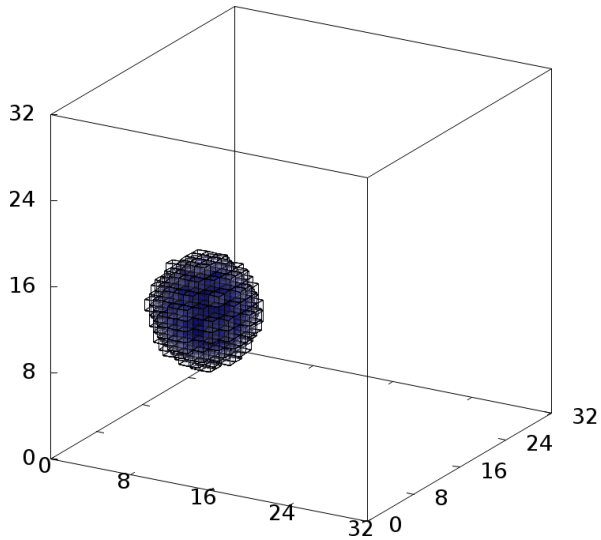
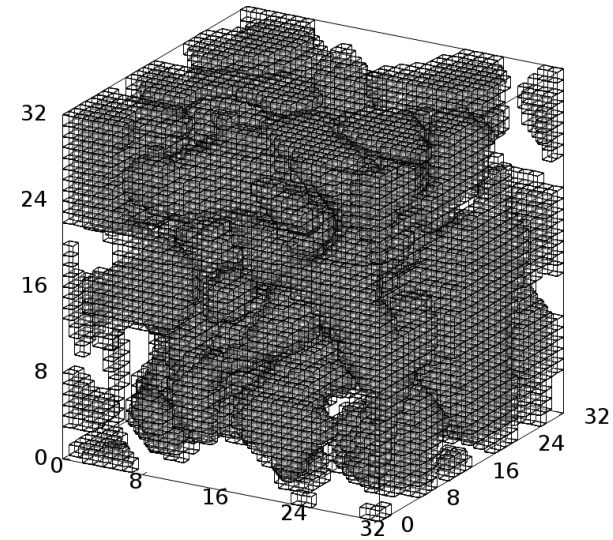
The QCD Anderson transition

with N_f=2+1+1 twisted mass Wilson quarks



The 36th Annual Symposium
on Lattice Field Theory

Lukas Holicki
Lorenz von Smekal
Ernst-Michael Ilgenfritz

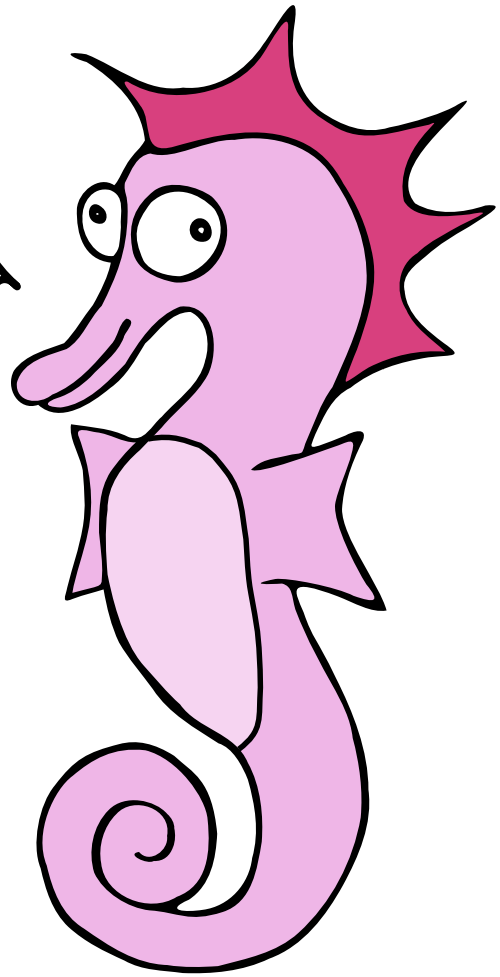


JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

LATTICE 18
East Lansing, MI, USA

Outline

- Introduction
- The temperature dependence of the mobility edge
- Localization and Polyakov loops
- Localization and Topology



See also talks by

Réka Á. Vig (Tuesday, 16:30) → The mobility edge vanishes at Tc (quenched)

Tamás G. Kovács (Tuesday, 16:50) → Non-interacting instanton-antiinstanton gas **does not** explain localization!

Anderson transitions

Seminal paper:
P. W. Anderson
Phys. Rev. 109, 1492 (1958)

Localization in ultracold atomic waves
Phys. Rev. Lett. 75, 4598 (1995)

Photon localization in a sample of titanium dioxide particles dispersed in polystyrene
J. Phys. A 38 10465 (2005)

Soundwave localization in elastic networks
Nature Physics 4, 945 - 948 (2008)

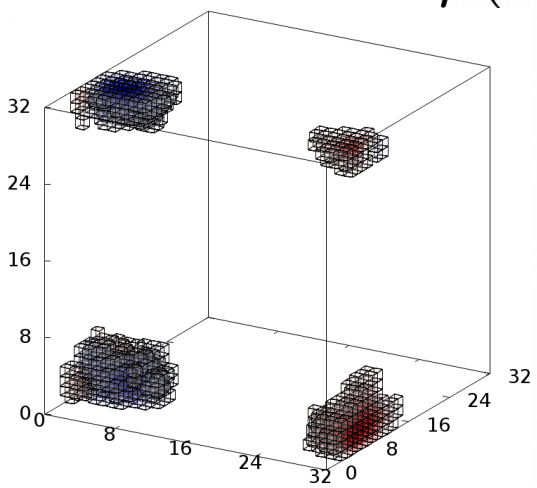
Localized Dirac eigenmodes in Two-Colour QCD
Phys.Rev.Lett. 104:031601 (2010)

⋮

Anderson transition	**QCD Anderson transition**
Hamiltonian H	Dirac operator D
Metal-insulator transition	Distinct spectral regimes
Vanishing zero T conductivity	Localization of low quark modes at $T > T_c$
Disorder / Impurities	**?**

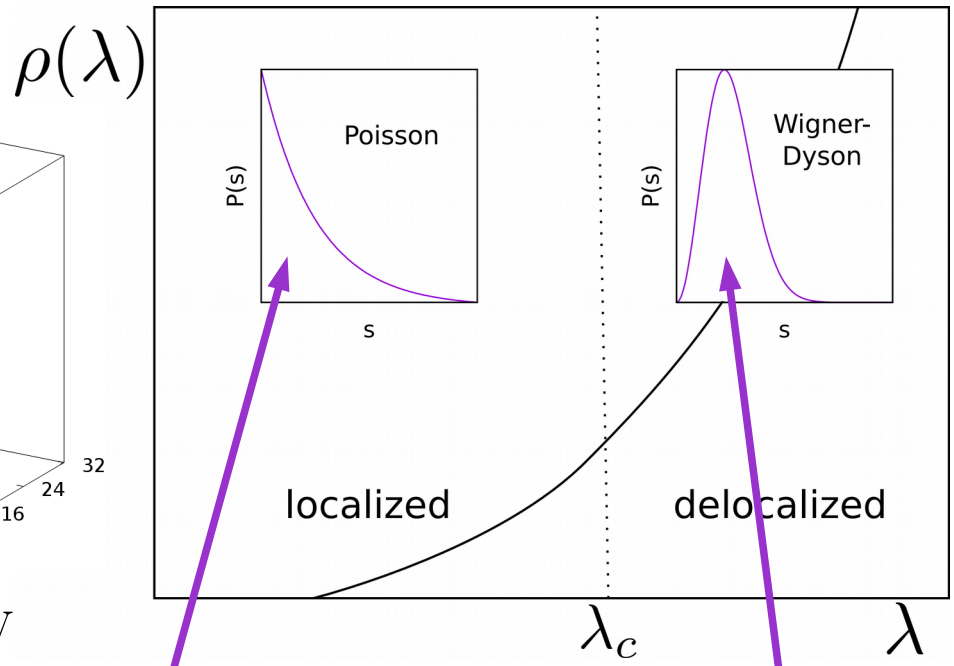
Anderson localization in QCD matter

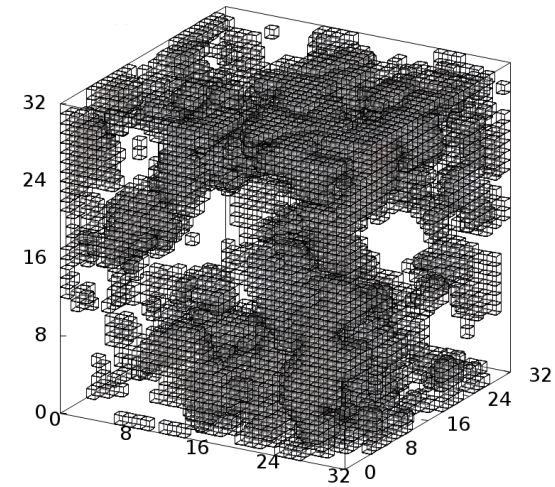
Quark localization in QCD



$\lambda = 1345.72$ MeV

$T > T_c$





$\lambda = 1593.65$ MeV

Uncorrelated level spacings

Random Matrix Theory

Level spacing statistics:

$$s_j = \frac{\lambda_{j+1} - \lambda_j}{\langle \lambda_{j+1} - \lambda_j \rangle}$$

$$\beta_D = 1 \leftrightarrow \text{ unitary}$$
$$\beta_D = 2 \leftrightarrow \text{ orthogonal}$$
$$\beta_D = 4 \leftrightarrow \text{ symplectic}$$

Lattice setup

Sea: $N_f = 2 + 1 + 1$ twisted mass fermions

Valence: Overlap fermions

Gauge: Iwasaki

Configurations were created by the
Twisted Mass at Finite Temperature Collaboration:

Burger, Hotzel, Müller-Preussker, Ilgenfritz, Lombardo
PoS Lattice2013 (2013) 153

Burger, Ilgenfritz, Lombardo, Müller-Preussker, Trunin
J.Phys.Conf.Ser. 668 (2016) no.1, 012092

Burger, Ilgenfritz, Lombardo, Müller-Preussker, Trunin
Nucl.Phys. A967 (2017) 880-883

A370 **512** modes / configuration

$N_s = 24 \qquad a = 0.0936 \text{ fm}$

$m_\pi = 364 \text{ MeV}$

$T = 210 \ldots 527 \text{ MeV} \approx 1.1 \ldots 2.9 T_c$

D370 **300** modes / configuration

$N_s = 32 \qquad a = 0.0646 \text{ fm}$

$m_\pi = 369 \text{ MeV}$

$T = 218 \ldots 1018 \text{ MeV} \approx 1.2 \ldots 5.5 T_c$

Fixed scale

~100 conf. / temperature

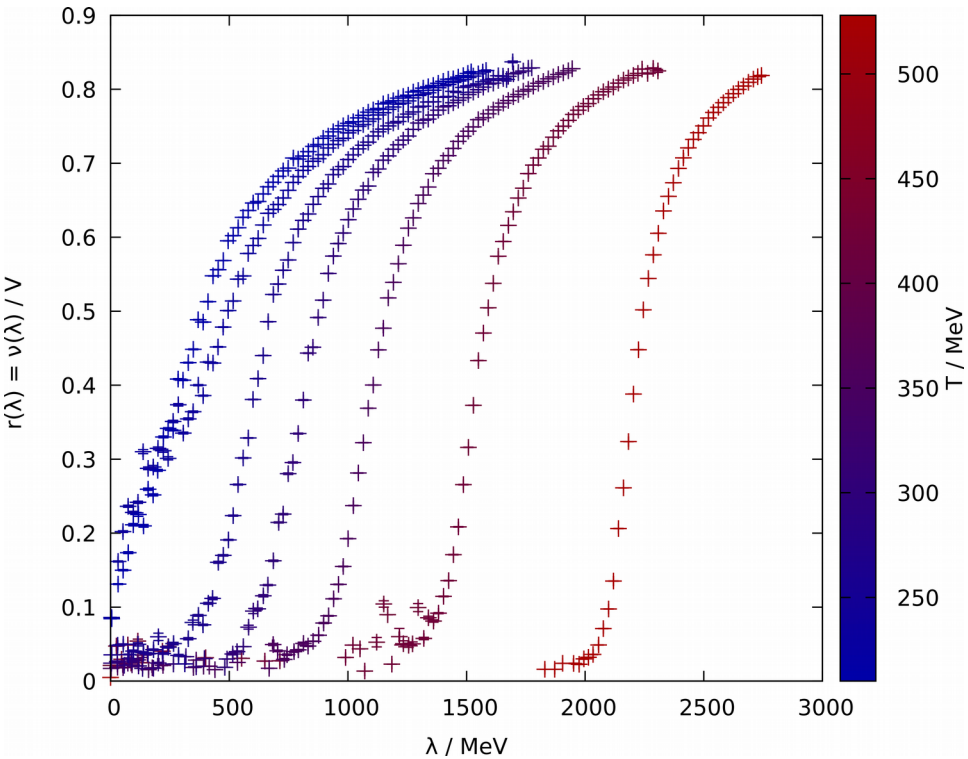
The mobility edge

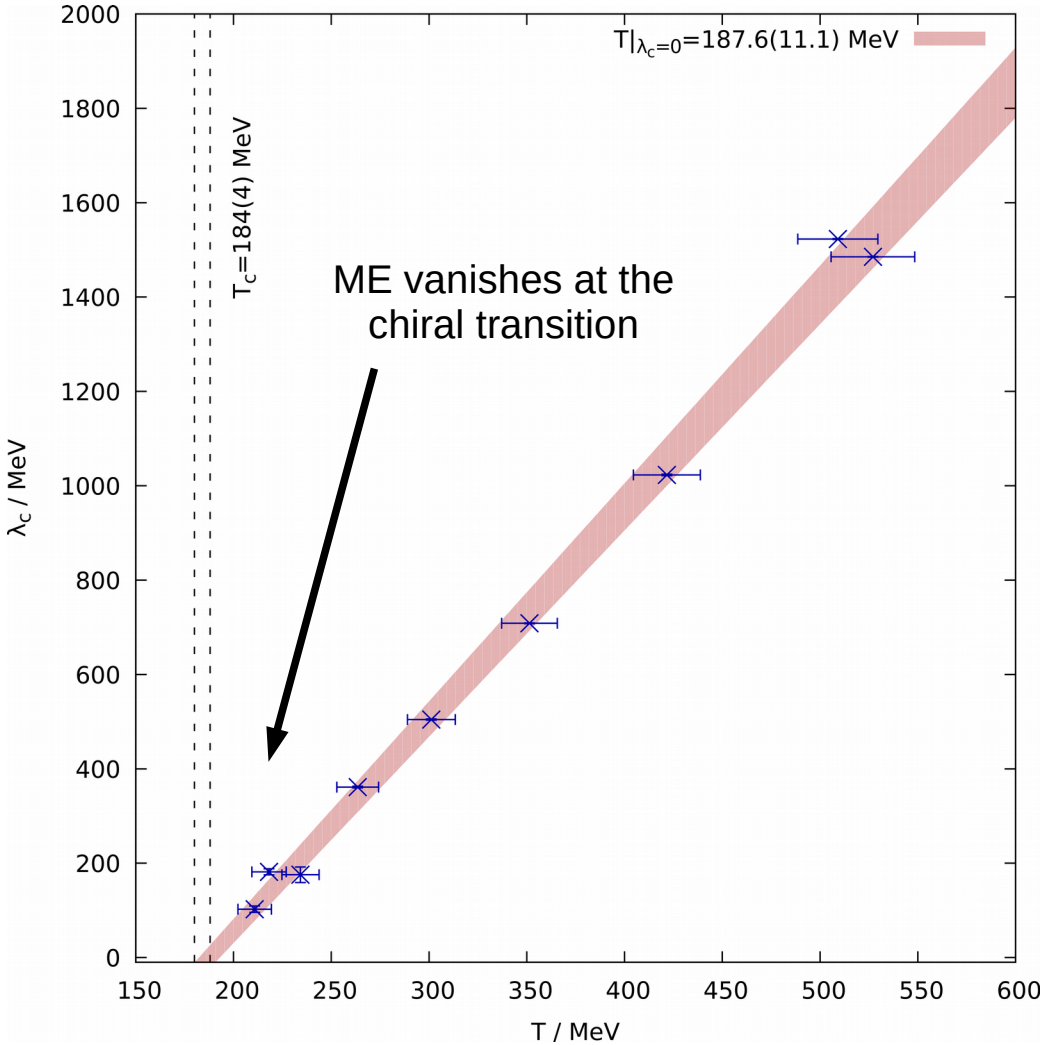
Inverse Participation Ratio (IPR):

$$\nu^{-1} = \int d^4x \langle \psi(x) | \psi(x) \rangle^2$$

Relative eigenvector volume:

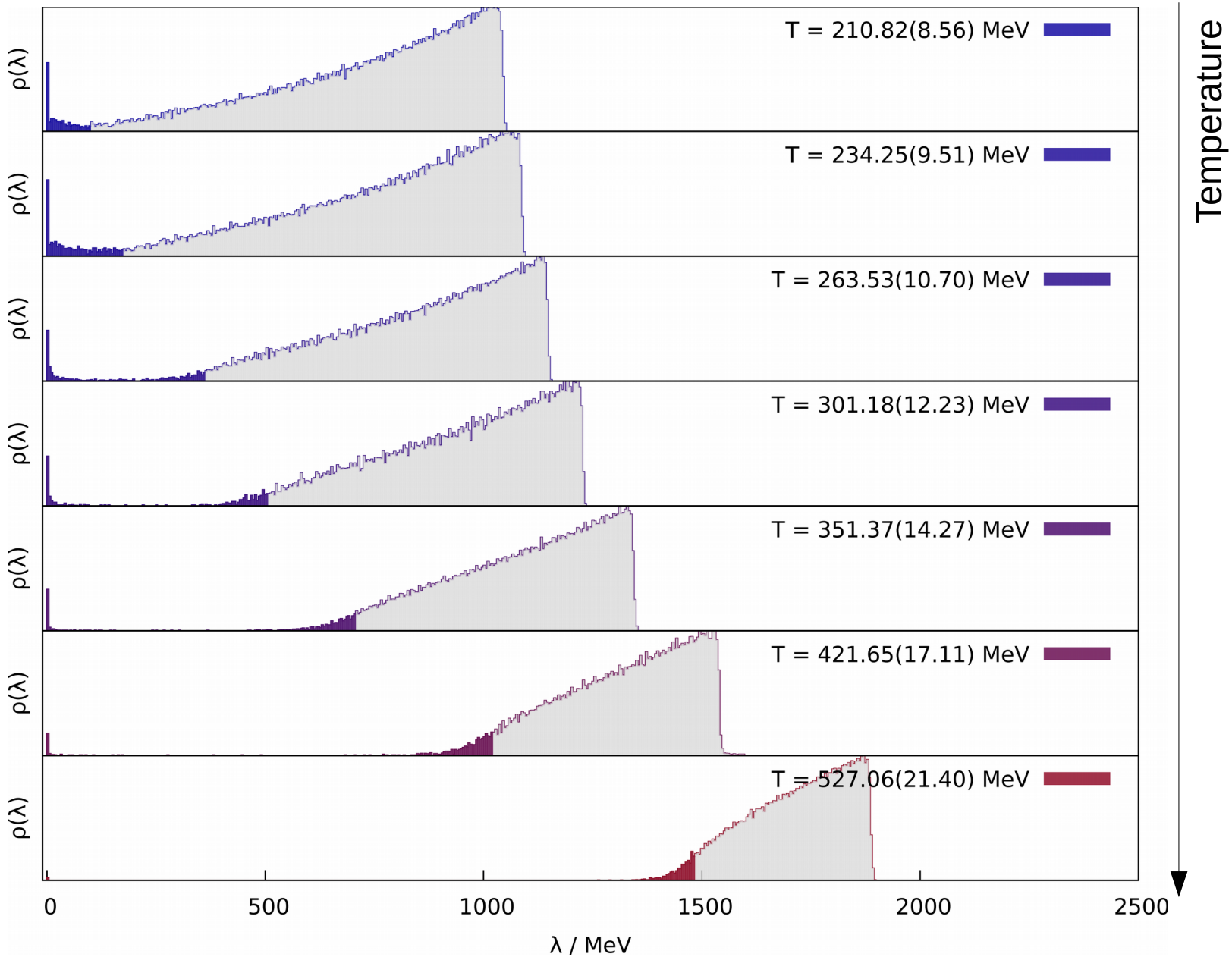
$$r(\lambda) = \frac{\nu(\lambda)}{V} \in (0, 1]$$





$$\lambda_c(T) \propto T$$

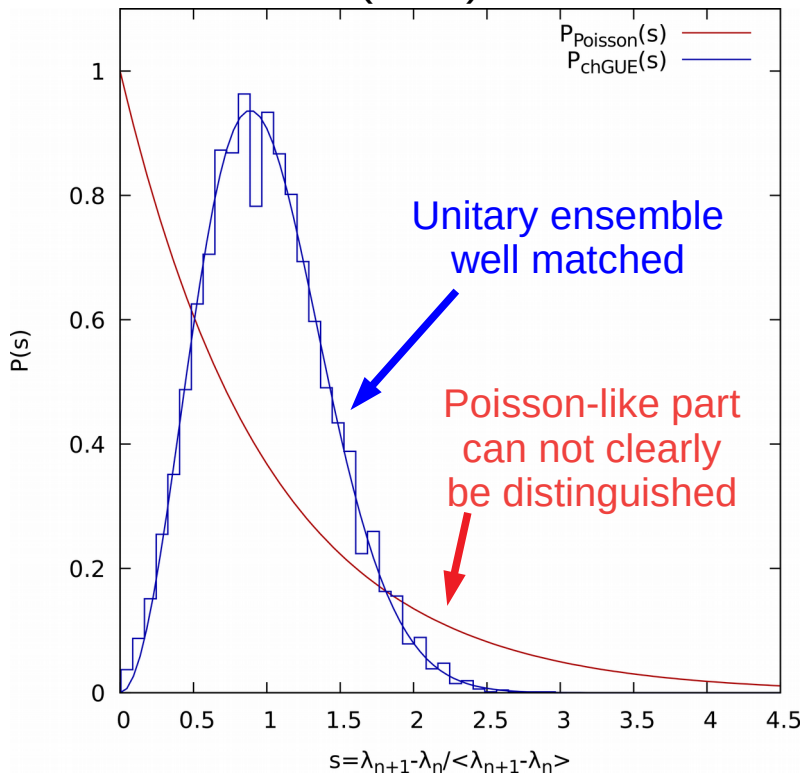
Banks-Casher gap $\langle\overline{\psi}\psi\rangle = -\pi \lim_{\lambda\mapsto 0} \lim_{m\mapsto 0} \lim_{V\mapsto\infty} \rho(\lambda)$



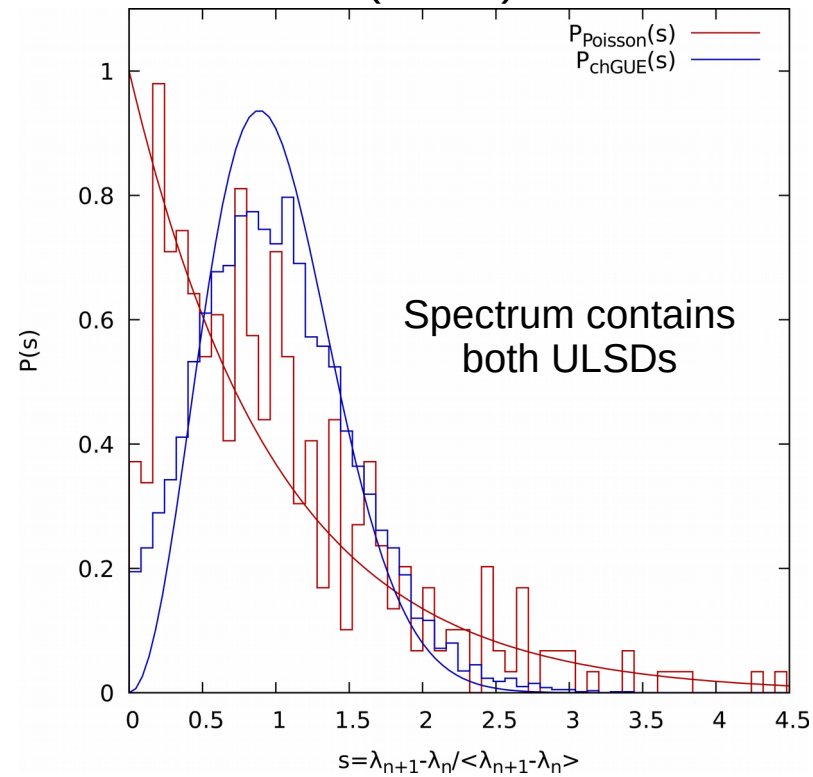
512
eigenmodes / configuration

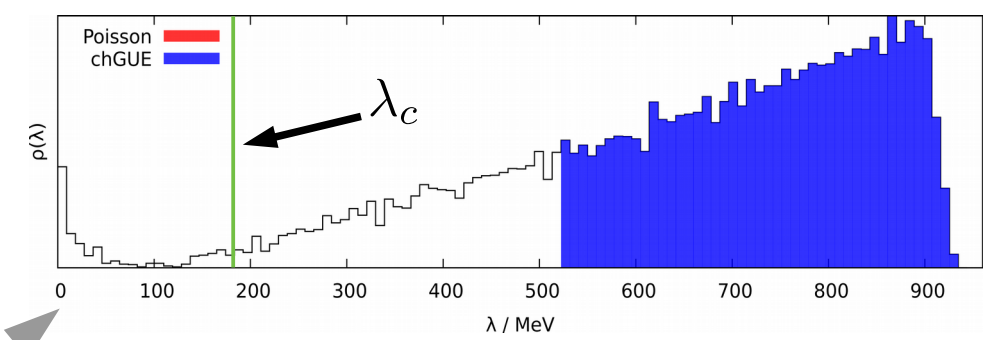
Unfolded level spacing distribution

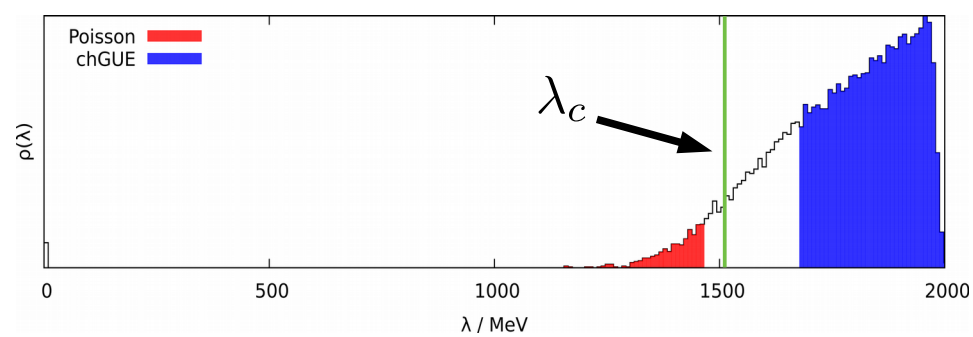
T = 218.19(8.79) MeV



T = 509.11(20.49) MeV



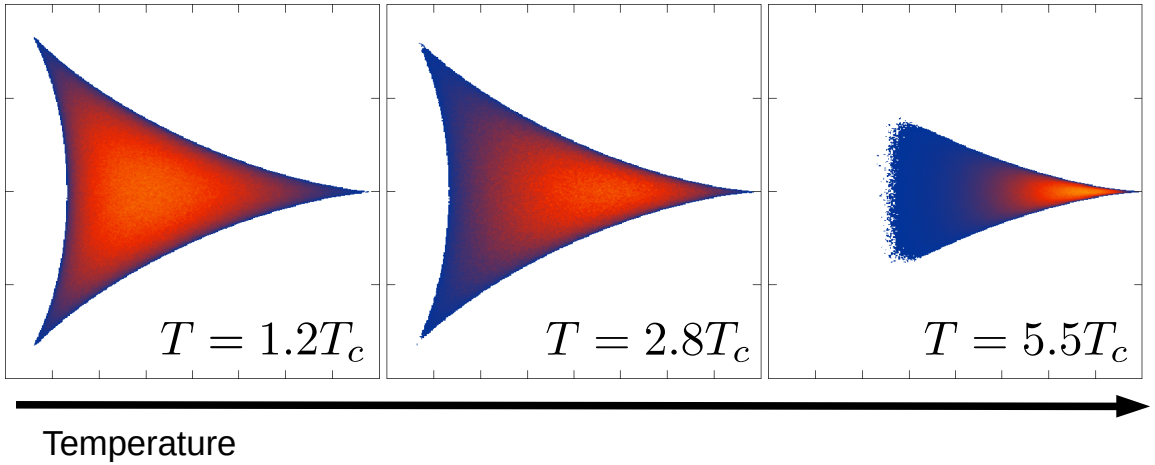




300
eigenmodes / configuration

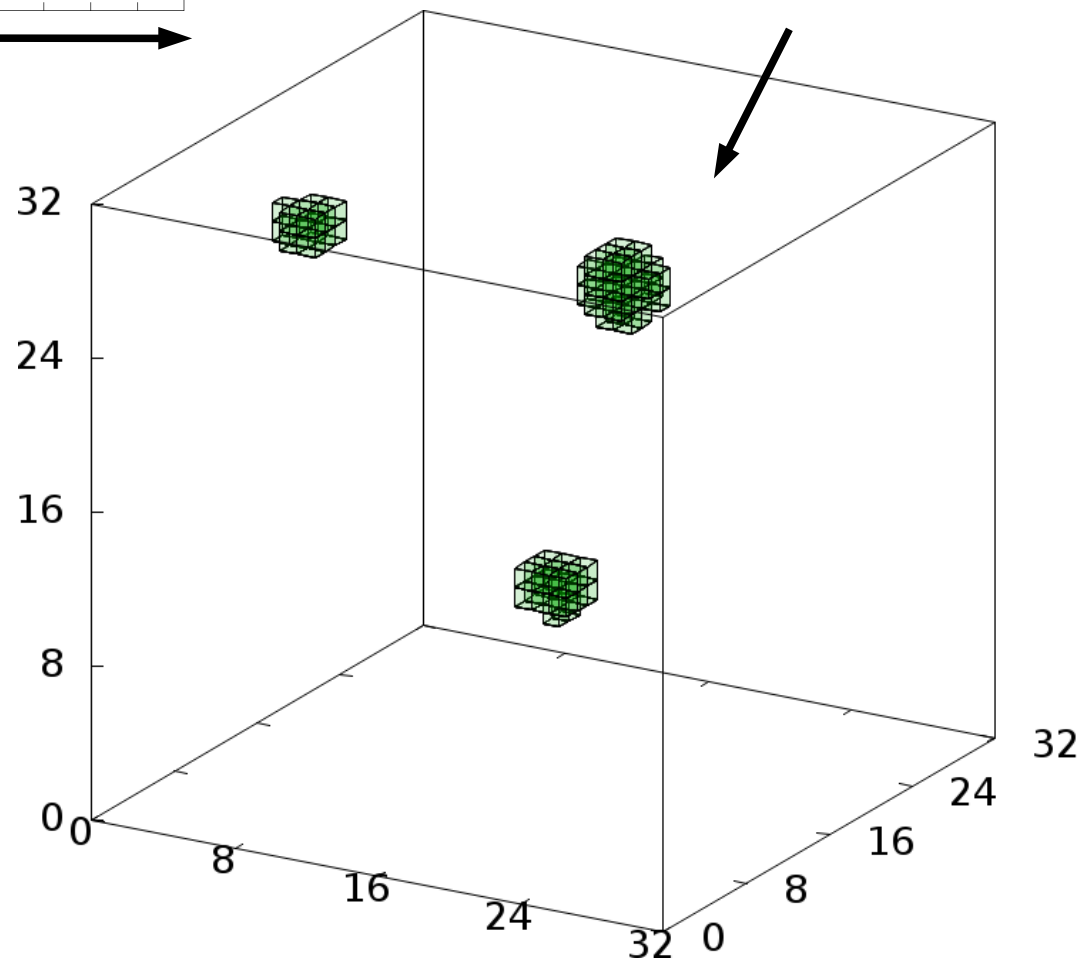
$$P_{\text{Poisson}}(s) = e^{-s} \mapsto P_{\text{chGUE}}(s) = \frac{32}{\pi^2} s^2 \exp\left(-\frac{4}{\pi} s^2\right)$$

Localization and Polyakov loops



$$l(x) = \frac{1}{N_c}\mathrm{tr}\left(e^{\int dt A_4(x)}\right)$$

Regions of small Re l(x) are clustered!



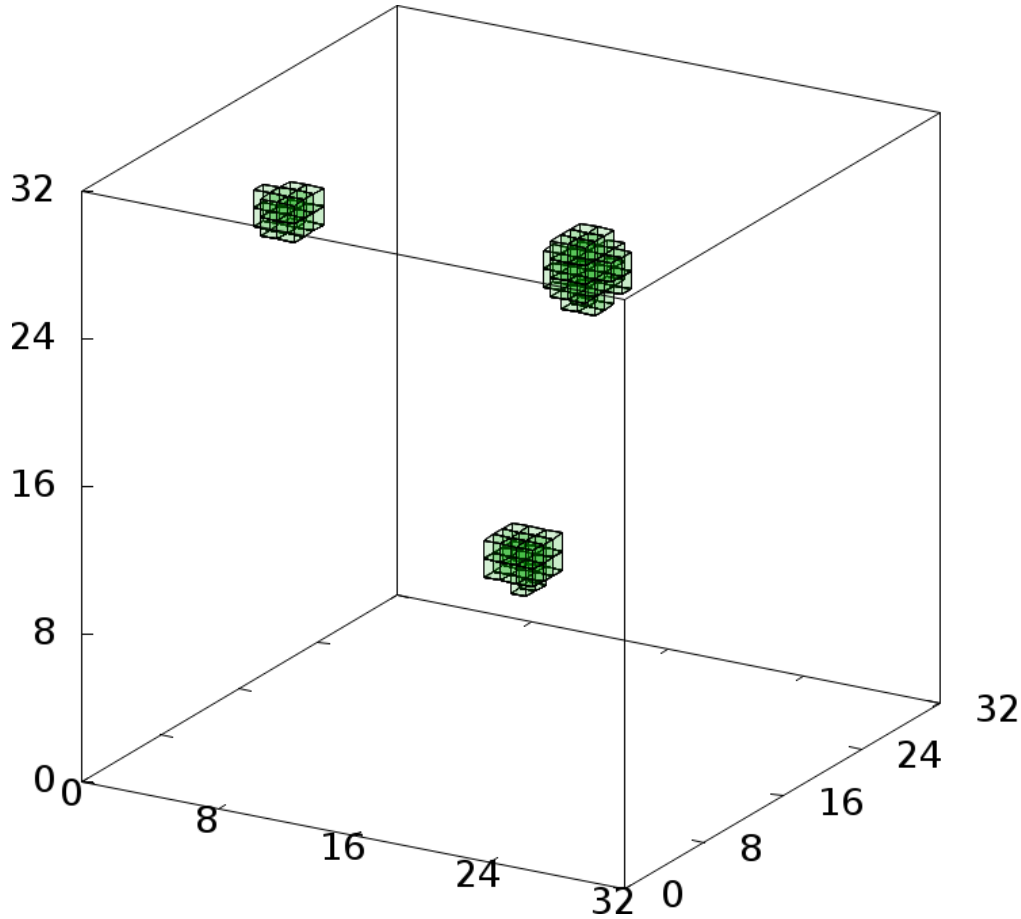
See also:

Bruckmann, Kovacs, Schierenberg
Phys.Rev. D84 (2011) 034505

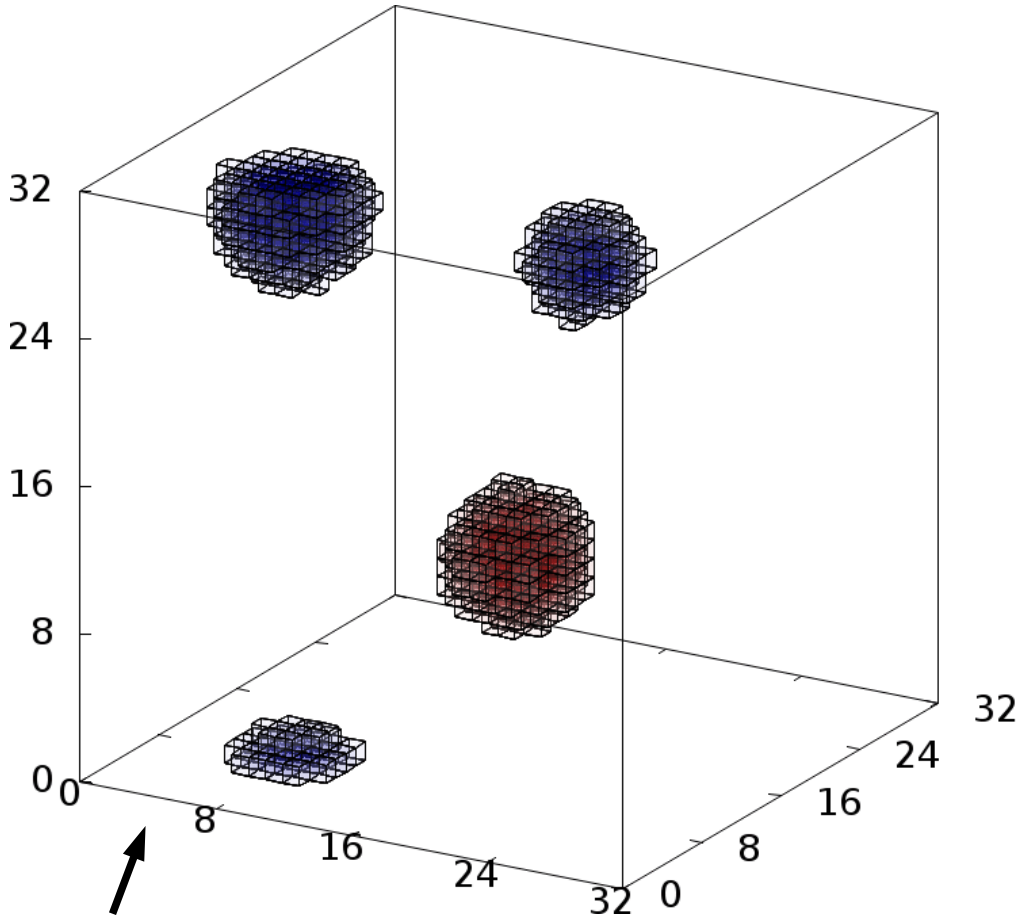
Giordano, Kovacs, Pittler
JHEP 1504 (2015) 112

Cossu, Hashimoto
JHEP 1606 (2016) 056

Localization and Polyakov loops



Clusters of small Re l(x)

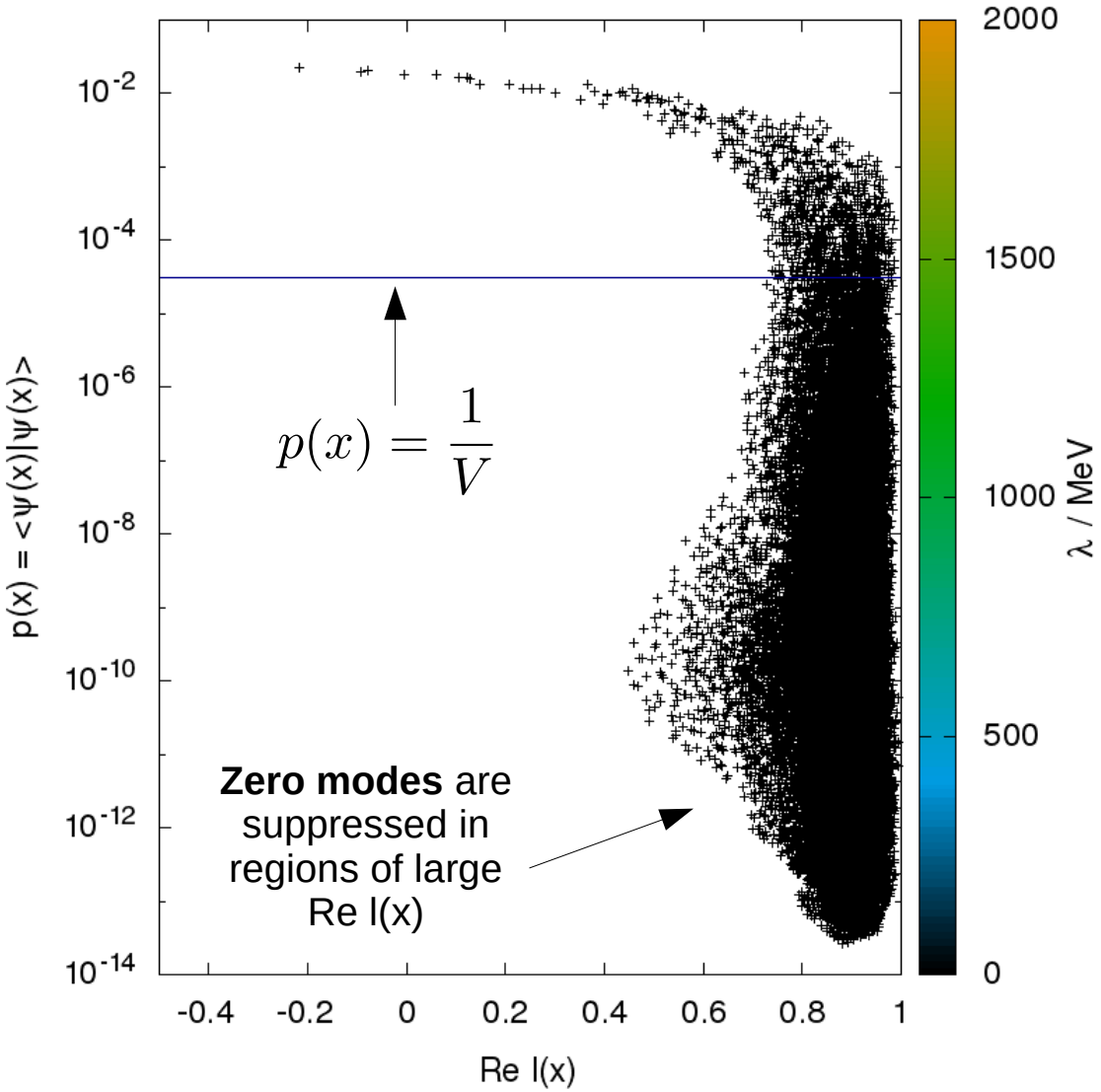


$\langle \psi(x) | \gamma_5 | \psi(x) \rangle$

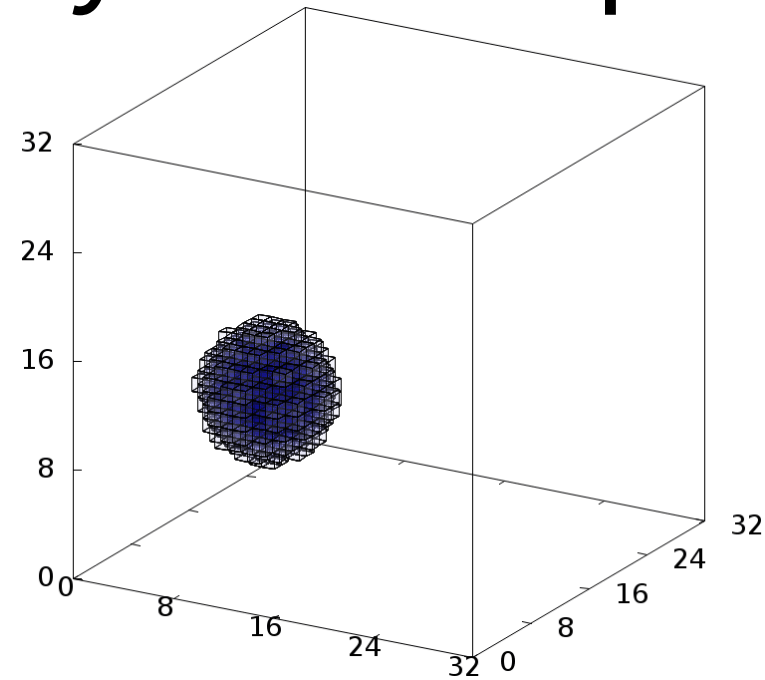
Zero modes in this configuration

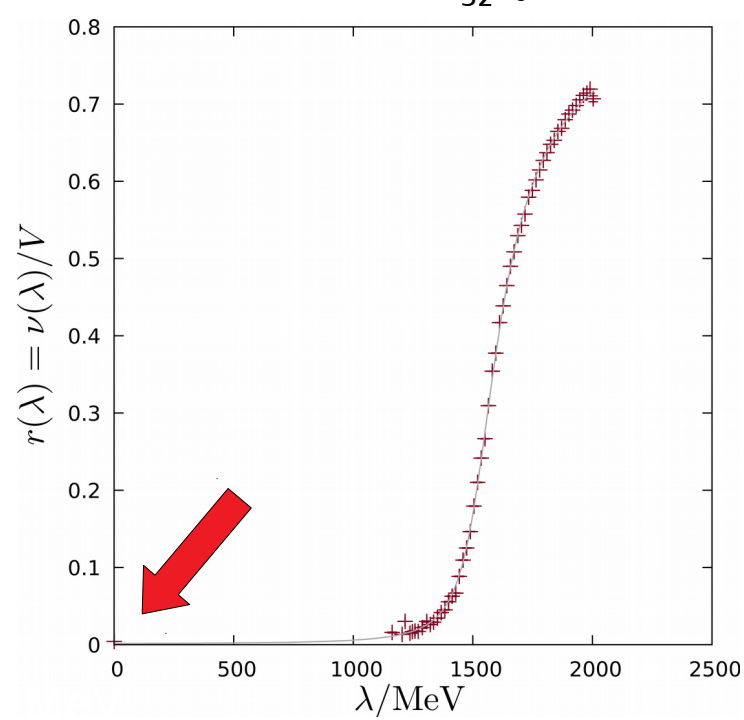
Localization and Polyakov loops

$\lambda = 0.0$ MeV



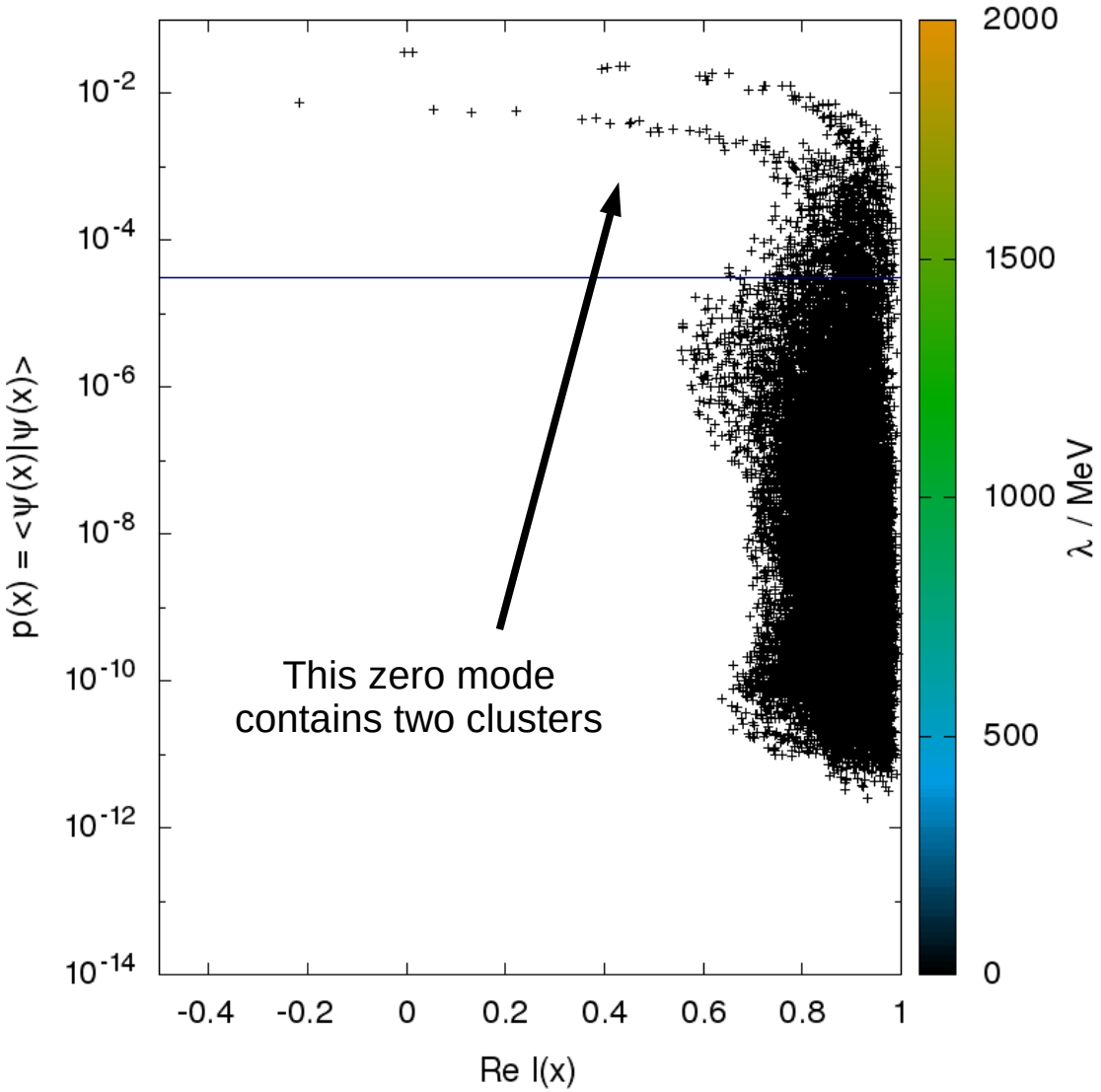
$T = 509.11(20.49)$ MeV



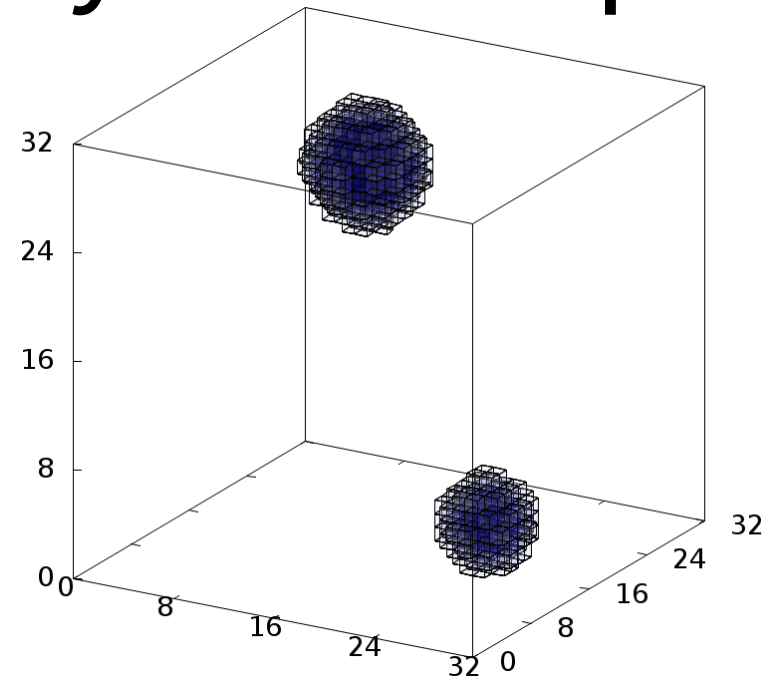


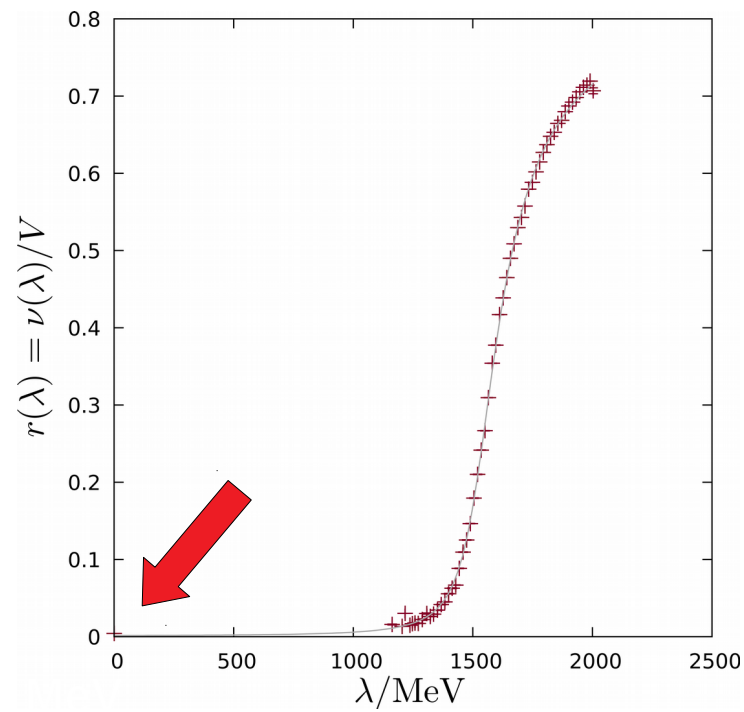
Localization and Polyakov loops

$\lambda = 0.0$ MeV



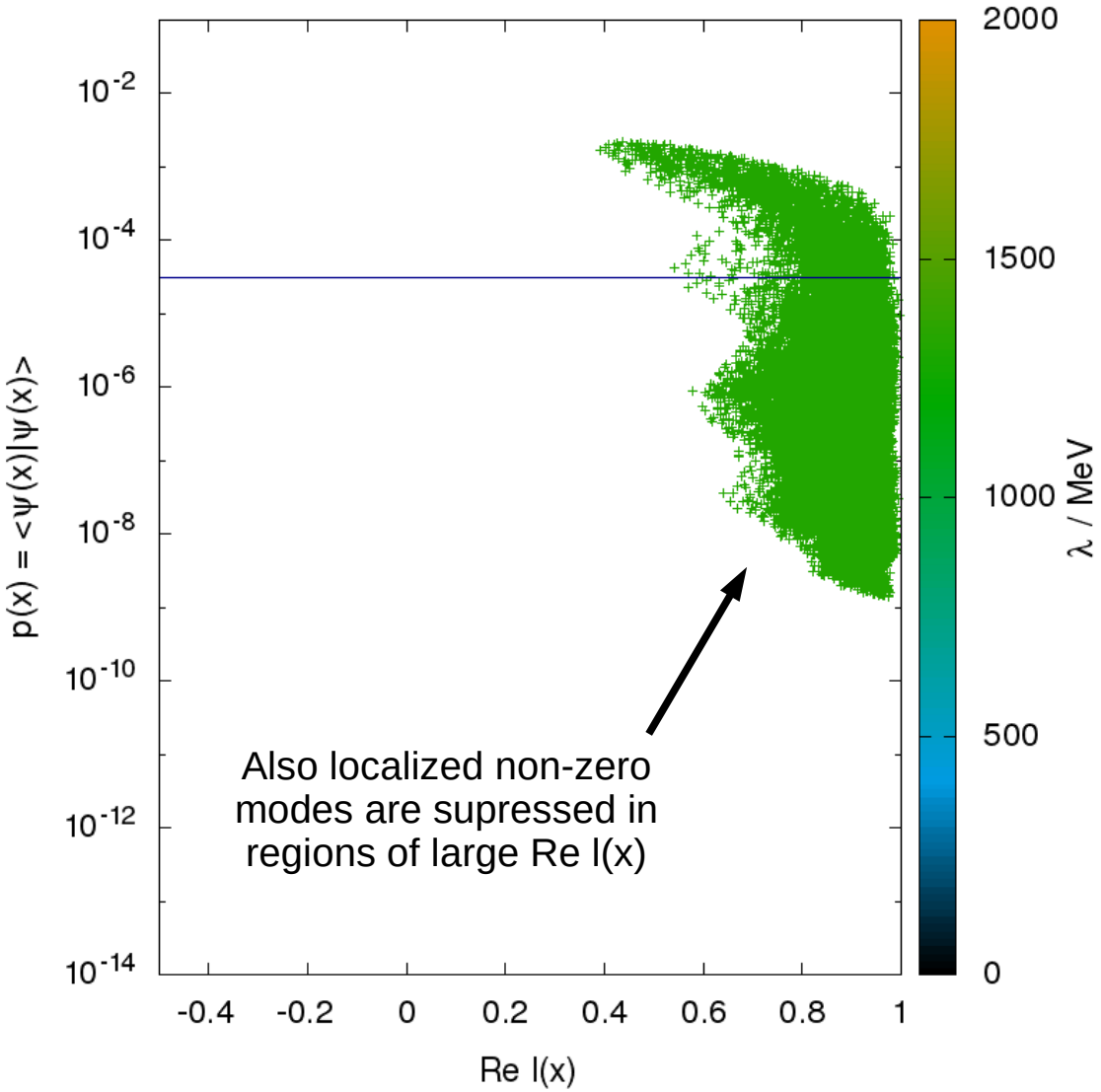
$T = 509.11(20.49)$ MeV



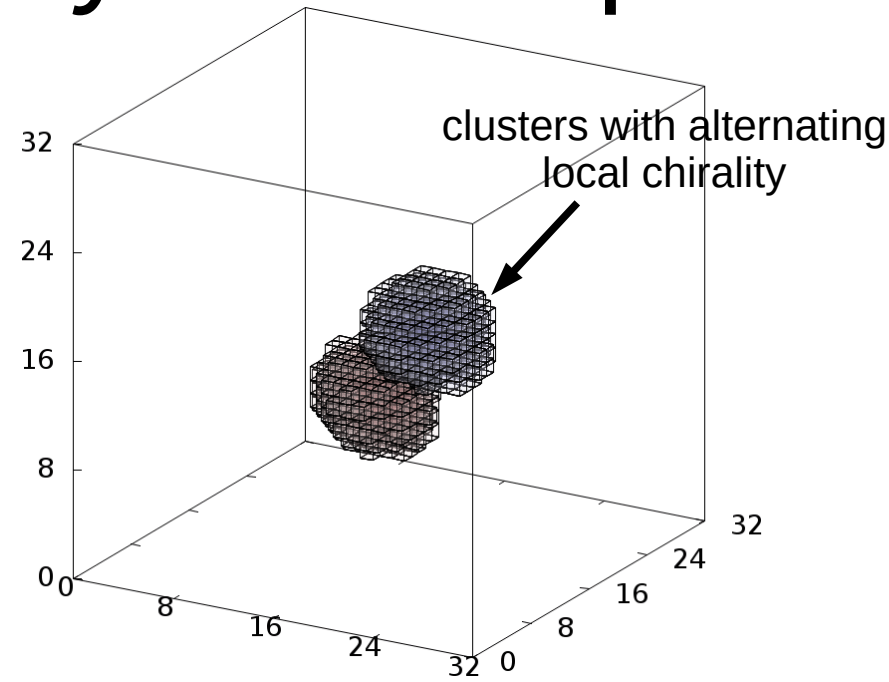


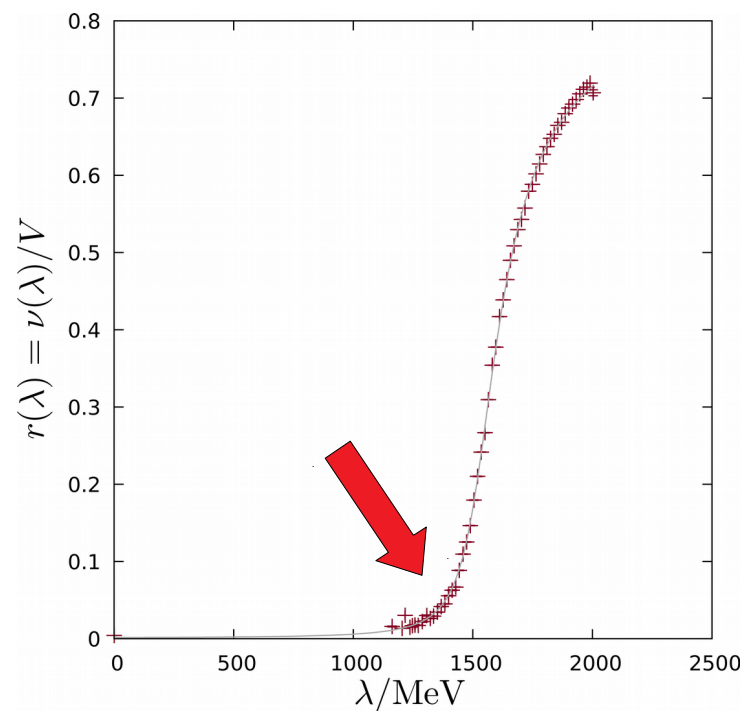
Localization and Polyakov loops

$\lambda = 1321.6 \text{ MeV} \approx 0.87\lambda_c$



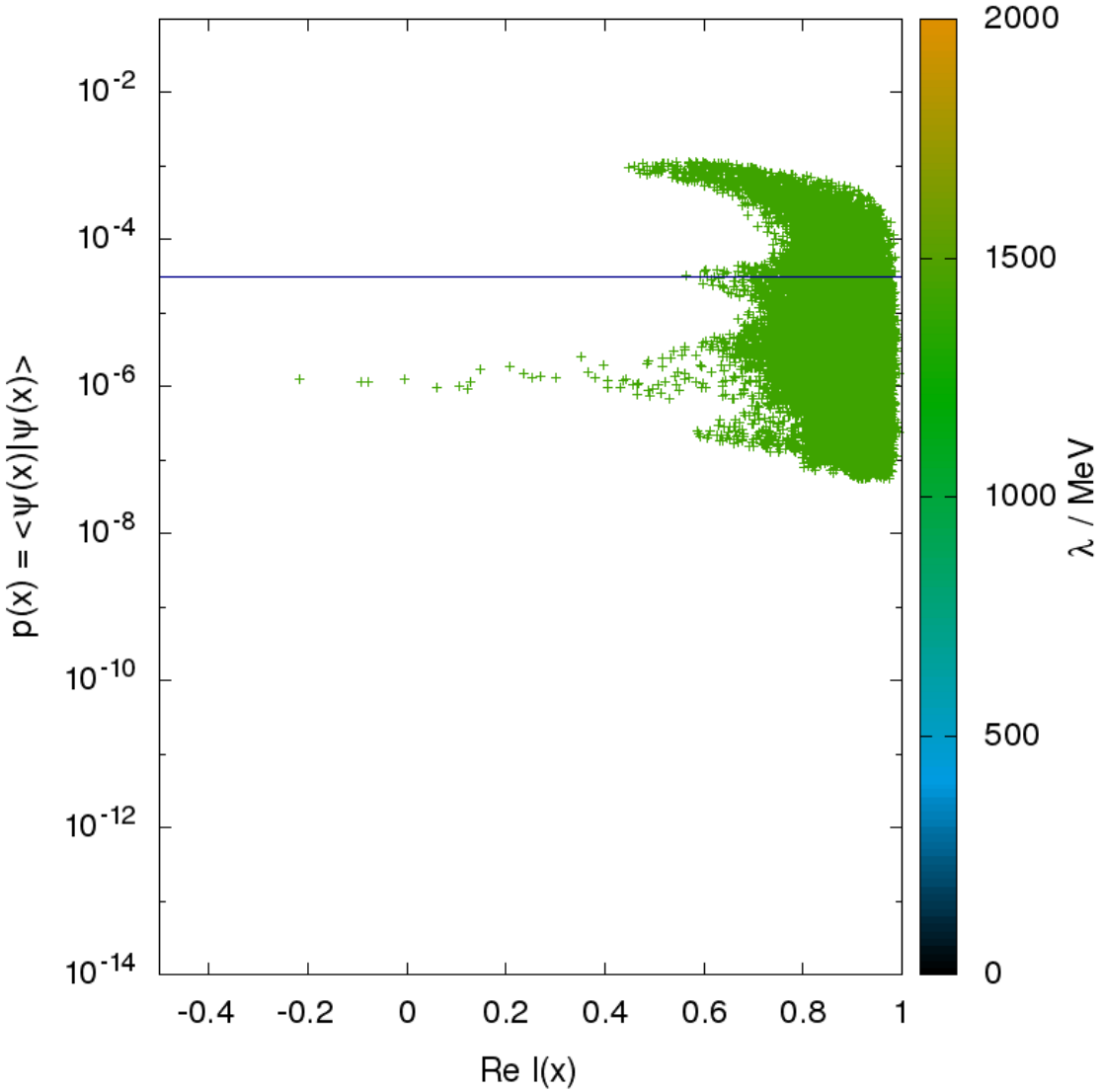
$T = 509.11(20.49) \text{ MeV}$

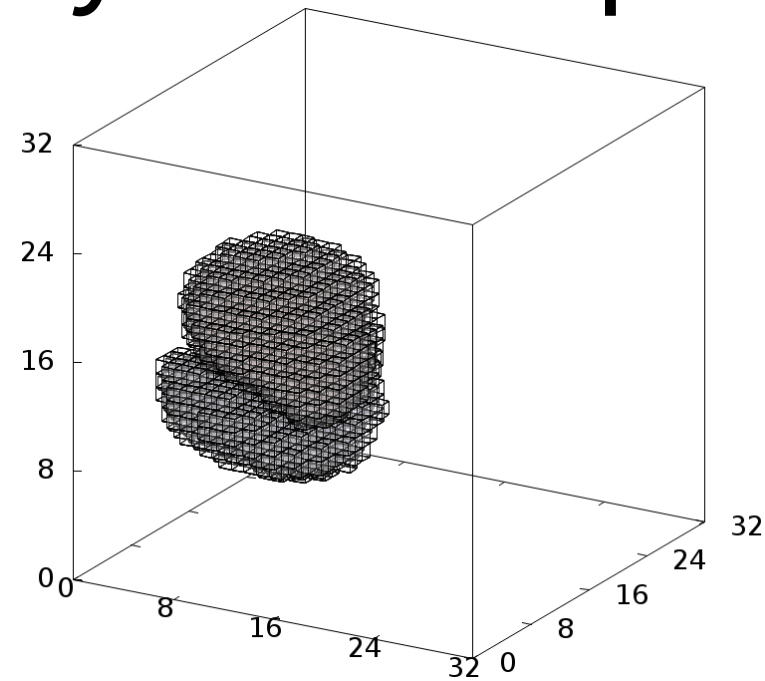


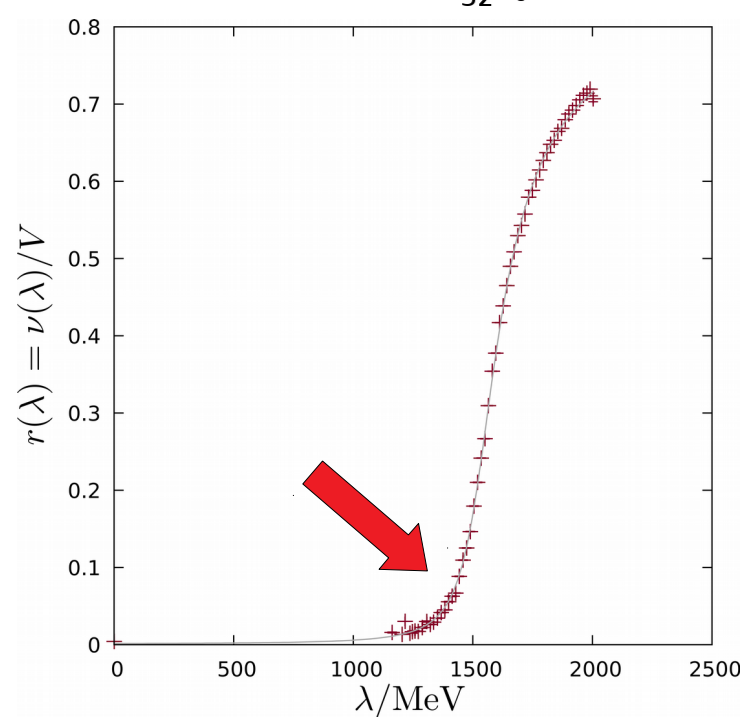


Localization and Polyakov loops

$\lambda = 1417.6\ \mathrm{MeV} \approx 0.93\lambda_c$



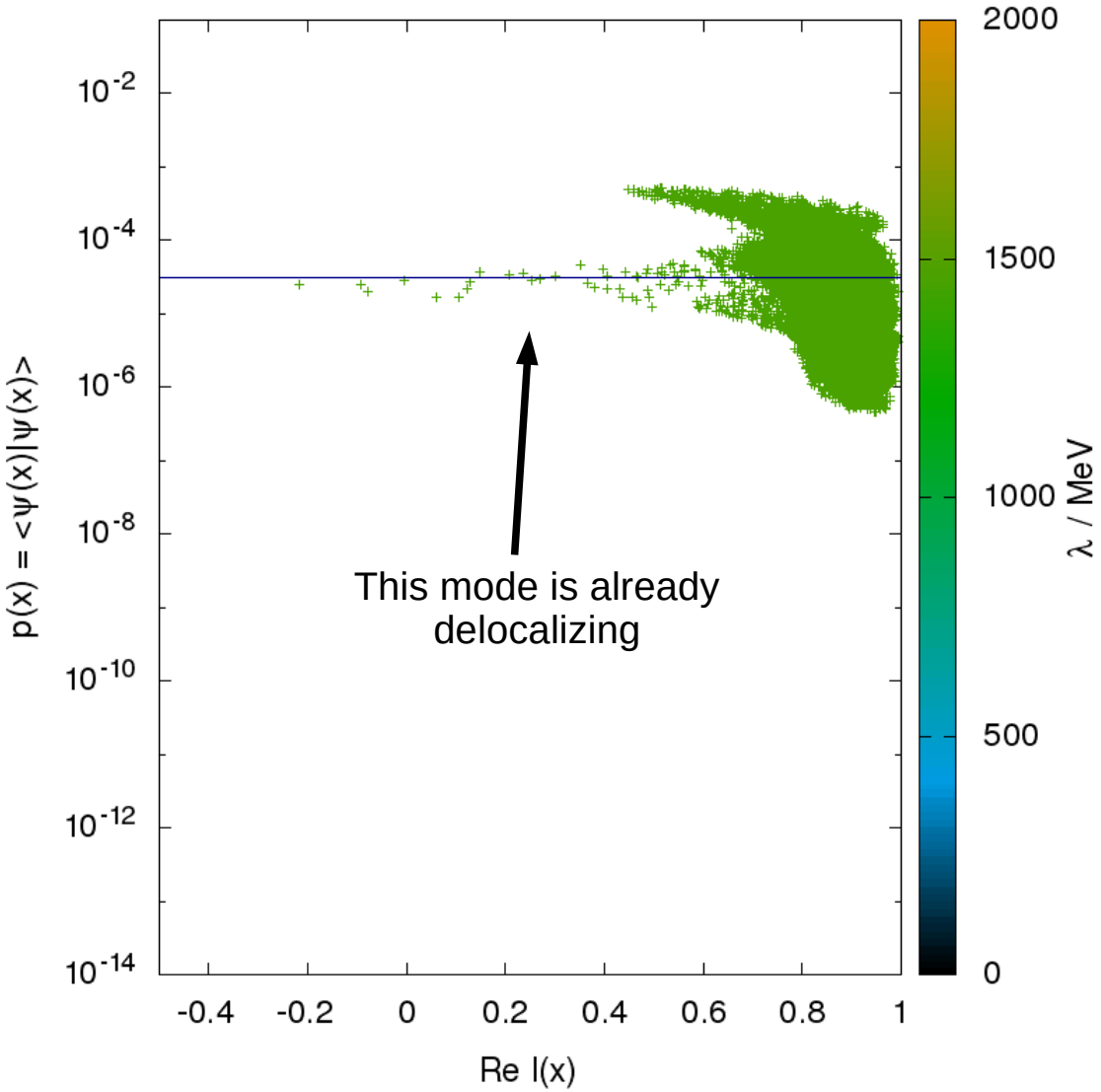


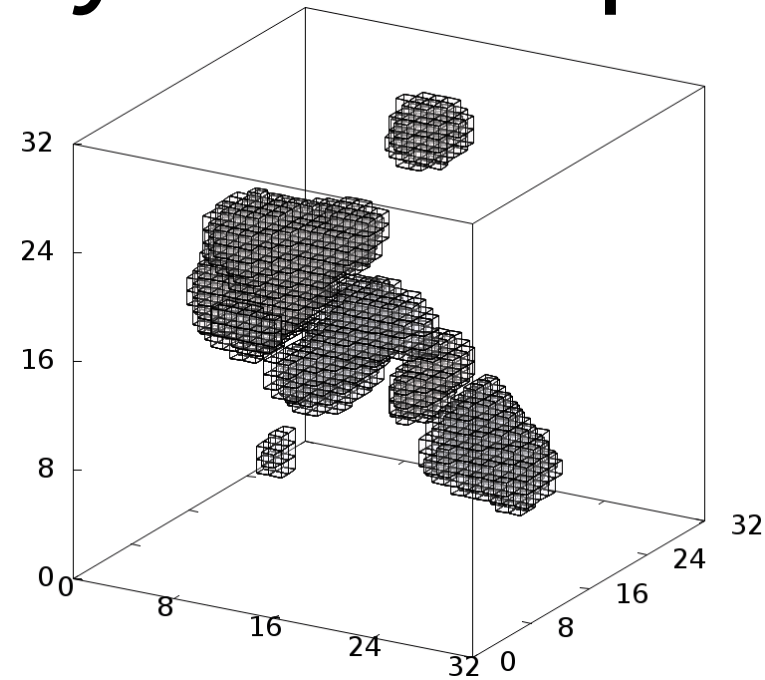


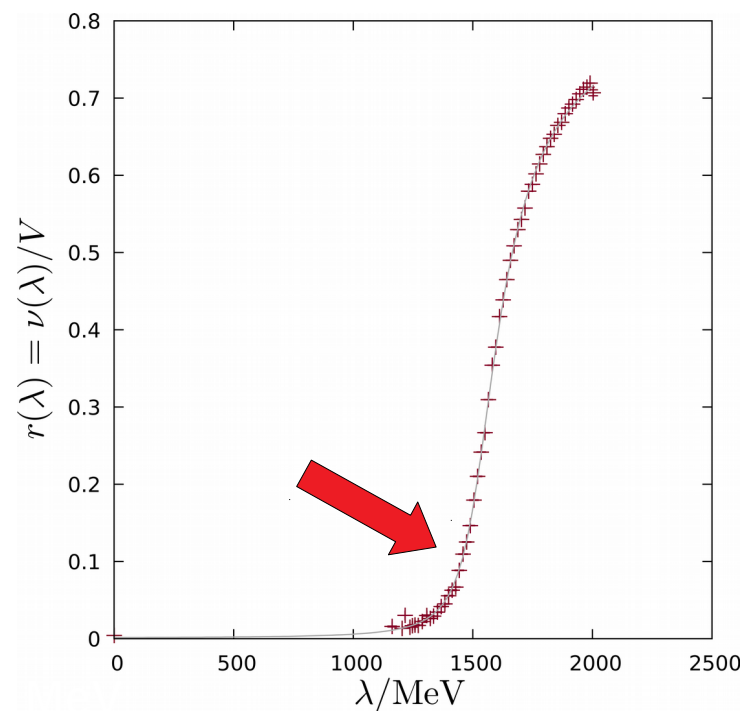
$T = 509.11(20.49)\ \mathrm{MeV}$

Localization and Polyakov loops

$\lambda = 1451.5\ \mathrm{MeV} \approx 0.95\lambda_c$



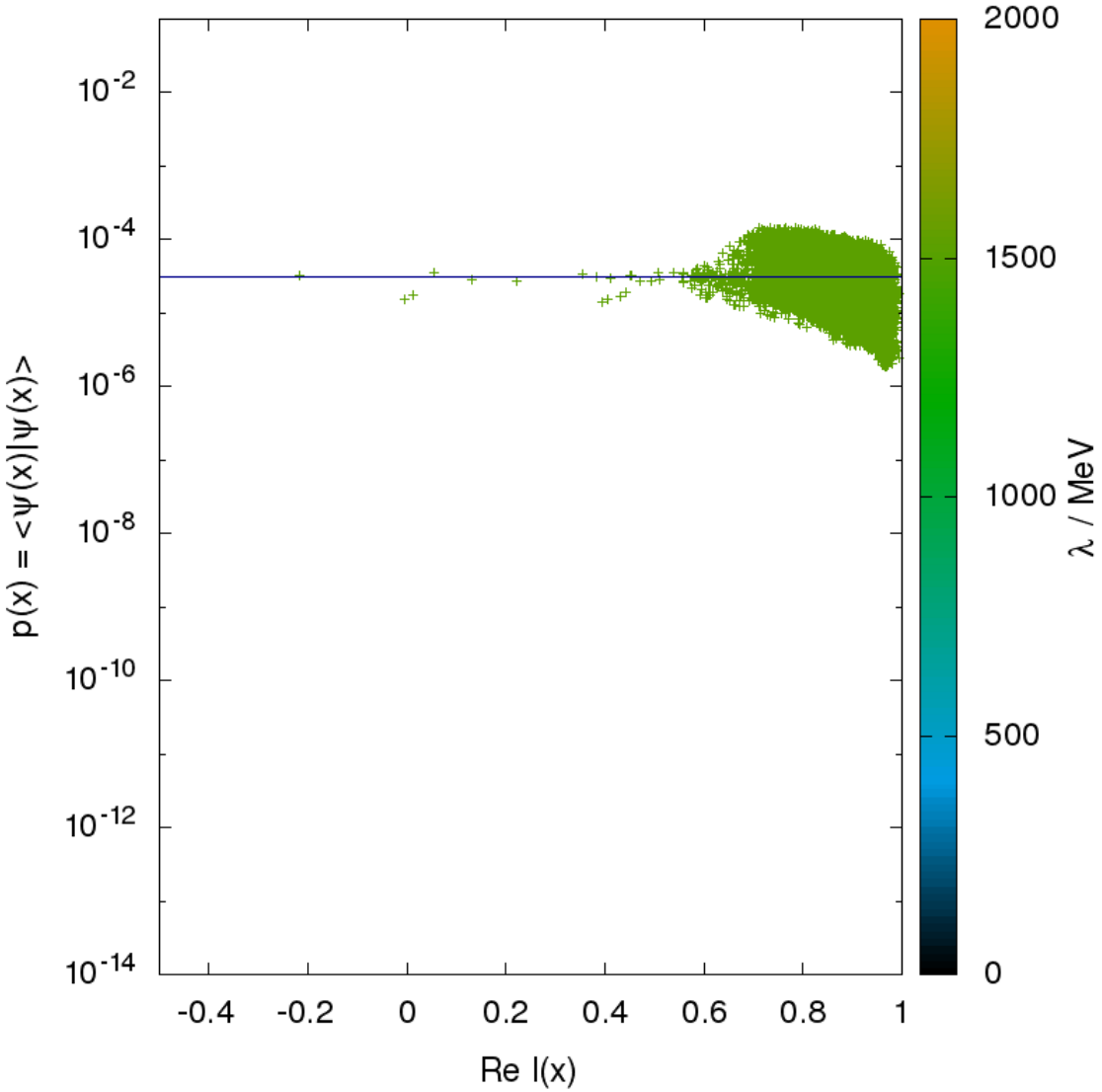


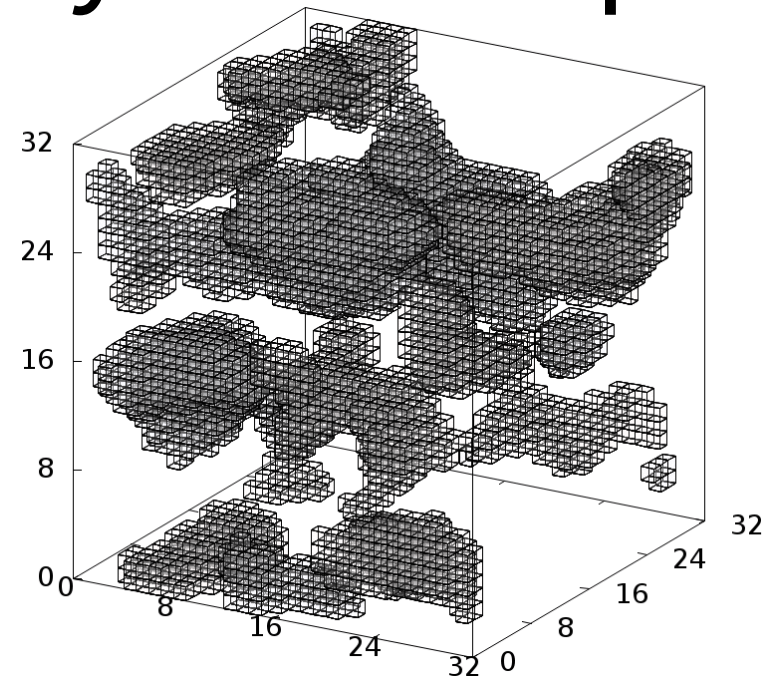


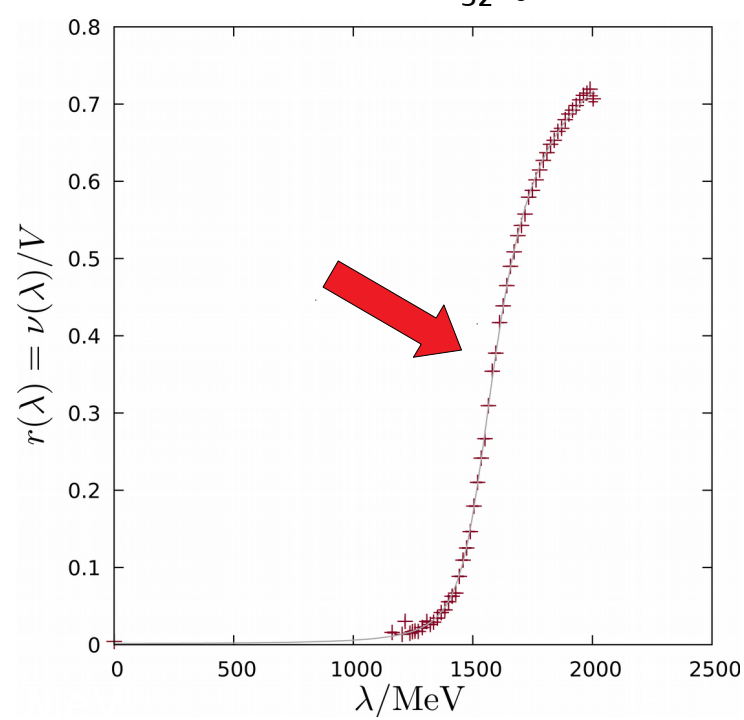
$T = 509.11(20.49)\ \mathrm{MeV}$

Localization and Polyakov loops

$\lambda = 1551.1 \text{ MeV} \approx 1.02\lambda_c$



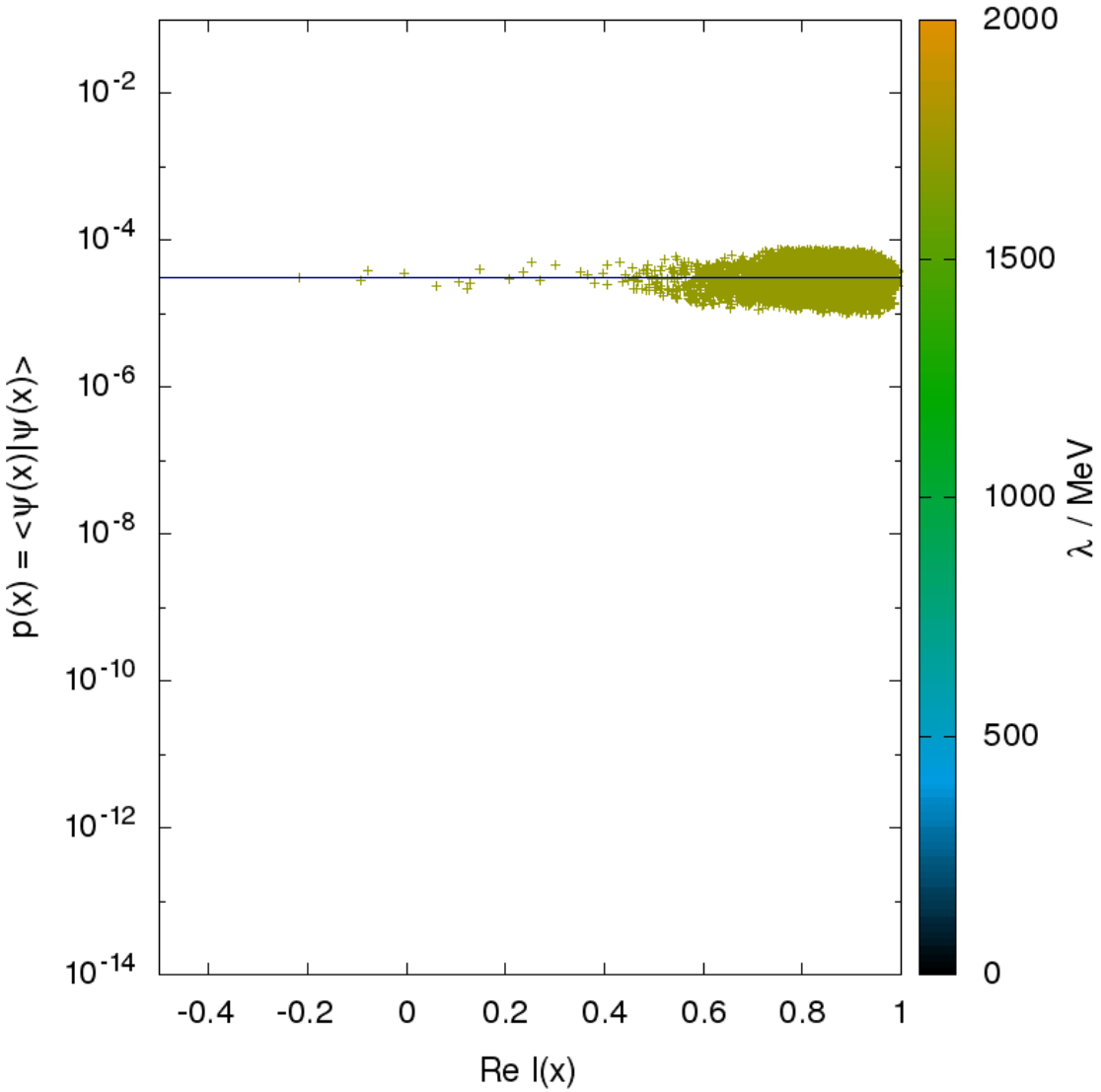


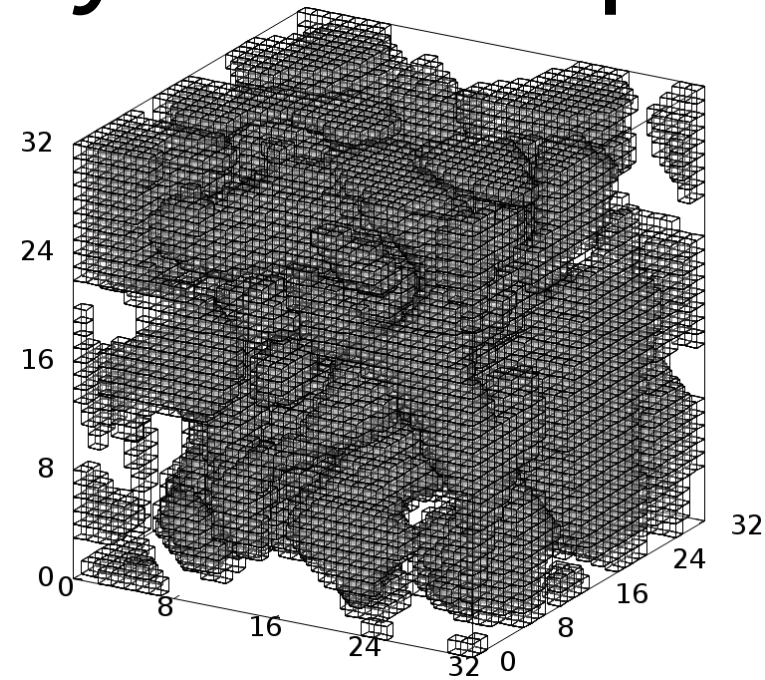


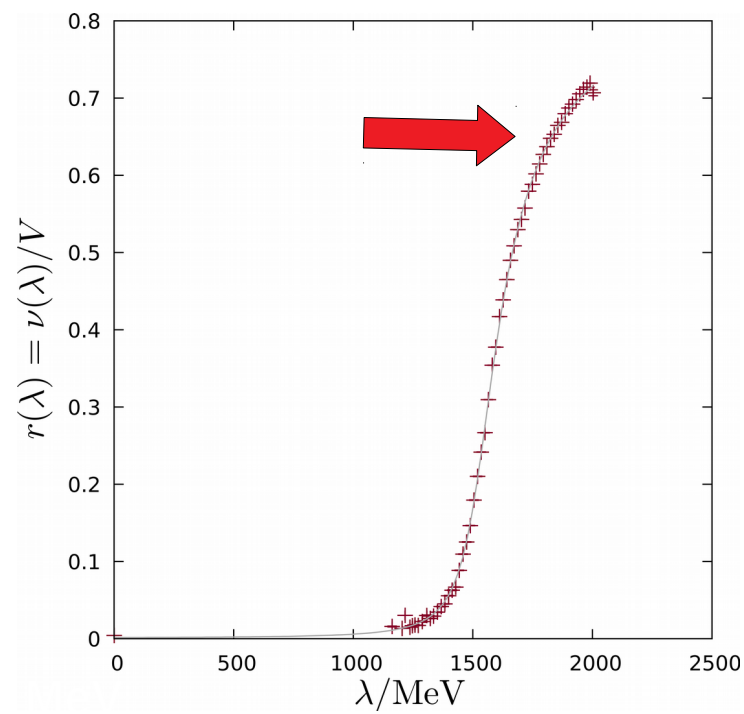
$T = 509.11(20.49) \text{ MeV}$

Localization and Polyakov loops

$\lambda = 1721.7 \text{ MeV} \approx 1.13\lambda_c$



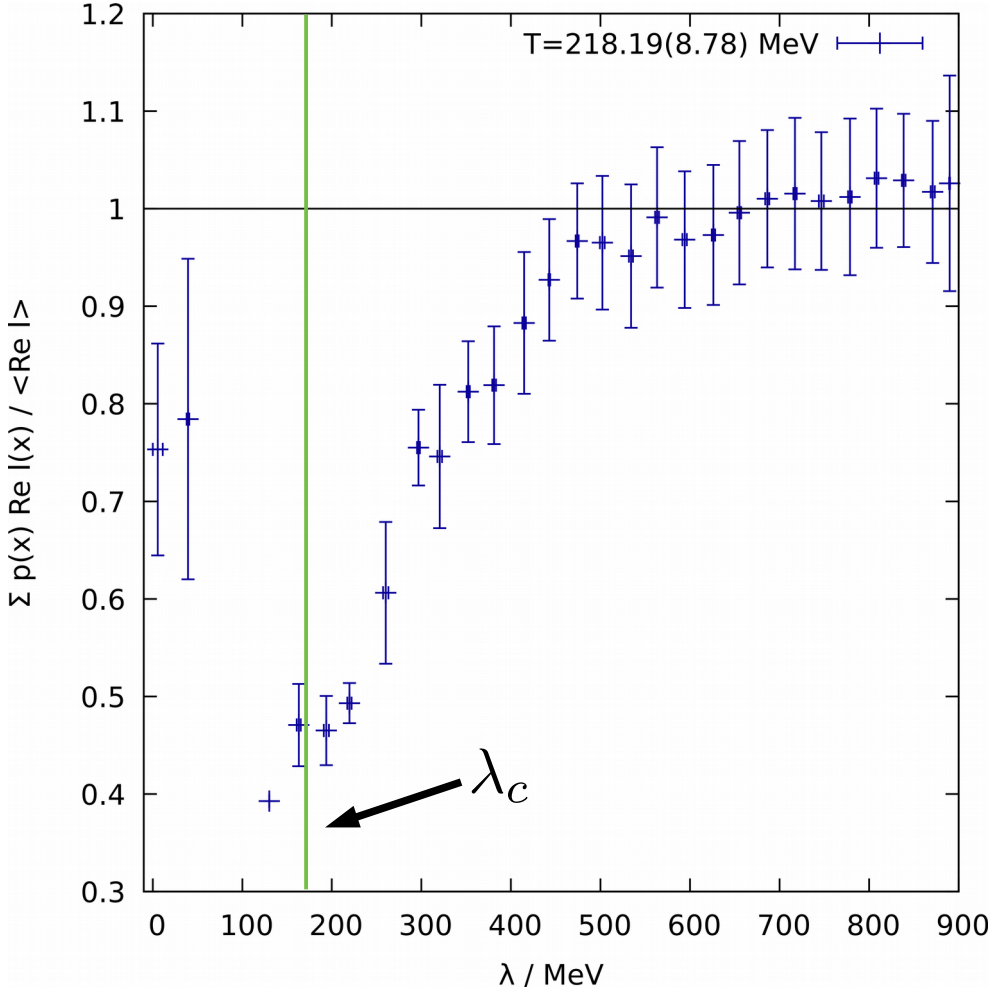


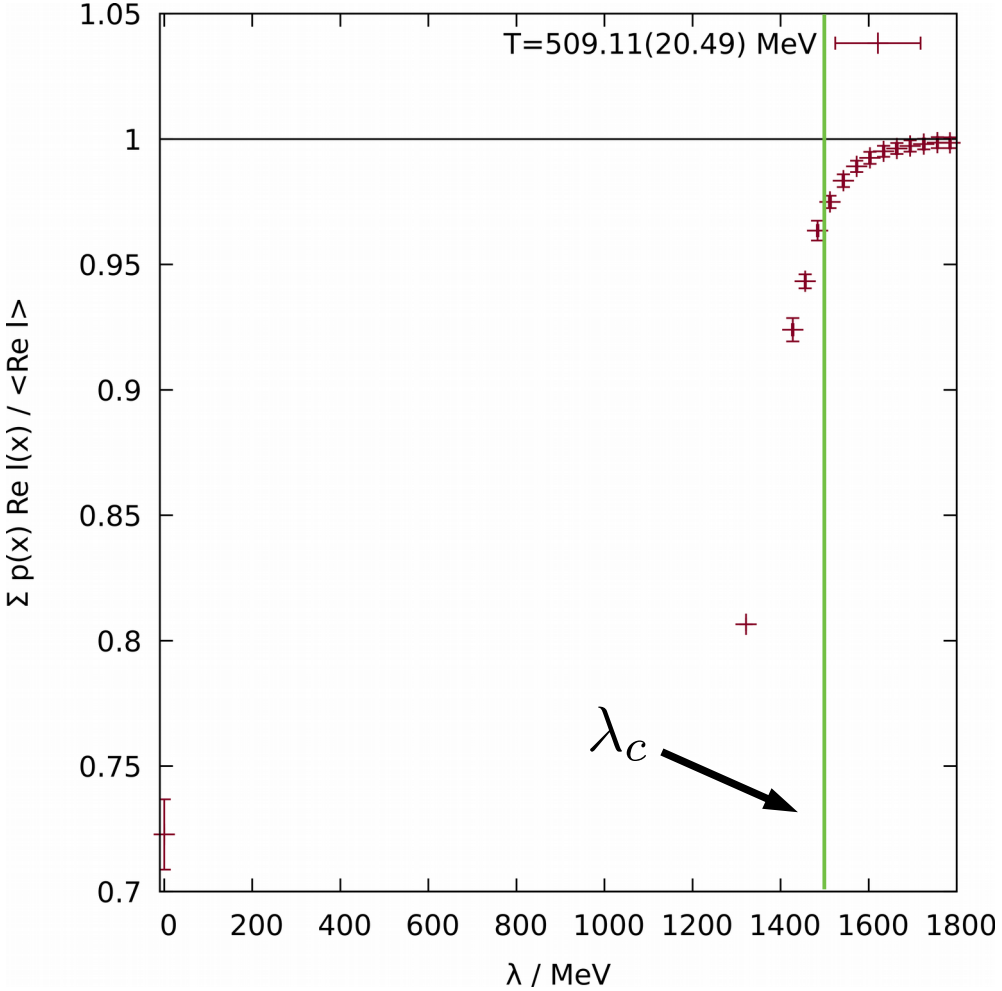


$T = 509.11(20.49) \text{ MeV}$

Localization and Polyakov loops

$$\frac{1}{\langle \mathrm{Re}l \rangle} \int d^d x \mathrm{Re}l(x) \langle \psi(x) | \psi(x) \rangle \begin{cases} = 1 & \text{for } p(x) = \frac{1}{V} \\ < 1 & \text{for localized modes} \end{cases}$$

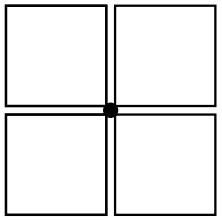




Localization and Topology

$$Q = \frac{1}{32\pi^2} \int d^4x q(x)$$

$$q(x) = \epsilon_{\mu\nu\rho\sigma} F^a_{\mu\nu}(x) F^a_{\rho\sigma}(x)$$

$F_{\mu\nu} = \frac{1}{4}\mathrm{Im}$ 

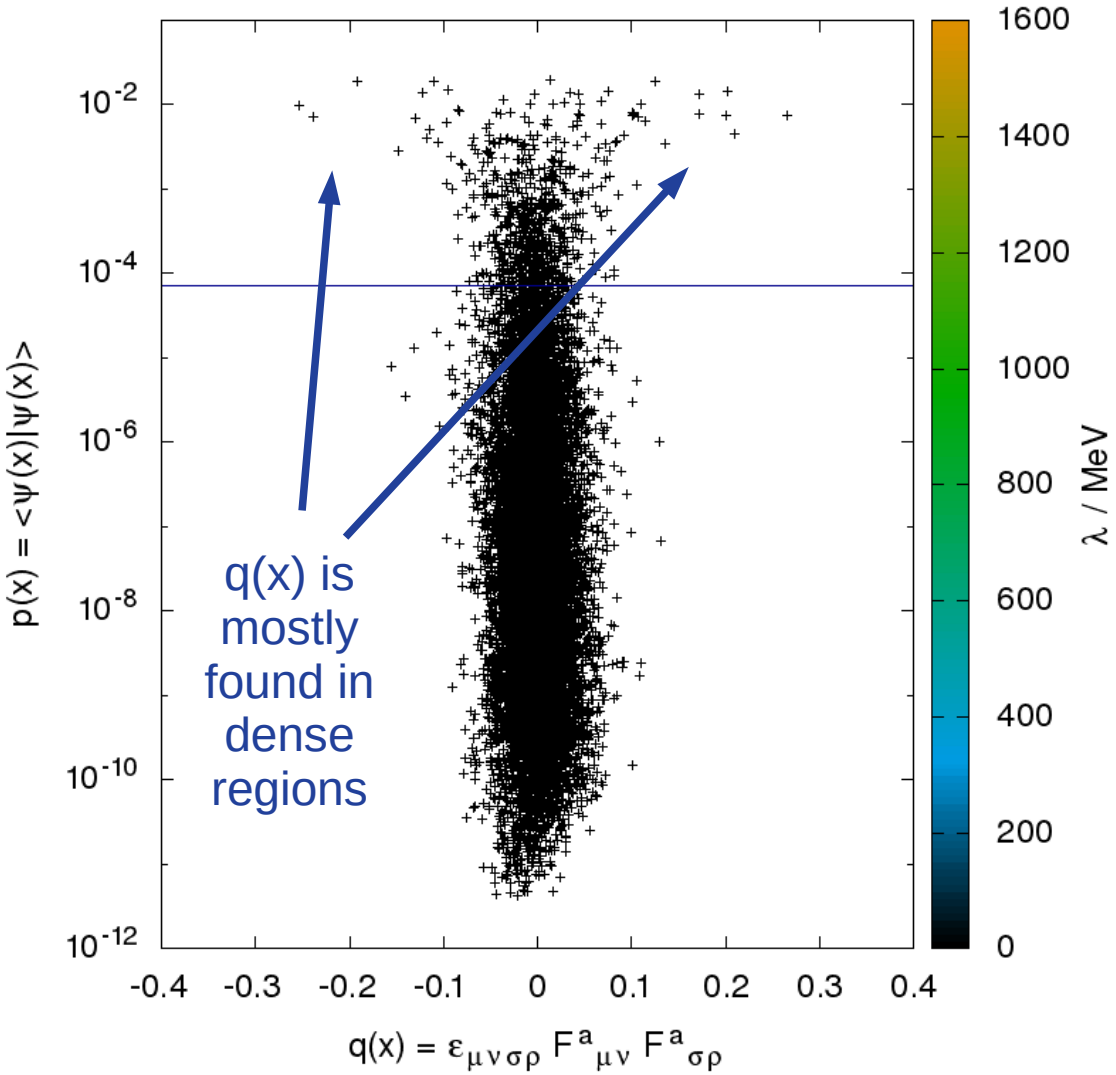
See also:

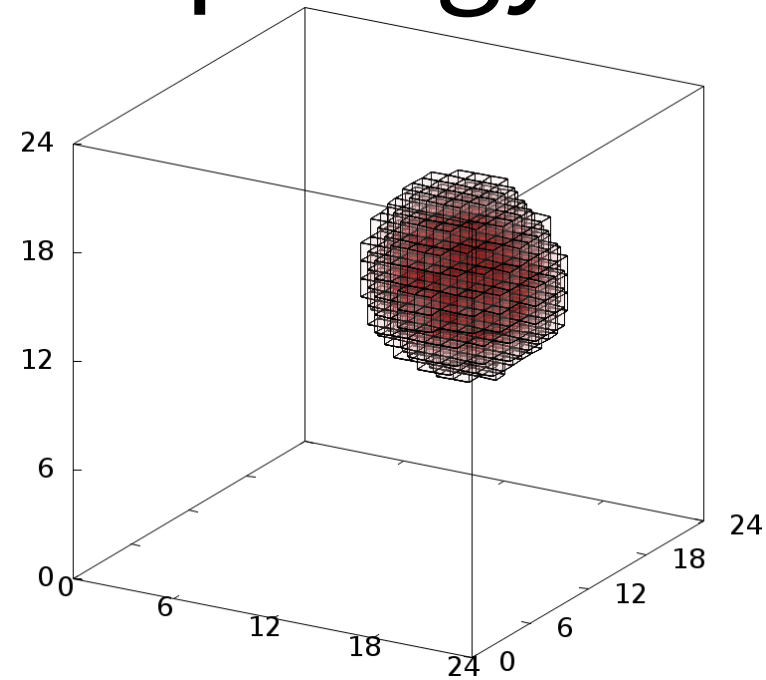
Kovacs, Vig
[Phys.Rev. D97 (2018) no.1, 014502]

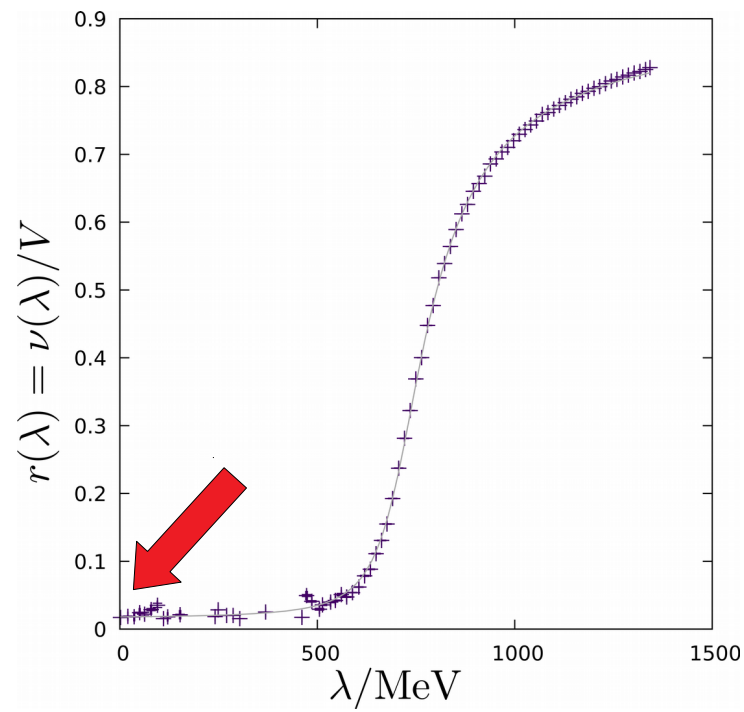
Localization can not be explained with a dilute instanton gas alone!

Localization and Topology

$\lambda = 0$ MeV





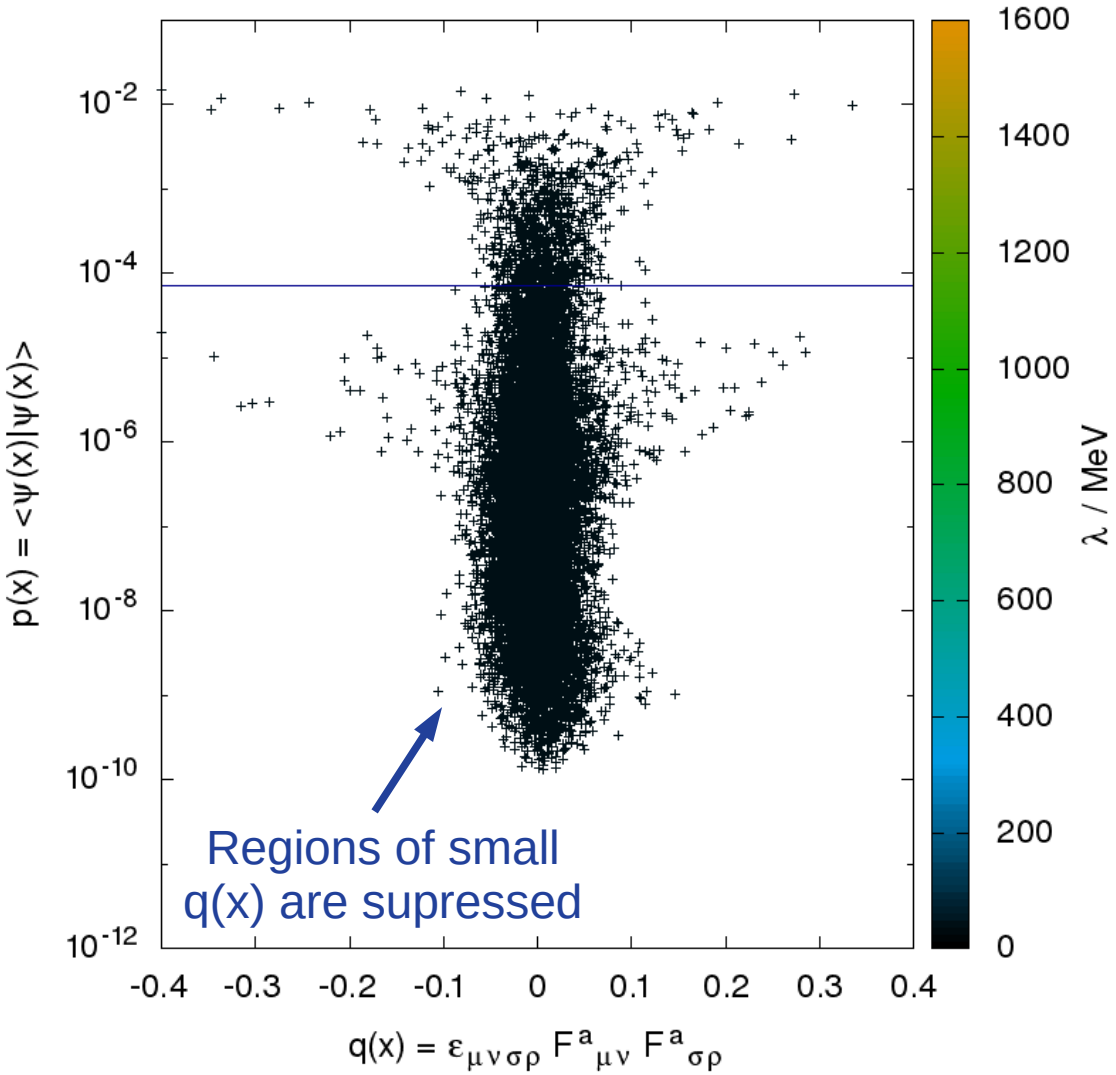


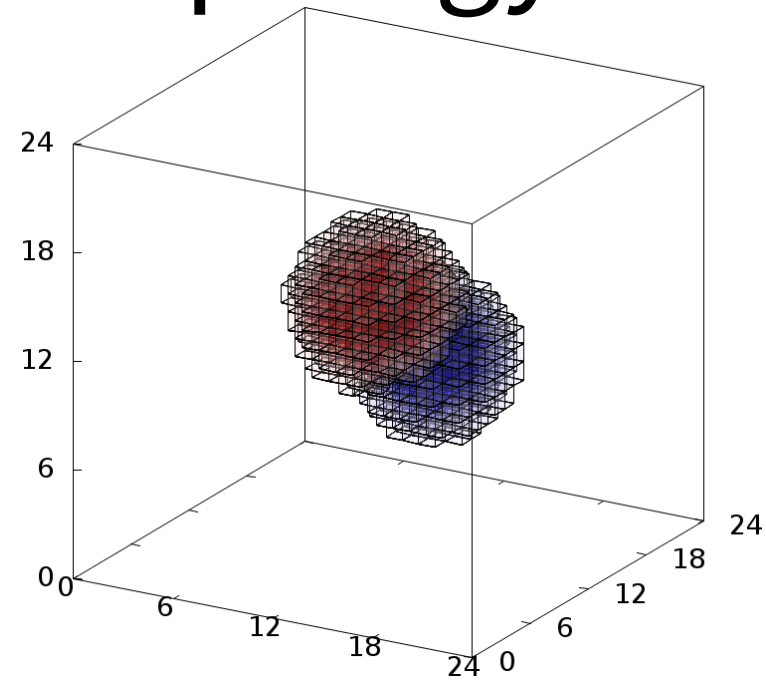
$T = 351.37(14.27)$ MeV

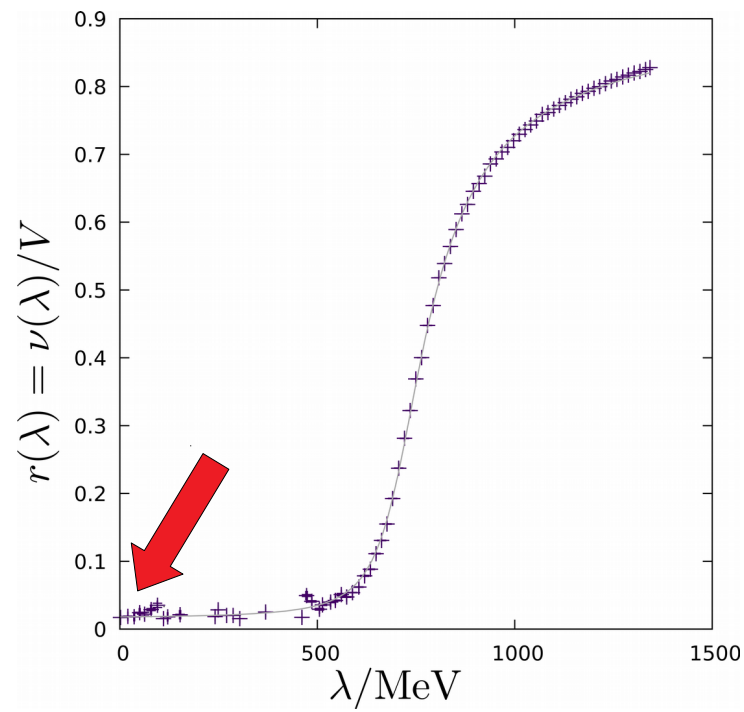
PRELIMINARY

Localization and Topology

$\lambda = 34.6\ \mathrm{MeV} \approx 0.05\lambda_c$





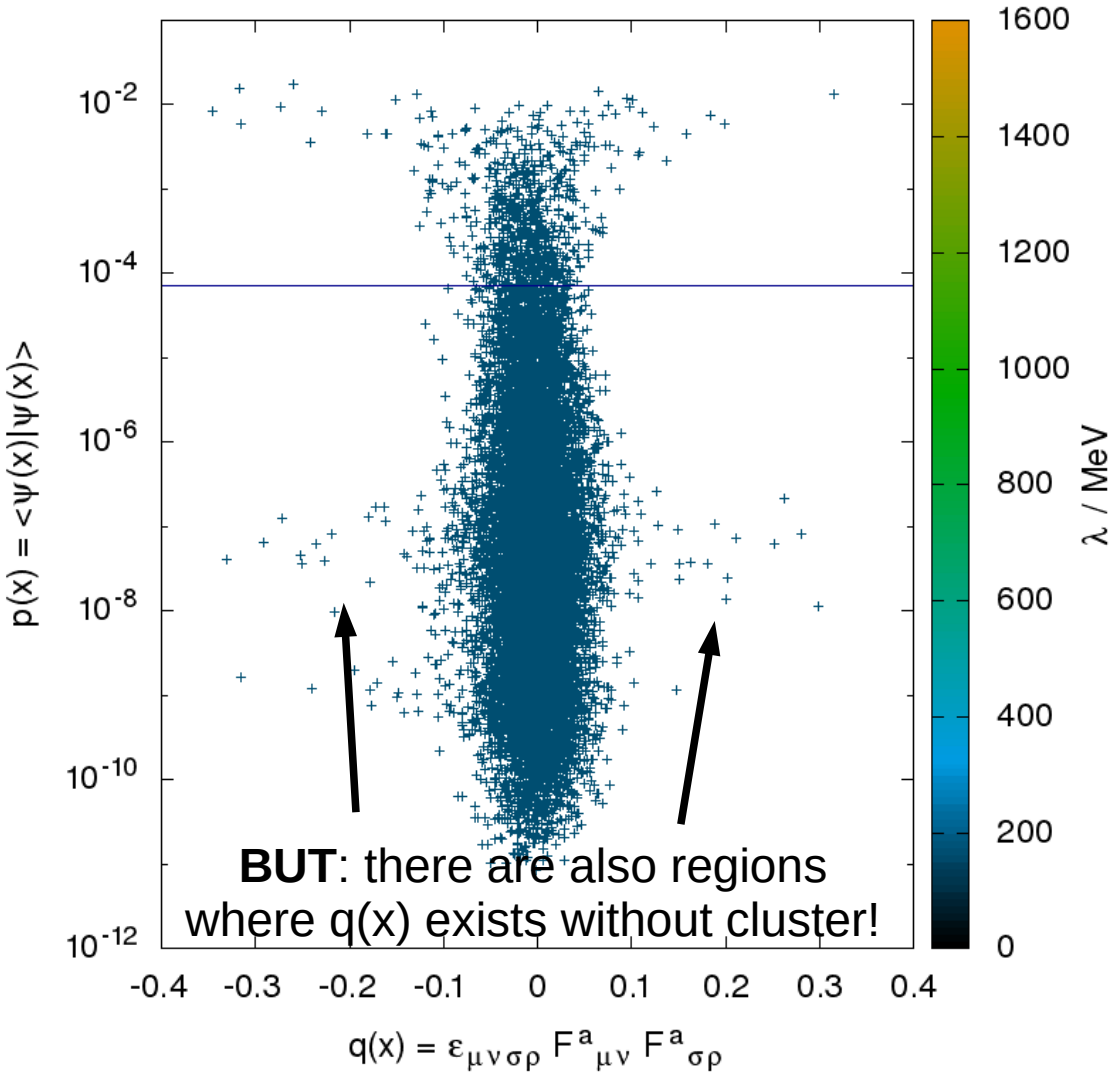


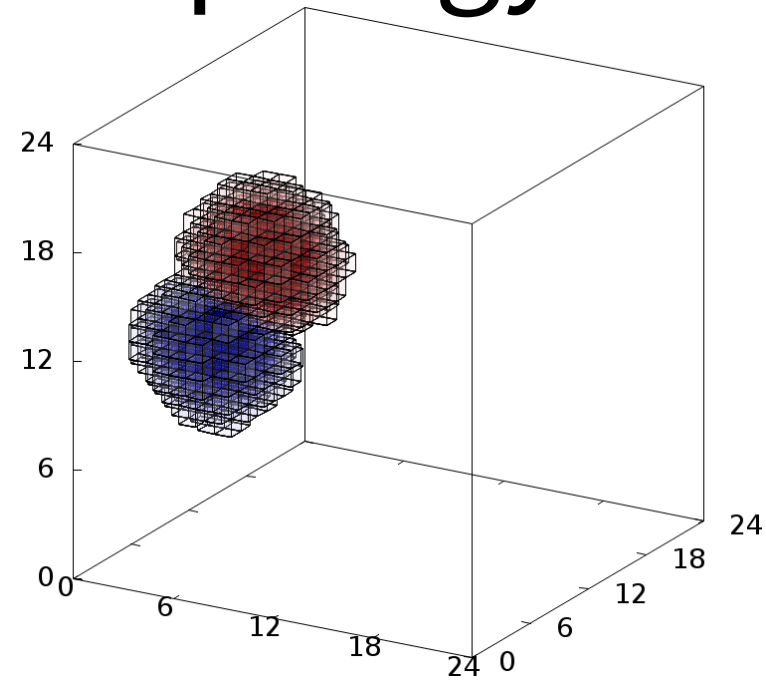
PRELIMINARY

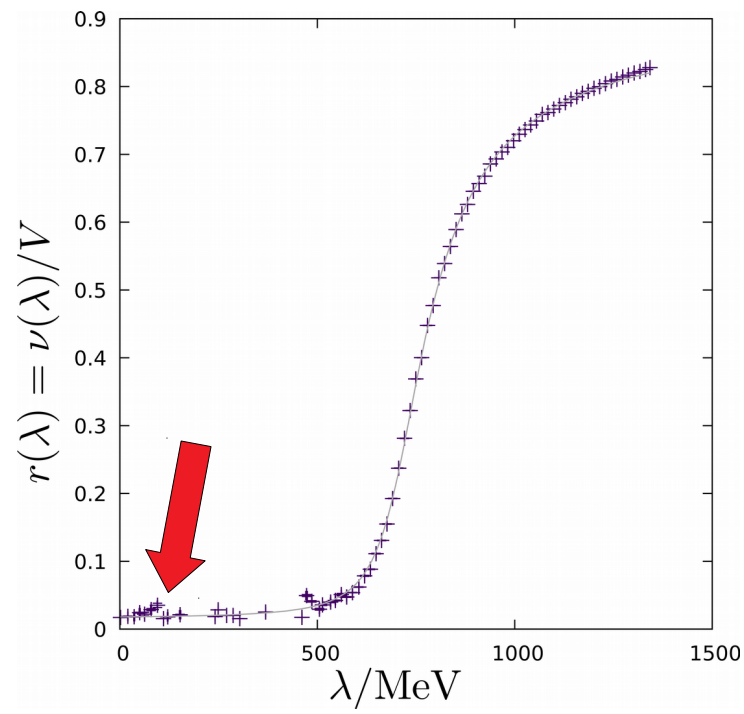
$T = 351.37(14.27)\ \mathrm{MeV}$

Localization and Topology

$\lambda = 167.1\ \mathrm{MeV} \approx 0.24\lambda_c$





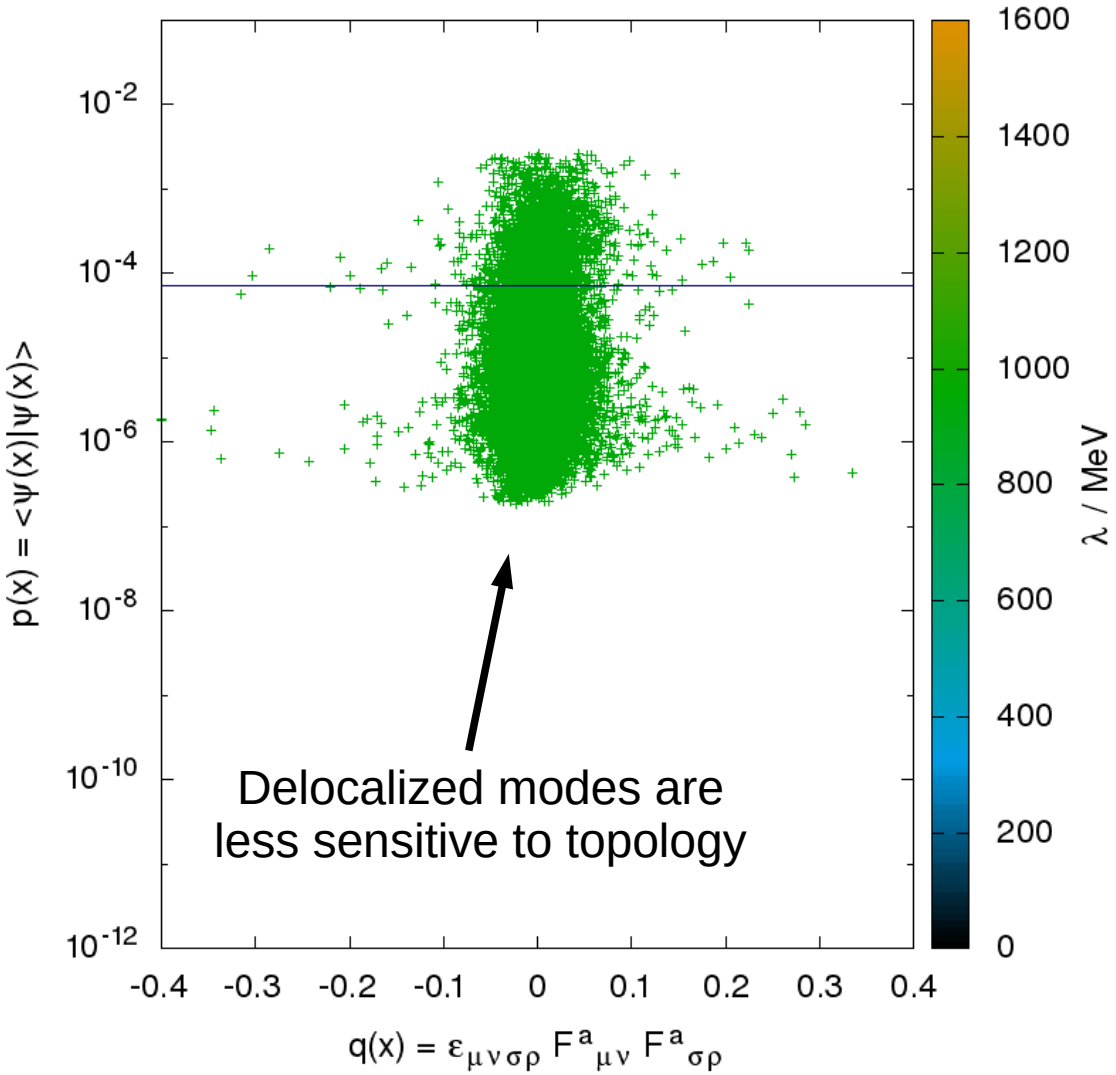


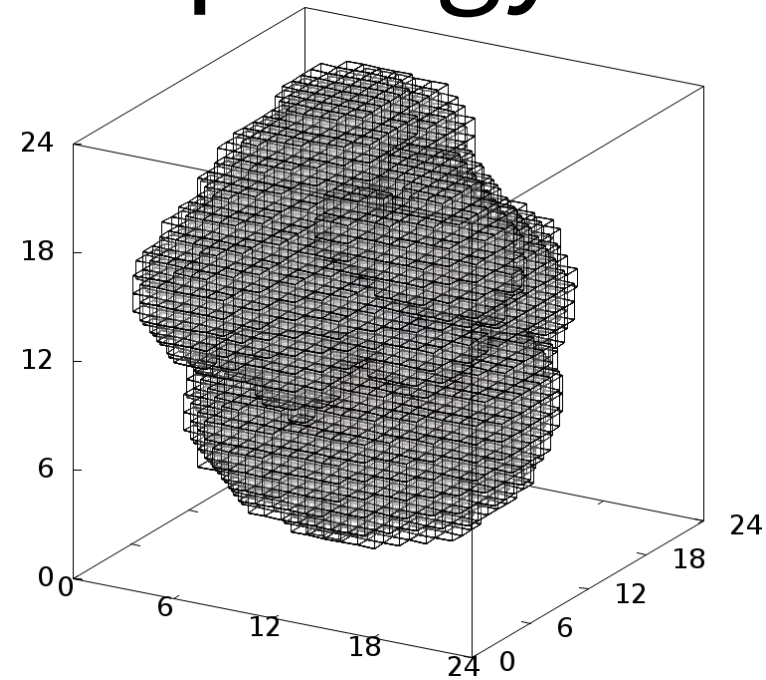
PRELIMINARY

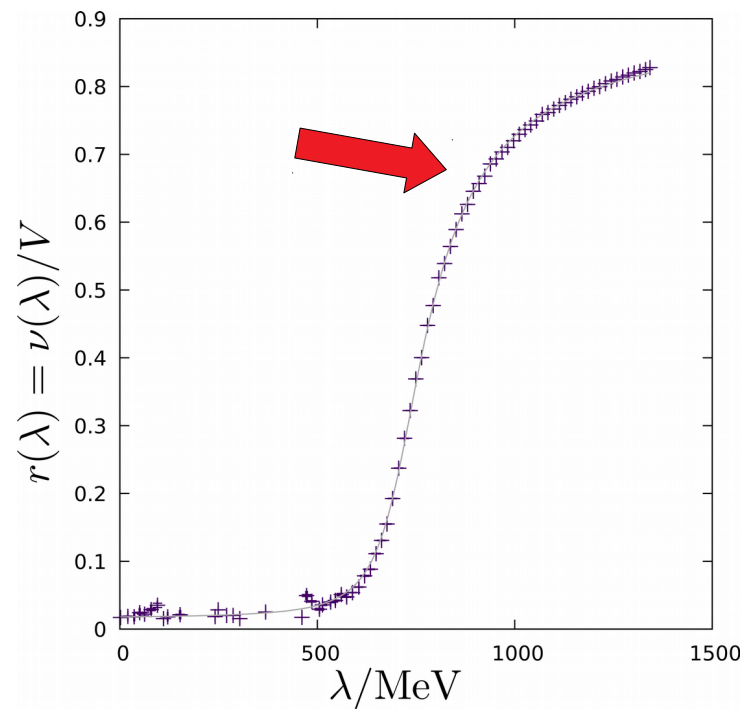
$T = 351.37(14.27)\ \mathrm{MeV}$

Localization and Topology

$\lambda = 936.6$ MeV $\approx 1.32\lambda_c$





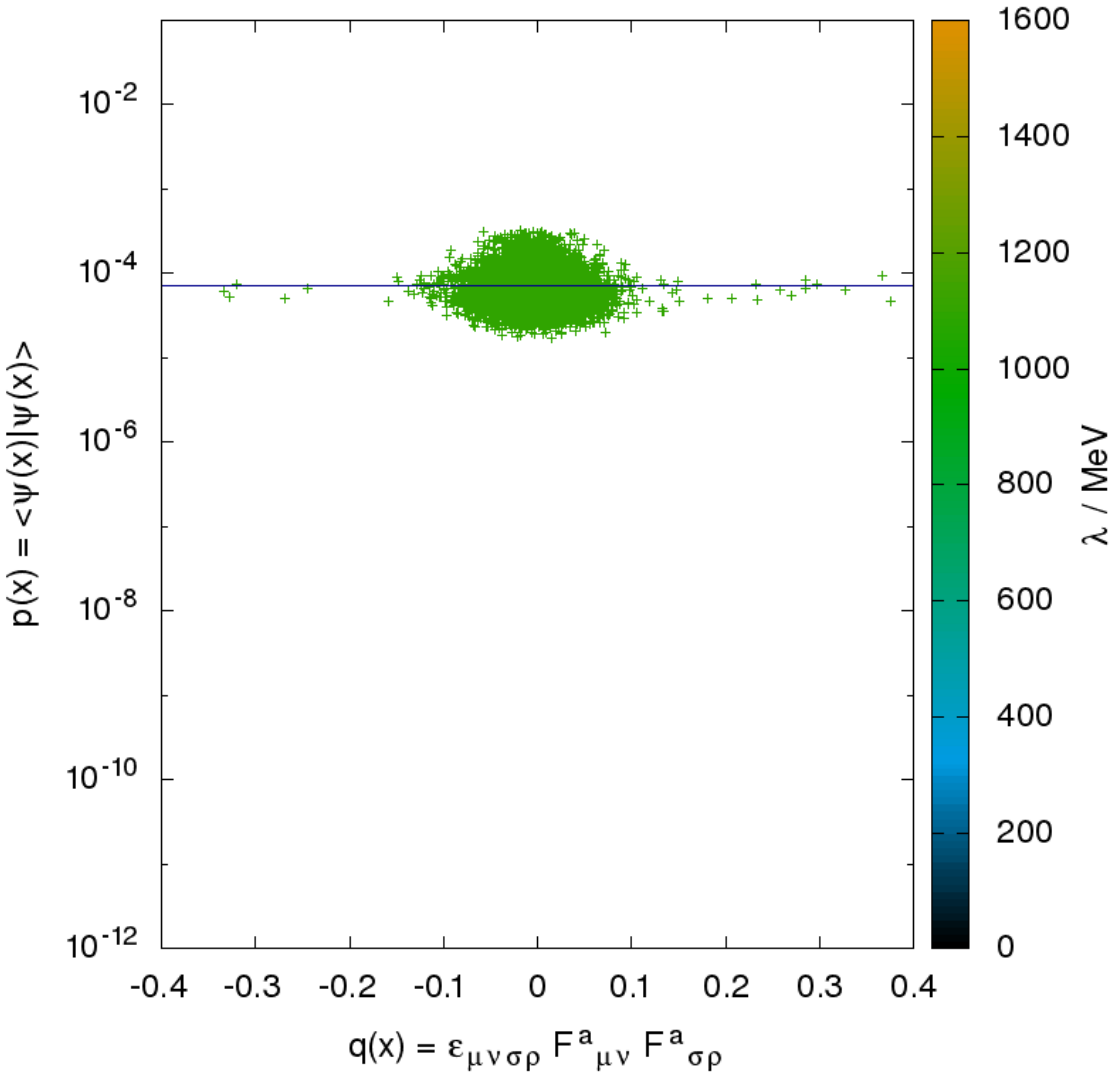


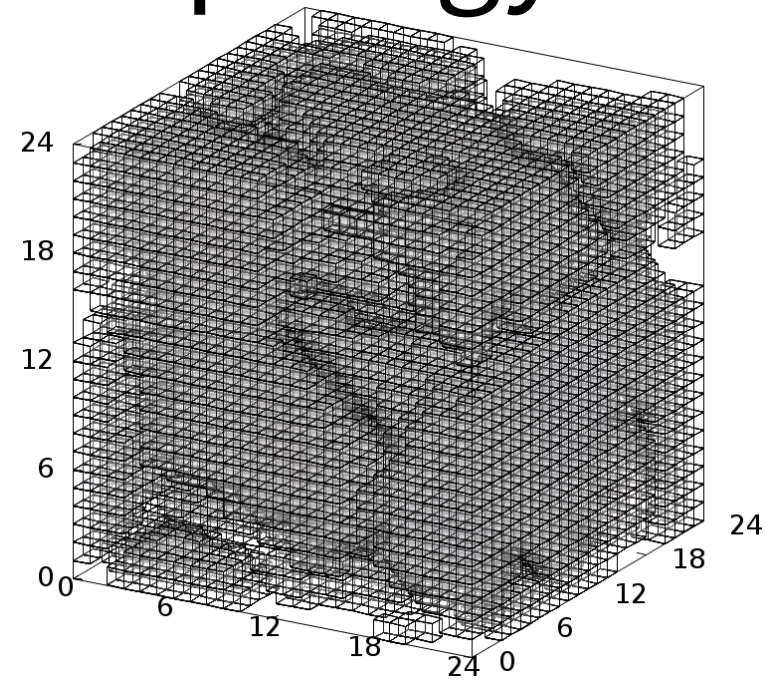
$T = 351.37(14.27)$ MeV

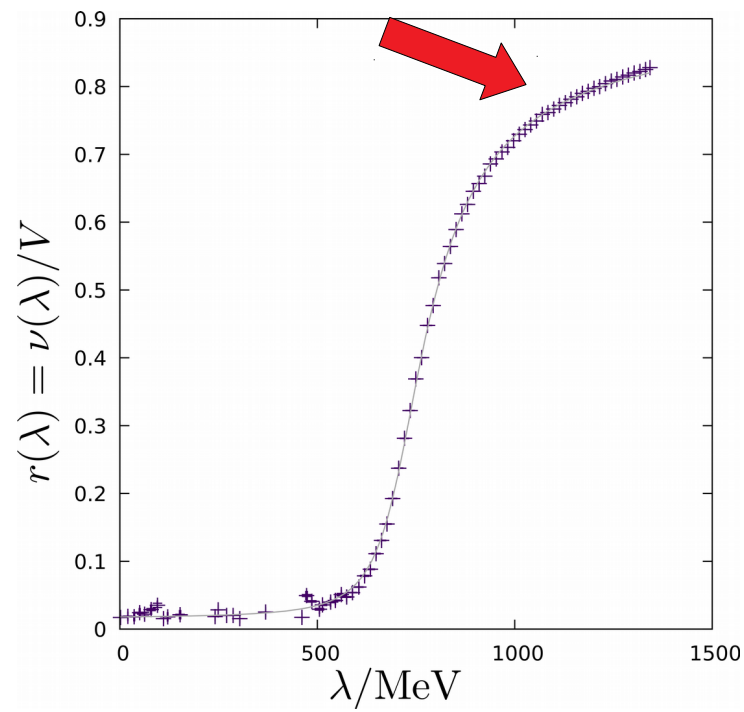
PRELIMINARY

Localization and Topology

$\lambda = 1083.2$ MeV $\approx 1.53\lambda_c$







PRELIMINARY

$T = 351.37(14.27)$ MeV

Summary

$\lambda_c \propto T$ and vanishes at the chiral transition

$\mathrm{Re}l(x)$ plays the role of the impurities in the Anderson model

Localized eigenmode clusters carry topological charge, but not all of Q is encoded in clusters!