Progress Towards Understanding the H-dibaryon from Lattice QCD

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Motivation for studying the H-dibaryon

Overview of recent results from the Mainz group
- $N_f = 2$ CLS ensembles with quenched strange quark
- Operator construction
- Use of Distillation

Preliminary results on $N_f = 2 + 1$ CLS ensembles

Future work
Motivation

- In 1977, Jaffe predicts deeply bound dibaryon with quark content $uuudss, J = 0, I = 0$
- Conclusive experimental evidence for such a state is still lacking
- Early quenched calculations disagree on existence of such a bound state
- More recent results with dynamical quarks from NPLQCD and HAL QCD disagree on the binding energy for $m_\pi \approx 800$ MeV
The Mainz Dibaryon Project

- In collaboration with:
  A. Francis, J. Green, P. Junnarkar, H. Wittig

- Recent results on $N_f = 2$ CLS ensembles with quenched strange quark (arXiv:1805.03966)
  - Main focus on two ensembles with $a = 0.0658$ fm and $L = 2.1$ fm
    - E1: $m_\pi = 960$ MeV, quenched $m_s = m_{u,d}$
    - E5: $m_\pi = 440$ MeV, quenched $m_s \approx m_s^{\text{phys}}$
  - Uses smeared point sources and Distillation
  - Finite volume analysis

- Recent extensions to $N_f = 3$
Interpolating Operators

• Hexaquark operators inspired by Jaffe’s bag model prediction:

\[ [rstuvw] = \epsilon_{ijk}\epsilon_{lmn}(s^i C\gamma_5 P_+ t^i)(v^l C\gamma_5 P_+ w^m)(r^k C\gamma_5 P_+ u^n) \]

• Can form singlet \( H^1 \) and 27-plet \( H^{27} \) flavor combinations

• Two-baryon operators:
  • Momentum-projected single-baryon operators

\[ B_\alpha(p, t)[rst] = \sum_x e^{-ip\cdot x}\epsilon_{abc}(s^a C\gamma_5 P_+ t^b)r^c_\alpha \]

• Form spin-zero and spin-one operators

\[ [B_1 B_2]_0(p_1, p_2) = B^{(1)}(p_1)C\gamma_5 P_+ B^{(2)}(p_2) \]
\[ [B_1 B_2]_i(p_1, p_2) = B^{(1)}(p_1)C\gamma_i P_+ B^{(2)}(p_2) \]

• Form \( SU(3) \) flavor operators from \( \Lambda\Lambda, \Sigma\Sigma, \) and \( N\Xi \)
Rotational Properties of Operators

• Python package using SymPy library to determine rotation properties

• Can very simply construct needed operators:

```python
u = QuarkField.create('u')
a = ColorIdx('a')
i = DiracIdx('i')
...
Delta = Eijk(a,b,c) * u[a,i] * u[b,j] * u[c,k]
```

• Project to definite momentum, and determine Little Group

```python
delta_ops = Operator(Delta, P([0,0,1]))
delta_op_rep = OperatorRepresentation(*delta_ops)
delta_op_rep.littleGroupContents()
# output: 6 G1 + 4 G2
```

• Supports multi-particle operators
Ground State for Singlet Channel on $E_1$ ($SU(3)$ Symmetric)

- Legend indicates sink operators
- Hexaquark operators noisier and slower ground-state saturation

\[ aE_{\text{eff}} \]

\[ t \text{ [fm]} \]

\[ E_1, \text{ singlet} \]

\[ H_{1,N}, H_{1,M} \]
\[ H_{1,N}, BB_{1,N,0} \]
\[ BB_{1,N,0}, BB_{1,N,1} \]
\[ BB_{1,N,0} \]
Adding Distillation to the Mix

- Use of point sources requires local operators at the source
- Leads to non-Hermitian correlator matrices

\[ \langle H(t)H^\dagger(0) \rangle \]

\[ \langle BB(t)H^\dagger(0) \rangle \]

- Add use of timeslice-to-all method
• Ensemble E1, ground state in singlet channel
• Better quality data at lower cost

\((E_{\text{eff}} - 2m_{\Lambda,\text{eff}}) \text{ [MeV]}\) vs. \(t \text{ [fm]}\)

Graph showing data points for Distillation and Point-to-all methods.
Finite Volume Analysis - Lüscher Method

- S-wave scattering phase shift:

\[ p \cot \delta_0(p) = \frac{2}{\sqrt{\pi}L\gamma} P_{00}(1, q^2), \quad q = \frac{pL}{2\pi}, \quad p^2 = \frac{1}{4}(E^2 - P^2) - m_\Lambda^2 \]

- Pole below threshold indicates a bound state

\[ A \propto \frac{1}{p \cot \delta_0(p) - ip} \]

\[ \Rightarrow p \cot \delta_0(p) = -\sqrt{-p^2} \]
Comparison to Other Collaborations

- Green are \( SU(3) \)-symmetric, and blue are \( SU(3) \) broken

\[
\begin{array}{c}
\Delta E \text{ [MeV]} \\
m_\pi \text{ [MeV]}
\end{array}
\]

- HAL QCD
- NPLQCD
- This work, distillation
- This work, FV-analysis
$N_f = 2 + 1$ CLS Ensembles

- Beginning extensions to CLS ensembles with $N_f = 2 + 1$ $O(a)$-improved Wilson fermions

- Initial results for the $SU(3)$-symmetric point, $m_\pi = m_K \approx 420$ MeV
  - U103 - $\beta = 3.40$, $24^3 \times 128$, open BCs
  - H101 - $\beta = 3.40$, $32^3 \times 96$, open BCs
  - B450 - $\beta = 3.46$, $32^3 \times 64$, periodic BCs
U103: $P^2 = 0$, $A_1^+$ irrep

![Graph showing $aE_{eff}$ vs $t/a$ for singlet, 27-plet, octet, and $2m_{\Lambda,eff}$](image_url)

- **Singlet** (blue dots)
- **27-plet** (red dots)
- **Octet** (green dots)
- **$2m_{\Lambda,eff}$** (black dots)

**U103 ($P^2 = 0, A_1^+$)**
B450: $P^2 = 0$, $A_1^+$ irrep

\[ 2m_{\Lambda, \text{eff}} \]

- singlet
- 27-plet
- octet

\[ aE_{\text{eff}} \]

\[ t/a \]

B450 ($P^2 = 0$, $A_1^+$)
H101: $SU(3)$ octet, $P^2 = 1$, $A_1$ irrep

\begin{align*}
\text{Level 0:} & \quad a_s E_{\text{fit}} = 1.0511(27) \\
& \quad \chi^2/\text{dof} = 0.61 \\
\text{Level 1:} & \quad a_s E_{\text{fit}} = 1.0724(30) \\
& \quad \chi^2/\text{dof} = 0.83 \\
\text{Level 2:} & \quad a_s E_{\text{fit}} = 1.1018(28) \\
& \quad \chi^2/\text{dof} = 0.81 \\
\text{Level 3:} & \quad a_s E_{\text{fit}} = 1.1262(25) \\
& \quad \chi^2/\text{dof} = 0.94 \\
\text{Level 4:} & \quad a_s E_{\text{fit}} = 1.1280(27) \\
& \quad \chi^2/\text{dof} = 0.94 \\
\text{Level 5:} & \quad a_s E_{\text{fit}} = 1.1604(41) \\
& \quad \chi^2/\text{dof} = 0.96
\end{align*}
Summary and Outlook

- Results for $N_f = 2$ ensembles shown
- Distillation substantially improves quality of data

Future Work

- Finalize $N_f = 3$ results
- Include $SU(3)$ broken ensembles
  - Coupled channels ($\Lambda\Lambda$, $N\Xi$, $\Sigma\Sigma$)
- Extensions to more ensembles
  - Cost scales as $N_{LapH}^4$
  - $N_{LapH}$ scales as $L^3$ for constant smearing radius
  - Investigate stochastic LapH
Questions?