The fate of axial U(1) in 2+1 flavor QCD towards the chiral limit

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July 24, 2018
The $U_A(1)$ puzzle

- Nature of chiral phase transition for QCD with two light quark flavors is not yet completely resolved.

- Usually symmetries determine the order parameter across a phase transition.

- However, for 2 light quark flavors anomalous $U_A(1)$ may affect the nature of phase transition.

- How severely $U_A(1)$ is broken can be answered non-perturbatively.

- We want to investigate whether or not the anomalous $U_A(1)$ symmetry in the flavor sector is effectively restored along with the chiral symmetry.
Observables sensitive to $U_A(1)$ breaking

- $U_A(1)$ is not an exact symmetry $\Rightarrow$ no unique order parameter!
- Instead, look at n-point correlation functions which become degenerate upon $U_A(1)$ rotation.
  Start with 2-point correlation functions.

[A. Bazavov et al., 2012]
[see also Sheng-Tai Li, Thursday 11.20h]
$U_A(1)$ breaking and QCD eigenvalue density

- Observable of interest is [Shuryak, 1994]

$$\chi_\pi - \chi_\delta = \int d^4 x \left[ \langle i\pi^+(x) i\pi^-(0) \rangle - \langle i\delta^+(x) i\delta^-(0) \rangle \right]$$

- Equivalently study $\rho(\lambda, m_f)$ of the Dirac operator [Cohen, 1995, Hatsuda & Lee, 1995]

$$\chi_\pi - \chi_\delta \xrightarrow{\nu \to \infty} \int_0^\infty d\lambda \frac{4m_f^2 \rho(\lambda, m_f)}{(\lambda^2 + m_f^2)^2}$$

- Possible scenarios:
  - $\lim_{m_f \to 0} \rho(0, m_f) \to 0 \Rightarrow U_A(1)$ trivially restored.
  - $\lim_{\lambda \to 0} \rho(\lambda, m_f) = \delta(\lambda)m_f^\alpha$ with $1 < \alpha < 2 \Rightarrow U_A(1)$ broken.
  - $\lim_{m_f \to 0} \rho(\lambda, m_f) \sim \lambda^3 \Rightarrow U_A(1)$ restored.
More on the eigenvalue spectrum and $U_A(1)$ breaking

- Looking at higher n-point correlation functions is also important!

- $\rho(\lambda, m_f)$ has been investigated analytically using chiral Ward identities of n-point functions of scalar and pseudo-scalar currents, assuming $\rho(\lambda, m_f)$ to be analytic in $m_f^2$
  
  [Aoki, Fukaya & Taniguchi, 2012].

- It was shown explicitly that $U_A(1)$ breaking is absent in upto six point correlation functions in the same scalar and pseudo-scalar sectors if $\rho(\lambda, m_f \to 0) \sim \lambda^3$.

- They found out that in the chiral limit $\rho(\lambda) \sim \lambda^n$ with $n > 2$. 
Summary of the results till now

- Previous studies at almost physical quark masses:
  Infrared part has both, non-analytic and analytic in $\lambda$
  [Sayantan Sharma et al.,2015].

- $\rho(\lambda, m_f) \sim \lambda^2$ at $T = 1.2 \, T_c \Rightarrow U_A(1)$ broken.

[Viktor Dick et al.,2015]
Gauge ensembles where generated within the Highly Improved Staggered Quark discretization scheme (HISQ) with 2+1 quark flavor.

We used the overlap Dirac operator to measure the low-lying eigenvalue spectrum.

98 eigenvalues per configuration.

<table>
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<th>$m_s/m_l$</th>
<th>$N_s^3 \times N_T$</th>
<th>$\beta$</th>
<th>$T/T_c$</th>
<th>#conf</th>
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<td>$32^3 \times 8$</td>
<td>6.445</td>
<td>1.05</td>
<td>94</td>
</tr>
</tbody>
</table>
The overlap operator is given as,

\[ D_{ov} = M \left[ 1 + \gamma_5 \text{sgn} (\gamma_5 D_W (-M)) \right] \]

\[ \text{sgn} (\gamma_5 D_W (-M)) = \gamma_5 D_W (-M) / \sqrt{D_W^\dagger (-M) D_W (-M)} \]

where \( D_W \) is the Wilson-Dirac operator with a negative mass parameter \( M \in [0, 2) \).

Eigenvalues of \( D_{ov}^\dagger D_{ov} \) are computed using the Kalkreuter-Simma (KS) Ritz algorithm.

Zero-modes were not measured. Only eigenvalues with positive or negative chirality have been measured.

Finally we perform a tuning of the valence overlap masses to the sea HISQ masses to get the renormalized observables.
Is topological tunneling sufficient?

$m_l = m_s/27$:

- $T = 0.97T_c$
- $T = 1.03T_c$
- $T = 1.09T_c$

$m_l = m_s/40$:

- $T = 0.99T_c$
- $T = 1.03T_c$
- $T = 1.05T_c$
Dirac eigenvalue spectrum

▶ Analytic part goes as $\lambda^\gamma$ with $\gamma \geq 1$.

▶ Non-analytic part reduces with temperature.

▶ Going towards smaller quark masses these features survive.

$m_t = m_s / 27$

$\rho(\lambda) m_s / T^4$

$m_t = m_s / 27$

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$$m_l = m_s/40$$

$$\rho(\lambda) m_s / T^4$$

- $T = 0.99T_c$
- $T = 1.03T_c$
- $T = 1.05T_c$
Renormalized $U_A(1)$ breaking parameter as a function of quark mass

- Valence overlap quark mass has been tuned to the HISQ sea quark masses by matching the renormalized quantity

$$\Delta = \frac{m_s \langle \bar{\Psi} \Psi \rangle_l - m_l \langle \bar{\Psi} \Psi \rangle_s}{T^4}$$

![Graph showing the renormalized $U_A(1)$ breaking parameter as a function of $T/T_c$. The graph includes two curves, one for $m_l = m_s/27$ and another for $m_l = m_s/40$. The $x$-axis represents $T/T_c$, and the $y$-axis represents $\frac{(\chi_\pi - \chi_\delta)}{m_s^2} T^4$. The graph shows a decrease in the renormalized quantity as $T$ increases.]
Conclusion

- $\rho(\lambda, m_f) \sim \lambda$ even when $m_f$ reduces from $m_s/27$ to $m_s/40$ for $T/T_c < 1.1$.

- Non-analytic part also survives when we reduce the mass.

- Both of them contribute to the breaking of $U_A(1)$ above $T_c$ after proper retuning of the valence quark masses and looking at renormalized quantities.

- We are looking at even smaller quark masses to check whether our conclusions survive in the chiral limit.