Phase structure of multiflavor gauge theories: Critical exponents of Fisher zeros near the endpoint

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Motivations and objectives

- Location of conformal window on multiflavor $SU(3)$ theories. Is 12 flavors conformal?
- How to study the IFRP? If the underlying continuum theory has a IRFP what to expect the lattice counterpart?
- For finite volume and non zero mass, 12 flavors has first order bulk transitions in $m \times \beta$ plane and second order phase transition at the end-point (Xiao-Yong Jin, Robert D. Mawhinney DOI 9789814566254 0011)
- Expectation: $m \neq 0$ destroys conformality
- Needs to be studied indirectly
- RG: Fixed points can be studied from critical exponents around critical points nearby, such as the mentioned endpoint

Previous work on complex RG flows

- Given a RG transformation $a \to ba$,

$$L \to L/b,$$

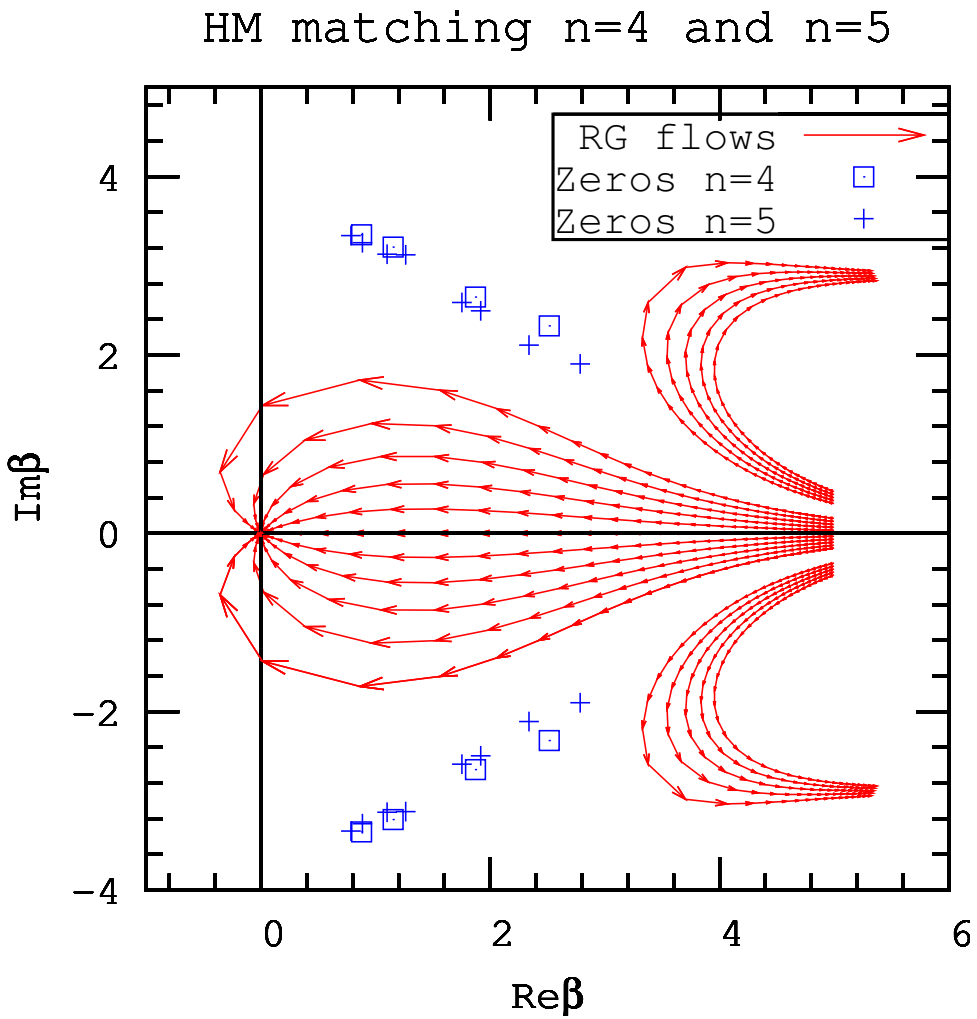
$$f_{sing} \to b^4 f_{sing}$$

$$f = -ln(Z)/V \Rightarrow Z_{sing} \to Z_{sing}$$

- The zeros of the partition function stay the same after RT
- Finite volume. f contains no singularities so we look at zeros in the complex $\beta = 6/g^2$ plane (Fisher zeros)
- Fisher zeros acts as separatrices of RG flows on complex plane
- (e.g. Denbleyker, A Et al Physical review letters. 104. 251601)

Previous work on complex RG flows

Fisher zeros acts as separatrices of RG flows on complex plane



(figure from same paper mentioned earlier)

Methodology

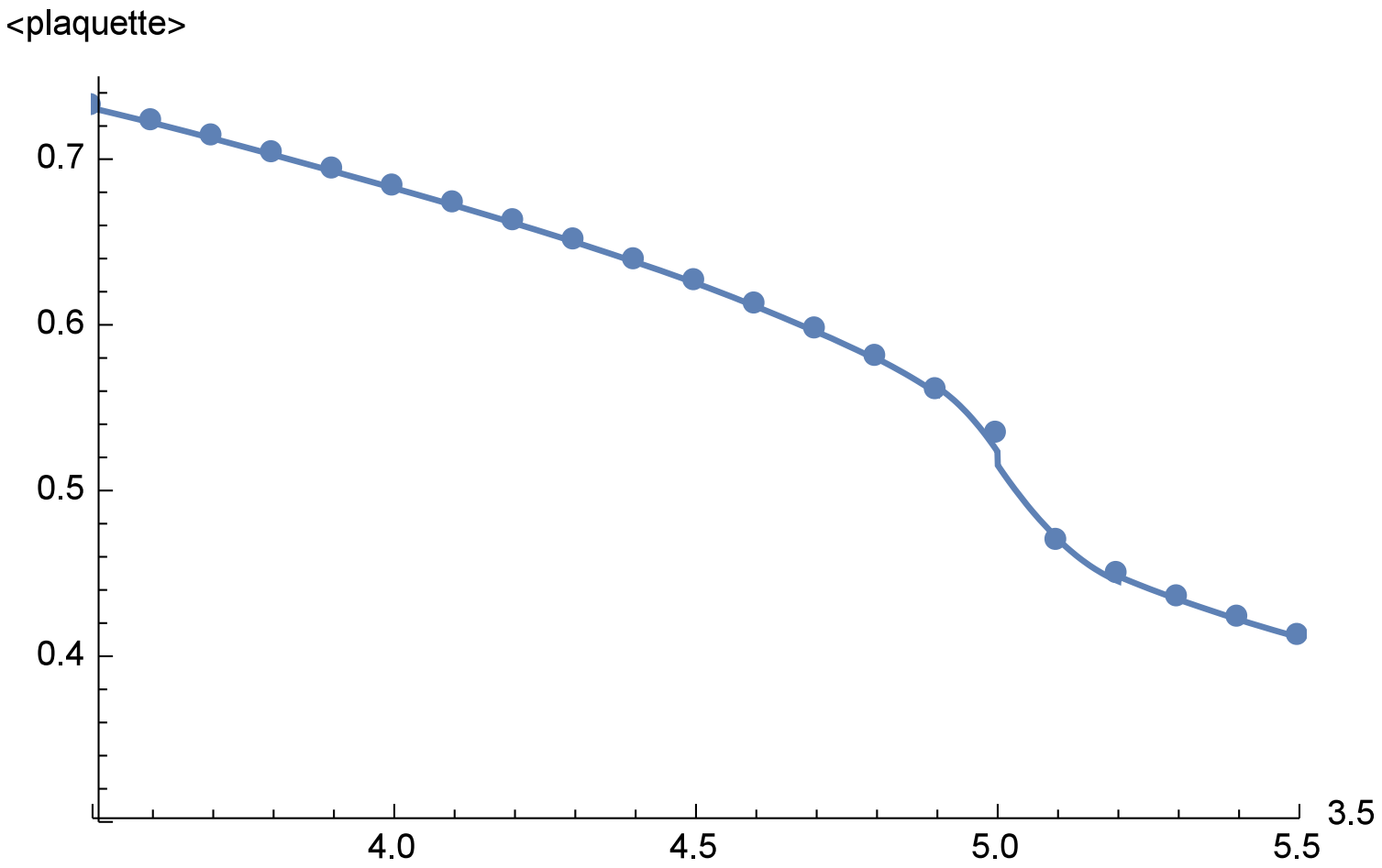
- Using Rational Hybrid Monte Carlo (code by Donald Sinclair) with unimproved staggered fermion action. Running on NERSC computing systems.
- Average plaquette $\langle U \rangle$ and chiral condensate $\langle \bar{\psi}\psi \rangle$ are measured
- Each simulation (5000 trajectories) gives information at vicinity of simulated β_0 can be used to obtain $Z(\beta_0 + \Delta\beta)$
- To connect the simulations we use the Ferrenberg-Swendsen algorithm

$$Z(\beta) = \int_0^{2N_p} dS n(S) e^{-\beta S}$$

$$n(S) = \frac{\sum_\alpha H_\alpha(S)/g_\alpha}{\sum_\alpha \left(e^{F_\alpha - \beta_\alpha S}\right)/g_\alpha} \quad , \quad e^{-F_\alpha} = \sum_S n(S) \Delta S e^{-\beta_\alpha S}$$

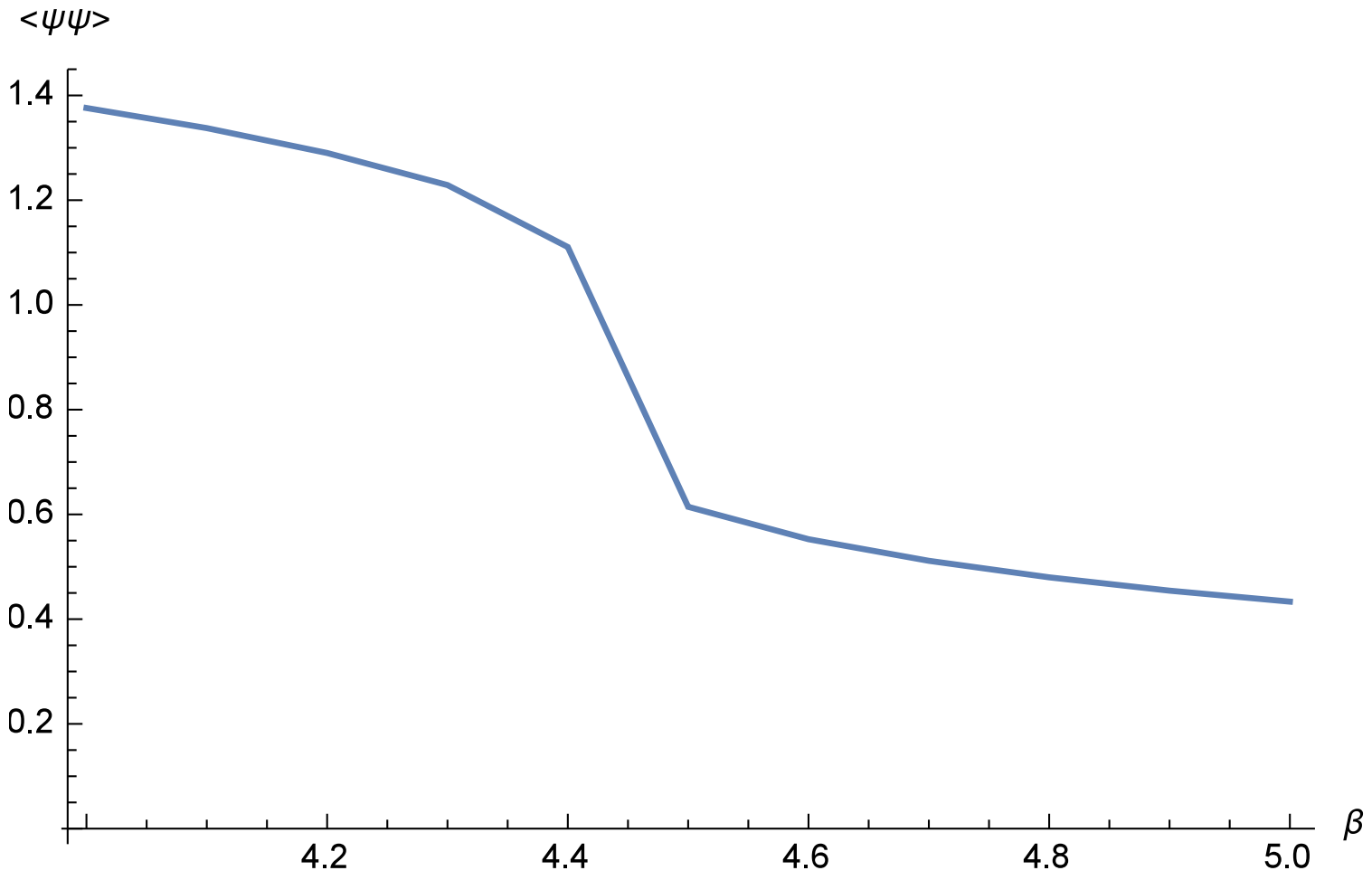
Convergence of Ferrenberg-Swendsen algorithm

Measured (points) versus calculated (curve) with resulting $n(s)$ $V = 4^4$,$m = 0.02$



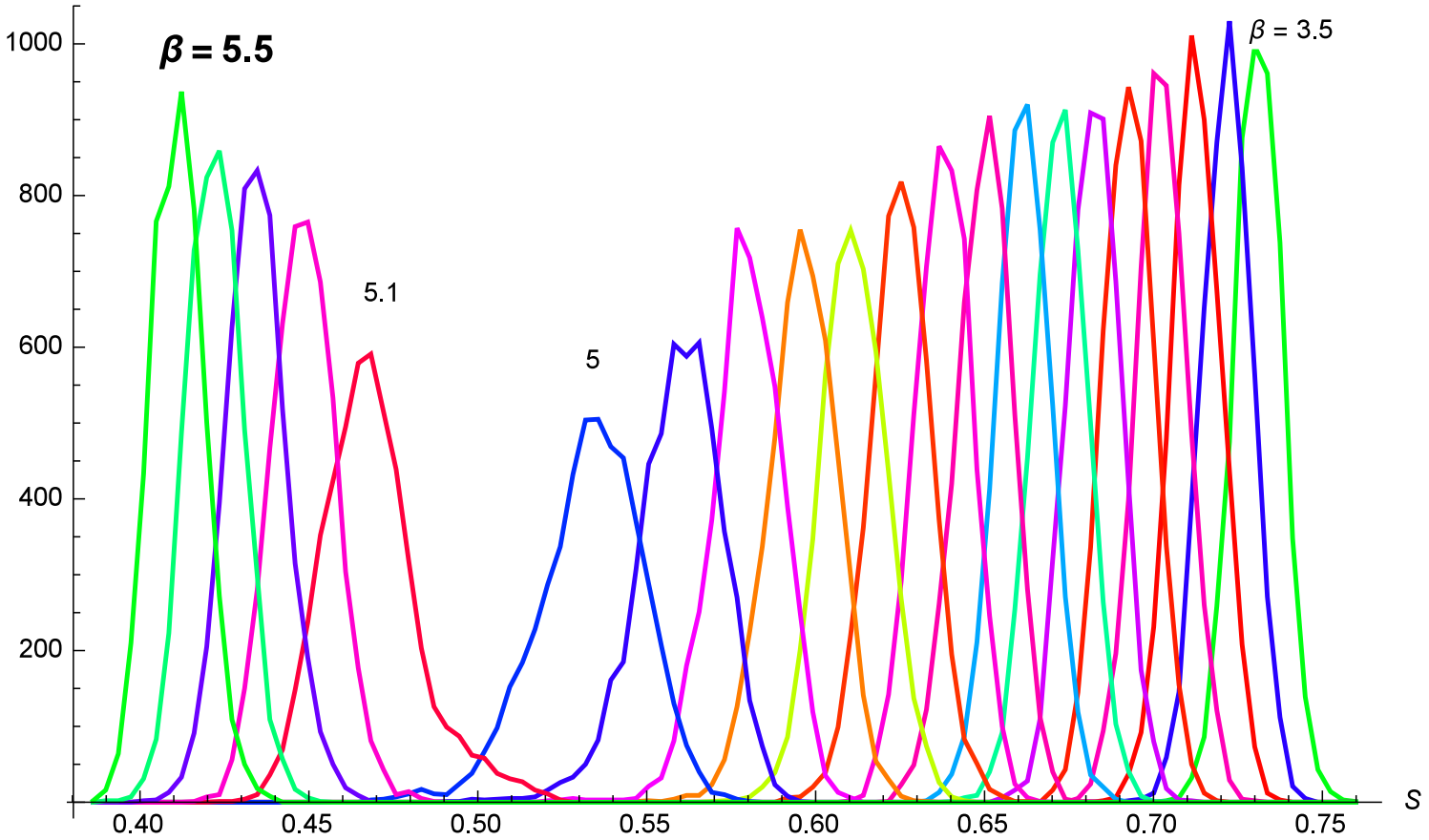
Chiral condensate

$V = 6^4$,$m = 0.1$



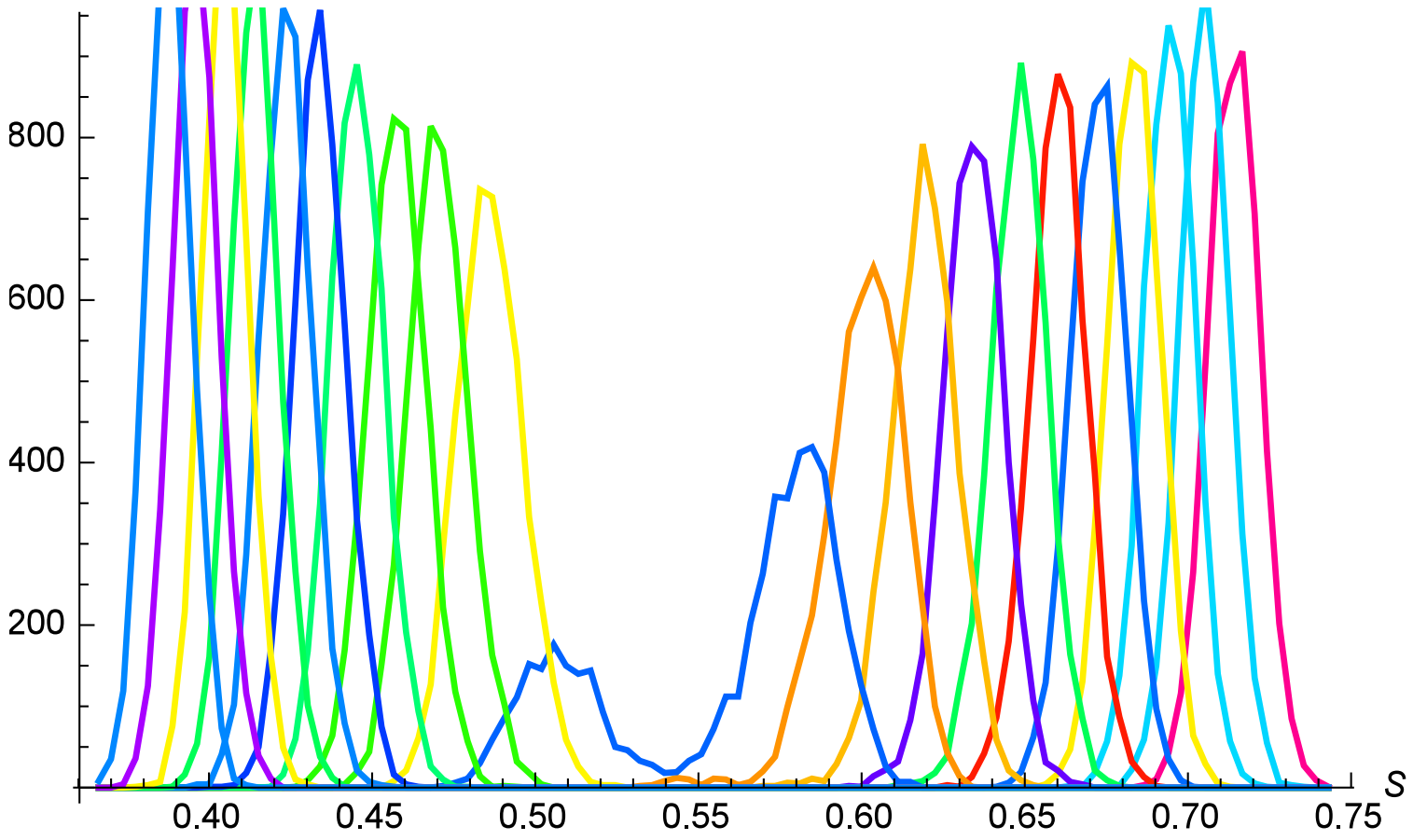
Binning of average plaquette

Histogram for $V = 4^4$,$m = 0.5$



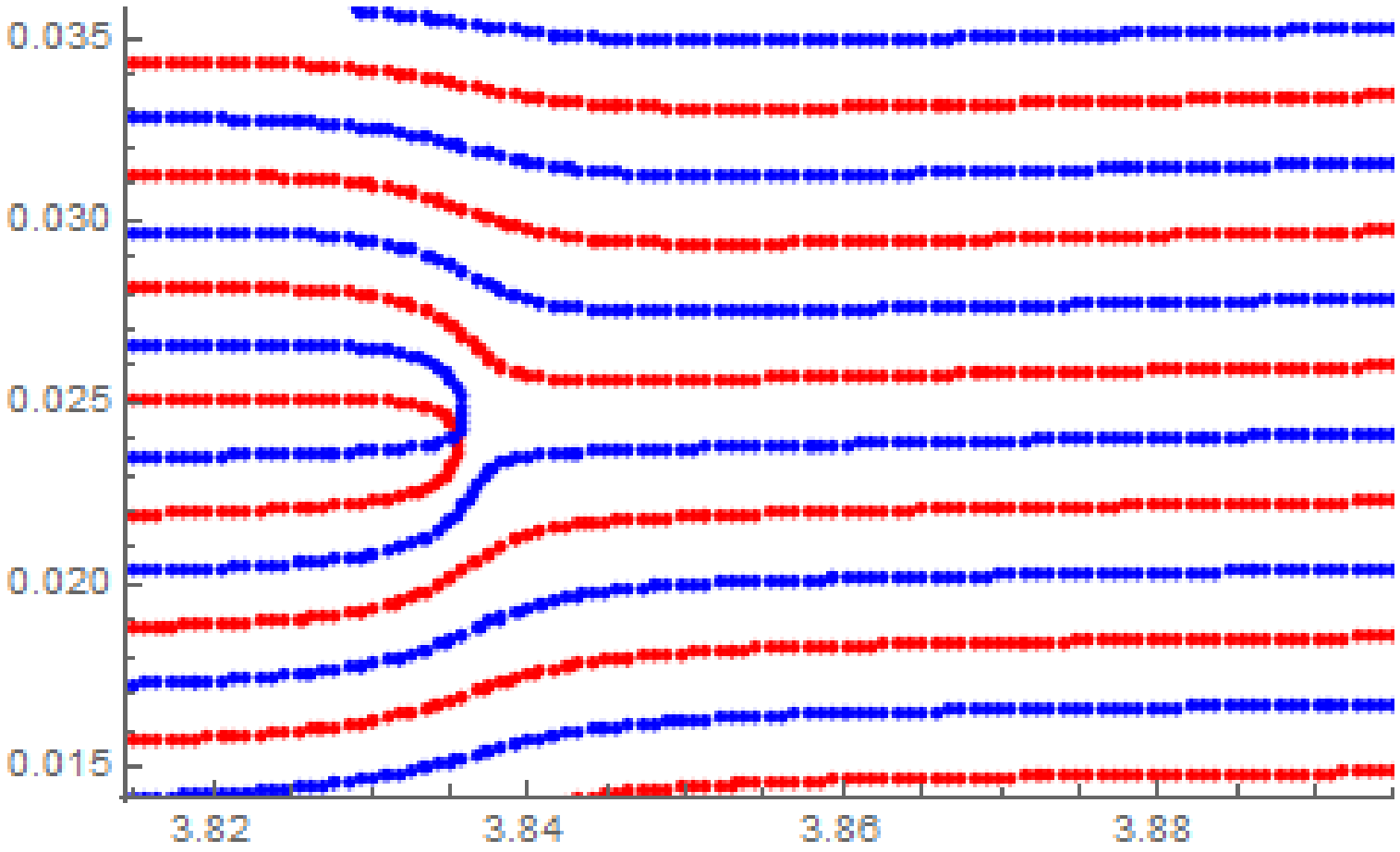
Binning of average plaquette

$V = 4^4$,$m = 0.02$



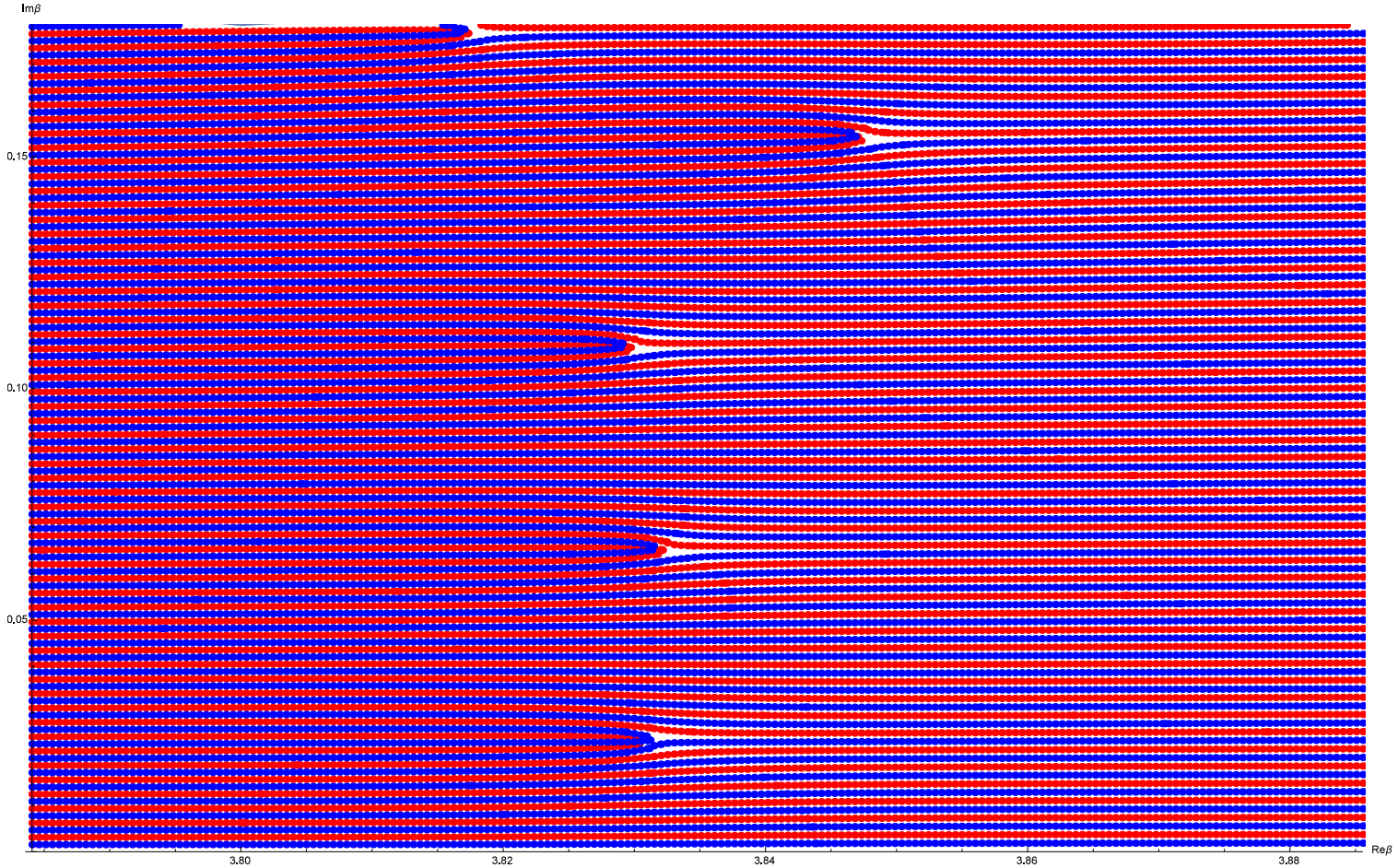
Fisher zeros

Intersection of $Re[Z] = 0$ and $Im[Z] = 0$



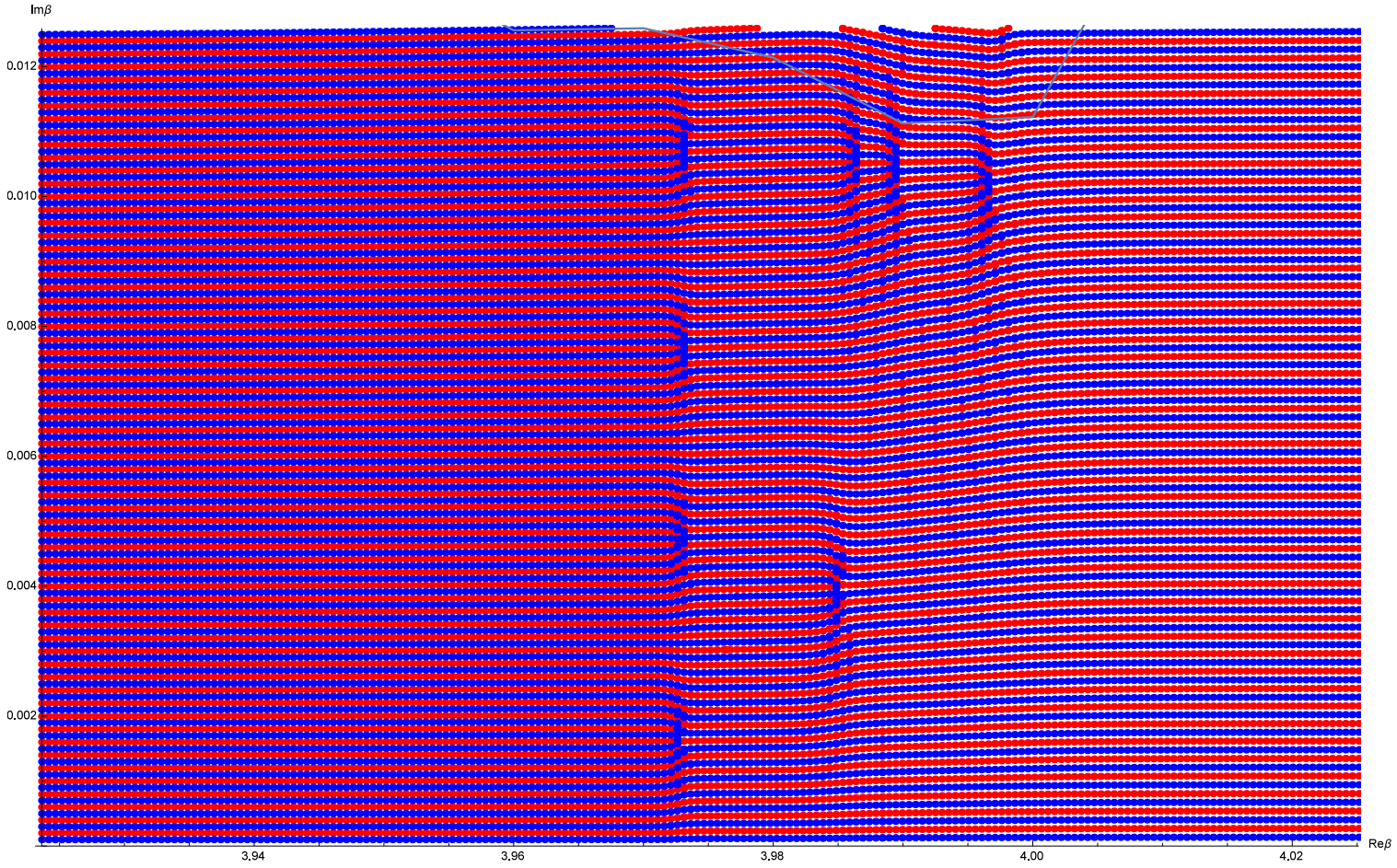
Fisher zeros

Wider view of zeros for $L = 4$, $m = 0.02$



Fisher zeros

Wider view of zeros for $L = 8$, $m = 0.02$ (same mass, higher volume)

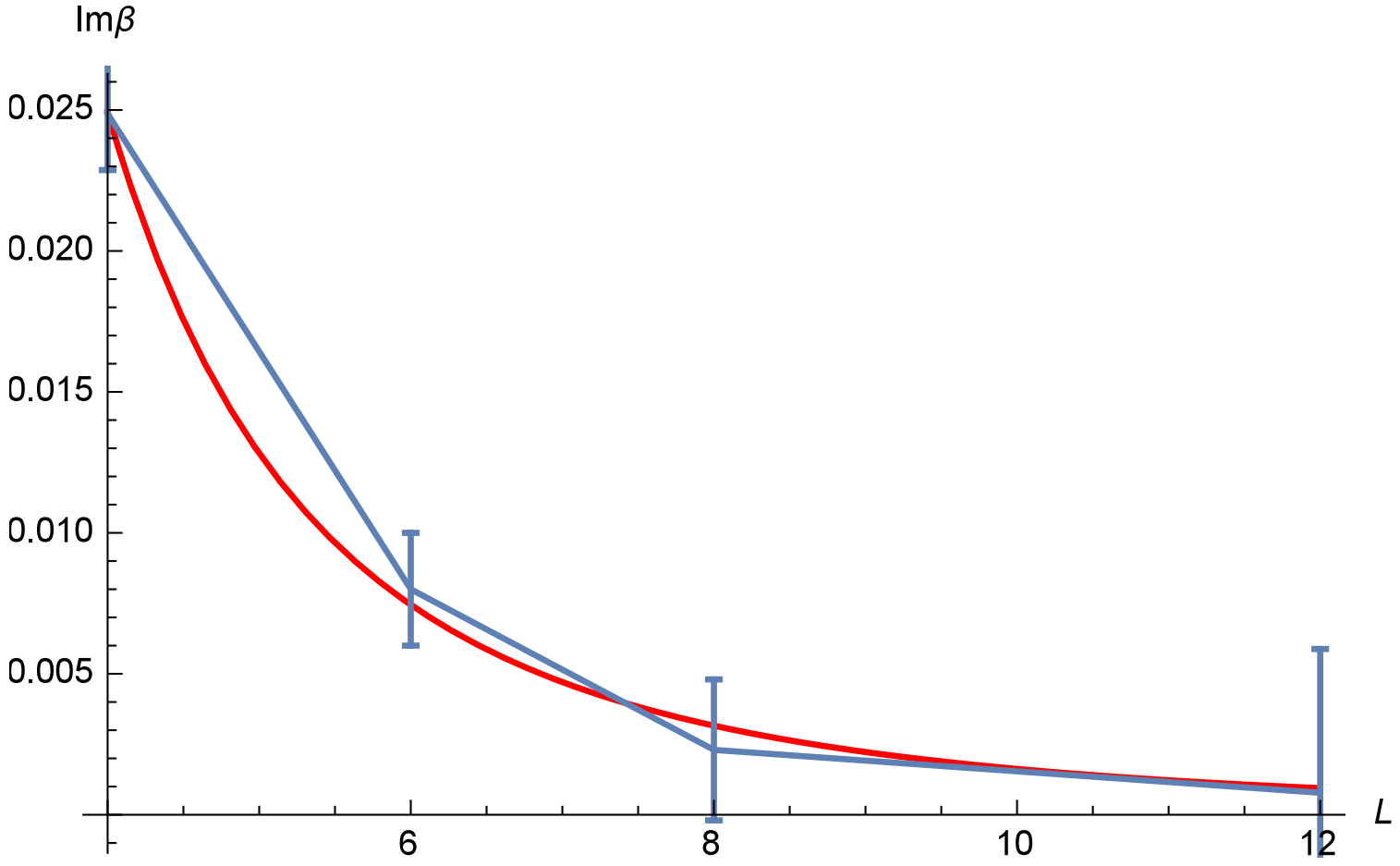


Pinching of Fisher zeros

- Zeros pinch the real axis as volume increases
- Lowest zero scales as L^{-4} for low mass indicating first order phase transition in infinite volume limit
- Near the endpoint in the $m \times \beta$ plane, scaling of lowest Fisher zero should reveal properties about relevant fixed point and its universality class
- (Denbleyker, A Et al Physical review letters. 104. 251601)

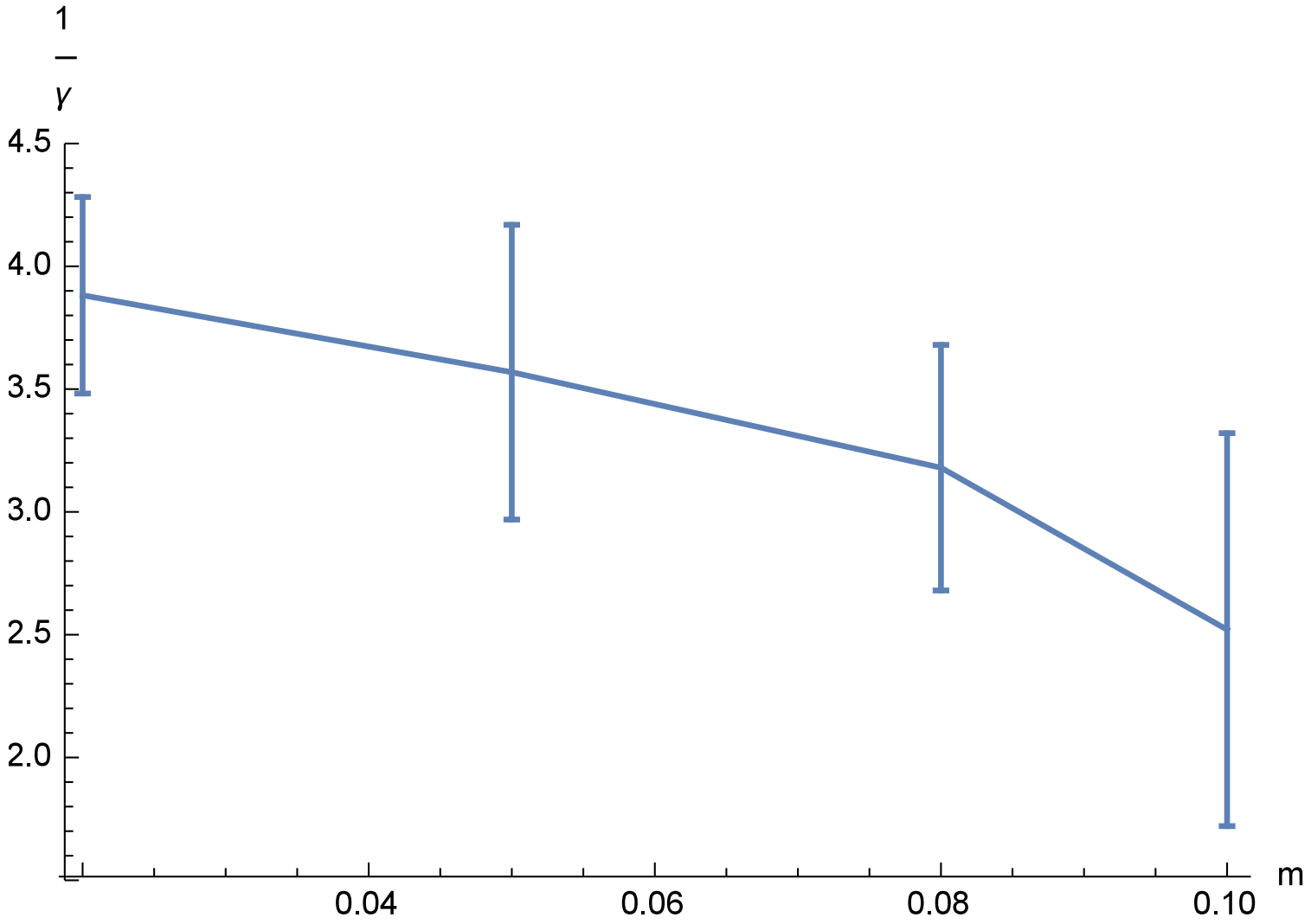
Critical exponents

$m = 0.1$, $\nu^{-1} = 3.05$
$Im\beta_f \propto L^{-3.05}$
(with preliminary error analysis)



Critical exponents

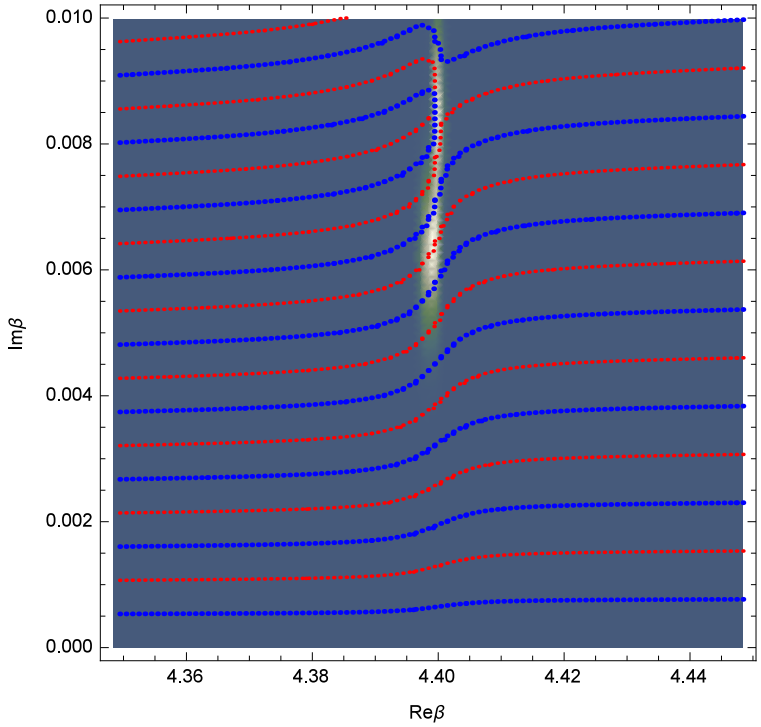
(with preliminary error analysis)



Is ν approaching 2?

Currently - Error analysis and more gathering of more data

- Zeros of L=12 require more time and are changing as we run more simulations
- Error analysis by bootstrap - resampling method of picking from the same data set



Linear sigma model with mass splitting

Previously (Y. Meurice PhysRevD.96.114507) a multiflavor sigma model was constructed with an anomaly term, reproducing a light sigma particle that behaves according to current lattice results for high N_f (remains lighter than pions) The model:

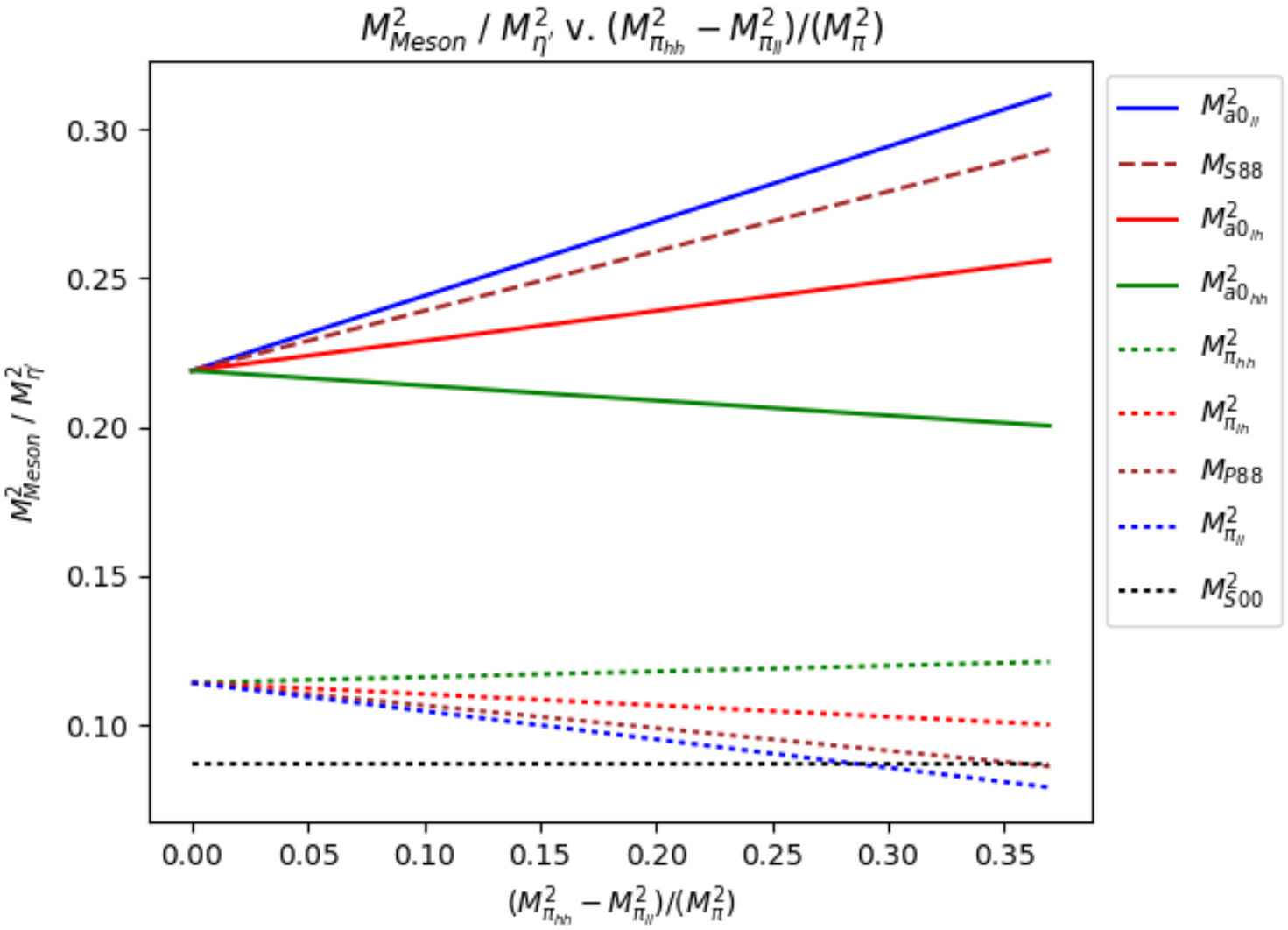
Lagrangian

- We define effective fields ϕ_{ij} which are $N_f \times N_f$ matrices transforming as $\bar{\psi}_{Rj}\psi_{Li}$ under $U(N_f)_L \otimes U(N_f)_R$. We use the parametrization,
  - $\phi = (S_\alpha + iP_\alpha)\Gamma^\alpha$, where the sum over $\alpha = 0, 1, ..., N_f^2 - 1$ is for a basis of $N_f \times N_f$ Hermitian matrices Γ^α such that:
  - $Tr(\Gamma^\alpha \Gamma^\beta) = (1/2)\delta^{\alpha\beta}$, and define $\Gamma_0 = I_{N_f \times N_f}/\sqrt{2N_f}$
- $\mathcal{L} = Tr(\partial_\mu \phi \partial^\mu \phi^\dagger) + V_0 + V_a + V_m$
- $V_0 = -\mu^2 Tr(\phi\phi^\dagger) + \frac{\lambda_\sigma - \lambda_{a0}}{2}(Tr(\phi\phi^\dagger))^2 + \frac{\lambda_{a0} N_f}{2} Tr((\phi\phi^\dagger)^2)$
- $V_a = -2(2N_f)^{N_f/2-2} X(det(\phi) + det(\phi^\dagger))$
- $V_m = -Tr(\mathcal{M}(\phi + \phi^\dagger))$

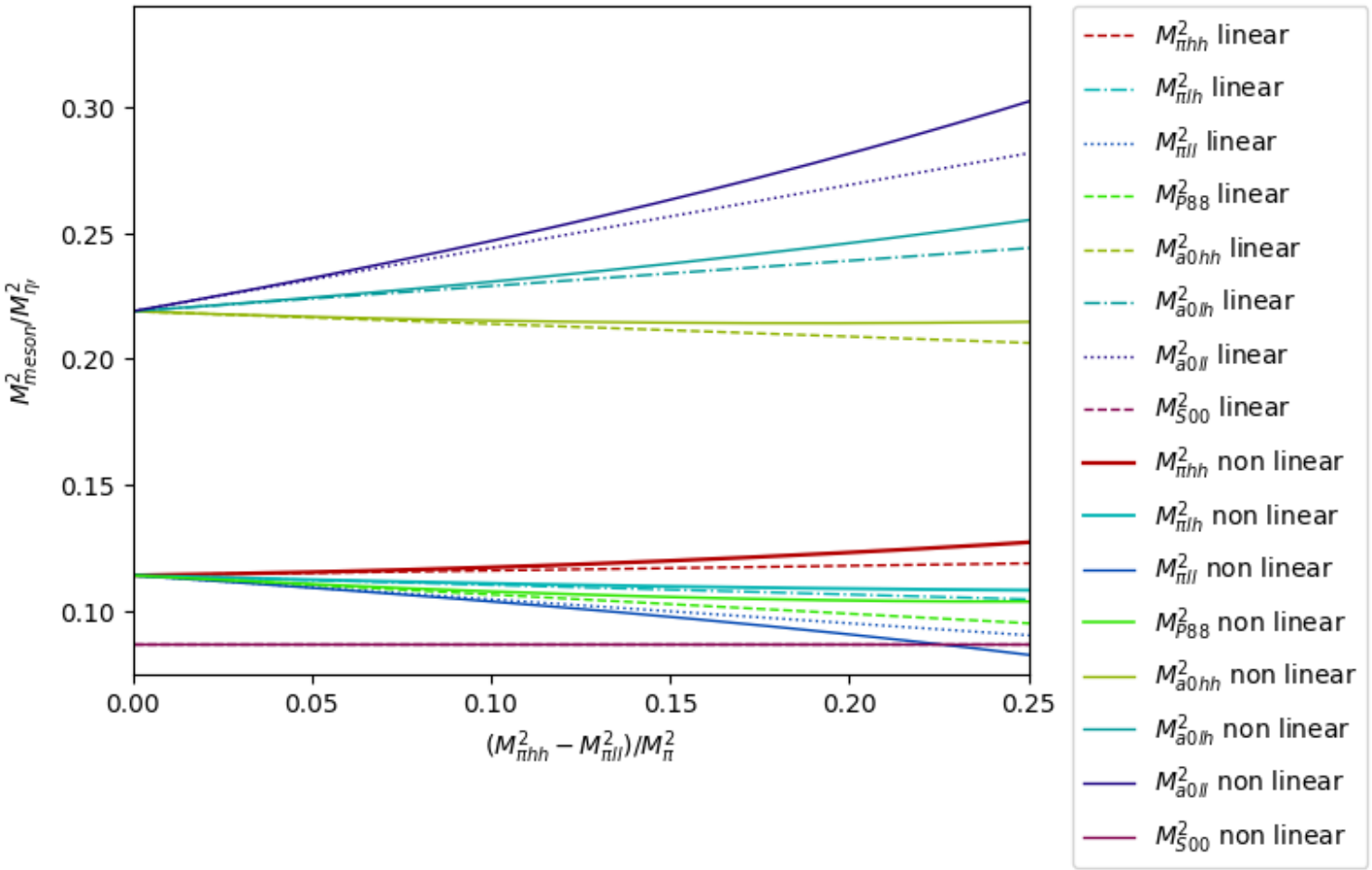
Mass spectrum

- Non-singlet pseudoscalar pectrum
  - $M^2_{\pi_{ll}} = \mathcal{C} + \frac{\lambda_{a0}}{2} v_1^2 - \frac{X}{N_f} v_1^{N_1-2} v_2^{N_2}$
  - $M^2_{\pi_{lh}} - M^2_{\pi_{ll}} = \left(\frac{\lambda_{a0}}{2} + \frac{X}{N_f} v_1^{N_1-2} v_2^{N_2-2}\right)(v_2 - v_1) v_2$
  - $M^2_{\pi_{hh}} - M^2_{\pi_{ll}} = \left(\frac{\lambda_{a0}}{2} + \frac{X}{N_f} v_1^{N_1-2} v_2^{N_2-2}\right)(v_2^2 - v_1^2)$
- Non singlet scalar spectrum
  - $M^2_{a0_{ll}} - M^2_{\pi_{ll}} = \lambda_{a0} v_1^2 + \frac{2X}{N_f} v_1^{N_1-2} v_2^{N_2}$
  - $M^2_{a0_{lh}} - M^2_{\pi_{lh}} = \lambda_{a0} v_1 v_2 + \frac{2X}{N_f} v_1^{N_1-1} v_2^{N_2-1}$
  - $M^2_{a0_{hh}} - M^2_{\pi_{hh}} = \lambda_{a0} v_2^2 + \frac{2X}{N_f} v_1^{N_1} v_2^{N_2-2}$

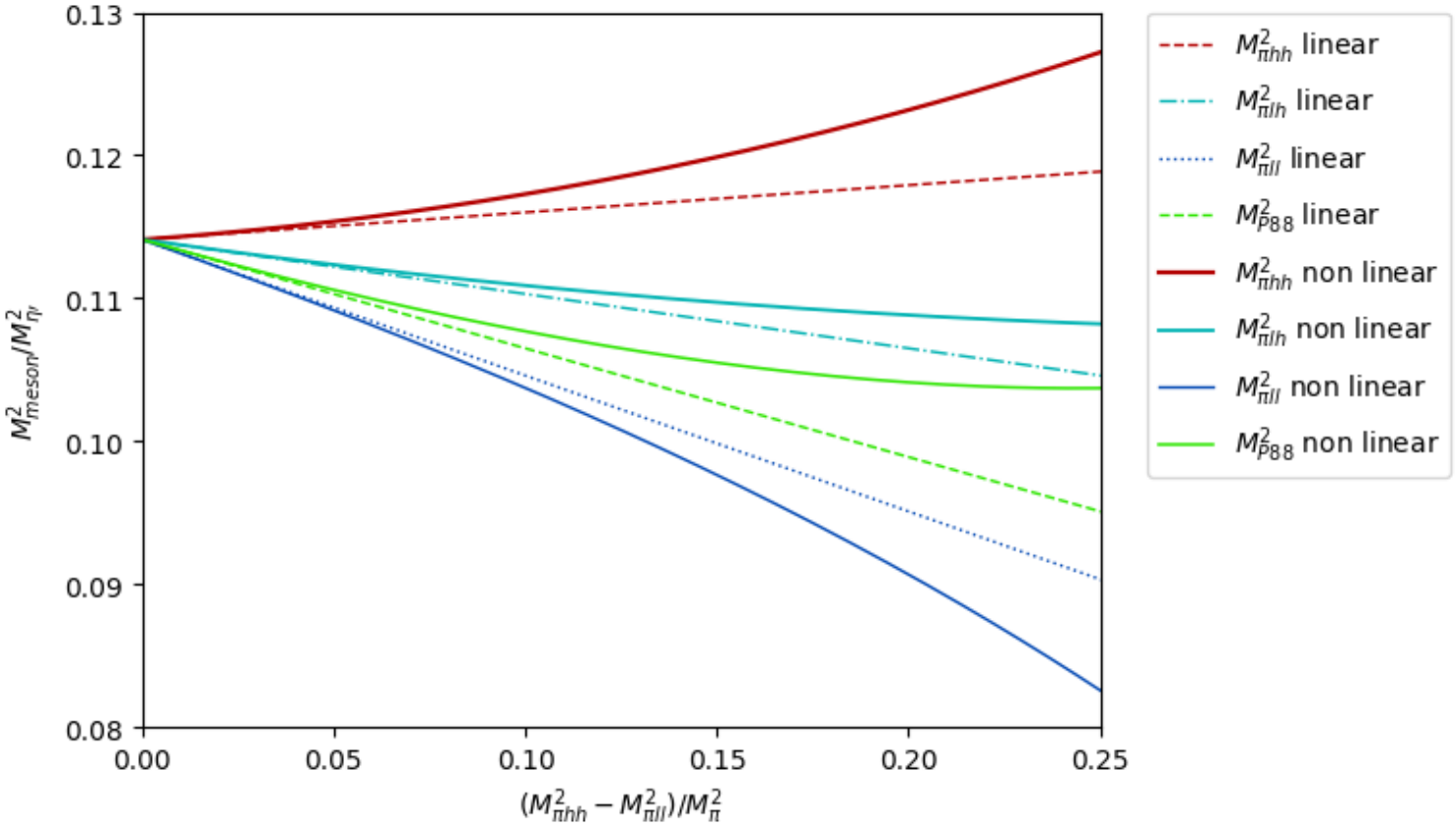
Linear


M^2_{Meson} / $M^2_{\eta'}$ v. $(M^2_{\pi_{hh}} - M^2_{\pi_{ll}})/(M^2_{\pi})$
0.30
0.25
0.20
0.15
0.10
M^2_{Meson} / $M^2_{\eta'}$
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
$(M^2_{\pi_{hh}} - M^2_{\pi_{ll}})/(M^2_{\pi})$
$M^2_{a0_{ll}}$
M_{S88}
$M^2_{a0_{lh}}$
$M^2_{a0_{hh}}$
$M^2_{\pi_{hh}}$
$M^2_{\pi_{lh}}$
M_{P88}
$M^2_{\pi_{ll}}$
M^2_{S00}

Non linear comparison - full spsctrum



Non linear comparison - pseudoscalars



Non linear comparison - scalars