Form factors for moments of correlation functions

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Motivation

**Lepton universality**

Charge radius puzzle

μ vs e Hydrogen radius in 5σ tension @ 4%.

**B-meson semileptonic decay**

4σ tension from Standard Model prediction.

**Origin of matter**

*CP-violation from neutrinos*

T2K measures maximal CPV in PMNS @ 2σ. Future experiments aim to improve this. Axial form factor @10% improves constraint.

Direct calculation of slope is interesting for a wide range of applications.
Overview of moment methods

*Issues with moment methods:* Odd moments on lattice yields wrong ground state.

*Some existing related works:*  
Slope of the Isgur-Wise function  
HVP - temporal moments  
Radii - expansion of lattice operators  
ETMC - position space method

Existing methods take derivatives at $q^2 = 0$

*Proposed method:* Method takes $\partial / \partial q^2$ generalized to all momenta
Ensemble and correlator overview

2+1 flavor JLab isotropic clover ensemble
\( a \sim 0.12 \) fm
\( m_\pi \sim 400 \) MeV
\( N_x^3 \times N_t = 32^3 \times 96 \) and \( 48^3 \times 96 \)

\( \sim 200 \) configurations \( \times 2 \) sources \([0, 48]\)
3-point correlator \( T_{\text{snk}} = 8 \) and \( 10 \)
Isovector vector form factor is studied

Goal of this study
Low statistics demonstration of the proposed method.
Study finite volume effects.
Assess whether this is a viable strategy for improving lattice calculations for form factors.
Kinematic setup

\[ t_{\text{snk}} = t \]
\[ t_{\text{src}} = 0 \]

\[ p_z = 0 \]
\[ q_z = k \]

\[ t_{\text{snk}} = T \]
\[ t_{\text{ins}} = t' \]
\[ t_{\text{src}} = 0 \]
Two-point correlators

Two-point correlator

\[ C_{2pt}(t) = \int d^3 x \langle N_{t,x} | N_{0,0}^{\dagger} \rangle e^{-ikx_i} \]

Two-point moment

\[ \frac{\partial}{\partial k^2} C_{2pt}(t) = \int d^3 x \langle N_{t,x} | N_{0,0}^{\dagger} \rangle \frac{-ix_i}{2k} e^{-ikx_i} \]

\[ \lim_{k^2 \to 0} C'_{2pt}(t) = \int d^3 x \langle N_{t,x} | N_{0,0}^{\dagger} \rangle \frac{-x_i^2}{2} \]

Only have even spatial moments
Three-point correlators

Three-point correlator

\[ C_{3\text{pt}}(T, t') = \int d^3 x d^3 x' \langle N_{t,x} | \Gamma_{t',x'} | N_{0,0}^\dagger \rangle e^{-i k x'_i} \]

Three-point moment

\[ \frac{\partial}{\partial k^2} C_{3\text{pt}}(T, t') = \int d^3 x d^3 x' \langle N_{T,x} | \Gamma_{t',x'} | N_{0,0}^\dagger \rangle \frac{-ix'}{2k} e^{-i k x'_i} \]

\[ \lim_{k^2 \to 0} C'_{3\text{pt}}(T, t') = \int d^3 x d^3 x' \langle N_{T,x} | \Gamma_{t',x'} | N_{0,0}^\dagger \rangle \frac{-x'^2}{2} \]

Moments are with respect to current insertion

Given correlators, moments are computationally free
**z-direction correlator plots**

- **Two-point**
  - Approx. 1400 MeV
  - Sum over $x, y, t$
  - Plot z-direction effective masses.
  - These effective mass have contamination from backward signal.

**Key message**

- FV effects are exp. suppressed

- Two-point FV corrections suppressed by mass of initial/final states.

- Three-point FV suppressed by meson mass at the current.

*For charge radius, 3pt has larger FV effects.*
Finite volume correction

**Correlator construction**
Spatial moments are applied to $X/2$ (half the box size), due to periodic boundary.

**What is the error for doing this? (Assume only ground state dominate past $X/2$)**
1) Missing the spatial sum (in one dimension) from $X/2$ to infinite.
2) Wrong moment applied to the backward propagating signal from 0 to $X$.

**Correction to 1)** (This piece is missing)
\[
\int_{X/2}^{\infty} x^2 e^{-Ex} \, dx = \left( \frac{X^2}{4E} + \frac{X}{E^2} + \frac{2}{E^2} \right) e^{-EX/2}
\]

**Correction to 2)** (This piece is extra)
\[
\int_{0}^{X/2} x^2 e^{E(x-X)} \, dx = \left( \frac{X^2}{4E} - \frac{X}{E^2} + \frac{2}{E^2} \right) e^{-EX/2} - \frac{2}{E^3} e^{-EX}
\]

**Total correction**
\[
\delta_{\text{FV}}(E, X) = \frac{2X}{E^2} e^{-EX/2} + \frac{2}{E^3} e^{-EX}
\]

**Relative correction**
\[
R_{\text{FV}}(E, X) = EX e^{-EX/2} + e^{-EX}
\]

**For a 1% relative correction** (some numbers for reference)
if $E \sim 800$ MeV a 3.6 fm box is needed, if $E \sim 600$ MeV a 4.8 fm box is needed.

A similar approach can be found for HVP @ PRD 96 034516 (2017)
Two-point fit functions

Two-point fit function

\[ C_{2\text{pt}}(t) = \sum_n \frac{Z_n Z_n^\dagger}{2E_n} e^{-E_n t} \]

Two-point moment fit function

\[ C'_{2\text{pt}}(t) = \sum_n C_{2\text{pt}}^m(t) \left( \frac{2Z'_{n}}{Z_n} - \frac{1}{2E_n^2} - \frac{t}{2E_n} \right) \]

Definitions

\[ Z_n = \langle N|n \rangle \quad E_n = \sqrt{M_n^2 + k^2} \]

Two-point constrains all parameters except \( Z'_{n} \)
Three-point fit functions

Three-point fit function

\[ C_{3pt}(t, t') = \sum_{n,m} \frac{Z_n(0) \Gamma_{nm}(k^2) Z_m^{\dagger}(k^2)}{4E_n(0) E_m(k^2)} e^{-E_n(0)(t-t')} e^{-E_m(k^2)t'} \]

Three-point moment fit function

\[ C'_{3pt}(t, t') = \sum_{n,m} C'_{3pt}^{nm}(t, t') \left( \frac{\Gamma'_{nm}}{\Gamma_{nm}} + \frac{Z'_m}{Z_m} - \frac{1}{2E_m^2} - \frac{t'}{2E_m} \right) \]

2pt and 3pt constraints all params. except slopes

2pt moment needed for smeared source/sink

3pt moment constrains slope of form factor
Two-point related fits

**Fit strategy**
2 state Bayesian simultaneous fit
2pt, 2pt deriv., 3pt, 3pt deriv. @ q=[0,1]

Due to low statistics, fit to as little data as possible to control covariance matrix estimation.

Red (q=0) Blue (q=1)
Red / Blue bands are the G.S. fit results.
Dotted gray line is fit region.
Three-point related fits

**Central result: Slope of form factors**
Plot 3pt derivative / 3pt minus linear contaminations.

At *fixed* src-snk separation, the remaining contamination is linearly increasing.

Therefore, this plot has *linearly increasing* excited state contamination.
Some preliminary results

Check dispersion relation with $Z$

$$Z_s(k^2) \equiv \frac{\langle N(k^2)|0 \rangle}{\sqrt{2E(k^2)}} = \frac{\langle N(0)|0 \rangle}{\sqrt{2E(k^2)}} (1 \pm O(ak))$$

Light gray region marks $O(ak)$ constraint
Dark gray region marks $O(\alpha_s ak)$ constraint

Connected Dirac radius
Consistent with other results.

Red and Blue bands are slopes extracted directly from LQCD calc.

Connected vector charge

Slope here corresponds to yellow star

CIPANP18 Sergey Syritsyn
LHP collaboration

Preliminary result from this talk
Summary and outlook

Requires little additional computation time.

Obtain slopes of matrix elements (radii).

Further constrains shape of form factors.

Possible path to precision radius calculation.