Computation of hybrid static potentials from optimized trial states in SU(3) lattice gauge theory

Christian Reisinger Owe Philipsen Marc Wagner
Stefano Capitani Lasse Mueller

Goethe University Frankfurt, Institute for Theoretical Physics

East Lansing, July 24, 2018
36th International Symposium on Lattice Field Theory
References

K.J.Juge, J.Kuti, C.Morningstar. Gluon excitations of the static quark potential and the hybrid quarkonium spectrum [arXiv:hep-lat/9709131]


Outline

1. Hybrid mesons
   - Quantum numbers
   - Lattice trial states

2. Optimization of trial states

3. Results
   - Lattice setup
   - Sets of optimized trial states
   - Effective potentials
   - Hybrid static potentials

4. Flux tube shapes

5. Outlook
Quantum numbers

Hybrid meson: meson with excitations in the gluon fields → exotic quantum numbers possible

- $\Lambda = 0, 1, 2, \ldots$, absolute angular momentum w.r.t separation axis
- $\epsilon = +, −$, eigenvalue of operator $P_x$, corresponding to reflection on the y-z-plane
- $\eta = g, u$, eigenvalue of operator $P \circ C$, the combination of parity and charge conjugation
Lattice trial states

Construction of trial states on the lattice:

- choose some non-trivial spatial path $S$ for a Wilson loop

- the state $|\psi_{\text{Hybrid}}\rangle_{S,\Lambda} = \sum_{k=0}^{3} \exp(i\Lambda k \pi/2) \mathcal{O}(k \pi/2) |\Omega\rangle$ has defined angular momentum $\Lambda$

- use projectors

$$P_{P_x,\epsilon} = \frac{1 + \epsilon P_x}{2} \quad P_{P \circ C, \eta} = \frac{1 + \eta P \circ C}{2}$$

to project $|\psi_{\text{Hybrid}}\rangle_{S,\Lambda}$ onto the subspace of eigenstates to $P_x, P \circ C$

...
Creation operators

- the state

\[ |\Psi_{\text{Hybrid}}\rangle_{S;\Lambda^\epsilon} = \mathcal{P}_\mathcal{P}_x,\epsilon \mathcal{P}_\mathcal{P}_C,\eta |\Psi_{\text{Hybrid}}\rangle_{S;\Lambda} \]

\[ = \bar{q}(\frac{-r}{2})a_{S;\Lambda^\epsilon}(\frac{-r}{2}, \frac{r}{2})q(\frac{r}{2}) |\Omega\rangle \]

has defined quantum numbers \( \Lambda^\epsilon \)

- \( \rightarrow \) generate creation operators

\[ a_{S;\Lambda^\epsilon} = \frac{1}{4} \sum_{k=0}^{3} \exp(i\Lambda k \pi/2) \hat{R}(k \pi/2) \]

\[ \times (S + \eta S_P + \epsilon S_{\mathcal{P}_x} + \epsilon \eta S_{\mathcal{P}_P \mathcal{P}_x}) \]

- ... which shape to choose?
Hybrid static potentials

- Compute hybrid static potentials from correlation functions

\[ \mathcal{W}_{S, S'; \Lambda^\epsilon} (r, t) = \langle \Psi_{\text{Hybrid}}(r, t) S; \Lambda^\epsilon | \Psi_{\text{Hybrid}}(r, 0) S; \Lambda^\epsilon \rangle \]

\[ \sim_{t \to \infty} \exp(-V_{\Lambda^\epsilon}(r)t) \]

- Usual problem: signal-to-noise ratio decreases exponentially for increasing \( t \)
- Find shapes which generate trial states with large ground state overlap
- Extract hybrid static potentials at region with larger signal-to-noise ratio
- To identify suitable shapes, we compute the effective mass

\[ V_{\text{eff}, S; \Lambda^\epsilon}(r, t) a = \ln \left( \frac{\mathcal{W}_{S, S'; \Lambda^\epsilon} (r, t)}{\mathcal{W}_{S, S'; \Lambda^\epsilon} (r, t + a)} \right) \]

at small separations \( t/a = 1, 2 \) and 100 gauge configurations for a large set of operators \( S \)
Operator set

Starting set of operators

- arrows represent straight lines of links
  - solid: length $\geq 1$
  - dotted: length $\geq 0$
- we vary
  - length of any straight line
  - placement on the separation axis
- colors mark paths of same length

$n$: number of distinct transformations (considering all rotations) $S, S_P, S_{P_x}, S_{PP_x}$
Optimizing operators

Example: optimizing $S_{I,1}$ with $x, z$ extensions $E_x, E_z$ for state $\Pi_u$

→ variations $S_{I,1}^{E_x, E_z}$

- variation of $E_x$

![Graph showing variations of $S_{I,1}$ with $r/a$]
Optimizing operators

Example: optimizing $S_{1,1}$ with $x, z$ extensions $E_x, E_z$ for state $\Pi_u$

→ variations $S_{1,1}^{E_x, E_z}$

- variation of $E_z$

![Diagram showing variations of $E_z$ at different $r/a$ values.]
Lattice setup

- Wilson plaquette action
  \[ S_g[U] = \frac{\beta}{3} \sum_n \sum_{\mu<\nu} \text{Re}\{\text{Tr} [1 - U_{\mu\nu}(n)]\} \]

- Lattice dimensions: \(24^3 \times 48\)
- \(\beta = 6.0 \rightarrow a \approx 0.093 \text{ fm}\)
- 5500 gauge configurations, generated using Chroma QCD
- check for autocorrelation using binning
- APE smearing of spatial links
  - \(\alpha_{\text{APE}} = 0.5\)
  - optimized \(N_{\text{APE}} \approx 20\)
Sets of optimized trial states

- Optimization of linear combinations too expensive
- Each operator is optimized independently for each sector $\Lambda_{\eta}^\epsilon$ and quark separation $r/a$
- Compute the correlation matrix $C(t)$ using a subset of 3 best operators for each sector

\[
\sum \sum g
\]

\[
S_{\text{III},1} = U_x^2 U_y^2 U_z^2 U_{-x}^2 U_{-y}^2 U_{-z}^2
\]

<table>
<thead>
<tr>
<th>$r/a$</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_z$</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

- Perform variational analysis by solving a generalized eigenvalue problem

\[
C(t)v_n(t, t_0) = \lambda_n(t, t_0)C(t_0)v_n(t, t_0)
\]
Effective potentials

Exemplary effective potentials for sector $\Sigma_u^-$

- previous results, non-optimized operators
- no real plateau visible, only crude guess possible

- effective potentials from optimized operators
- plateau reached much earlier, allowing for fit in region with favorable signal-to-noise ratio
Hybrid mesons

Optimization of trial states

Results

Flux tube shapes

Outlook

$r_0 (V(r) - V_0(2r_0))$

$r_0 (V(r) - V(2r_0))$

Previous results

On the lattice field strength tensor corresponds to plaquette

\[
P_{\mu\nu} = \text{Tr} \left[ e^{igaF_{\mu\nu}} \right] \Rightarrow \text{Tr} \left( F_{\mu\nu}^2 \right) \approx \frac{2}{g^2 a^2} (2 - P_{\mu\nu})
\]

\[\Rightarrow E^2 \text{ and } B^2 \text{ are gauge invariant quantities}\]

\[
\Delta B_j^2 \equiv \langle B_j(\vec{x})^2 \rangle_{Q\bar{Q}} - \langle B_j^2 \rangle_{\text{vac}} = \frac{2}{g^2 a^2} \left[ \langle P_{kl} \rangle - \frac{\langle W \cdot P_{kl}(T/2, \vec{x}) \rangle}{\langle W \rangle} \right]
\]

\[
\Delta E_j^2 \equiv \langle E_j(\vec{x})^2 \rangle_{Q\bar{Q}} - \langle E_j^2 \rangle_{\text{vac}} = \frac{2}{g^2 a^2} \left[ \frac{\langle W \cdot P_{0j}(T/2, \vec{x}) \rangle}{\langle W \rangle} - \langle P_{0j} \rangle \right]
\]
hybrid static potential flux tube profile on mediator axis

Very recently flux tubes for hybrid static potentials have been investigated for the first time in

with discrepancies to our work.
The only major difference in the computations were SU(3) rather than SU(2) lattice gauge theory.

In contrary to this work, our studies involved treating the E- and B-field components separately which led to results consistent with analytic approaches in

The next step will be to compute chromoelectric and chromomagnetic field strength components in SU(3) Lattice gauge theory.
Outlook

- Use the obtained hybrid static potentials to solve Schrödinger equations and obtain hybrid meson spectra
- computations using a smaller lattice spacing
- QCD configurations
Thank you!