is finite
- Hydrodynamical calculations describes the data
- Lattice QCD EOS is used as input
- QGP is thermalized and strongly interacting

Evidence for strongly interacting QGP

Proton Proton collision

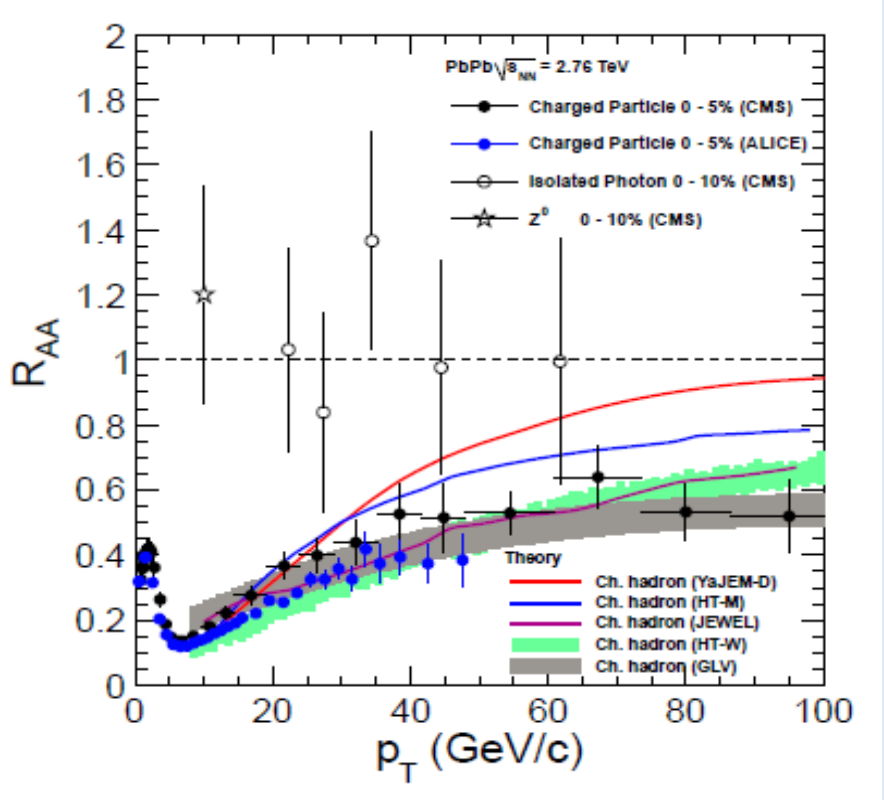


Nucleus-Nucleus collision



Nuclear Modification Factor (R_{AA}):

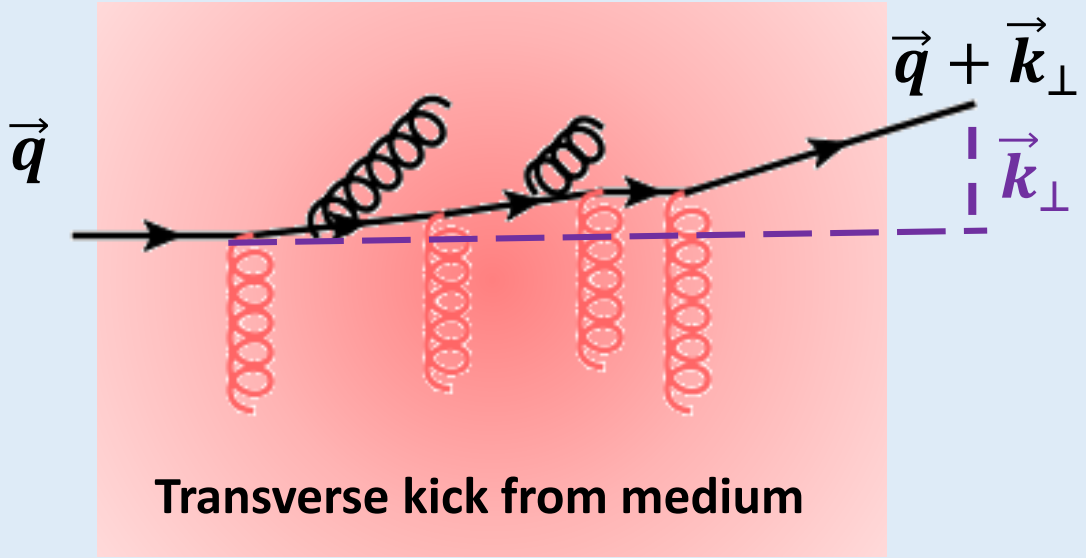
$$R_{AA} \equiv \frac{d^2N^{AA}/dydp_\perp}{d^2N^{pp}/dydp_\perp \times \langle N_{coll}^{AA}\rangle}$$



Mueller et al., Ann. Rev. Nucl. Part. Sci. 62, 361 (2012)

Transport parameter $\hat{q}$ and leading hadron suppression

Leading parton going through medium



Transport parameter $\hat{q}$: Average transverse momentum change per unit length

$$\hat{q}(\vec{r}, t) = \frac{< k_\perp^2 >}{L}$$

$\hat{q}$ is Input parameter to full model calculation

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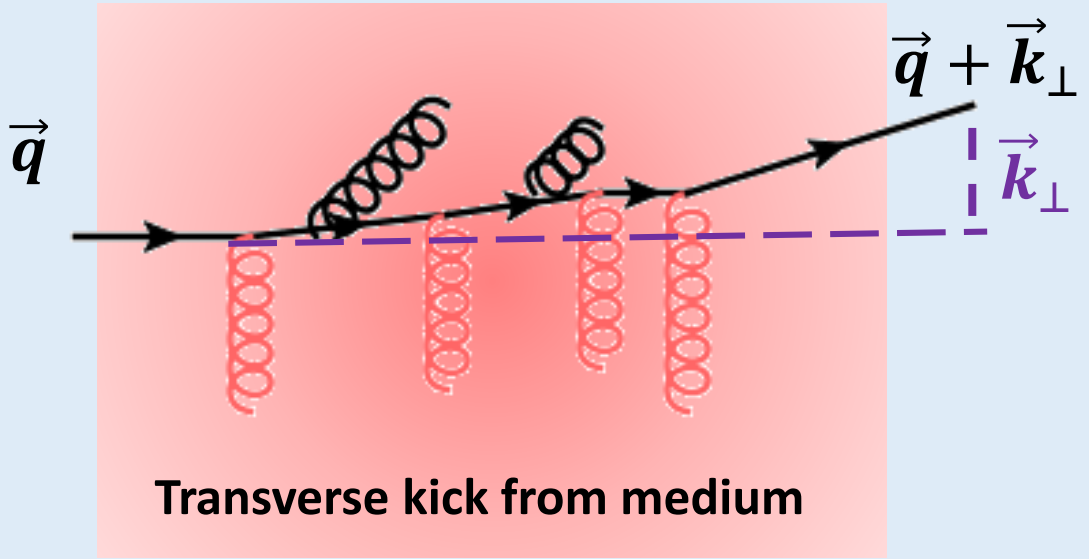
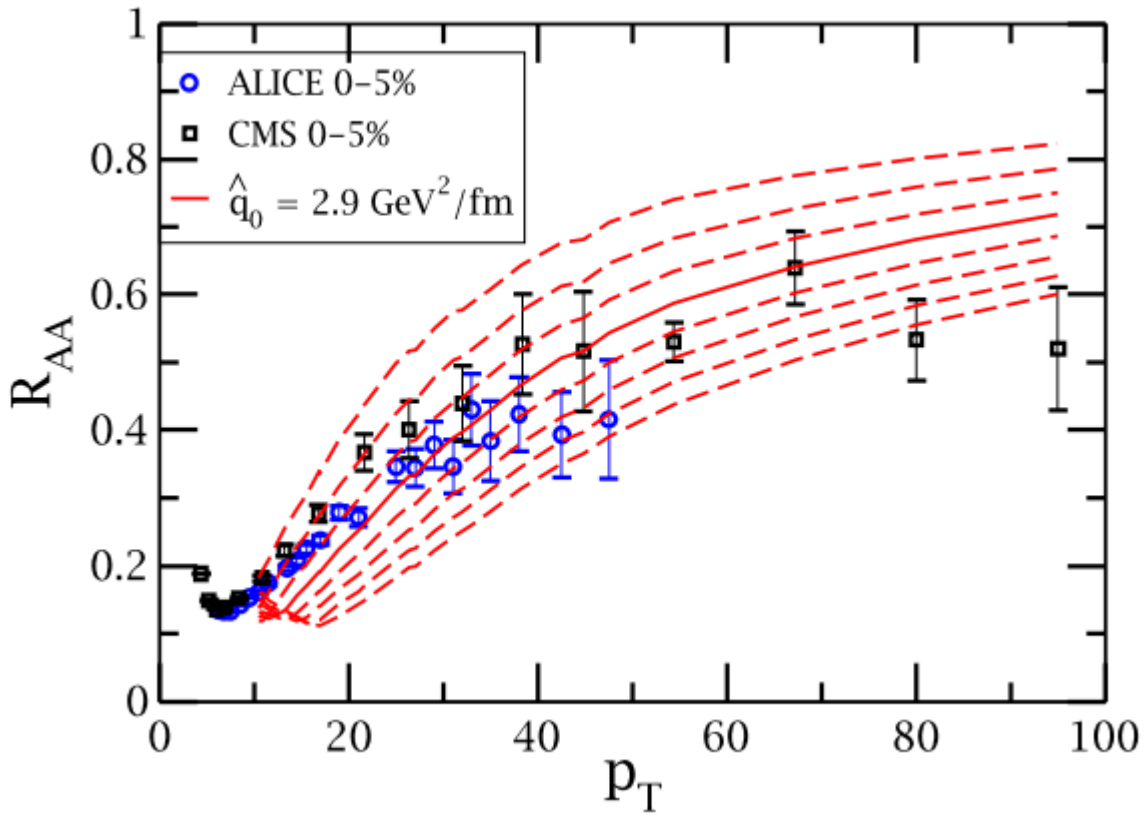


JET Collaboration(Burke et al. 2014)



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Transport parameter $\hat{q}$

Based on fit to the experimental data

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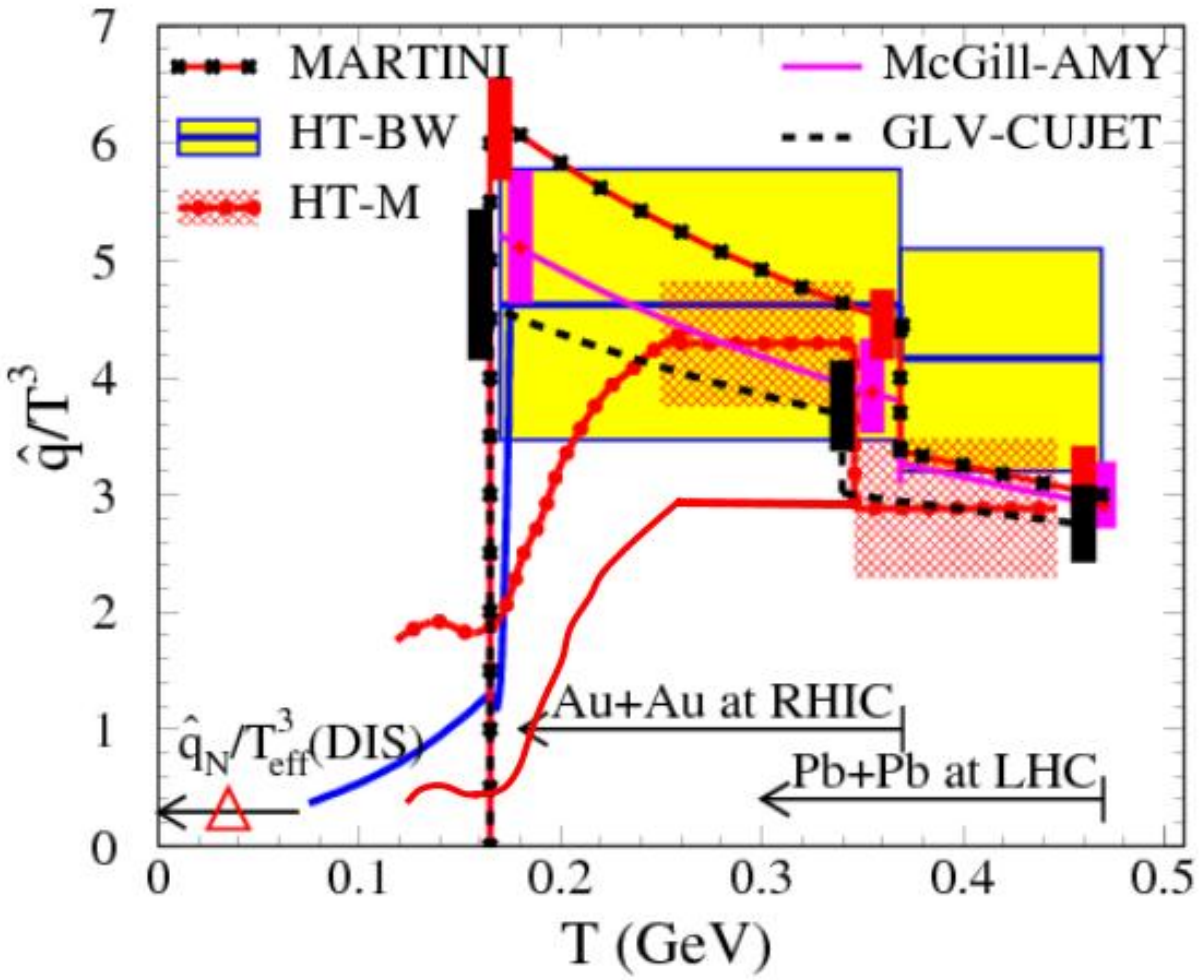
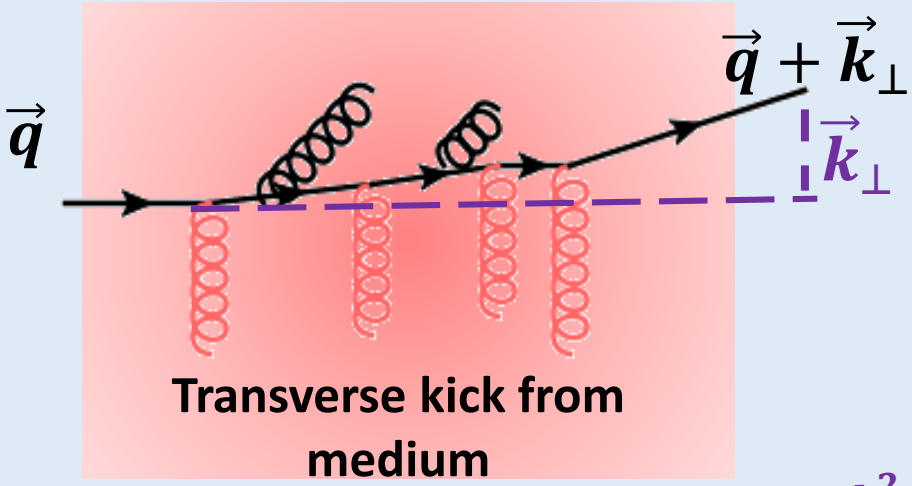


QGP is locally thermalized and highly non-perturbative



Transport parameter $\hat{q}(\vec{r}, t) = \frac{<k_\perp^2>}{L}$

First principles calculation: Lattice QCD to compute $\hat{q}$

Light-cone coordinates

❖ **Minkowski coordinate**

Four vector $p = (p^0, p^1, p^2, p^3)$

Off-shellness $p^2 = (p^0)^2 - (p^1)^2 - (p^2)^2 - (p^3)^2$

❖ **Light-cone coordinate**

Four vector $p = (p^+, p^-, p_\perp)$

$$p^+ = \frac{(p^0 + p^3)}{\sqrt{2}}$$

$$p^- = \frac{(p^0 - p^3)}{\sqrt{2}}$$

$$p_\perp = (p_\perp^1, p_\perp^2)$$

Off-shellness $p^2 = 2p^+p^- - p_\perp^2$

Examples: Particle traveling in $+P_z$ direction : $P^+ \gg P^-;\ P_\perp = 0$

Lattice formulation of $\hat{q}$

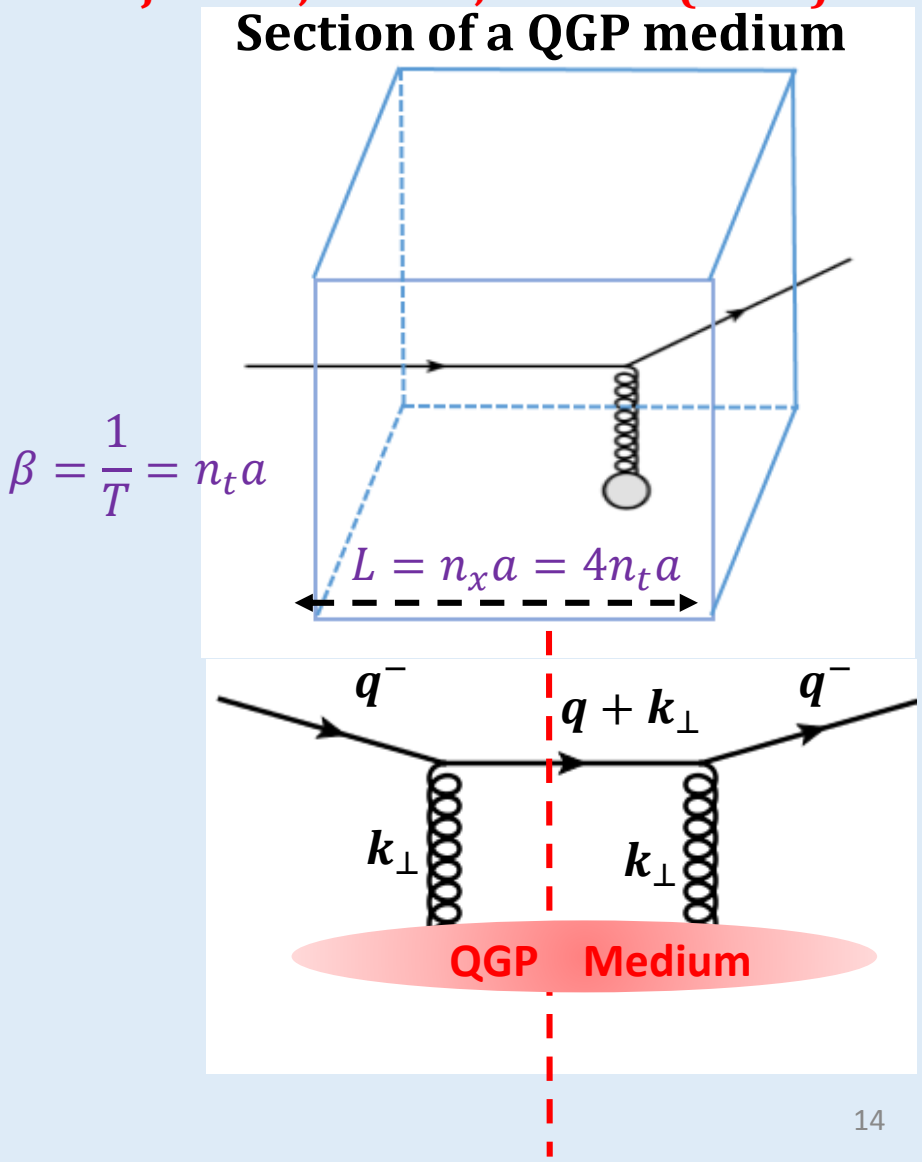
A. Majumder, PRC 87, 034905 (2013)

- Simplest process: A leading quark propagating through **hot plasma (gluons only)** at temperature T

$$\boldsymbol{q} = \left(\mu^2/2q^-, q^-, 0\right) = \left(\lambda^2, 1, 0\right)Q; \quad \textit{Hard scale} = Q; \lambda \ll 1$$

$$\boldsymbol{k} = \left(k^+, k^-, k_\perp 0\right) = \left(\lambda^2, \lambda^2, \lambda\right)Q; \textit{Glauber gluon}$$

- Life time of quark, $\tau \geq 4n_t a = \frac{4}{T}$



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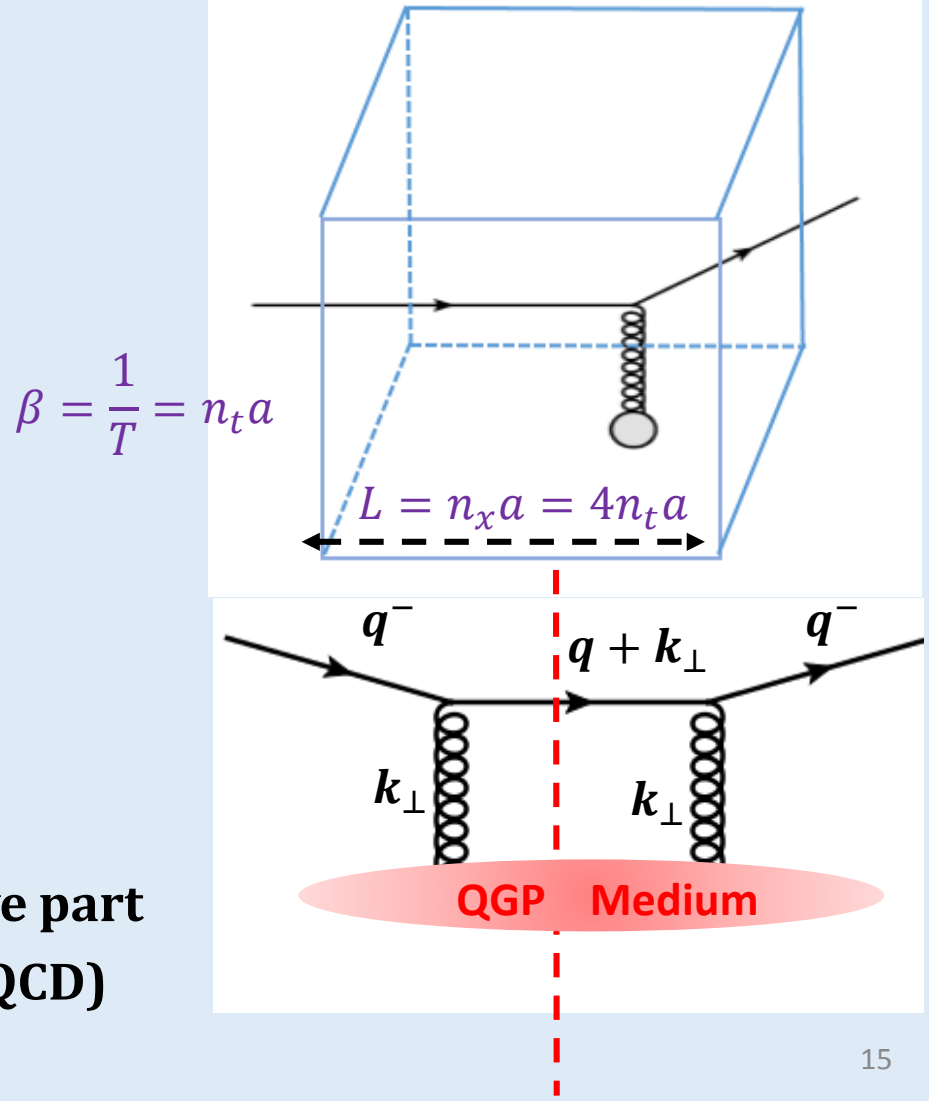
- Life time of quark, $\tau \geq 4n_t a = \frac{4}{T}$

$$\boxed{\hat{q}(\vec{r}, t) = \sum_k k_\perp^2 \frac{Disc[W(k)]}{L}}$$

$$\boxed{\hat{q} = \frac{4\pi^2\alpha_s}{N_c}\int \frac{dy^- d^2y_\perp}{(2\pi)^3}\, d^2k_\perp e^{-i\frac{k_\perp^2}{2q^-}y^- + i\vec{k}_\perp.\vec{y}_\perp} \times \langle M|F_\perp^{+\mu}(y^-, y_\perp)F_{\perp\mu}^{+}(0)|M\rangle}$$

Non-perturbative part (Lattice QCD)

Section of a QGP medium



Constructing a more general expression as $\hat{Q}$

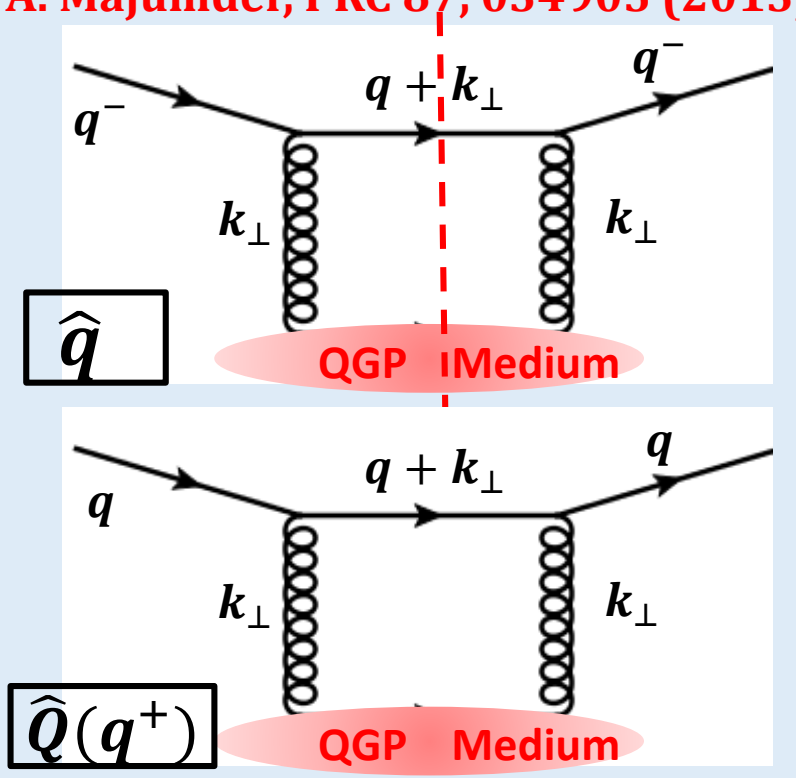
A. Majumder, PRC 87, 034905 (2013)

❖ **Physical form of $\hat{q}$**

$$\hat{q} = \frac{4\pi^2\alpha_s}{N_c}\int \frac{dy^- d^2y_\perp}{(2\pi)^3}\, d^2k_\perp e^{-i\frac{k_\perp^2}{2q^-}y^- + i\vec{k}_\perp.\vec{y}_\perp} \langle M|F_\perp^{+\mu}(y^-, y_\perp)F_{\perp\mu}^{+}(0)|M\rangle$$

❖ **General form of $\hat{q}$:** with q^- *is Fixed*; $q_\perp = 0$; q^+ *is variable*

$$\hat{Q}(q^+) = \frac{4\pi^2\alpha_s}{N_c}\int \frac{d^4y d^4k}{2\pi^4} e^{iky}\, 2q^- \frac{\langle M|F_\perp^{+\mu}(0)\, F_{\perp\mu}^{+}(y)|M\rangle}{(q+k)^2 + i\epsilon}$$



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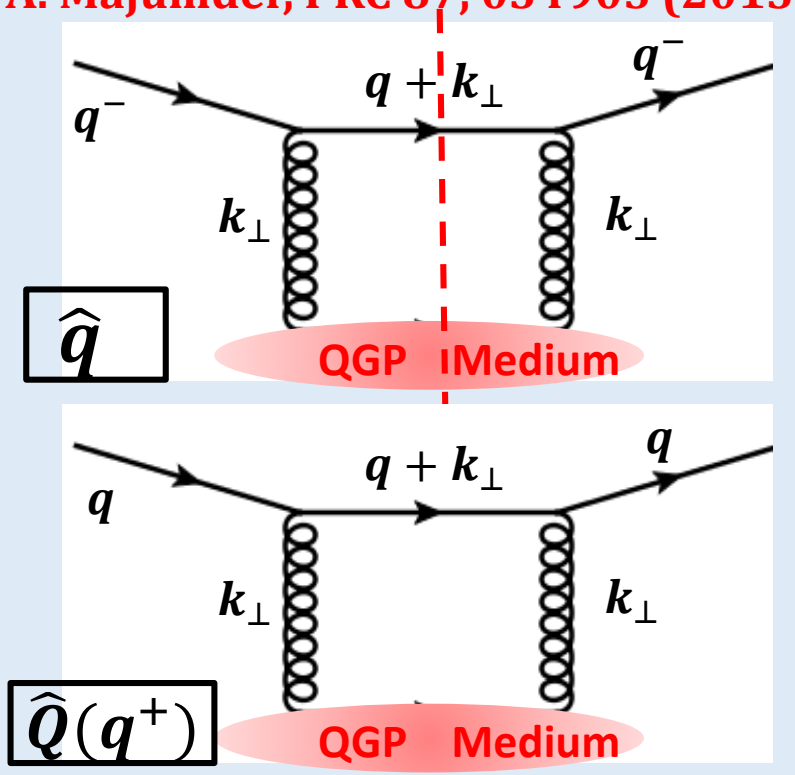
1) When $q^+ \sim T$

$$\mathrm{Disc}\left[\hat{Q}(q^+)\right]_{at\ q^+\sim T} = \hat{q}$$



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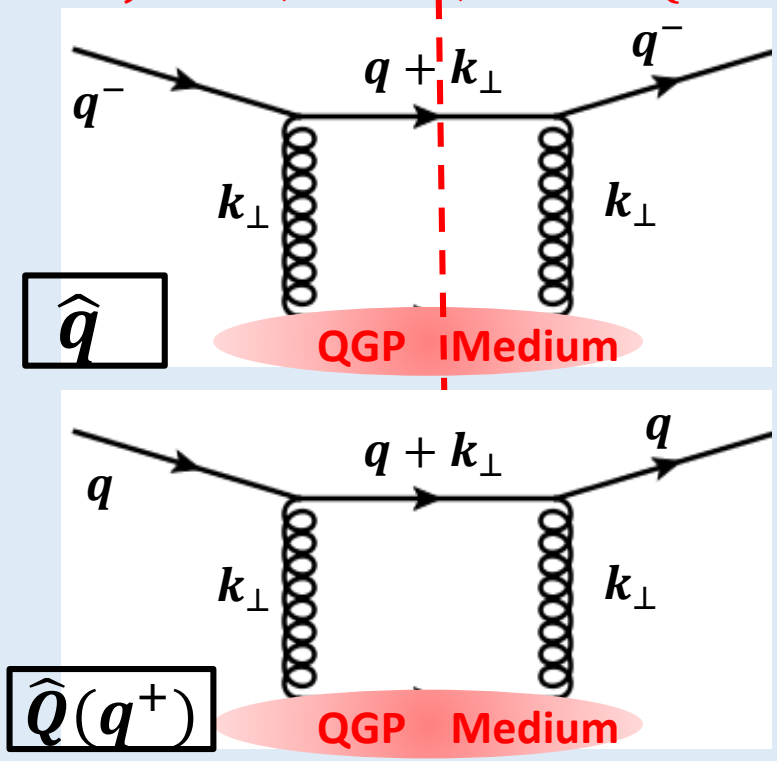

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$$\frac{1}{(q+k)^2} \simeq \frac{1}{-2q^-q^- + 2q^-(k^+ - k^-)} = -\frac{1}{2(q^-)^2}\left[1 - \left(\frac{k^+ - k^-}{q^-}\right)\right]^{-1} = -\frac{1}{2(q^-)^2}\left[1 - \left(\frac{\sqrt{2}k_z}{q^-}\right)\right]^{-1} = -\frac{1}{2(q^-)^2}\left[\sum_{n=0}^{\infty}\left(\frac{\sqrt{2}k_z}{q^-}\right)^n\right]$$

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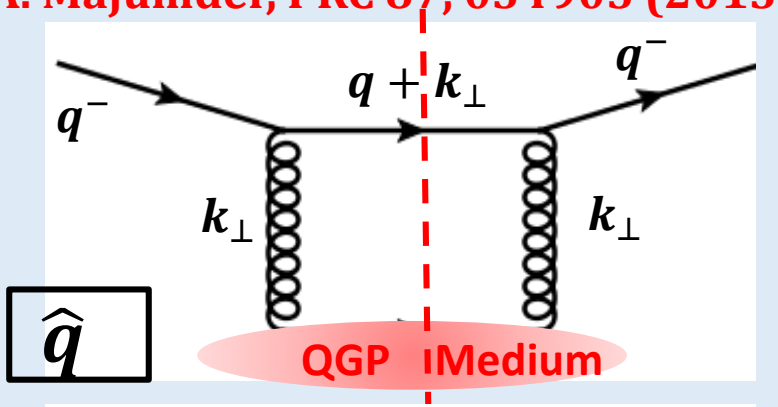

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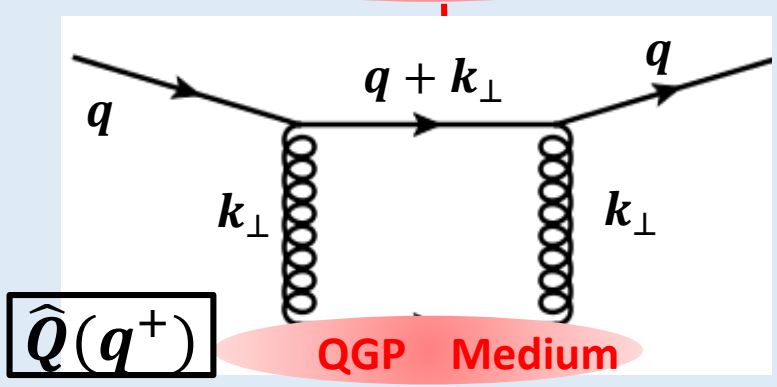

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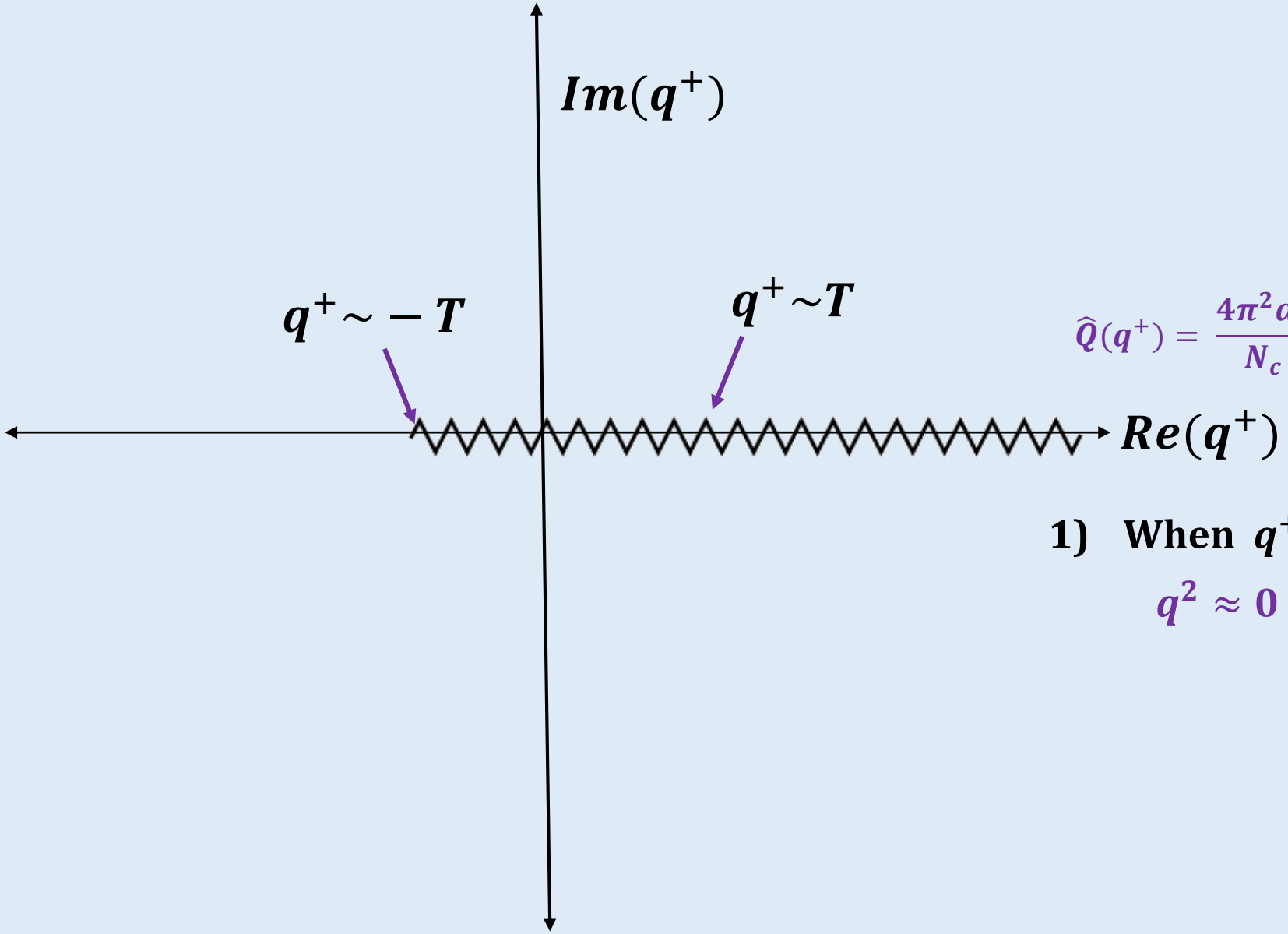
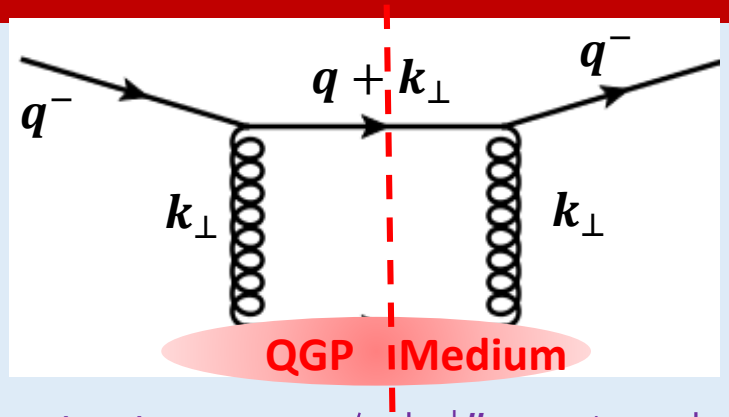
$$\hat{Q}(q^+ = -q^-) = \frac{4\pi^2\alpha_s}{N_c\, q^-}\langle M|F_\perp^{+\mu}(0)\sum_{n=0}^{\infty}\left(\frac{i\sqrt{2}D_z}{q^-}\right)^n F_{\perp\mu}^{+}(0)\,|M\rangle$$

Extracting $\hat{q}$ through analytic structure of $\hat{Q}(q^+)$





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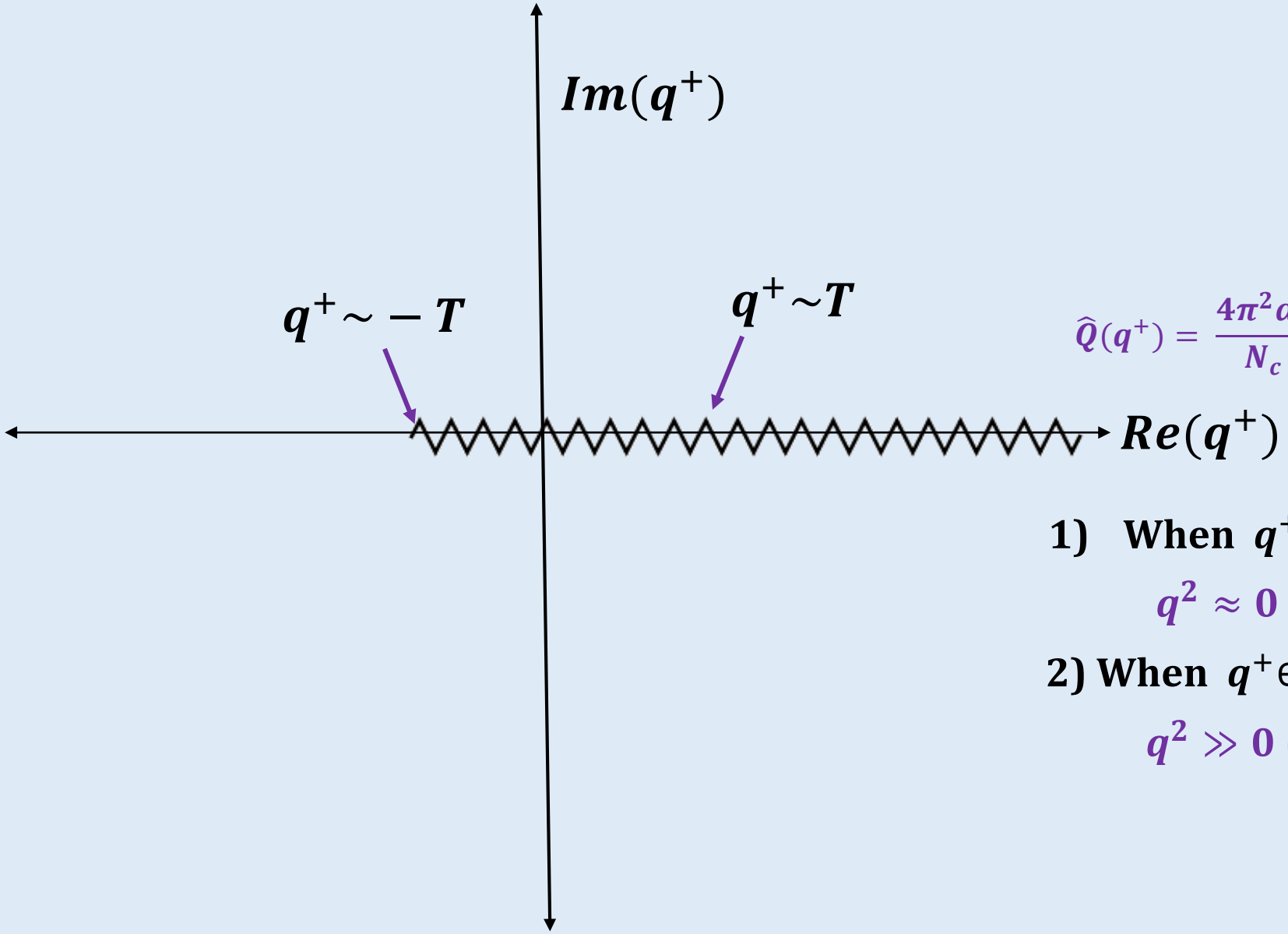
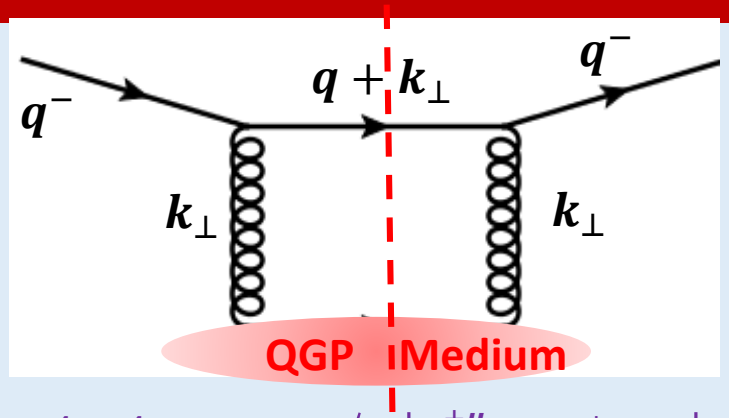
1) When $q^+ \in \#[-T, T]$

$q^2 \approx 0$ (in-medium scattering)

Extracting $\hat{q}$ through analytic structure of $\hat{Q}(q^+)$





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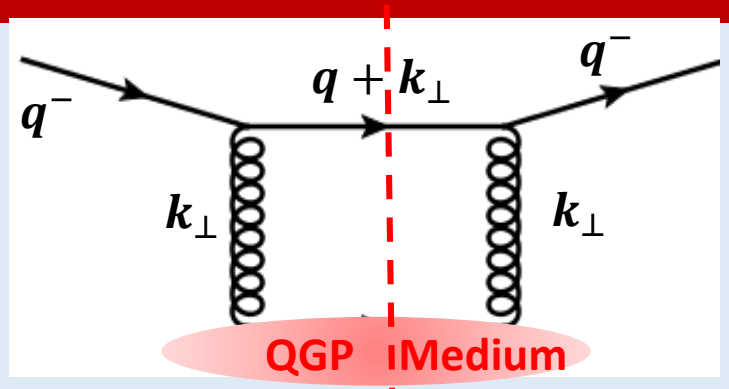
$q^2 \approx 0$ (in-medium scattering)

2) When $q^+ \in [\#T, +\infty)$

$q^2 \gg 0$ (Bremsstrahlung radiation)

Extracting $\hat{q}$ through analytic structure of $\hat{Q}(q^+)$



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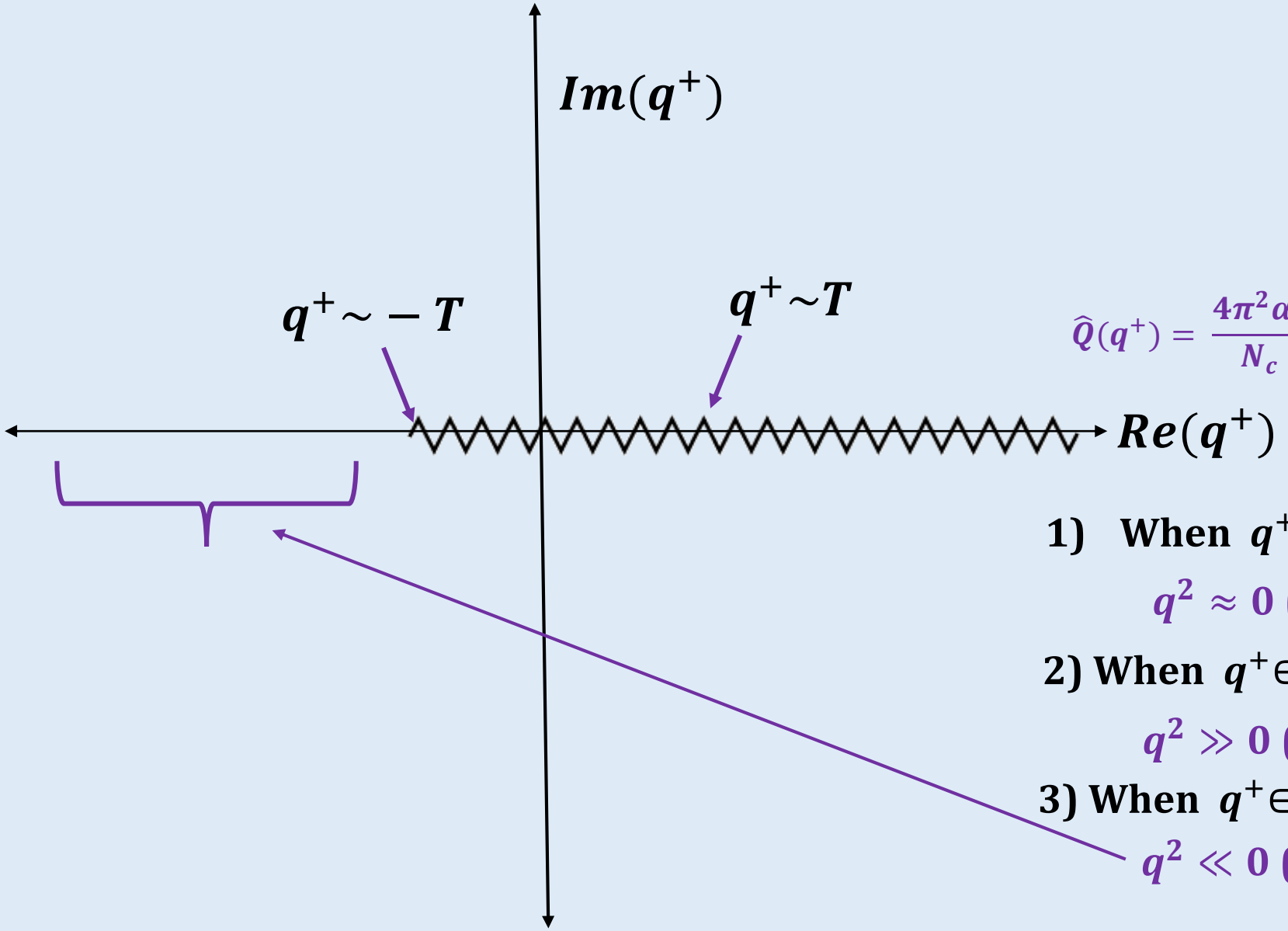

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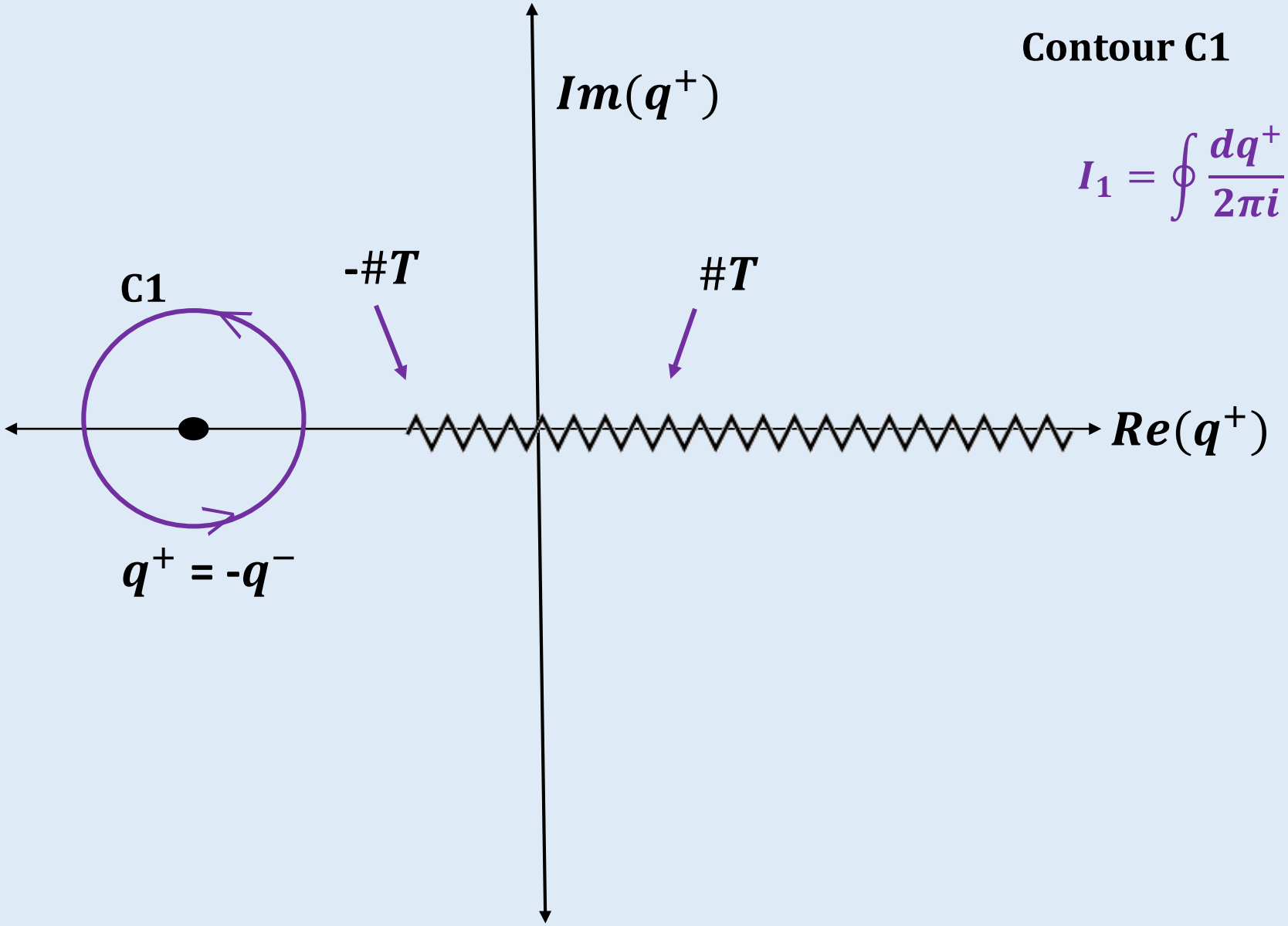
3) When $q^+ \in (-\infty, -\#T]$

$q^2 \ll 0$ (Space-like) ; $\lim_{q^+ \to -\infty} \mathrm{Disc}[\hat{Q}(q^+)] = 0$

Extracting $\hat{q}$ through analytic structure of $\hat{Q}(q^+)$



Contour C1

$$I_1 = \oint \frac{dq^+}{2\pi i} \frac{\hat{Q}(q^+)}{(q^+ + q^-)} = \hat{Q}(q^+ = -q^-)$$

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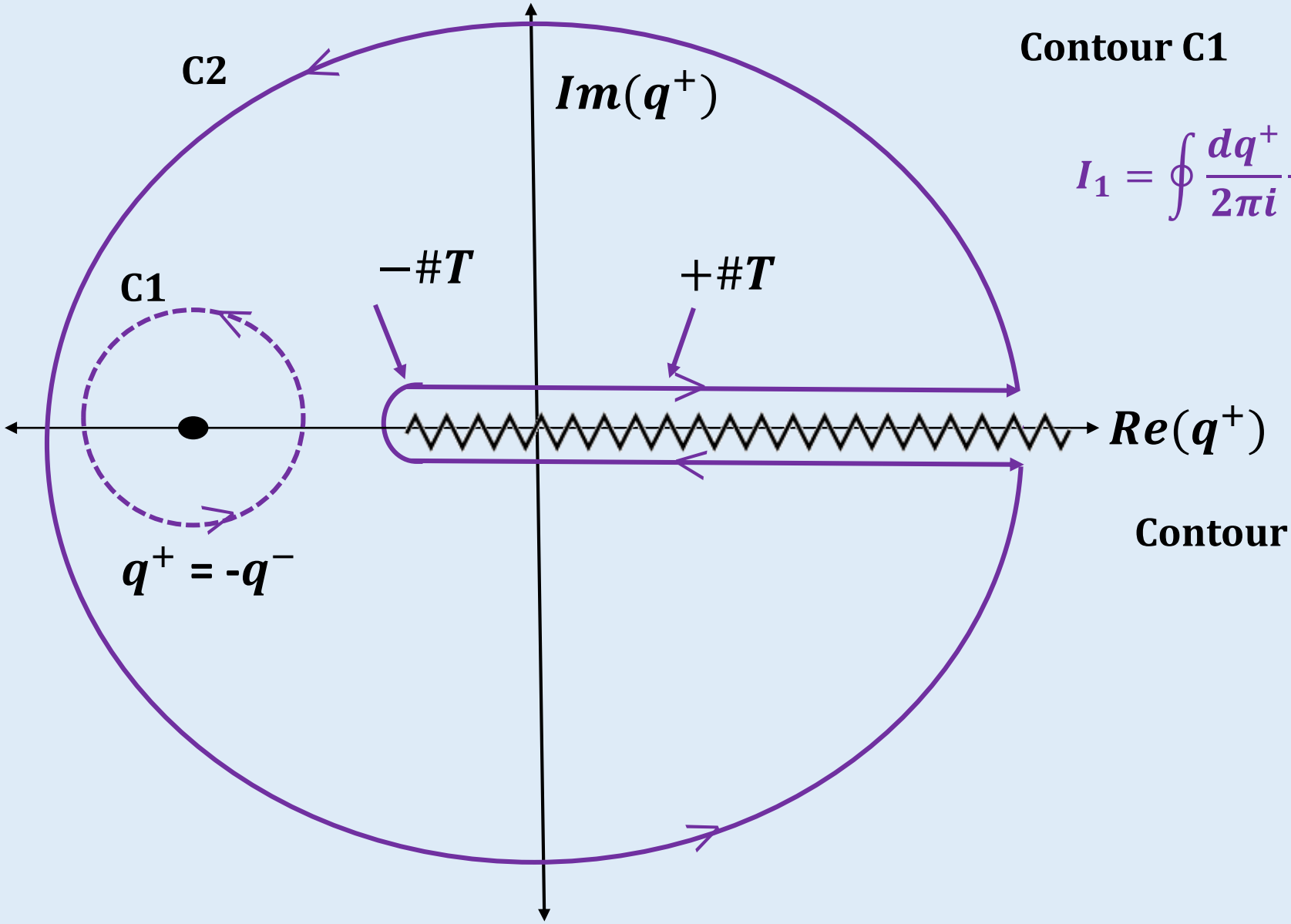

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Contour C2: On stretching it to infinity

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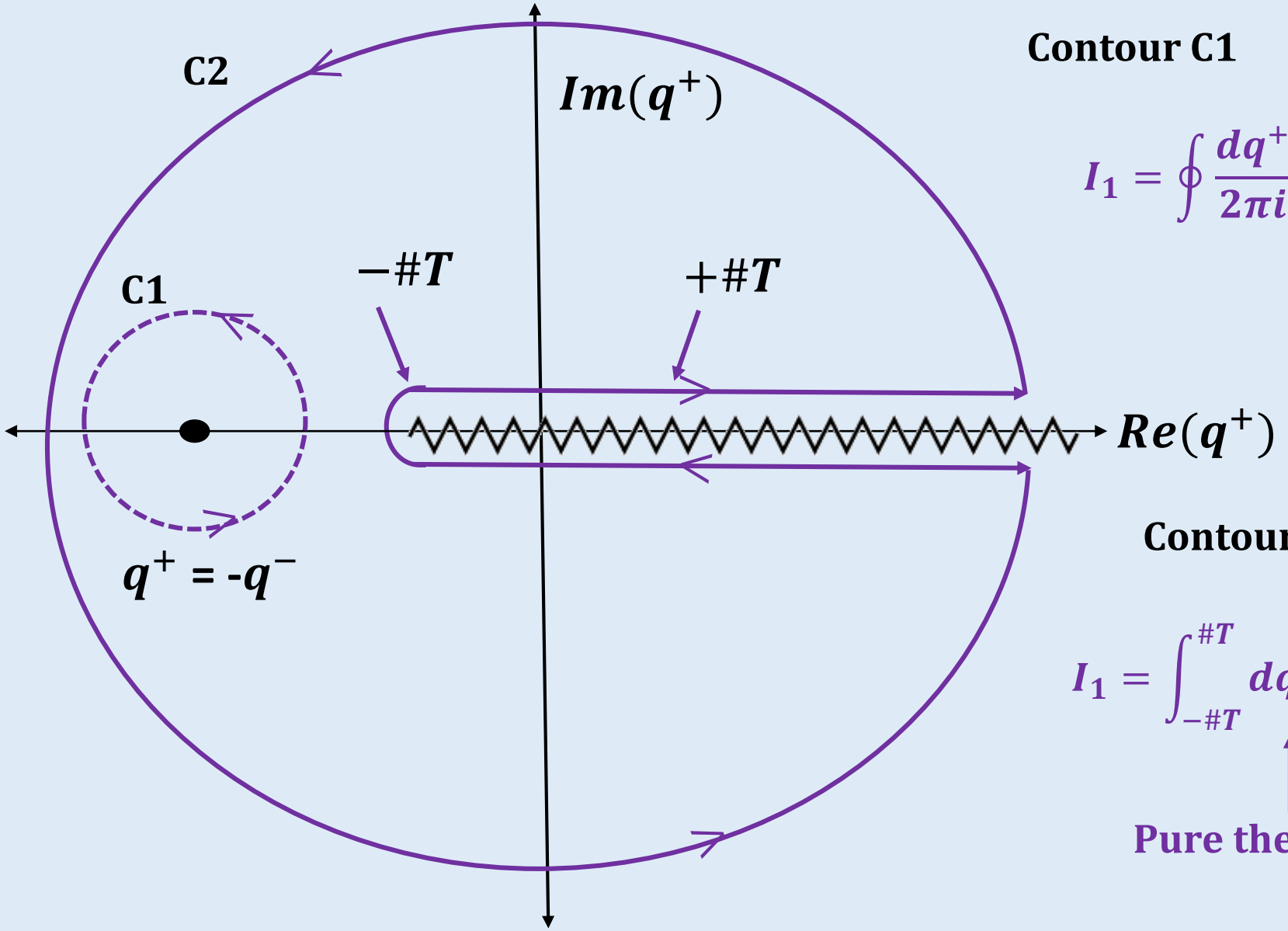

Contour C1

$$I_1 = \oint \frac{dq^+}{2\pi i} \frac{\hat{Q}(q^+)}{(q^+ + q^-)} = \hat{Q}(q^+ = -q^-)$$

Contour C2: On stretching it to infinity

$$I_1 = \int_{-\#T}^{\#T} dq^+ \frac{\hat{q}(q^+)}{q^+ + q^-} + \int_0^\infty dq^+ \frac{Disc[\hat{Q}(q^+)]}{q^+ + q^-}$$

Pure thermal part **Pure Vacuum part**

$\hat{q}$ as a series of local operators

❖ **Physical form of $\hat{q}$ at LO:**

$$\hat{q} = \frac{4\sqrt{2}\pi^2\alpha_s}{N_c T}\langle M|F_{\perp}^{+\mu}(0)\sum_{n=0}^{\infty}\left(\frac{i\sqrt{2}D_z}{q^-}\right)^n F_{\perp\mu}^{+}(0)\,|M\rangle_{(Thermal-Vacuum)}$$

$\hat{q}$ as a series of local operators

Xiangdong Ji, PRL 110, 262002 (2013)
Parton PDF operator product expansion with D_z derivatives

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Rotating to Euclidean space:

$$x^0 \to -ix^4; \quad A^0 \to iA^4$$

$$\Longrightarrow \quad F^{0i} \to iF^{4i}$$

LO operators:

$$\sum_{i=1}^{2} Trace\left[F^{3i}F^{3i} - F^{4i}F^{4i}\right] + 2i\sum_{i=1}^{2} Trace\left[F^{3i}F^{4i}\right]$$

Uncrossed operator **Crossed operator**

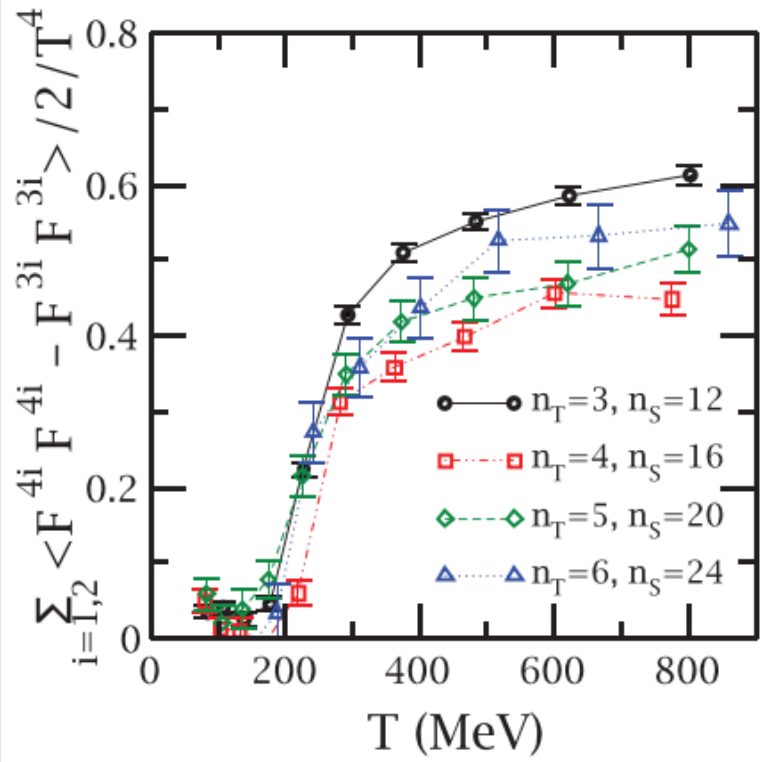
LO operators with $D_z$derivative:

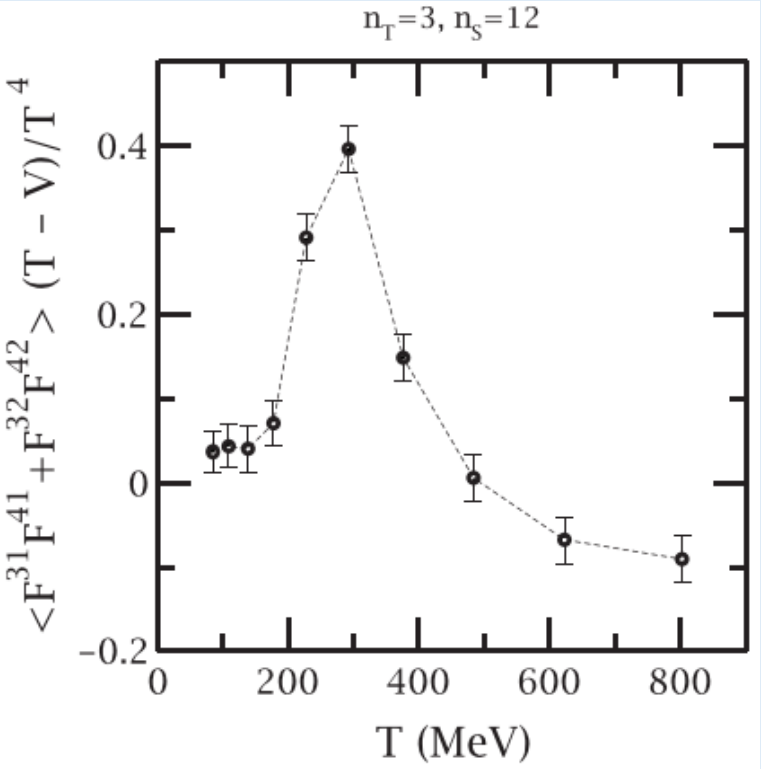
$$\sum_{i=1}^{2} Trace\left[F^{3i}D_zF^{3i} - F^{4i}D_zF^{4i}\right] + i\sum_{i=1}^{2} Trace\left[F^{3i}D_zF^{4i} + F^{4i}D_zF^{3i}\right]$$

Operators in quenched SU(2) plasma

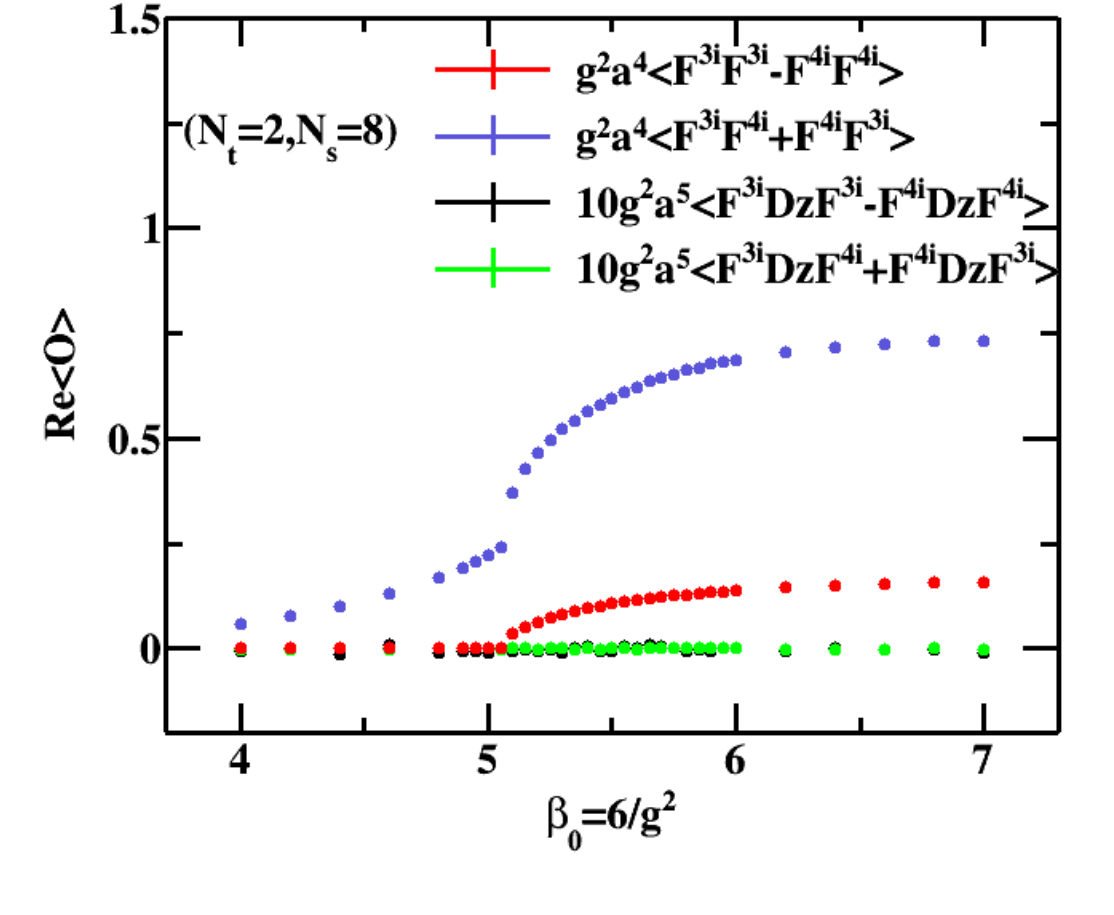
- **Average over 5000 configuration**
- **Transition temperature $T_c \in [170, 350]$ MeV**
- **Crossed correlator is small for T ~ 400 MeV**

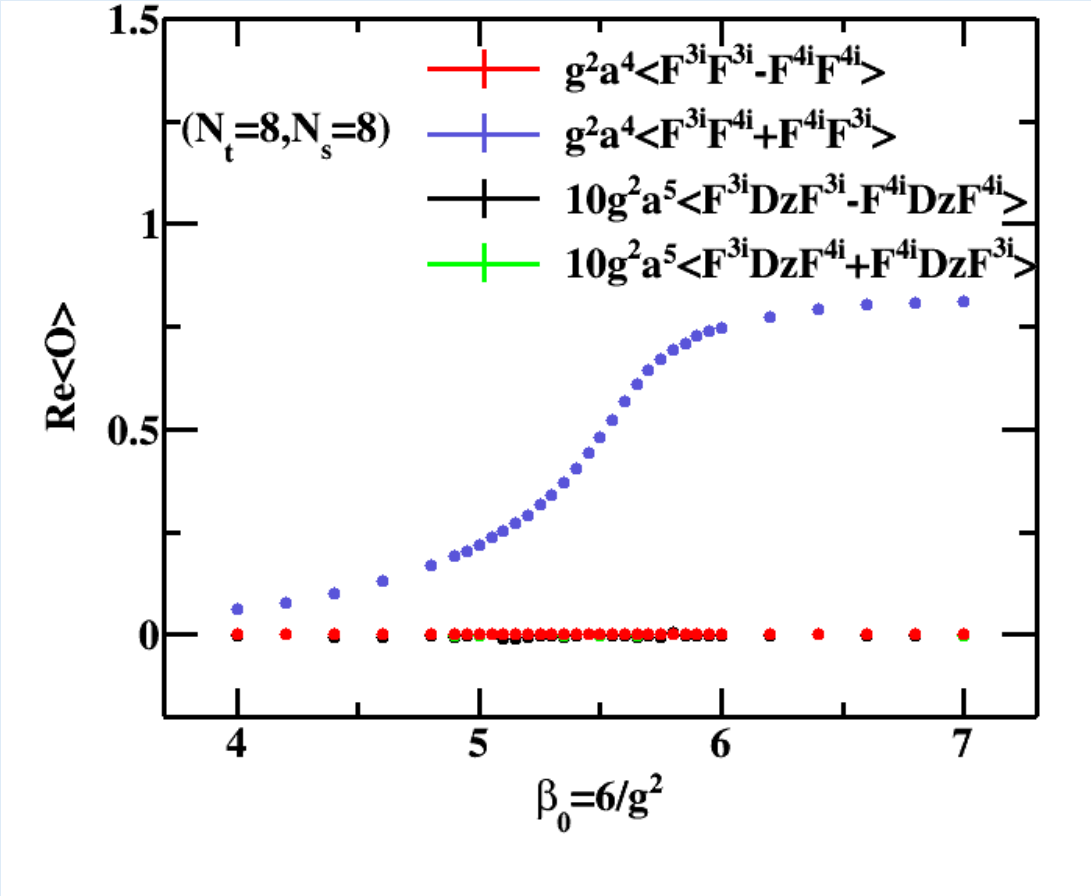
A. Majumder, PRC 87, 034905 (2013)





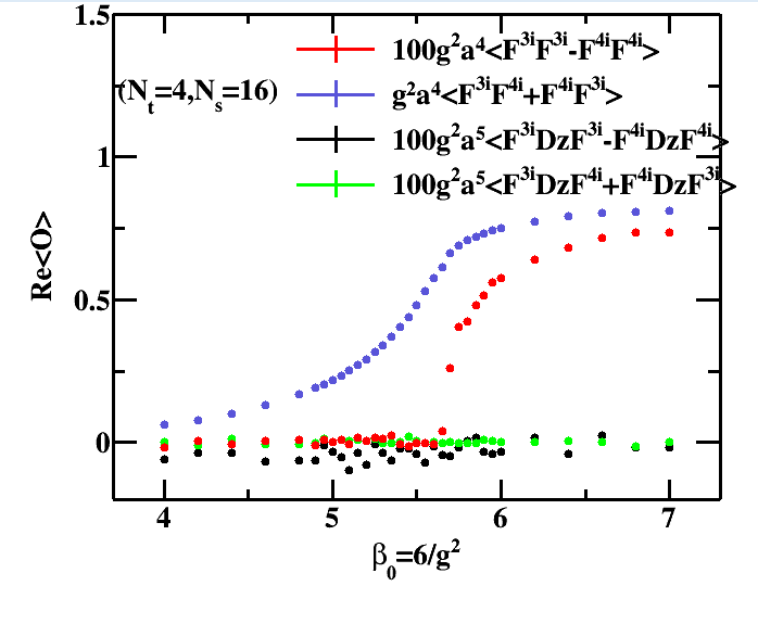
Operators in quenched SU(3) plasma

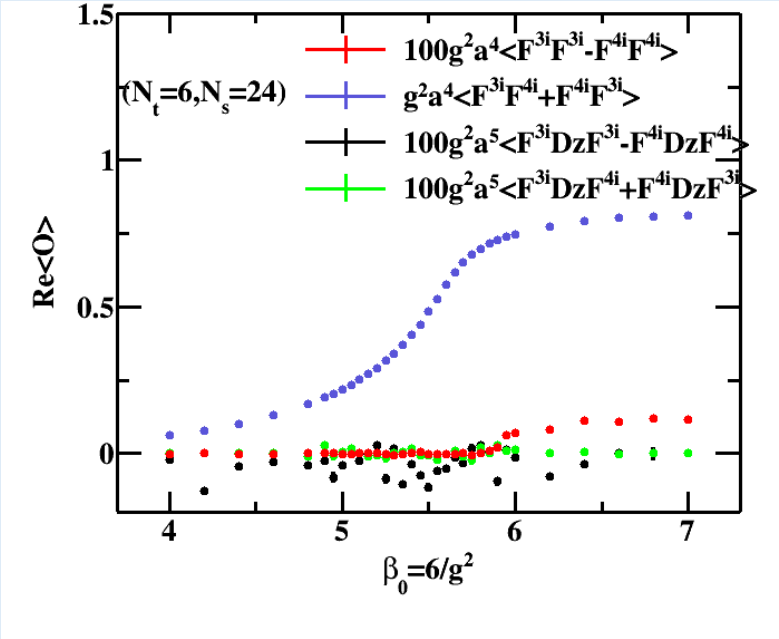


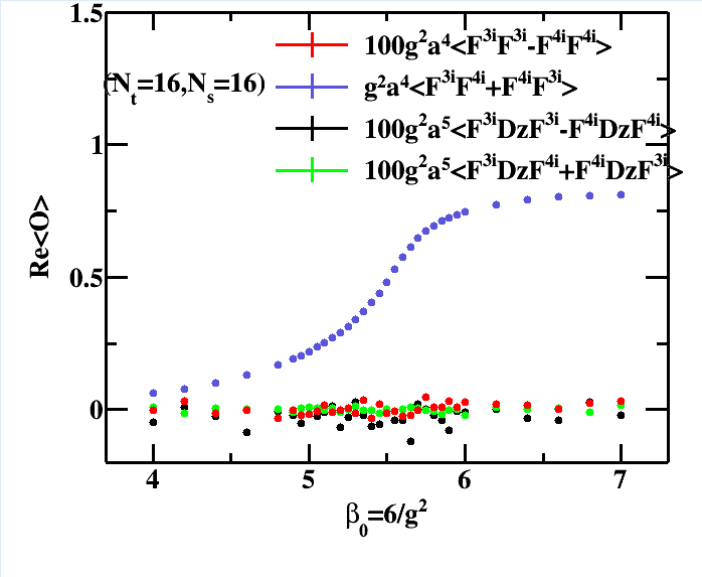


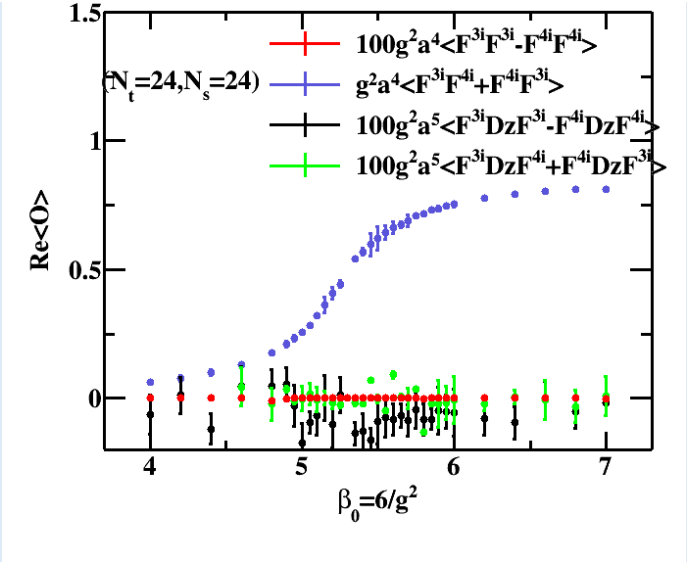
Preliminary
(in collaboration with Chiho Nonaka)

Operators in quenched SU(3) plasma



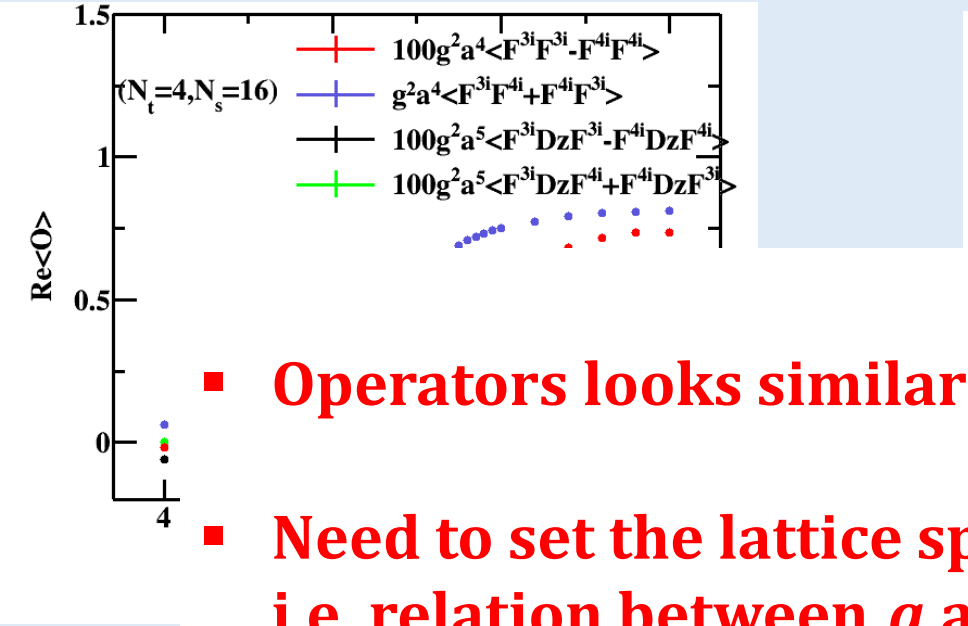


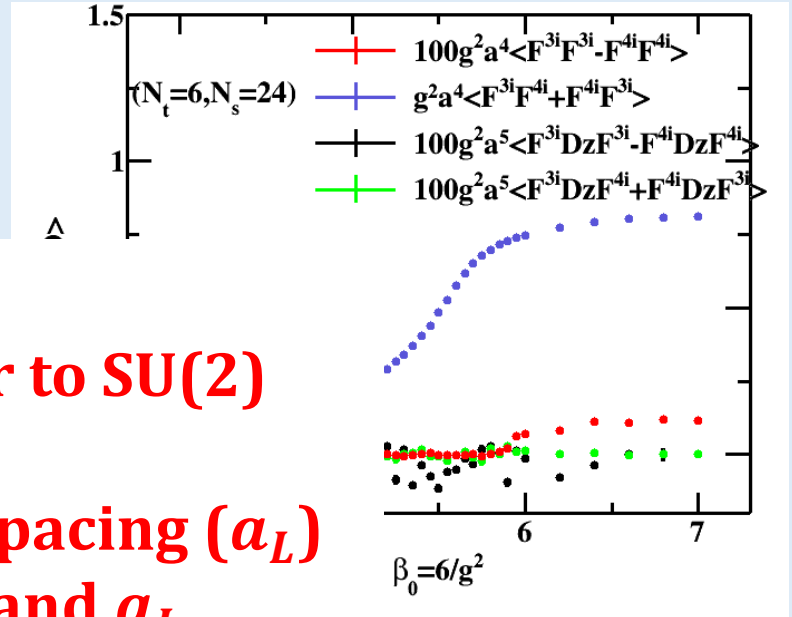




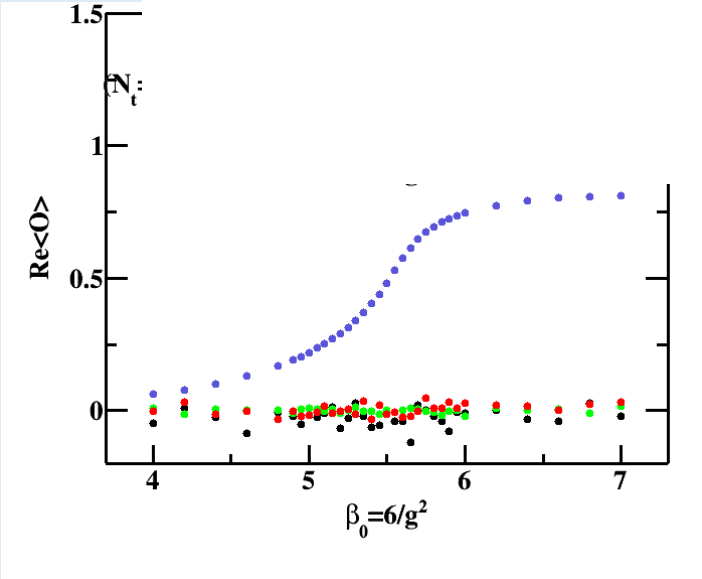
Preliminary (in collaboration with Chiho Nonaka)

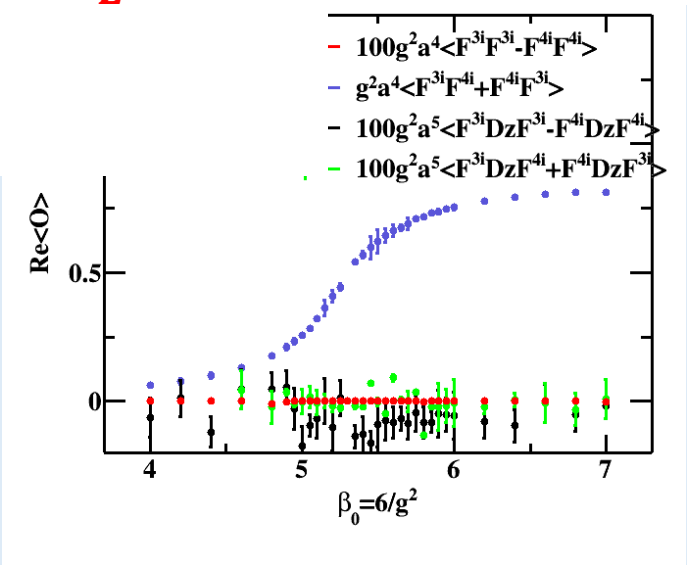
Operators in quenched SU(3) plasma





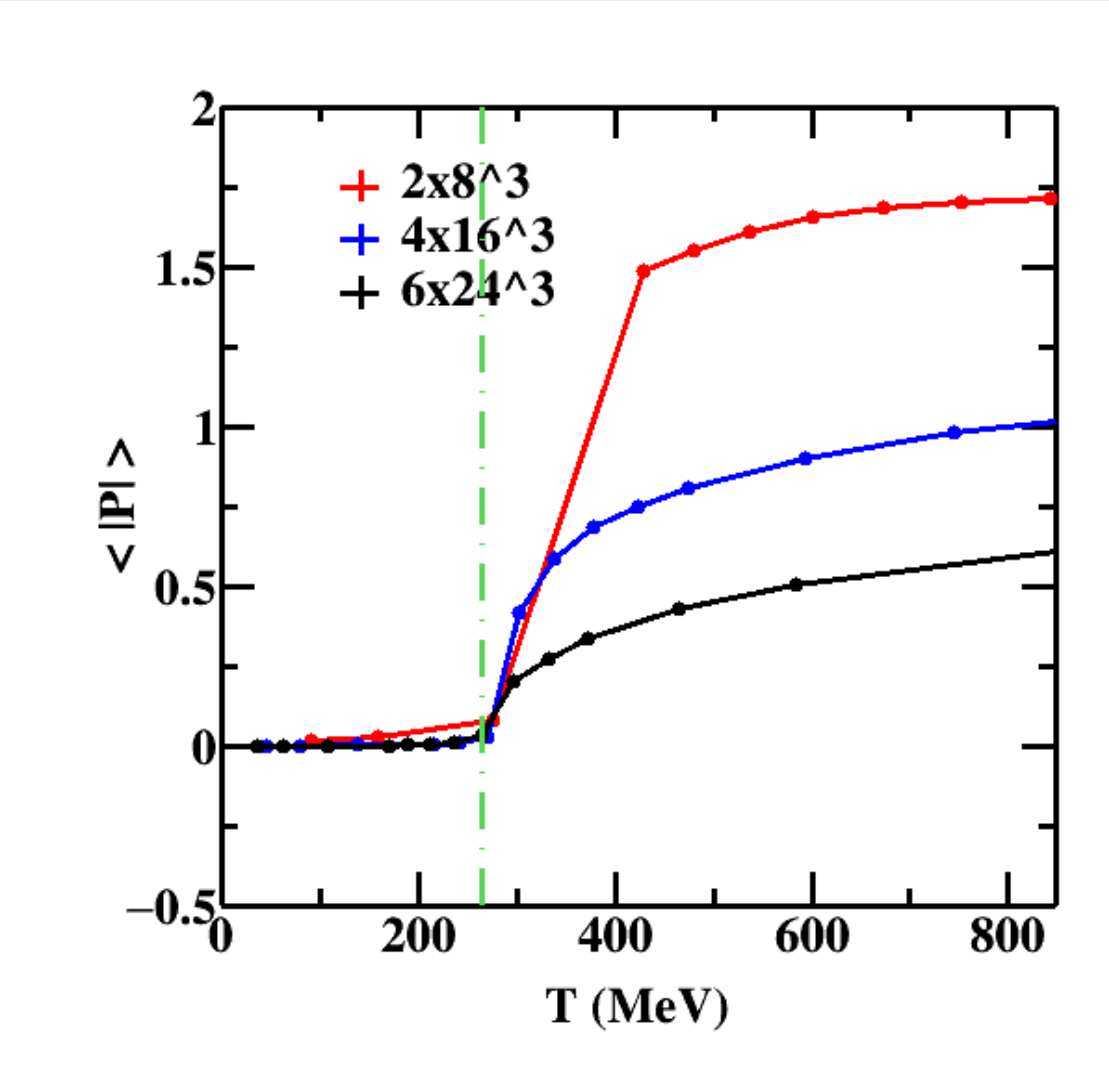
- **Operators looks similar to SU(2)**
- **Need to set the lattice spacing (a_L) i.e. relation between g and a_L**





Preliminary (in collaboration with Chiho Nonaka)

Scale setting on the lattice using Polyakov loop



- **Expectation value of Polyakov loop:**

$$P = \frac{1}{n_x n_y n_z} tr\left[\sum_{\vec{r}} \prod_{n=0}^{n_t-1} U_4(na,\vec{r})\right]$$

- **Two loop beta function**

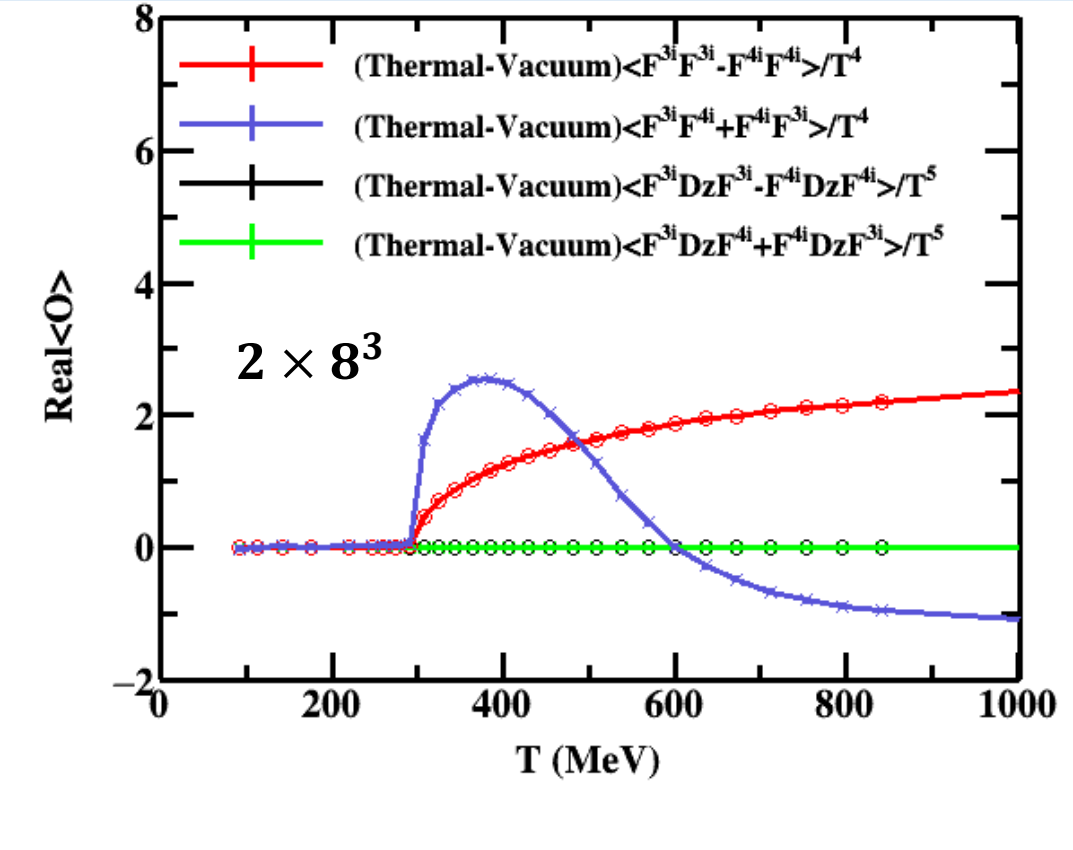
$$a_L = \frac{1}{\Lambda_L}\left(\frac{11}{16\pi^2 g^2}\right)^{-\frac{51}{121}} exp\left(-\frac{8\pi^2}{11g^2}\right)$$

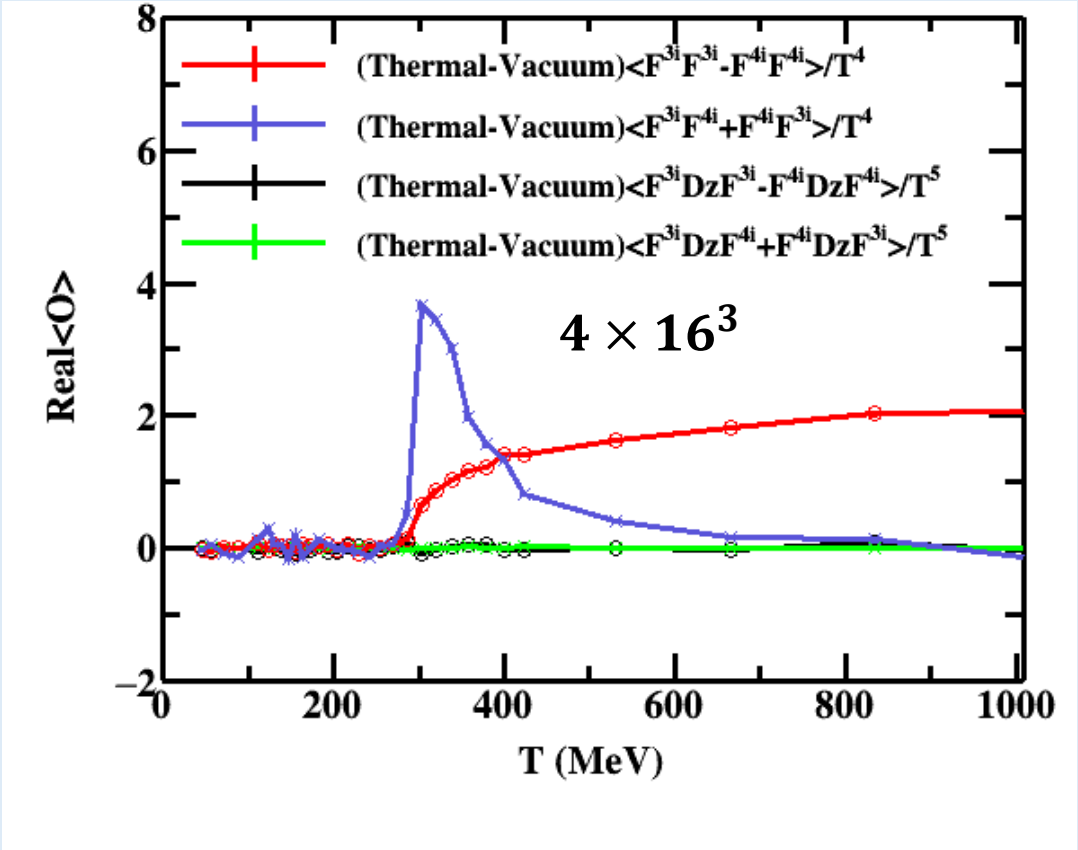
$$Temperature, T = \frac{1}{n_t a_L}$$ (Pure SU(3))

- **Nonperturbative correction**

$$Tune\ \frac{T_c}{\Lambda_L}\ is\ independent\ of\ g$$

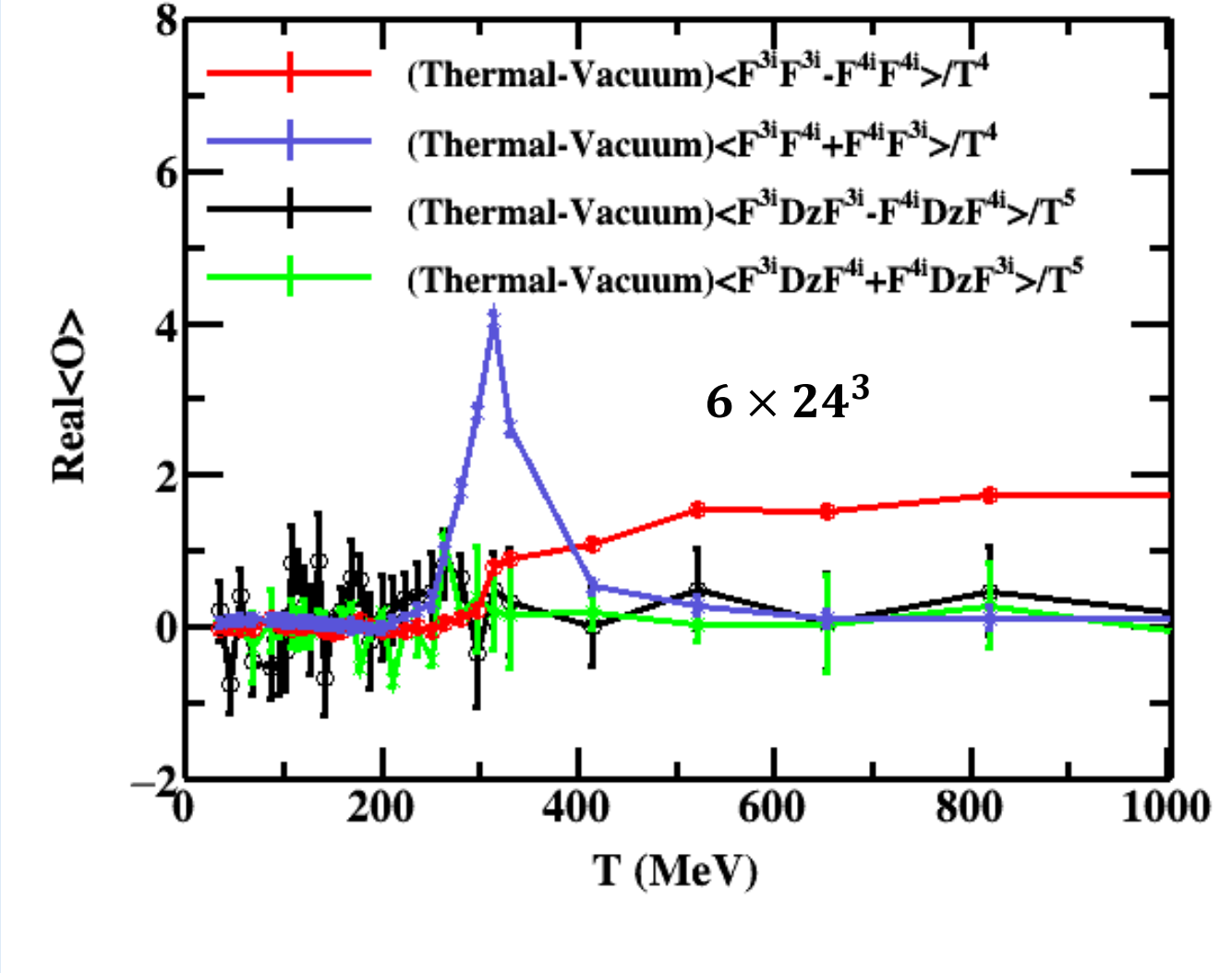
Real part of FF correlator in quenched SU(3)





- **Uncrossed correlator is dominant at high temperature**
- **Crossed correlator goes to zero at high temperature**
- **Correlator with Dz derivative are suppressed**

Real part of FF correlator in quenched SU(3)



- **Uncrossed correlator is dominant at high temperature**
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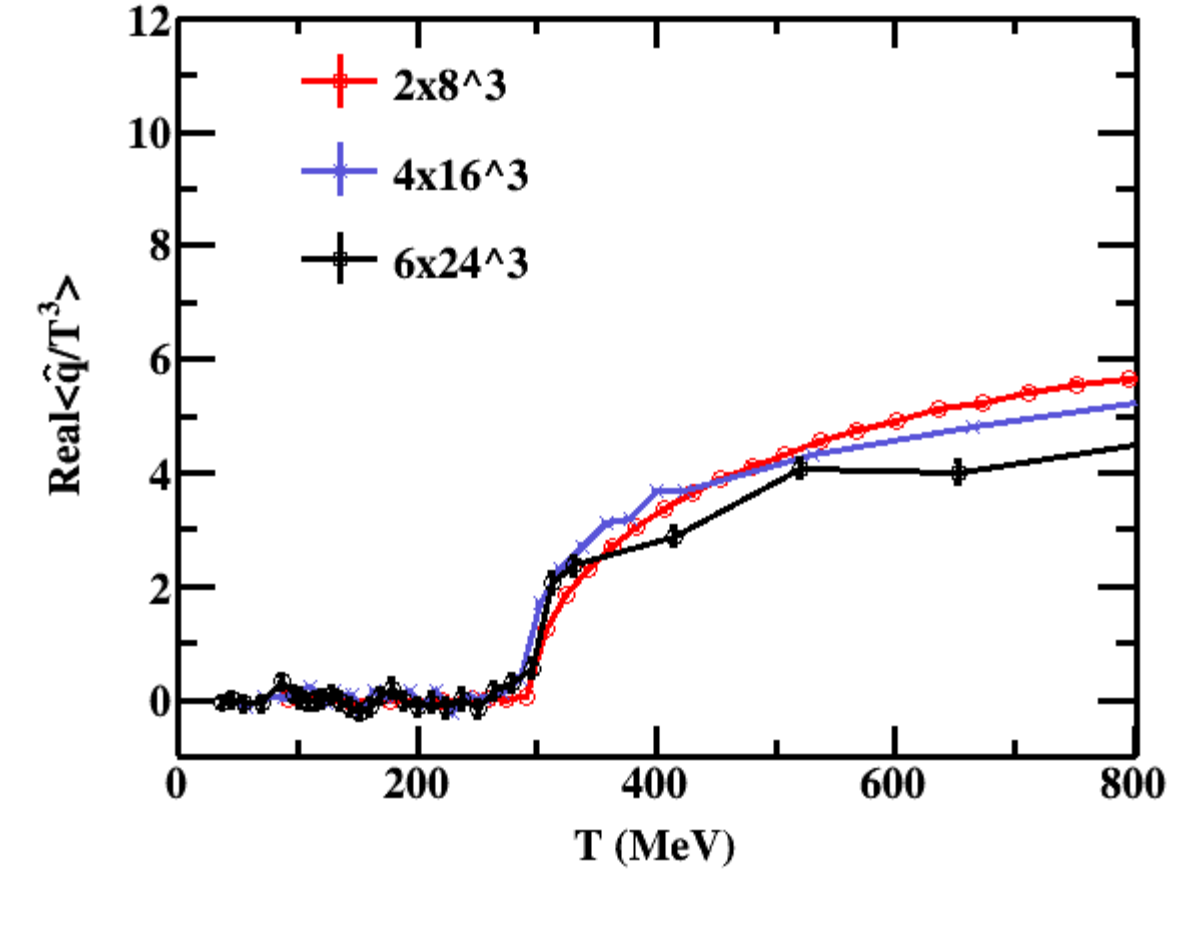
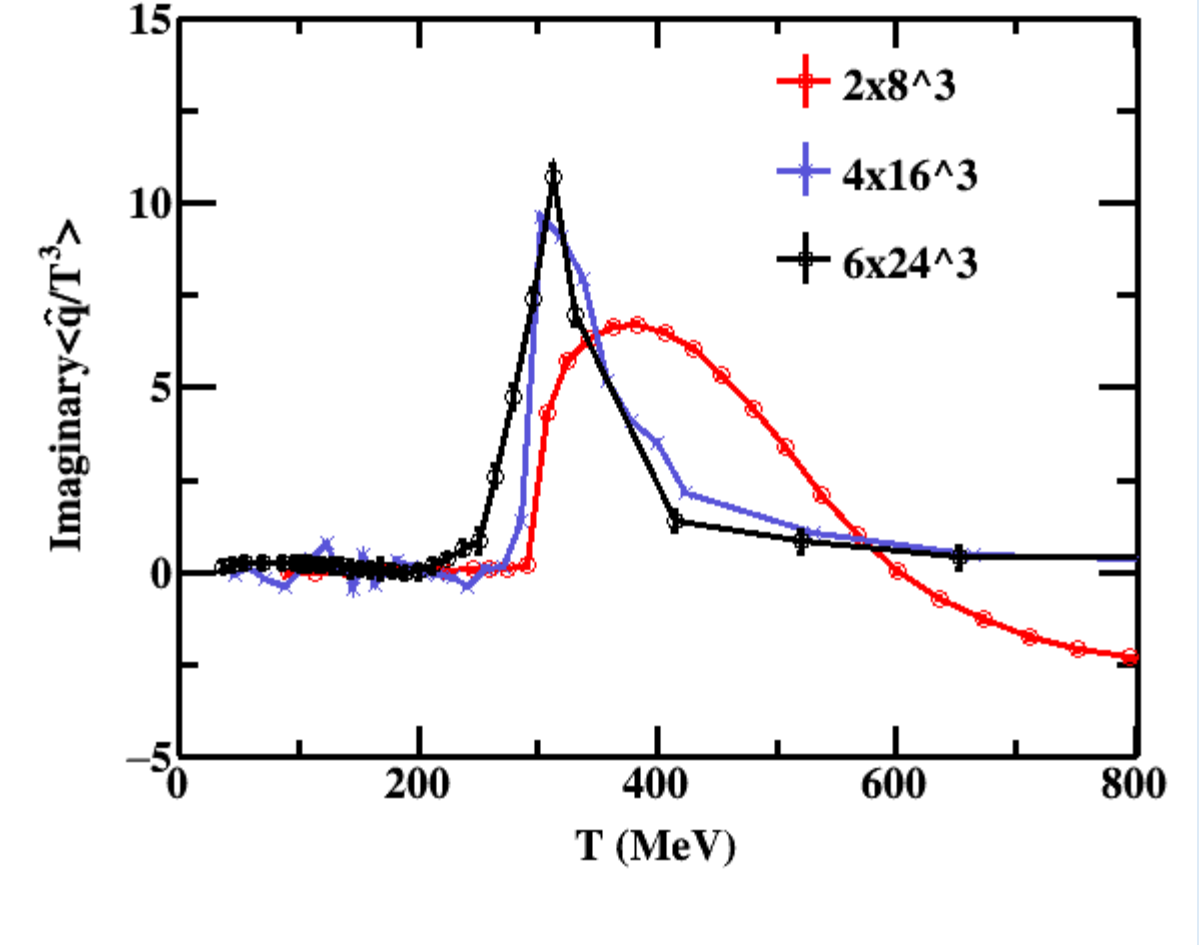
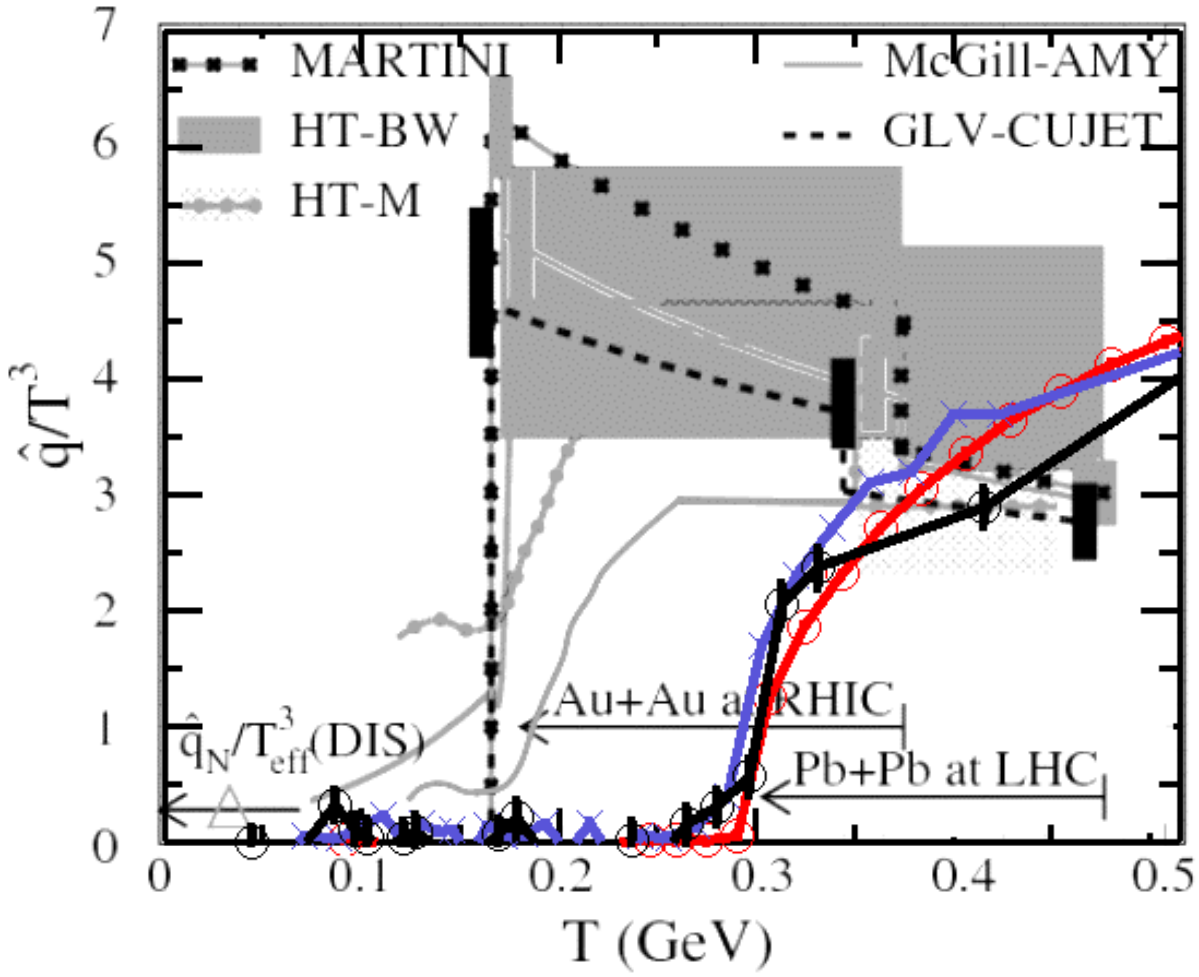
$\hat{q}$ in quenched SU(3) plasma





$\hat{q}$ in quenched SU(3) plasma

JET Collaboration(Burke et al. 2014)



Quenched SU(3) Lattice

- 2x8^3
- 4x16^3
- 6x24^3

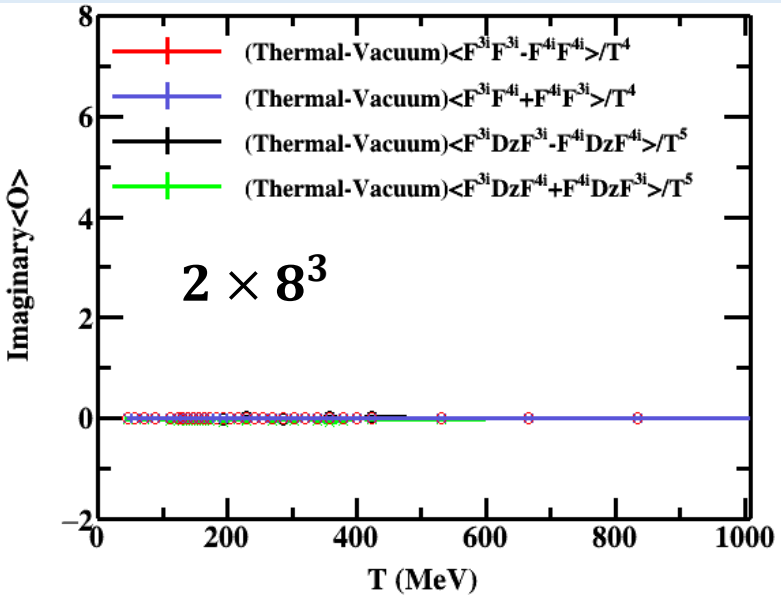
Summary and Future work

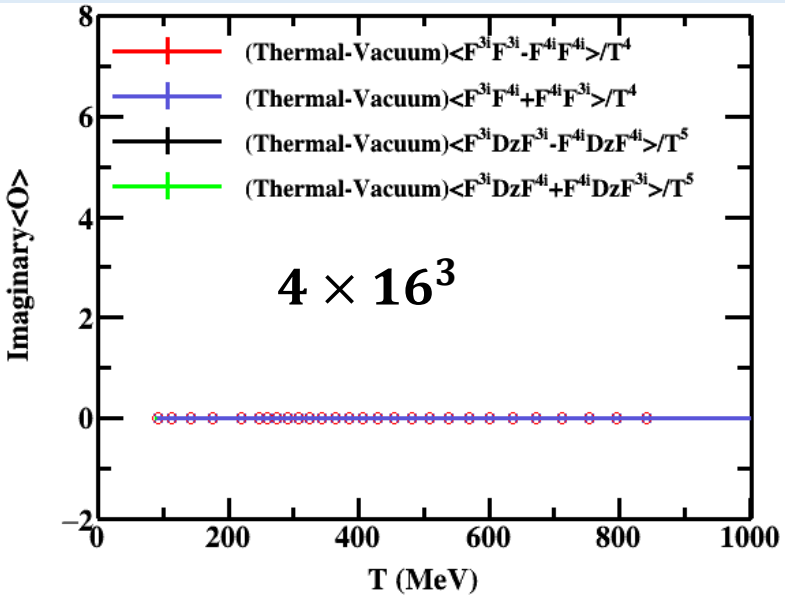
- First calculation of $\hat{q}$ on SU(3) quenched plasma
  - Analytic continuation to deep Euclidean space and expressed as local operators
  - Scale setting using perturbative loop beta function with non-perturbative correction using Polyakov loop.
- Temperature dependence of $\hat{q}$
  - Real part of $\hat{q}$ goes as T^3 for $T > 400$ MeV
  - Real part of $\hat{q}$ shows scaling behavior (Nt=2,4 and 6)
  - Imaginary part goes to 0 for $T > 400$ MeV
- Future work
  - Extend calculation using Improved Action and bigger lattice size
  - Include radiation diagram contributions
  - Extend to unquenched plasma (QGP)

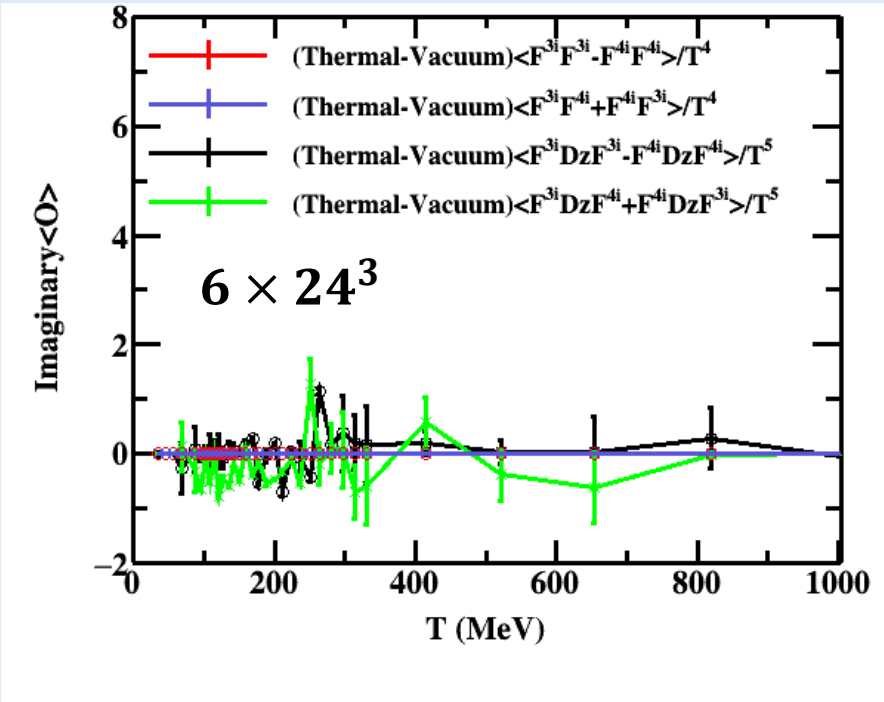
Thanks to group members and colleagues

- ❖Abhijit Majumder
- ❖Chiho Nonaka (Nagoya University)
- ❖ShanShan Cao, Yasuki Tachibana and Chathuranga Sirimana

Imaginary part of FF correlator in quenched SU(3)







- **Imaginary part of FF correlator does not contribute**