Sextet sigma particle or dilaton?
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with the Lattice Higgs Collaboration  (LHC)

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• walking $\beta$-functions? $\chi$SB phase? which models are potential dilaton candidates?

• linear $\sigma$-model with low mass $m_\pi \gtrsim m_\sigma$ requires extensions $\rightarrow$ dilaton?

• dilaton signatures in the $p$-regime of the sextet model 2016 Bernardfest: Shamir explains to me the problem with capturing the dilaton: NDA does not work. 2017 BU workshop: while I we are struggling with the sextet analysis, Appelquist et al.: it works for $nf=8$ anyway. Hmmm ….

• dilaton signatures in the $\mathcal{E}$-regime?

• Simulating the effective potential of the composite scalar
testing scale-dependent BSM gauge couplings and $\beta$-functions:

- with established $\chi$SB, sextet model closest to CW in explored range of $\beta$-function
- $n_f=10$ is not conformal in explored $\beta$-function range of our analysis  
  Dani Nogradi
- $n_f=12$ is not conformal in explored $\beta$-function range of our analysis  
  new check here
- $n_f=13$ is conformal  
  Kieran Holland
- sextet SU(2) flavor group simplest with light 0++ scalar  — dilaton analysis
LatHC PLB B779 (2018) 230-236 arXiv:1710.09262 confirmed with new updated results:

L=32 -> L=64 step at three tuned $g^2$ targets adds further evidence against nf=12 IRFP

staggered “non-universality argument” based on 3d spin models is misguided

New Boulder-BU poster with DW fermions so far is not in contradiction with our Nf=12 analysis
light 0++ scalar and spectrum sextet model LatHC

July, 2013 Mainz: LatHC sextet 0++ presented

0++ is tracking the Goldstone pion $m_\pi^2 \geq m_\sigma^2$

Light scalar below mass 700 GeV to be described by linear sigma model?
**chiPT difficulties?**

- sextet
- B and F inconsistency

\[ M^2 = 2Bm \cdot (1 + c_{NLO}m + c_{\log} \log(m)) \]

- \( B = 3.562 \pm 0.085 \)
- \( c_{NLO} = 110 \pm 41 \)
- \( c_{\log} = 26.5 \pm 8.3 \)

\[ \chi^2/\text{dof} = 0.27 \]

\[ F = F^*(1 + c_{\log} \log(m) + c_{NLO}m) \]

- \( F = 0.0125 \pm 0.0016 \)
- \( c_{NLO} = -9.1e+02 \pm 84 \)
- \( c_{\log} = -270 \pm 3.6e+02 \)

\[ \chi^2/\text{dof} = 0.31 \]

- inputs from \( 32^3 \times 64 \) - \( 56^3 \times 96 \) volume range
- m fit range: \( 0.0015 - 0.004 \)

- taste broken goldstone spectrum
- 6 parameters describe 10 data
- B and F are inconsistent in the fits
- cutoff effect is “input”
chiPT difficulties? sextet B and F consistency forced but light scalar ignored

rooted staggered chiral perturbation theory

$N_f = 2$ sextet rep $\beta = 3.20$

$32^3 \times 64$ to $56^3 \times 96$ volumes

$M^2_{\pi} / 2m$

$B = 1.30 \pm 0.47$
$F = 0.0184 \pm 0.0032$
$\Lambda_3 = 0.412 \pm 0.011$
$\Lambda_4 = 0.594 \pm 0.054$
$C_M = 1.96 \pm 1$
$C_F = -0.528 \pm 0.02$

$m$ fit range: $0.0015 - 0.005$

composite plot of pngb spectra in taste reps A,T,V,I

$\beta = 3.20$ inputs: $32^3 \times 64$, $48^3 \times 96$, $56^3 \times 96$

- taste broken goldstone spectrum
- 6 parameters describe 10 data
- B and F are inconsistent in the fits
- cutoff effect is “input”
Fitted range is not “conformal mass deformation”
\( \gamma \) is inconsistent between M and F fits!
linear $\sigma$-model in simulations in low mass range with $m_\pi \approx m_\sigma$ requires extension

$SU(2) \otimes SU(2) \sim O(4)$ for sextet model

$$L = \frac{1}{2} \left( \partial_\mu \pi \right)^2 + \frac{1}{2} \left( \partial_\mu \sigma \right)^2 - \frac{1}{2} \mu^2 \left( \sigma^2 + \pi^2 \right) + \frac{4}{3} g \left( \sigma^2 + \pi^2 \right)^2 - \varepsilon \sigma$$

$$L = \frac{F^2}{4} \text{tr} \left( \partial_\mu \Sigma \partial_\mu \Sigma^\dagger \right) - \frac{F^2 M^2}{4} \text{tr} \left( \Sigma + \Sigma^\dagger \right)$$

$m_\sigma^2 \geq 3m_\pi^2$ tree level relation

Pion field $\Sigma = e^{i \pi_\alpha \tau_\alpha / F}$ with $\tau_\alpha$ Pauli matrices,

tree level pion mass $M^2 = 2Bm$

$$M_\pi^2 = M^2 \left\{ 1 - \frac{M^2}{216 \pi^2 F^2} \bar{I}_3 + O(M^4) \right\}$$

$$F_\pi = F \left\{ 1 + \frac{M^2}{16 \pi^2 F^2} \bar{I}_4 + O(M^4) \right\}$$

$$\bar{I}_3 = \frac{16 \pi^2}{g_R} - \ln \frac{M_\pi^2}{M_R^2} - \frac{14}{3}$$

$$\bar{I}_4 = \frac{8 \pi^2}{g_R} - \ln \frac{M_\pi^2}{M_R^2} - \frac{1}{3}$$

$$\frac{1}{2} \frac{M_\pi^2}{16 \pi^2 F^2} \bar{I}_3 \approx 0.5 \Rightarrow \frac{M_\sigma}{M} > \sqrt{2} \text{ with the condition } \frac{M_\sigma}{F} = 2$$

Similar condition from $\bar{I}_4 = \frac{8 \pi^2}{g_R} - \ln \frac{M_\pi^2}{M_R^2} - \frac{1}{3}$

$m_\sigma^2 \geq 2m_\pi^2$ 1-loop relation

generalize linear O(4) $\sigma$-model in low mass $m_\pi \geq m_\sigma$ range

$\rightarrow$ nonlinear $\sigma$-model, perhaps dilaton?
extended EFT of $\sigma$-$\pi$ entanglement in the BSM Higgs sector:

$$L = \frac{1}{2} \partial_{\mu} h \partial^{\mu} h - V(h) + \frac{v^2}{4} \left( D_{\mu} \Sigma D^{\mu} \Sigma \right) \left( 1 + 2a \frac{h}{v} + b \frac{h^2}{v^2} + b_3 \frac{h^3}{v^3} + \ldots \right)$$

$$\Sigma = e^{\pi \tau^a h/v} \text{ with } \tau^a \text{ Pauli matrices}$$

$$V(h) = \frac{1}{2} m_h^2 \cdot h^2 + d_3 \left( \frac{m_h^2}{2v^2} \right) \cdot h^3 + d_4 \left( \frac{m_h^2}{8v^2} \right) \cdot h^4 + \ldots$$

$\sigma$-model limit (SM): $a = b = d_3 = d_4 = 1$ (or more relaxed in $\chi$SB framework)

dilaton model limit: $a = b^2$, $b_3 = 0$ scale symmetry breaking set by $f_d$ (in far IR $\chi$SB can be triggered)

$M_\pi$, $F_\pi$, $M_\sigma$ are calculated to 1-loop: extended SU(2) flavor chiral dynamics

We have been analyzing the small pion mass region in the $M_\pi = 0.07$ - 0.015 range of the $p$-regime, also targeting the $\varepsilon$-regime

linear sigma model limit in of $\chi$PT $p$-regime simulations requires very small pion masses

$m_\pi \ll m_\sigma$ not reached in $p$-regime simulations
Low energy effective theory of the $\sigma(x)$ dilaton field and the $\pi^a(x)$ Goldstone bosons separated from the higher resonance states with SU(2) flavor in sextet model:

$$L = \frac{1}{2} \partial_\mu \chi \partial_\mu \chi - V_d(\chi) + \frac{f_\pi^2}{4} \left( \frac{\chi}{f_d} \right)^2 \text{tr} \left[ \partial_\mu \Sigma^\dagger \partial_\mu \Sigma \right] - \frac{f_\pi^2 m_\pi^2}{4} \left( \frac{\chi}{f_d} \right)^4 \text{tr} \left( \Sigma + \Sigma^\dagger \right)$$

$$\gamma = 3 - \gamma \text{ where } \gamma \text{ is the mass anomalous dimension}$$

$$\chi(x) = f_d e^{\sigma(x)/f_d} \text{ describes the dilaton field } \sigma(x)$$

pion field $\Sigma = e^{i\pi^a \tau^a / f_\pi}$ with $\tau^a$ Pauli matrices, tree level pion mass $m_\pi^2 = 2Bm$

$$V_{d1} = \frac{m_d^2}{2 f_d^2} \left( \frac{\chi^2}{2} - \frac{f_d^2}{2} \right)^2 \text{ relevant deformation of IRFP theory}$$

$$V_{d2} = \frac{m_d^2}{16 f_d^2} \chi^4 \left( 4 \ln \frac{\chi}{f_d} - 1 \right) \text{ nearly marginal deformation}$$

Golterman and Shamir
Appelquist et al.
Matsuzaki and Yamawaki
LatKMI …

Golterman and Shamir
Appelquist et al.
notation for comparison

we adapt Appelquist et al. notation for comparison

two different dilaton potentials illustrate scope of the analysis

Appelquist et al. test $N_f=8$ fundamental rep and fit obsolete sextet data with paper and pencil!

test $V \sim \chi^p$ for large $\chi$

dictionary for the effective dilaton theory coupled to Goldstone pions:

\begin{align*}
 f_\pi & \quad \text{Goldstone decay constant} \\
 m_\pi = 2B & \quad \text{Goldstone pions} \\
 f_d & \quad \text{dilaton decay constant} \\
 m_d & \quad \text{dilaton mass} \\
\end{align*}

chiral limit $\Rightarrow$

\begin{align*}
 F_\pi & \quad \text{with fermion mass deformations } m \\
 M_\pi & \quad \text{} \\
 F_d & \quad \text{} \\
 M_d & \quad \text{ } \\
\end{align*}
How do we test dilaton theory? General scaling laws:

- Golterman and Shamir
- Appelquist et al. nf=8 tests

$F_d$, minimum of dilaton potential after fermion mass $m$ is turned on:

for $V_{d1}$ potential:

$$\left( \frac{F_{d1}^2}{f_{d1}^2} \right)^{2-y/2} \frac{2yF_{d1}^2}{f_{d1}^2} \left( \frac{m_d^2}{m_{d1}^2} \right)$$

for $V_{d2}$ potential:

$$\left( \frac{F_{d2}^2}{f_{d2}^2} \right)^{2-y/2} \ln \left( \frac{F_{d2}^2}{f_{d2}^2} \right) \frac{2yF_{d2}^2}{f_{d2}^2} \left( \frac{m_d^2}{m_{d2}^2} \right)$$

$F_{\pi}^2$ and $M_{\pi}^2$ at finite fermion mass $m$:

$$\left\{ \begin{array}{l}
\frac{F_{\pi}^2}{f_{\pi}^2} = \frac{F_d^2}{f_d^2} \\
\frac{M_{\pi}^2}{m_{\pi}^2} = \left( \frac{F_d^2}{f_d^2} \right)^{y/2-1}
\end{array} \right\} \Rightarrow M_{\pi}^2 \left( \frac{F_{\pi}^2}{f_{\pi}^2} \right)^{1-y/2} = 2B_{\pi} \left( \frac{f_{\pi}^2}{f_d^2} \right)^{1-y/2} \cdot m$$

scaling test I: non-chiPT $M_{\pi}^2$ and $F_{\pi}^2$

independent of dilaton potential!

now we test this in the sextet model
scaling test (independent of the shape of the dilaton potential):
\[(aM_\pi)^2 \cdot (aF_\pi)^{2-y} = C \cdot am \text{ with } C = 2aB_\pi (af_\pi)^{2-y}\]
\[\gamma = 3 - y \text{ mass anomalous dimension (what scale?)}\]
FSS and covariance matrix are used in the fits
for checks: Bayesian posterior y distribution is determined from
Markov Chain Monte Carlo on the Maximum Likelihood Function

full analysis in Ricky Wong talk

all is well? cutoff-dependent \( \gamma^* \) ? if not, what \( \gamma \) scale?
mass anomalous dimension $\gamma$ from Dirac spectrum: sextet data

- Chebyshev expansion of mode number
- infinite volume limit from FSS
- $m \to 0$ chiral limit at fixed $a$
- $a \to 0$ continuum limit

SU(3) $N_f = 2$ sextet

\[ \gamma(g^2, a^2/t_0) = \gamma(g^2, a=0) + c \cdot a^2/t_0 \]

$g^2 = 6.7$ massless fermions in continuum limit
scaling test II: from $V(\chi) \approx \chi^p$ large $\chi$ asymptotic shape of the dilaton potential: $(aM_\pi)^2 \cdot (aF_\pi)^{2-p} = B$

covariance matrix is used in the fits shown cross check: p and B generated from ensembles of Fpi and Mpi at each m in Markov Chain Monte Carlo of the exact Maximum Likelihood Function without covariance matrix approx.

full analysis in Ricky Wong talk

these fits are failing (what did "the other sextet analysis" do?)
control of loop effects?
cutoff effects? not prime suspect missing dilaton potential terms?
limited FSS at Q=0? not prime suspect
what is the definition and fit consistency of $y$ and $\gamma$?
dilaton “decoupling” in the $\varepsilon$-regime

\[ L = \frac{1}{2} \partial_\mu \chi \partial_\mu \chi - V(\chi) + \frac{f_\pi^2}{4} \left( \frac{\chi}{f_d} \right)^2 \text{tr} \left[ \partial_\mu \Sigma^\dagger \partial_\mu \Sigma \right] - \frac{f_\pi m_\pi^2}{4} \left( \frac{\chi}{f_d} \right)^y \text{tr} \left( \Sigma + \Sigma^\dagger \right) \]

\[ L = \frac{1}{2} \partial_\mu \chi \partial_\mu \chi - V(\chi) - \frac{f_\pi^2 m_\pi^2}{4} \left( \frac{\chi}{f_d} \right)^y \text{tr} \left( \Sigma + \Sigma^\dagger \right) \]

in RMT dynamics:

$m_\pi \neq 0$ mixed $\varepsilon -$ regime (Damgaard-Fukaya)

pion light and dilaton "heavy"

$m_\pi = 0$ limit: dilaton is decoupled from pion zero mode in $\varepsilon$ regime

- Mixed regime RMT analysis:
six quarks earlier in p-regime this is changed now spectrum in $\varepsilon$-regime

- taste breaking is handled in same framework
James Osborne worked out

- thanks to James for the discussions and the opportunity for checking our software on the output of his code
dilaton “decoupling” in the $\varepsilon$-regime

- simulations of the $\varepsilon$-regime are set up and running:
  - staggered stout fermions at our medium fine lattice spacing
  - dropping down from our lightest pion mass $m^*a \sim 0.07$ in the $p$-regime
  - one order of magnitude (2 orders of magnitude in the fermion mass)
  - $64^4$ lattice size running in the $m=0.001 - 0.00001$ fermion mass range
  - $M_{\pi^*L} \sim 0.5$ !!
  - $F^*L \sim 1$ is our projection - important for $\varepsilon$-regime expansion
constraint effective potential

\[
\exp(-\Omega U_\Omega(\Phi)) = \prod_x \int d\phi(x) \delta \left( \Phi - \frac{1}{\Omega} \sum_x \phi(x) \right) \exp(-S[\phi])
\]

scalar field \( \phi(x) \) elementary, or source of composite operator

\[
P(\Phi) = \frac{1}{Z} \exp(-\Omega U_\Omega(\Phi)), \quad Z = \int d\Phi' \exp(-\Omega U_\Omega(\Phi'))
\]

probability distribution of order parameter in finite volume \( \Omega \)


\[
\frac{dU_{\text{eff}}}{d\Phi} = m^2 \Phi + \frac{1}{6} \lambda \langle \phi^3 \rangle_\Phi - N_F y \langle \bar{\psi} \psi \rangle_\Phi, \quad \langle \bar{\psi} \psi \rangle_\Phi = \langle \text{Tr}(D[\phi]^{-1}) \rangle_\Phi
\]

HMC at fixed zero momentum mode of the scalar field

composite 0++ scalar emergent from NJL: equivalent Yukawa model

\[
\dot{\phi}(x,t) = \pi(x,t),
\]

\[
\dot{\pi}(x,t) = - \left[ \frac{\partial S_{\text{eff}}}{\partial \phi(x,t)} - \frac{1}{\Omega} \sum_y \frac{\partial S_{\text{eff}}}{\partial \phi(y,t)} \right]
\]

\[
\frac{1}{\Omega} \sum_y \phi(y,t) = \Phi, \quad \sum_y \pi(y,t) = 0
\]
constraint effective potential

\[ \frac{dU_{\text{eff}}}{d\Phi} \]

- 1-loop renorm PT with cut-off
- 1-loop renorm PT without cut-off
- simulations

Can we extend the analysis to gauge theories?

jk, Kieran Holland
LatHC

can be extended to four-fermion operator and more general BSM models
Summary:

• sextet model is consistent with $\chi_{SB}$ from all angles we looked at
• general EFT approach will change the $\chi_{PT}$ analysis
• dilaton EFT is a new fresh look
• dilaton signatures are problematic in sextet model
• sources of the problem?
• missing dilaton terms? scale dependent $\gamma(\lambda)$? loop control?
• the $\varepsilon$-regime (RMT) is new opportunity for general EFT signatures!
• constraint effective potential method