New lattice interaction and spectrum of light and medium-mass nuclei

Ning Li

FRIB/NSCL, Michigan State University

In collaboration with S. Elhatisari, E. Epelbaum, D. Lee, B.-N. Lu, and Ulf-G. Meissner

Outline:

- Motivation
- Chiral EFT
- New Lattice Formulation
- Spectrum of Nuclei
- Summary
Motivation:

Nuclear Lattice EFT combing the lattice method, Monte Carlo simulation and Chiral EFT have been used in recent years to describe the structure and scattering of atomic nuclei.


However, the treatment of nuclear forces at higher orders in Chiral EFT expansion is more difficult on the lattice due to the breaking of rotational invariance introduced by the nonzero lattice spacing. Fitting the LECs of the short-range lattice interactions to the empirical phase shifts can introduce significant uncertainties.
Chiral EFT organizes the nuclear forces in powers of \( \left( \frac{\Lambda}{\Lambda} \right)^n \),

- LO: \( \left( \frac{\Lambda}{\Lambda} \right)^0 \)
- NLO: \( \left( \frac{\Lambda}{\Lambda} \right)^2 \)
- N2LO: \( \left( \frac{\Lambda}{\Lambda} \right)^3 \)
- N3LO: \( \left( \frac{\Lambda}{\Lambda} \right)^4 \)

E. Epelbaum, H. Hammer, Ulf.-G. Meissner, Rev. Mod. Phys. 81, 1773 (2009); 
New lattice formulation

Pair annihilation operators with specific quantum numbers

\[ [a(n)a(n')]_{S,S_z,I,I_z}^{s_{NL}} = \sum_{i,j,i',j'} a_{i,j}^{s_{NL}}(n) M_{i,i'}(S, S_z) M_{j,j'}(I, I_z) a_{i',j'}^{s_{NL}}(n') \]

\[ P_{S,S_z,L,L_z,I,I_z}^{2M,s_{NL}}(n) = \left[ a(n) \nabla^2 R_{L,L_z}^* (\nabla) a(n) \right]_{S,S_z,I,I_z}^{s_{NL}} \]

\[ O_{S,L,J,J_z,I,I_z}^{2M,s_{NL}}(n) = \sum_{S_z,L_z} <S, S_z, L, L_z | J, J_z > P_{S,S_z,L,L_z,I,I_z}^{2M,s_{NL}}(n) \]


Courtesy Dean Lee
At LO(Q0), there are two independent S-wave operators, one is for 1S0 (S=0, I=1) and the other one is for 3S1 (S = 1, I=0)

\[ V_{1S_0,Q^0}(n) = \sum_{l_z=-1,0,1} \left[ O^{0,\text{SNL}}_{0,0,0,0,1,l_z}(n) \right]^\dagger \ O^{0,\text{SNL}}_{0,0,0,0,1,l_z}(n) \]

\[ V_{3S_1,Q^0}(n) = \sum_{J_z=-1,0,1} \left[ O^{0,\text{SNL}}_{1,0,1,J_z,0,0}(n) \right]^\dagger \ O^{0,\text{SNL}}_{1,0,1,J_z,0,0}(n) \]

Higher-order operators are constructed in a similar way. NLO(O2), 7 more operators, N3LO(Q4), 15 more operators.

The operators constructed in this way only survive in particular channels. This advantage simplifies the fitting procedure significantly.

OPEP:

\[ V_{\text{OPE}} = - \frac{g_A^2}{8 F_\pi^2} \sum_{n,n',S',S,I} : \rho_{S',I}(n') f_{S',S}(n' - n) \rho_{S,I}(n) : \]

\[ f_{S',S}(n' - n) = \frac{1}{L^3} \sum_q \exp \left[ -i q \cdot (n' - n) - b_\pi (q^2 + M_\pi^2) \right] \frac{q_S q_{S'}}{q^2 + M_\pi^2} \]

Deuteron property

At distance $r$ beyond the range of the interaction, the radial wave function for the deuteron in the 3S1 channel behaves as

$$u(r) = A_S e^{-\gamma r}$$

In the 3D1 channel the deuteron behaves as

$$w(r) = \eta A_S \left[ 1 + \frac{3}{\gamma r} + \frac{3}{(\gamma r)^2} \right] e^{-\gamma r}$$
TABLE II. The deuteron properties and S-wave parameters calculated with the full NN interaction up to chiral order $O(Q^4)$ using $a = 0.99$ fm. The error bars we quote in this table indicate uncertainties from the fitting procedure only.

<table>
<thead>
<tr>
<th></th>
<th>LO</th>
<th>NLO</th>
<th>N^2LO</th>
<th>N^3LO</th>
<th>Empirical</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_d$ (MeV)</td>
<td>2.2246 ± 0.0002</td>
<td>2.224575 ± 0.000016</td>
<td>2.224575 ± 0.000025</td>
<td>2.224575 ± 0.000011</td>
<td>2.224575(9)[24]</td>
</tr>
<tr>
<td>$A_d$ (fm^{-1/2})</td>
<td>0.8662 ± 0.0007</td>
<td>0.8772 ± 0.0003</td>
<td>0.8777 ± 0.0004</td>
<td>0.8785 ± 0.0004</td>
<td>0.8846(9)[25]</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.0212 ± 0.0000</td>
<td>0.0258 ± 0.0001</td>
<td>0.0257 ± 0.0002</td>
<td>0.0254 ± 0.0001</td>
<td>0.0256(4)[26]</td>
</tr>
<tr>
<td>$Q_d$ (fm^2)</td>
<td>0.2134 ± 0.00000</td>
<td>0.2641 ± 0.0016</td>
<td>0.2623 ± 0.0023</td>
<td>0.2597 ± 0.0013</td>
<td>0.2859(3)[27]</td>
</tr>
<tr>
<td>$r_d$ (fm)</td>
<td>1.9660 ± 0.0001</td>
<td>1.9548 ± 0.0005</td>
<td>1.9555 ± 0.0008</td>
<td>1.9545 ± 0.0005</td>
<td>1.97535(85)[28]</td>
</tr>
<tr>
<td>$a_3 S_1$</td>
<td>5.461 ± 0.000</td>
<td>5.415 ± 0.001</td>
<td>5.421 ± 0.002</td>
<td>5.417 ± 0.001</td>
<td>5.424(4)[29]</td>
</tr>
<tr>
<td>$r_3 S_1$</td>
<td>1.831 ± 0.0003</td>
<td>1.759 ± 0.002</td>
<td>1.760 ± 0.003</td>
<td>1.758 ± 0.002</td>
<td>1.759(5)[29]</td>
</tr>
<tr>
<td>$a_1 S_0$</td>
<td>-23.8 ± 0.1</td>
<td>-23.69 ± 0.05</td>
<td>-23.8 ± 0.2</td>
<td>-23.678 ± 0.038</td>
<td>-23.748(10)[29]</td>
</tr>
<tr>
<td>$r_1 S_0$</td>
<td>2.666 ± 0.001</td>
<td>2.647 ± 0.003</td>
<td>2.69 ± 0.02</td>
<td>2.647 ± 0.004</td>
<td>2.75(5)[29]</td>
</tr>
</tbody>
</table>
The theoretical uncertainty for some observable $X(p)$ at order $N^{m\text{LO}}$ and momentum $p$ is defined as

$$\Delta X^{N^{m\text{LO}}}(p) = \max \left( Q^{m+2} |X^{\text{LO}}(p)|, Q^m |X^{\text{LO}}(p) - X^{\text{NLO}}(p)|, \ldots, Q^1 |X^{N^{m-1}\text{LO}}(p) - X^{N^m\text{LO}}(p)| \right)$$


We show, in the next four pages, the neutron-proton scattering phase shifts for four different lattice spacings, $a = 1.97$ fm, 1.64 fm, 1.32 fm and 0.99 fm.
Lattice spacing $a = 1.97$ fm:
Lattice spacing $a = 1.64 \text{ fm}$:
Lattice spacing $a = 1.32$ fm:

Graphs showing the relationship between $\delta$ and $p_{\text{rel}}$ for various states: $\delta^{(1\text{S}_0)}$, $\delta^{(3\text{S}_1)}$, $\delta^{(3\text{P}_0)}$, $\delta^{(3\text{P}_1)}$, and $\delta^{(3\text{P}_2)}$. Each graph plots $\delta$ degrees against $p_{\text{rel}}$ in MeV, with error bars indicating the uncertainty at different $p_{\text{rel}}$ values. The graphs compare theoretical predictions (NLO, N2LO, N3LO) with lattice data.
Lattice spacing $a = 0.99$ fm:
NP phase shifts, $\alpha = 1.97$ fm.
Nuclei binding

With the new lattice interaction, we calculate the binding energy of the light and medium-mass nuclei using the Auxiliary Field Monte Carlo simulation.

S. Elhatisari, et al, work in progress
Summary

- We have proposed a new lattice formulation of the chiral NN force which is easily decomposed into partial channels. These new operators only survive in particular channels. This advantage simplifies the fitting procedure very much. The phase shifts with these new interactions are more accurate than the previous lattice calculations.

- Using these new interactions, we did some Monte Carlo simulations for the light and medium-mass nuclei. The binding energy are very promising.
Thanks!