Roper State from Overlap Fermion

- Discrepancy in various lattice calculations
- Fitting methods: variation vs sequential empirical Bayes method
- Chiral dynamics: Multi-hadrons from single hadron interpolater – $\eta \pi$ ghost state

QCD Collaboration:

M.Y. Sun, G. Wang, Y. Chen, T. Draper, M. Gong, K.F. Liu, J. Liang, R. Suffian, Y.B. Yang

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Many Facets of Roper Resonance

Theory:

- Quark potential model prediction is 100-200 MeV too high (Liu and Wong, 1983, Capstick and Isgur, 1986)
- Skyrmion can accommodate it as a radial excitation (J. Breit and C. Nappi, 1984, Liu, Zhang, Black, 1984; U. Kaulfuss and U. Meissner, 1985)
- Suggestion as a pentaquark (Krewald 2000); as a member of the antidecuplet (Jaffe, Wilczek, 2003)
- Perhaps a hybrid (Barnes, Close, etc. 1983)
- Lattice calculations

(Build of PDG--1440 MeV)
Quenched Lattice Calculations of Roper
Roper on the lattice

4 issues about lattice calculations:

- Radial excitation or pentaquark state?
- Dynamical fermions
- Variation vs Bayesian fitting
- Chiral dynamics
Roper
Radial excitation? $q^4q$ State?

- Roper is seen on the lattice with three-quark interpolation field.
- Weight:

$$|<0|O_N|R>|^2 > |<0|O_N|N>|^2 > 0 \quad \text{(point source, point sink)}$$

$$\sum O_N(x) \quad \sum O_N(x)$$

Point sink \quad Wall source

$$<0|O_N(0)|N> \quad <N| \sum \psi(x) \sum \psi(y) \sum \psi(z)|0 > > 0$$

However,

$$<0|O_N(0)|R> \quad <R| \sum \psi(x) \sum \psi(y) | \sum \psi(z)|0 > < 0$$

1S

2S
Roper and Nucleon Wavefunctions at $m_{\pi} = 438$ MeV

$O_{RN} = 0.59$
Dynamical Fermions
Variation on JLab anisotropic $24^3 \times 128$ Clover lattice ($m_{\pi} = 390$ MeV, $a = 0.12$ fm)

4 smearing sizes, the largest $<r^2>^{1/2} = 0.86$ fm

$$M_R = 1.92(6) \text{ GeV}$$

3 smearing sizes, the largest $<r^2>^{1/2} = 0.39$ fm

$$M_R = 2.19(11) \text{ GeV}$$
Variation on $24^3 \times 64$ DWF lattice with overlap valence ($m_T = 330\text{ MeV}, a = 0.111\text{ fm}$)

GEVP with projected correlator

$$\tilde{C}(t) = U^T C(t) U, \quad U = [u_1, u_2]$$

$$\tilde{C}(t) v_n(t, t_0) = \lambda(t, t_0) \tilde{C}(t_0) v_n(t, t_0)$$
Check source size dependence

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<th>Method</th>
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Why such a difference between clover and overlap fermion?

- Not due to fitting algorithm -- variation agrees with SEB for both clover and overlap fermions

- Chiral dynamics?
  - Dynamical coupled-channel model predicts couplings to \( N\pi, N\eta \) and \( N\pi\pi \) brings down the bare \( N \) by ~ 400 MeV.
  - Higher Fock space components needed in experimental electroexcitation amplitude of Roper.
  - Bethe-Salpeter wavefunctions of Roper and nuclear are less orthogonal as pion mass decreases.
  - Parity reversal of Roper and \( S_{11} \) might be due to meson exchanges between quarks (Glozman and Riska)
Isovector scalar correlator in quenched approximation

Ghost would-be $\eta\pi$ state

$$-W_{\eta\pi} (1 + m_\pi t) e^{-2m_\pi t}$$

Indication of the strength of coupling to multihadrons with one hadron interpolation field.
Comparison of would-be $\eta\pi$ ghost state on quenched lattices

$a = 0.12$ fm, pion mass = 296 MeV  
$a = 0.09$ fm, pion mass ~ 280 MeV
Minima of ghost state for overlap and Wilson fermions

Ratio (overlap to Wilson) $\sim 7.6$

Ratio (overlap to Wilson) $\sim 3.9$
N to Nπ, Nη, Nππ coupling

Using both $q^3$ and $q^4\bar{q}$ operators have not see the Roper below 1.65 GeV.

C.B. Lang et al., 1610.01422
A.L. Kiratidis et al., 1704.08816
Summary

- SEB method and variational approach give consistent results separately for the clover and overlap fermions, but the Roper from clover fermion is ~ 300 MeV higher than that of overlap.

- Model and experimental electroexcitation suggest large higher Fock space in Roper.

- Compare the ghost would-be $\eta\pi$ states of Wilson and overlap fermions on quenched lattices and found large discrepancy at $a = 0.12$ and $0.09$ fm.

- Chiral symmetry for clover fermion may be restored below $a \sim 0.06$ or $0.04$ fm(?). Clover calculation at these small lattice spacings may be the final solution to the `Roper Puzzle'.
Backup
Evidence of $\eta'N$ GHOST State in $S_{11} (1535)$ Channel

$S_{11}$ Correlation Functions

$m_{\pi} = 0.342(6)$
$m_{\pi} = 0.290(6)$
$m_{\pi} = 0.248(7)$

$m_{\pi} = 0.212(7)$
$m_{\pi} = 0.196(7)$
$m_{\pi} = 0.181(8)$

Time-Slices ($t$)

$W > 0$
$W < 0$
**N* spectrum in LQCD & dynamical coupling**

**Lattice N* states (m_π=396MeV)**

Dynamics of P_{11}-states:
The bare state at ~1750 MeV through coupling to inelastic channels generates 2 poles below 1400 MeV. They are identified with the “Roper” resonance.

LQCD finds states as predicted in SU(6)xO(3)

*R. Edwards, J. Dudek, D. Richards, S. Wallace, PRD84, 074508 (2011)*