Updated spectroscopy for SU(3) with eight fundamental flavors

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Motivation/context

- So far LHC $\rightarrow$ Higgs and nothing else - but the Standard Model is still incomplete: $\nu$ mass, dark matter, Higgs naturalness

- BSM theories with strong dynamics (strongly-coupled gauge interactions) are interesting, need lattice to fully understand. Gauge group, fermion content can (and often does!) vary from QCD.

- Role of lattice here is not high precision! Main contributions from lattice are:
  - **Relating LECs** in terms of a handful of underlying parameters
  - **Revealing dynamical surprises**: LECs which are unexpectedly large/small, or possibly even that the EFT we guessed originally may not be correct

$$\mathcal{L}_{\text{EFT}} = c_1 + c_2 + c_3 + c_4 + \ldots$$

$$\mathcal{L}_{\text{UV}} = a + b$$
SU(3) with $N_f=8$

Why this theory?

- “Near-conformal”: lattice results so far hint this theory is very near the conformal transition

- SU(3) + fundamental rep: can use highly optimized codes available for QCD, adjusting the number of fermions is easy

- No particular pheno model connection in mind, although some attempts have been made\textsuperscript{1,2} (assumptions about low-energy EFT…)

\textsuperscript{1} S. Matsuzaki and K. Yamawaki, arXiv:1201.4722
\textsuperscript{2} LatKMI collaboration, arXiv:1302.6859
Main result is **light σ state for $N_f=8$**, degenerate with pions!

*(New this analysis: added statistics, new lightest 0++ result, refined analysis procedure.)*
Lattice setup and ensembles

- nHYP staggered fermions with fundamental-adjoint plaquette action
- High statistics for disconnected diagrams - determination of 0++ scalar
- Wilson flow scale \( t_0 \) used for scale setting. (Note: strong dependence on \( a m \))
- Finite volume: explicit tests show small effects, \( M_\pi L \sim 5 \) or larger for all ensembles used in main analysis
- No continuum extrapolation of results yet; taste breaking indicates \( a \sim 0.15 \) fm or so.

(4-flavor ensembles mainly as cross-check of methods; expected to be “QCD-like”.)
Fitting procedure

- Standard procedure for connected two-point functions; all fits use one pair of excited states ("2+2" model)

\[ C_{\text{eff}}(t) \equiv C(t)e^{E_0t} \]

- Fit-range systematic scan: dotted error bars show variation with \( t_{\text{max}} \), all fits with \( p>0.1 \) considered. Sys err from significant \((1\sigma)\) difference of mean.

"Best fit" from variant of the MILC criterion:

\[ C \equiv \frac{p \times N_{\text{dof}}}{\sum_n (\sigma_n/\mu_n)^2} \]

**Fully automated procedure** with conservative error from fit range choice.
Extracting the sigma mass

- For flavor-singlet states (like $\sigma$), propagator from source/sink to itself does not vanish $\rightarrow$ double-trace (disconnected) diagrams:

$$S(t) \sim \frac{N_f}{2} \left( G(0,0) - G(0,t) \right) - G(t,0) + G(0,t)$$

$$\equiv 2D(t) - C(t)$$

- We use stochastic estimation (6 U(1) wall sources/config) with dilution (suppresses off-diagonal noise correlations) to reduce noise in disconnected correlator. Fermionic operator only.

- In practice, fit finite difference correlator $\sim[D(t+1) - D(t)]$. Removes $t$-independent constant, which is annoying to fit; also reduces autocorrelation significantly.
- Joint fit to \( S=2D-C \) and \( D \), both of which contain \( M_{0^{++}} \) but have different systematics:

\[
S(t) = a_d \cosh \left[ M_{0^{++}} \left( N_t/2 - t \right) \right] + b_d (-1)^t \cosh \left[ M_{\eta_{sc}} \left( N_t/2 - t \right) \right]
\]

\[
D(t) = \frac{4}{N_f} \left\{ S(t) - (-C(t)) \right\} = a_d e^{-M_{0^{++}} t} - a_c e^{-M_{a_0} t} + (-1)^t \left\{ b_d e^{-M_{\eta_{sc}} t} - b_c e^{-M_{\pi_{sc}} t} \right\}
\]

Same procedure based on MILC criterion for best-fit selection and fit-range systematic error assignment.

Updated Spectroscopy: SU(3), Nf=8
Joint fits tend to be more stable with respect to t-range. (Note opposite sign of excited states in S vs. D correlators, left!)

Above: comparison of joint fit (green) to individual S, D fits for 0++

Joint extraction gives smaller error than just taking difference between S, D results as systematic
Vector Meson Saturation

• Saturation of vector channel by a single resonance ($\rho$) gives a phenomenological model of low-energy quantities, based on rho mass and width.

• VMS works well in QCD (~10%) for some things, e.g. KSRF\(^1\) relations:

$$F_\rho = \sqrt{2} F_\pi, \quad g_{\rho\pi\pi} = \frac{M_\rho}{\sqrt{2} F_\pi},$$

\[\Gamma_\rho \approx \frac{g_{\rho\pi\pi}^2 M_\rho}{48\pi} \approx \frac{M_\rho^3}{96\pi F_\pi^2}\]

large-$N_c$:

$$\frac{M_\rho}{F_\pi} \sim \frac{1}{\sqrt{N_c}}$$

• What happens in other strongly-coupled models?

• **Left:** Test of one KSRF relation ($F_\rho/F_\pi$), nice agreement, little mass dependence. Other relation gives $g_{\rho\pi\pi}$ from $M_\rho/F_\pi$ (convention: $F_\pi \sim 93$ MeV in QCD.)

(Thamm, Torre and Wulzer, 1502.01701)

$M_\rho \rightarrow g_\rho f$

(note different convention: $M_\rho = g_\rho f$)

- **Right:** $g_{\rho\pi\pi}$ leads directly to collider constraints on $\rho$ production in composite Higgs models. Lattice greatly reduces the available parameter space!

band using rough $g_{\rho\pi\pi}$ from QCD/lattice
Summary

• Vector-meson saturation seems to work well beyond QCD, and \( g_{\rho\pi\pi} \sim 6 \) (in my convention) is fairly insensitive to fermion mass/number.

• Light 0++ scalar, degenerate with pions, reliably established; comparison to \( N_f=4 \) helps. Joint S/D fit and marginalizing over fit range improve extraction of 0++ mass.

• What is the low-energy EFT? Adding more mass points, observables to discriminate, future work will compare model predictions.
Backup
Staggered taste breaking

- $N_f=8$ shown; $N_f=4$ looks similar, but somewhat smaller splitting.

- Pion taste splitting is on the order of 20%-30%; compare to 15%-20% splittings seen by the MILC collaboration for $a\sim0.12\text{fm}$ in QCD.
Finite-volume effects?

- Explicit tests for volume dependence on heavier masses reveal no large effects for any of the states we consider.

- No explicit tests available at lighter fermion mass, but all results shown in our central analysis satisfy $M_{\pi}L>5.3$. 

\[
\sqrt{8\ell_0 M_X} = \begin{cases} 
\infty & 16 \\
8 & 5 \\
4 & 1 \end{cases} \quad \begin{cases} 
\infty & 17 \\
9 & 6 \\
4 & 1 \end{cases}
\]

\[am_f = 0.0075, \quad am_f = 0.00889\]
Topological charge

- Good evolution of topological charge seen on all ensembles, even the largest $L=64$ (shown above, two streams.)
Scale setting

- Wilson flow + clover operator ("WWC") to determine $t_0$
- Dependence of flow scale on bare parameters much stronger for $N_f=8$ (left: fixed bare coupling $\beta$)
- Mass-dependent scale setting could leverage this for continuum extrapolation, but requires good understanding of $t_0$ in appropriate EFT...
Hyperscaling spectrum?

- Ratios of spectrum results are fairly constant for Nf=8; does that indicate infrared conformality \(\rightarrow\) “hyperscaling”?

- “Hyperscaling” predicts all states scale with same power-law dependence on \(m_f\), so ratios are constant.

- Closer look at more precise ratios reveals some movement: left, \(M_\rho/M_\pi\) shows some hint of expected behavior as \(m_f\rightarrow 0\).

- Larger question: what is the right chiral effective theory that describes our results? Subject of future work.
Scalar ($\sigma$, $0^{++}$) meson

\[ M_H (8t_0)^{1/2} \]

\[ M_\pi (8t_0)^{1/2} \]

(LSD preliminary)
- QCD looks very different at heavy mass!

\[ \kappa \text{ Dependence of Effective Masses} \]

with disconnected diagram
- \( m_{\text{molec}} a \)
- \( m_{\text{con}} a \)

connected diagram

- Tetra (plateau ?)
- Disconnected diagram is important.
- small overlap to ground state of molecule

\[ (m_\pi a)^2 \]

\[ 2m_\pi a \]

\[ \frac{1}{\kappa} \]
0++ on lightest 4f ensemble

Large error dominated by systematics (left plot).

Possible contamination by $2\pi$ state may be allowed by light $\pi$ mass + relatively heavy 0++? To be investigated in future work.