Gluon structure of spin-1 meson as it becomes unstable using variationally optimized operators
Collaborators

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Hadron structure in QCD

• How to correctly describe a QCD state?
  e.g. $|n> = c_0 q\bar{q} + c_1 q\bar{q}q\bar{q} + c_2 q\bar{q}g + \ldots$

• Which one to choose? It there a hierarchy?

• Quantifying internal structure might help

• GPDs (Generalized Parton Distributions)

• Encode 3D distribution of partons in hadrons
Gluonic structure

- Gluons are essential in hadron and nuclear structure
- Gluon GPDs harder to measure experimentally than quark
- Better understanding of gluons goal of current and future experiments (GlueX, EIC)
Gluon GPDs in Lattice QCD

• Moments of GPDs (GFFs) directly related to matrix elements calculable on the lattice for **bound states**

• Analogous quark functions (e.g form factors) have been studied extensively in Lattice QCD

• Gluon GFFs less understood. In recent years they have been investigated for pion, phi, nucleon and nuclei

• Goal: understand the gluonic structure of unstable states (resonances)
Exotic mesons

• Spin addition and symmetry transformation of fermion wavefunctions

• Not all $J^P_C$ combinations can be built from $q\bar{q}$

• However multiple experimental candidates (e.g. $\pi_1(1600)$ at COMPASS)

• Possible explanations: Tetraquark states, meson molecules, glueballs, hybrids

Dudek et al 1004.4930
Hybrids $q\bar{q}g$

- Flux tube model (Isgur & Paton PRD 31 (1965), 2910)
  → supported by Lattice (Bali et al PRD 62 (2000) 054503):

- Gluonic flux tube between quark and antiquark

- Excitations of tube results in hybrid states, some exotic

- Model predicts allowed $J^{PC}$ and decay modes
Hybrids $q\bar{q}g$

From GlueX Collaboration (presentation to PAC30)

GlueX search for light hybrids

Meyer & Swanson arXiv:1502.07276
First case study: $\rho$ meson

- Mellin moments of GPDs related to gluonic operator matrix elements

\[
\tilde{O}_{\mu_1 \ldots \mu_n} = S[G_{\mu_1,\alpha} \hat{D}_{\mu_3} \ldots \hat{D}_{\mu_n} G^\alpha_{\mu_2}]
\]  
Spin-independent

\[
\tilde{O}_{\mu_1 \ldots \mu_n} = S[\tilde{G}_{\mu_1,\alpha} \hat{D}_{\mu_3} \ldots \hat{D}_{\mu_n} \tilde{G}^\alpha_{\mu_2}]
\]  
Helicity

\[
O_{\nu_1 \nu_2 \mu_1 \ldots \mu_n} = S[G_{\nu_1,\mu_1} \hat{D}_{\mu_3} \ldots \hat{D}_{\mu_n} G_{\nu_2,\mu_2}]
\]  
Transversity

- Subduce into appropriate irreps of hypercubic rotation/reflection symmetry

- Lowest twist ($n=2$) decomposition into Lorentz structure for spin-1 meson gives 7 spin-independent, 7 helicity + 8 transversity GFFs

- Each structure function defines a gluonic radius (slope at zero momentum transfer)
Forward limit spin independent GFFs

\[ \tilde{O}_{\mu_1\mu_2} = G_{\mu_1\alpha}G_{\mu_2}^\alpha - \frac{1}{4} g_{\mu_1\mu_2} G_\alpha^\alpha \]

Traceless part of gluon energy momentum tensor

\[ < p E' | S[\tilde{O}_{\mu_1\mu_2}] | p E > = S[M^2 E_{\mu_1} E_{\mu_2}] B_{2,1} + S[(E \cdot E') p_\mu p_\nu] B_{2,2} \]

Directly related to gluon momentum fraction in forward limit

On lattice proportional to 3-pt/2-pt function ratio:

\[ \frac{< O_n \tilde{O}^{E,\Lambda}_{\mu_1\mu_2} O_n^\dagger > - < O_n O_n^\dagger > < \tilde{O}^{E,\Lambda}_{\mu_1\mu_2} >}{< O_n O_n^\dagger >} \]

- \( \Lambda \) denotes the hypercubic irrep that the operator has been reduced to. Two convenient choices: \( \tau_{1}^{(3)} \tau_{3}^{(6)} \) (Goeckeler et al PRD 54 (1996))

- Can mix with quark operator of same dimension. However mixing has been shown to be 10% (Alexandrou et al PRD 96 (2017) 054503)
Stable heavy $\rho$ spin-indep GFFs

$24^3 \times 64$

$m_\pi \sim 450\ MeV$

Detmold, Pefkou, Shanahan PRD 95 (2017) 114515
Variationally optimized interpolators

- Use large basis of definite spin operators of form $\bar{\psi} \Gamma \mathbf{D} \ldots \mathbf{D} \psi$

- Reduce into cubic irreps if at rest or little group if in flight

- Optimized interpolators look like $\Omega_n^\dagger = \sum_i w_i^n O_i^\dagger$ where the weights are proportional to the eigenstates of the generalized eigenvalue problem $C(t)v^n = \lambda_n(t)C(t_0)v^n$

- Allows to probe in theory any state $\rightarrow$ spectra + structure functions of bound states

- Lüscher: infinite volume spectra of resonances
Results from Hadron Spectrum Collaboration

Wilson, Briceno, Dudek, Edwards, Thomas PRD 92, 094502 (2015)

Dudek et al PRD 82, 034508 (2010)
spin-indep $\rho$ matrix element

With variationally optimized operators

$$m_\pi \sim 700 \text{ MeV}$$

$$24^3 \times 128$$

$$\frac{a_s}{a_t} \sim 3.5$$

$$\tau_1^{(3)}$$
GFFs of unstable $\rho$

- Asymptotic state $\rightarrow$ Corresponds to complex valued pole
- No such thing in finite volume
- Several studies have looked into solving that problem
- In particular we will use the Briceno-Hansen formalism (non-perturbatively maps lattice matrix elements to infinite volume amplitudes)
- Necessary elements: $\pi$ elastic FFs, $\pi\pi$ matrix elements
- For more details see Alessandro Baroni's talk
Short term goals

• Calculate the infinite volume matrix element of $\rho$ resonance using BH formalism

• Extract gluonic GFFs and renormalize results

• Parallel efforts on renormalization of gluonic operators by Yang et al (arxiv:1805.00531).

• Compare with stable $\rho$ GFFs to interpret e.g. gluon momentum fraction similarities and differences

• What happens to the gluon radius?
Long term goals

• Expand formalism to exotic states (hybrids)

• Predict 3D picture of gluonic structure of states with explicit gluonic degrees of freedom. How does it compare with conventional states?

• Combine with experimental efforts to get better understanding of QCD and nature
THANK YOU