On two-flavor QCD(adj): a journey from the small to large

MOHAMED ANBER

Lewis & Clark College

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WITH E. POPPITZ (U OF TORONTO)

Outline

- Overview, why do we do Yang-Mills over a (small) circle?
- Spectrum of QCD(adj) with two flavors on a circle.
- Spectrum of QCD(adj) on R^4 via 't Hooft anomaly matching conditions and Continuity between the small and large.

Overview

- Analytically tractable confining theories are scarce; any new method that sheds light on the problem is welcomed.
- AdS/CFT provides a novel analytical (circumventing) tool. This method mostly applies to the large-N limit.
- 4-D models do exist: Seiberg–Witten (SW) theory. Dyons and monopoles, confinement in the IR.
- SW strings, mesons, and baryons are very different from those of Yang-Mills! (SW vacuum breaks the Weyl symmetry).

Overview

- Yet, over the past decade a new paradigm has emerged: Yang-Mills on $R^3 \times S^1$ (dates back to Seiberg and Witten 1996, Davies, Hollowood, Khoze, Mattis 1999, Dorey, Parnachev 2000).

- Incarnation around 2007 (Unsal and Yaffe).

- Many works over the past decade, very promising results.

Aitken, MA, Argyres, Bergner, Cherman, Li, Kanazawa, Misumi, Piemonte, Poppitz, Simic, Schaefer, Shifman, Shuryak, Sulejmanpasic, Tanizaki, Thomas, Vairhinos, Voloshin, Unsal, Yaffe, Zhitnitsky …

Overview

- Disclaimer: Yang-Mills on $R^3 \times S^1$ (QCD-like) is not nature. It is, however, the closest **theory** to nature we know of.

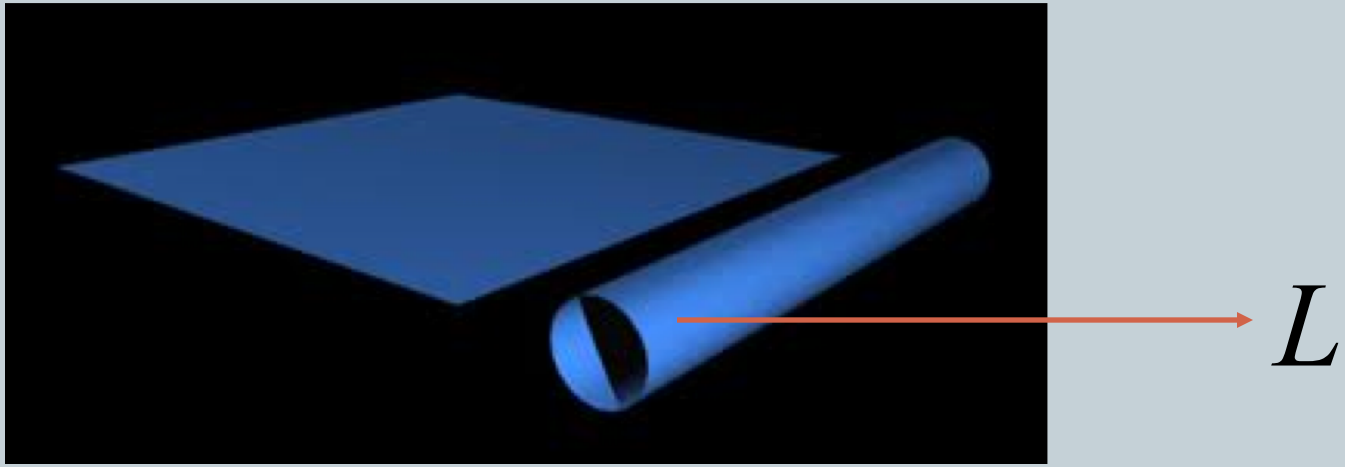
- It shares many essential features with QCD:
  - It has a center symmetry
  - Instantons, monopoles, and bions M. Unsal 2007
  - Asymptotic freedom and confinement in the IR
  - String spectrum M.A., Vito Pellizzani 2017; E. Poppitz, Shalchian 2017
  - Nature of phase transition
    E. Poppitz, T. Schafer, M. Unsal 2012;
    M.A., E. Poppitz, B. Teeple 2014;
    M.A., S. Collier, E. Poppitz, S. Strimas-Mackey, B. Teeple 2013;
    G. Bergner, S. Piemonte 2014

Overview

- This is a very encouraging sign for a possible connection (continuity) between QCD-like and QCD.
- SYM is continuously connected on the circle (Witten's index)
- 't Hooft anomaly matching can provide one more handle to connect the small with large circle theories beyond SYM.

$SU(N_c = 2)$ Yang-Mills on $R^3 \times S^1$ with two adjoint Weyls

QCD(adj)



QCD(adj) on a Circle

- QCD-like theories on $R^3 \times S^1$ are **not** theories at finite temperature.

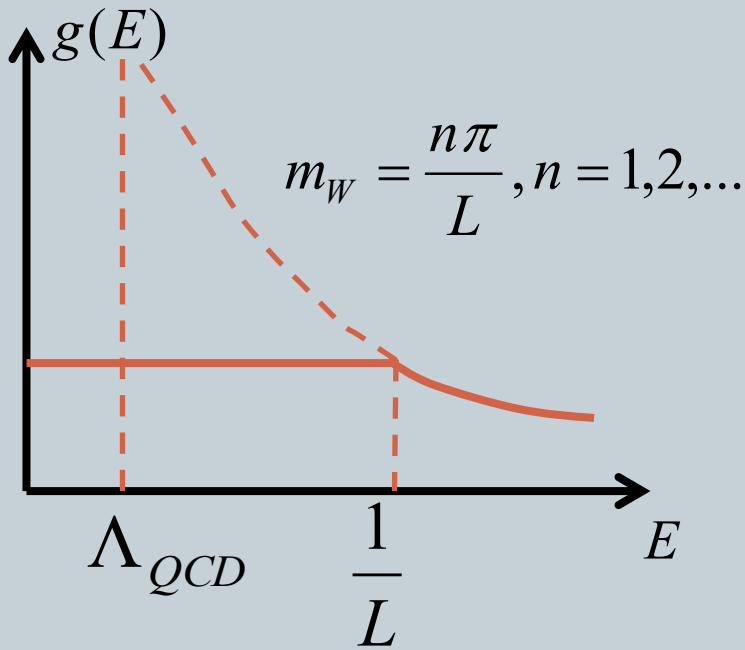
$$S = \frac{1}{g^2} \int_{R^3 \times S^1} \frac{1}{2} \mathrm{tr} \underbrace{\left[F_{\mu\nu}^2 \right]}_{SU_C(2)} + i \mathrm{tr} \underbrace{\left[\bar{\lambda}^I \bar{\sigma}_\mu D_\mu \lambda^I \right]}_{SU_f(2) \times U_A(1)}$$

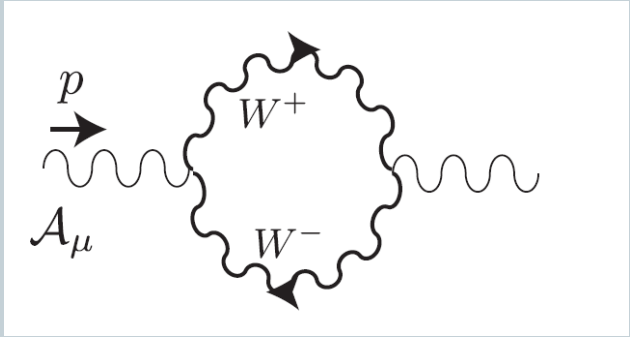
- In a BPST instanton background $U_A(1) \to Z_{2n_f N_c}^{d\chi} = Z_8^{d\chi}$

$$\left.\begin{aligned} A_\mu(t,x,y,z) &= A_\mu(t,x,y,z+L) \\ \lambda^I(t,x,y,z) &= \lambda^I(t,x,y,z+L) \end{aligned}\right\} \textbf{periodic boundary Conditions}$$

QCD(adj) on a Circle

- If we take $L\Lambda_{QCD} << 1$ then the theory is in a weakly coupled regime.





- Integrate a tower of KK excitations $\Rightarrow$ $\underbrace{V(A_3)}_{V_{\min}\left(A_3=\frac{\pi}{L}\right)}$

QCD(adj) on a Circle

- Center-symmetric vacuum $\mathrm{tr}_F\left[\mathrm{Exp}\left(i\oint_{S^1} A_3\right)\right]=0$
- The 3-D effective theory is abelian: $SU(2)\to U(1)$
- We use the abelian duality: $F_{\mu\nu}\sim\frac{g^2}{L}\varepsilon_{\mu\nu\alpha}\partial_\alpha\sigma$

$$S_{3D}=\int_{R^3}\frac{g^2}{L}\left(\partial_\mu\sigma\right)^2+\underbrace{\frac{L}{g^2}i\overline{\chi^I}\overline{\sigma}_\mu\partial_\mu\chi^I}_{\chi^I\equiv\lambda^{3I}}$$

- χ is $SU_f(2)$ doublet (uncharged under $U(1)$)

QCD(adj) on a Circle

- The theory also admits bions (correlated monopole events):

$$B(x) \equiv e^{-\frac{8\pi^2}{g^2}} e^{2i\sigma(x)}$$

- Under $Z_8^{d\chi} : \sigma \to \sigma + \pi$ (from monopole vertex)

- Summing up the bions:

$$\ell_{3D} \sim \left(\partial_\mu \sigma\right)^2 + B(x) + \overline{B}(x) = \left(\partial_\mu \sigma\right)^2 + e^{-\frac{8\pi^2}{g^2}} \underbrace{\cos\left(2\sigma\right)}_{\text{mass gap}}$$

QCD(adj) on a Circle

- The theory confines in R^3: the theory has a 1-form Z_2^C center symmetry:

Center symmetry is preserved

$$\mathrm{tr}\left[\exp\left(i\oint_{\text{in } R^3\text{plane}} A_\mu\right)\right] \xrightarrow[\text{1-form}]{\underbrace{Z_2^C}} \pm\mathrm{tr}\left[\exp\left(i\oint_{\text{in } R^3\text{plane}} A_\mu\right)\right] \quad \mathrm{tr}\left[\exp\left(i\oint_{\text{in } R^3\text{plane}} A_\mu\right)\right] = 0$$

- The theory has two vacua: $\sigma = 0, \sigma = \pi$

$$Z_8^{d\chi} \to Z_4^{d\chi} \quad \text{(domain walls)}$$

QCD(adj) on a Circle

- Summary of QCD(adj) on a ciercle :

- Global symmetries: $SU_f(2) \times Z_8^{d\chi} \times \underbrace{Z_2^C}_{1-\text{form}}$

- Spectrum: $SU_f(2)$ fermion doublet (singlet under $U(1)$)

$$Z_8^{d\chi} \to Z_4^{d\chi} \text{ (DW) and the theory confines}$$

QCD(adj) on R^4

- No analytical tools are available. Lattice studies are not conclusive. A. Athenodorou, E. Bennett, G. Bergner, B. Lucini, 2014

- Yet, one can use ‘t Hooft anomaly matching conditions to learn about the possible spectrum.

  Y. Frishman, A. Schwimmer, T. Banks, S. Yankielowicz, 1981;
  C. Csaki, H. Murayama, 1997

- It has always been assumed that the IR theory:

  (i) breaks $SU(2) \to SO(2)$ or (ii) is within the conformal window

QCD(adj) on R^4

- ‘t Hooft anomalies provide one more scenario that is continuously connected to the small circle.

- Disclaimer: ‘t Hooft anomalies are not inconsistencies within the theory. They are kind of probes: what if questions!

- Given **a global symmetry** G of a theory, we may try to gauge G. If there is an obstruction in gauging it, this should be true at all scales.

QCD(adj) on R^4

- Global symmetries of QCD(adj): $SU_f(2) \times Z_8^{d\chi} \times \underbrace{Z_2^C}_{1-\text{form}}$

- The new proposal of the IR spectrum:
- One $SU_f(2)$ doublet (baryon, singlet under $SU_C(2)$), broken $Z_8^{d\chi} \to Z_4^{d\chi}$, and confined fundamental probes

$$\mathcal{O}^{(1)} \equiv \left(\epsilon_{\alpha\beta}\lambda_i^{\alpha a}\lambda_j^{\beta a}\right)\left(\epsilon_{\alpha'\beta'}\lambda_{i'}^{\alpha' a'}\lambda_{j'}^{\beta' a'}\right)\epsilon^{ii'}\epsilon^{jj'}$$

$$Z_8^{d\chi} : O^{(1)} \to -O^{(1)}$$

$$SU_c(2), SU_f(2) : O^{(1)} \to O^{(1)}$$

$$\underbrace{\mathcal{O}^{(2)\,\gamma}_{[ij]k} \equiv \epsilon_{\alpha\beta}\lambda_{[i}^{\alpha\, a}\lambda_{j]}^{\beta\, b}\lambda_k^{\gamma c}\epsilon^{abc}}_{\text{baryon}}$$

QCD(adj) on R^4

- This spectrum is identical to the one on a small circle.
- We need to check that it is consistent with ‘t Hooft anomalies.
- Since the anomalies are RG invariant, we compute them in the UV. The IR spectrum must saturate the anomalies.

QCD(adj) on R^4

	$SU_c(2)$	$SU_f(2)$	$\mathbb{Z}_8^{d\chi}$	$\mathbb{Z}_4^{d\chi}$
λ_i^α	adj	$\square$	1	1
$\mathcal{O}^{(1)}$	1	1	4	$4 \equiv 0$
$\mathcal{O}^{(2)\,\gamma}_{[ij]k}$	1	$\square$	3	3

	UV	IR
$SU_f(2) - SU_f(2) - SU_f(2)$	Witten anomaly (3Weyls)	Witten anomaly (1 Weyl)
$SU_f(2) - SU_f(2) - Z_8^{d\chi}$	$N_c^2 - 1 = 3$	3
$G - G - Z_8^{d\chi}$	6	6

QCD(adj) on R^4

- There is also a mixed 't Hooft anomaly between $Z_8^{d\chi}$ and 1-form Z_2^C D. Gaiotto, A. Kapustin, Z. Komargodski,N. Seiberg 2017
- To see it:

(1) $Z_8^{d\chi} : \lambda \to e^{i\frac{2\pi}{8}}\lambda, \qquad [D\lambda] \to [D\lambda] e^{i\frac{2\pi}{8}\underbrace{(8)}_{2n_f N_c} Q_T}$

(2) turn on 't Hooft fluxes (gauging Z_2^C) $\Rightarrow$ $Q_T = \frac{1}{N_c}$

QCD(adj) on R^4

- The $Z_2^C - Z_2^C - Z_8^{d\chi}$ anomaly is saturated by the existence of two vacua (probed by $O^{(1)}$)
- The theory also preserves its Z_2^C center symmetry
- Thus, we obtain the same spectra of QCD(adj) on a circle.
- Such scenario can be examined in future lattice simulations (however, see also the proposal of Cordova and Dumitrescu: the spectrum should be supplemented with emergent Z_2 gauge symmetry)

Conclusion

- QCD-like theories on a circle provide a new means to understand nonperturbative phenomena in confining gauge theories.
- Combining this method with ‘t Hooft anomaly can give nontrivial perspective on strongly coupled gauge theories.
- Our proposal shows that QCD(adj) is more complex than what has been thought.