The QCD Anderson transition

with $N_f=2+1+1$ twisted mass Wilson quarks
Outline

- Introduction
- The temperature dependence of the mobility edge
- Localization and Polyakov loops
- Localization and Topology

See also talks by

Réka Á. Vig (Tuesday, 16:30)

Tamás G. Kovács (Tuesday, 16:50)

The mobility edge vanishes at $T_c$ (quenched)

Non-interacting instanton-antiinstanton gas **does not** explain localization!
Anderson transitions

Seminal paper:
P. W. Anderson
Phys. Rev. 109, 1492 (1958)

<table>
<thead>
<tr>
<th>Anderson transition</th>
<th>QCD Anderson transition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hamiltonian H</td>
<td>Dirac operator D</td>
</tr>
<tr>
<td>Metal-insulator transition</td>
<td>Distinct spectral regimes</td>
</tr>
<tr>
<td>Vanishing zero T conductivity</td>
<td>Localization of low quark modes at $T &gt; T_c$</td>
</tr>
<tr>
<td>Disorder / Impurities</td>
<td>?</td>
</tr>
</tbody>
</table>

Localization in ultracold atomic waves

Photon localization in a sample of titanium dioxide particles dispersed in polystyrene

Soundwave localization in elastic networks

Localized Dirac eigenmodes in Two-Colour QCD

Anderson localization in QCD matter
Quark localization in QCD

Level spacing statistics:

\[ s_j = \frac{\lambda_{j+1} - \lambda_j}{\langle \lambda_{j+1} - \lambda_j \rangle} \]

Uncorrelated level spacings

Random Matrix Theory

\[ \beta_D = 1 \leftrightarrow \text{unitary} \]
\[ \beta_D = 2 \leftrightarrow \text{orthogonal} \]
\[ \beta_D = 4 \leftrightarrow \text{symplectic} \]
Lattice setup

Sea: \( N_f = 2 + 1 + 1 \) twisted mass fermions

Valence: Overlap fermions

Gauge: Iwasaki

Configurations were created by the Twisted Mass at Finite Temperature Collaboration:

Burger, Hotzel, Müller-Preussker, Ilgenfritz, Lombardo
PoS Lattice2013 (2013) 153

Burger, Ilgenfritz, Lombardo, Müller-Preussker, Trunin

Burger, Ilgenfritz, Lombardo, Müller-Preussker, Trunin

A370 512 modes / configuration
\[ N_s = 24 \quad a = 0.0936 \text{ fm} \]
\[ m_\pi = 364 \text{ MeV} \]
\[ T = 210 \ldots 527 \text{ MeV} \approx 1.1 \ldots 2.9T_c \]

D370 300 modes / configuration
\[ N_s = 32 \quad a = 0.0646 \text{ fm} \]
\[ m_\pi = 369 \text{ MeV} \]
\[ T = 218 \ldots 1018 \text{ MeV} \approx 1.2 \ldots 5.5T_c \]

Fixed scale
\(~100\) conf. / temperature
The mobility edge

Inverse Participation Ratio (IPR):
\[ v^{-1} = \int d^4x \langle \psi(x) | \psi(x) \rangle^2 \]

Relative eigenvector volume:
\[ r(\lambda) = \frac{\nu(\lambda)}{V} \in (0, 1] \]

\[ \lambda_c(T) \propto T \]
Banks-Casher gap \( \langle \bar{\psi} \psi \rangle = -\pi \lim_{\lambda \to 0} \lim_{m \to 0} \lim_{V \to \infty} \rho(\lambda) \)

\( \rho(\lambda) \)

- \( T = 210.82(8.56) \text{ MeV} \)
- \( T = 234.25(9.51) \text{ MeV} \)
- \( T = 263.53(10.70) \text{ MeV} \)
- \( T = 301.18(12.23) \text{ MeV} \)
- \( T = 351.37(14.27) \text{ MeV} \)
- \( T = 421.65(17.11) \text{ MeV} \)
- \( T = 527.06(21.40) \text{ MeV} \)

Temperature

512 eigenmodes / configuration

\( \lambda / \text{MeV} \)
Unfolded level spacing distribution

T = 218.19(8.79) MeV

- Unitary ensemble well matched
- Poisson-like part can not clearly be distinguished

\[ T = 509.11(20.49) \text{ MeV} \]

- Spectrum contains both ULSDs

\[
P_{\text{Poisson}}(s) = e^{-s} \implies P_{\text{chGUE}}(s) = \frac{32}{\pi^2} s^2 \exp \left( -\frac{4}{\pi} s^2 \right)
\]

300 eigenmodes / configuration
Localization and Polyakov loops

\[ l(x) = \frac{1}{N_c} \text{tr} \left( e^{\int dt A_4(x)} \right) \]

Regions of small Re \( l(x) \) are clustered!

See also:

Bruckmann, Kovacs, Schierenberg
Phys.Rev. D84 (2011) 034505

Giordano, Kovacs, Pittler
JHEP 1504 (2015) 112

Cossu, Hashimoto
JHEP 1606 (2016) 056
Localization and Polyakov loops

Clusters of small $\text{Re } l(x)$

$\langle \psi(x) | \gamma_5 | \psi(x) \rangle$

Zero modes in this configuration
Localization and Polyakov loops

$\lambda = 0.0 \text{ MeV}$

Zero modes are suppressed in regions of large $\text{Re } I(x)$

$T = 509.11(20.49) \text{ MeV}$
Localization and Polyakov loops

\[ \lambda = 0.0 \text{ MeV} \]

\[ T = 509.11(20.49) \text{ MeV} \]
Localization and Polyakov loops

\[ \lambda = 1321.6 \text{ MeV} \approx 0.87 \lambda_c \]

\[ T = 509.11(20.49) \text{ MeV} \]

Also localized non-zero modes are suppressed in regions of large Re l(x)

Clusters with alternating local chirality
Localization and Polyakov loops

$\lambda = 1417.6 \text{ MeV} \approx 0.93\lambda_c$

$T = 509.11(20.49) \text{ MeV}$
Localization and Polyakov loops

\[ \lambda = 1451.5 \text{ MeV} \approx 0.95 \lambda_c \]

This mode is already delocalizing

\[ T = 509.11(20.49) \text{ MeV} \]

\[ r(\lambda) = \nu(\lambda)/V \]
Localization and Polyakov loops

\[ \lambda = 1551.1 \text{ MeV} \approx 1.02\lambda_c \]

\[ T = 509.11(20.49) \text{ MeV} \]
Localization and Polyakov loops

\[ \lambda = 1721.7 \text{ MeV} \approx 1.13 \lambda_c \]

\[ T = 509.11(20.49) \text{ MeV} \]
Localization and Polyakov loops

\[ \frac{1}{\langle \text{Rel} \rangle} \int d^d x \text{Rel}(x) \langle \psi(x) | \psi(x) \rangle \begin{cases} = 1 & \text{for } p(x) = \frac{1}{V} \\ < 1 & \text{for localized modes} \end{cases} \]
Localization and Topology

\[ Q = \frac{1}{32\pi^2} \int d^4 x q(x) \]

\[ q(x) = \epsilon_{\mu\nu\rho\sigma} F^a_{\mu\nu}(x) F^a_{\rho\sigma}(x) \]

<table>
<thead>
<tr>
<th>Kovacs, Vig</th>
</tr>
</thead>
</table>

 Localization can not be explained with a dilute instanton gas alone!
Localization and Topology

$\lambda = 0 \; \text{MeV}$

$q(x) = \varepsilon_{\mu \nu \sigma \rho} F_{\mu \nu}^a F_{\sigma \rho}^a$

$q(x)$ is mostly found in dense regions

$T = 351.37(14.27) \; \text{MeV}$
Localization and Topology

\[ \lambda = 34.6 \text{ MeV} \approx 0.05 \lambda_c \]

\[ q(x) = \epsilon_{\mu \nu \sigma \rho} F^a_{\mu \nu} F^a_{\sigma \rho} \]

\[ T = 351.37(14.27) \text{ MeV} \]
Localization and Topology

\[ \lambda = 167.1 \text{ MeV} \approx 0.24\lambda_c \]

**BUT**: there are also regions where \( q(x) \) exists without cluster!

\[ q(x) = \varepsilon_{\mu\nu\sigma\rho} F_{\mu\nu}^a F_{\sigma\rho}^a \]

\[ T = 351.37(14.27) \text{ MeV} \]
Localization and Topology

\[ \lambda = 936.6 \text{ MeV} \approx 1.32\lambda_c \]

Delocalized modes are less sensitive to topology

\[ q(x) = \epsilon_{\mu\nu\sigma\rho} F_{\mu\nu}^a F_{\sigma\rho}^a \]

\[ T = 351.37(14.27) \text{ MeV} \]
Localisation and Topology

\[ \lambda = 1083.2 \text{ MeV} \approx 1.53 \lambda_c \]

\[ q(x) = \epsilon_{\mu \nu \sigma \rho} F^a_{\mu \nu} F^a_{\sigma \rho} \]

\[ T = 351.37(14.27) \text{ MeV} \]
Summary

\[ \lambda_c \propto T \] and vanishes at the chiral transition

\[ \text{Rel}(x) \] plays the role of the impurities in the Anderson model

Localized eigenmode clusters carry topological charge, but not all of Q is encoded in clusters!