Sextet sigma particle or dilaton?
2018

with the Lattice Higgs Collaboration (L_{at}HC)

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36th International Symposium on Lattice Field Theory

East Lansing, MI, USA July 22-28, 2018

based on: DOI:10.1103/PhysRevD.94.091501, arXiv:1712.08594, arXiv:1710.09262, arXiv:1711.04833,arXiv:1711.05299, PLB B779 (2018) 230-236

- walking β-functions? χSB phase? which models are potential dilaton candidates?

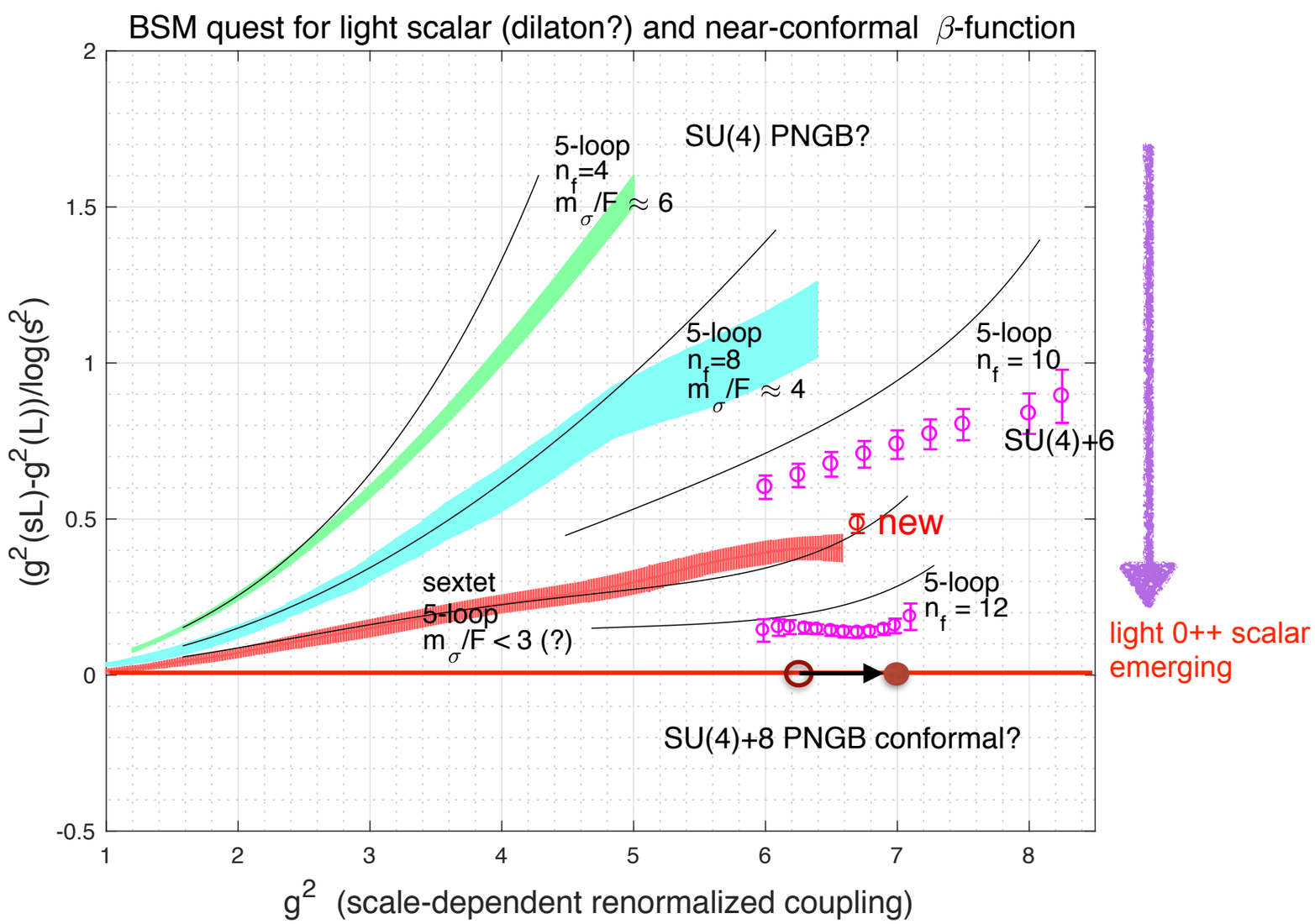
- linear σ-model with low mass $m_\pi \gtrsim m_\sigma$ requires extensions ➙ dilaton?

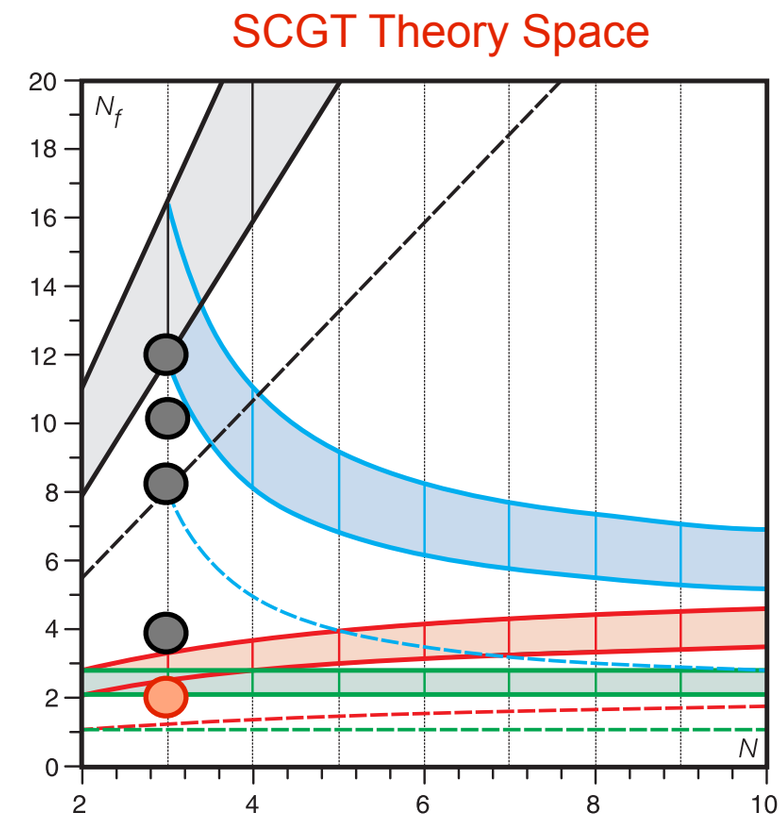
- dilaton signatures in the p-regime of the sextet model 2016 Bernardfest: Shamir explains to me the problem with capturing the dilaton: NDA does not work. 2017 BU workshop: while Iwe are struggling with the sextet analysis, Appelquist et al.: it works for nf=8 anyway. Hmmm ….

- dilaton signatures in the $\mathcal{E}$-regime ?

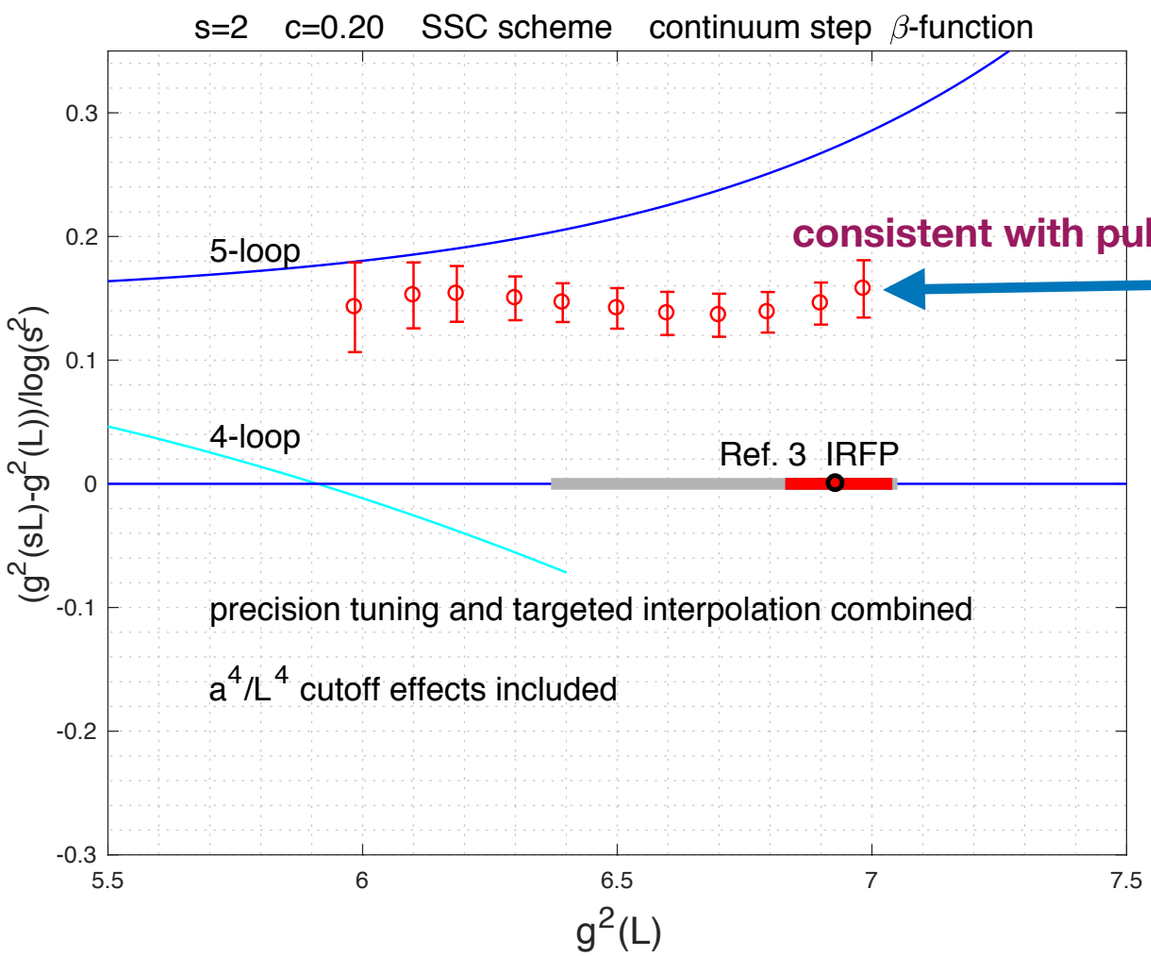
- Simulating the effective potential of the composite scalar

testing scale-dependent BSM gauge couplings and β-functions:

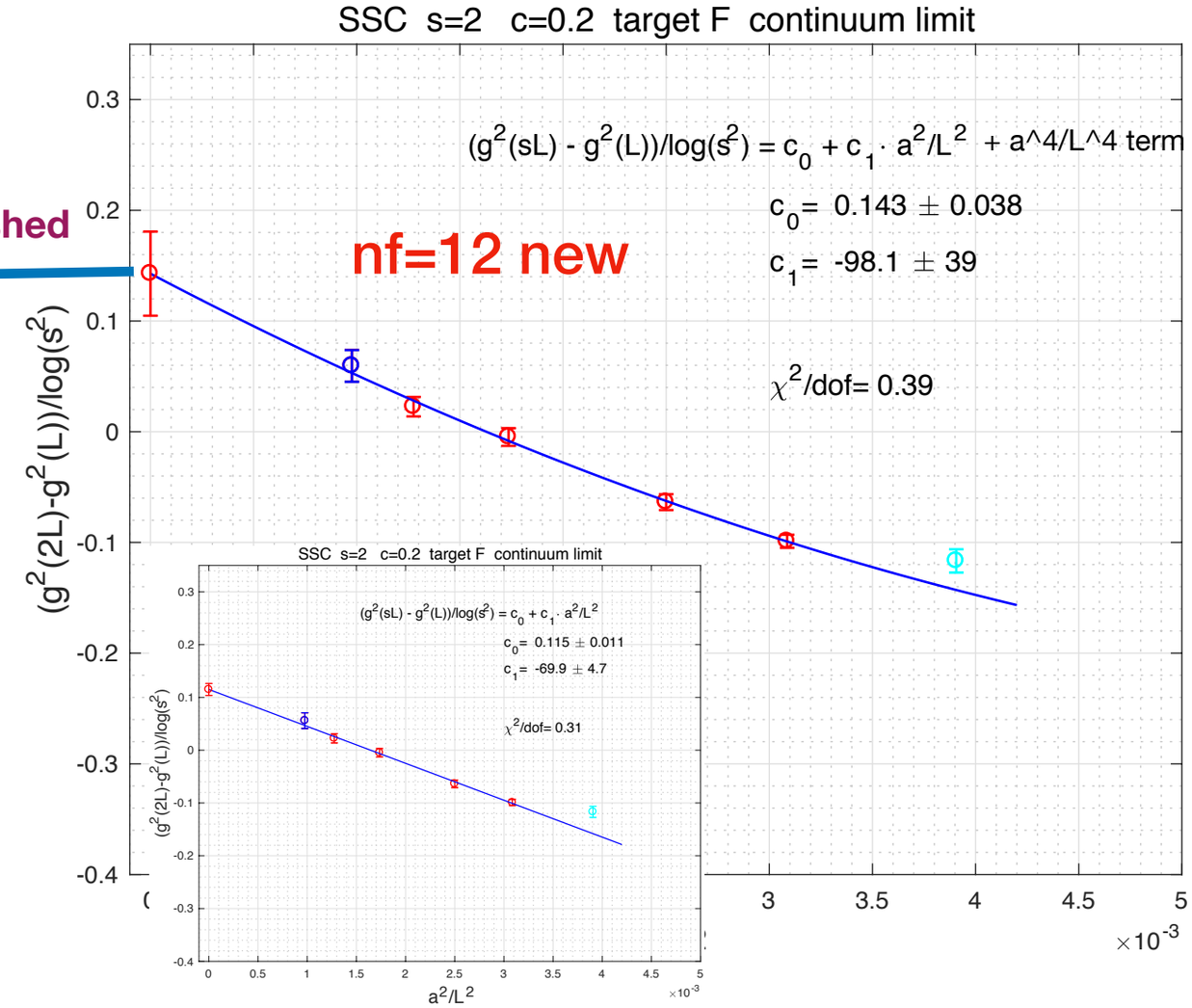


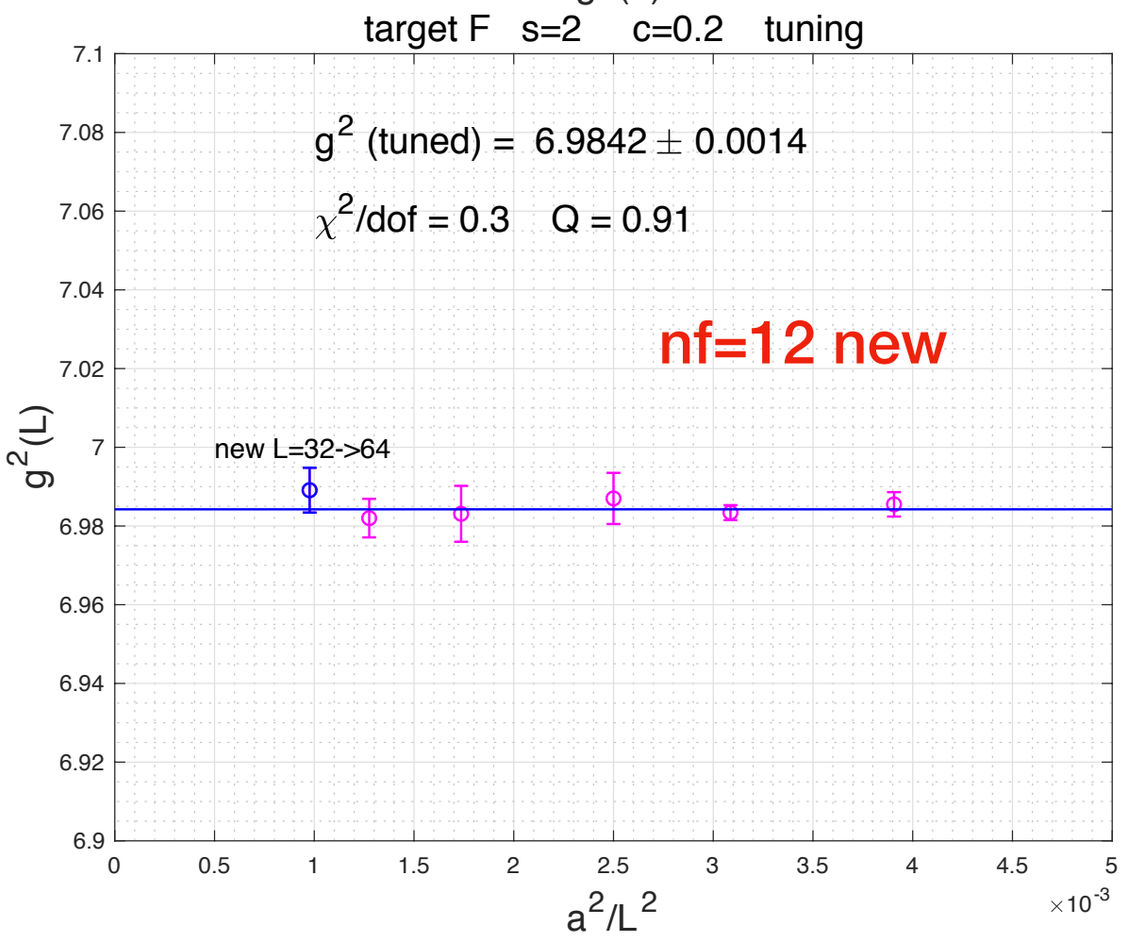


- with established χSB, sextet model closest to CW in explored range of β-function
- nf=10 is not conformal in explored β-function range of our analysis Dani Nogradi
- nf=12 is not conformal in explored β-function range of our analysis new check here
- nf=13 is conformal Kieran Holland
- sextet SU(2) flavor group simplest with light 0++ scalar — dilaton analysis



consistent with published





LatHC PLB B779 (2018) 230-236 arXiv:1710.09262
confirmed with new updated results:

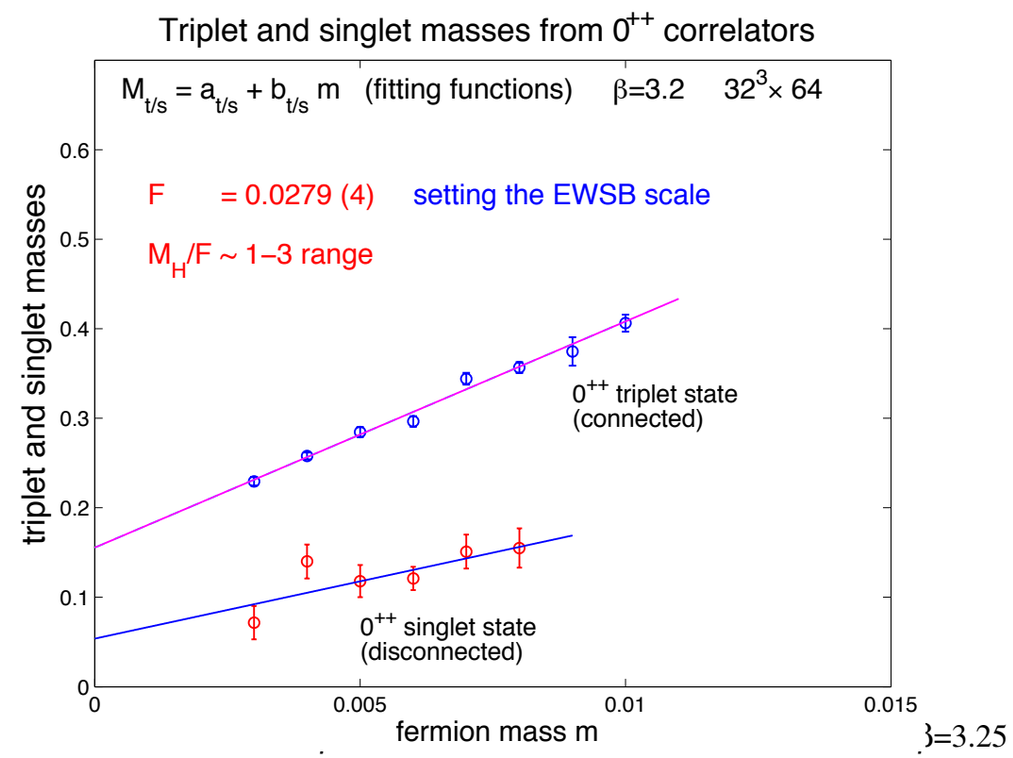
L=32 -> L=64 step at three tuned g^2 targets adds further evidence against nf=12 IRFP

staggered "non-universality argument" based on 3d spin models is misguided

New Boulder-BU poster with DW fermions so far is not in contradiction with our Nf=12 analysis

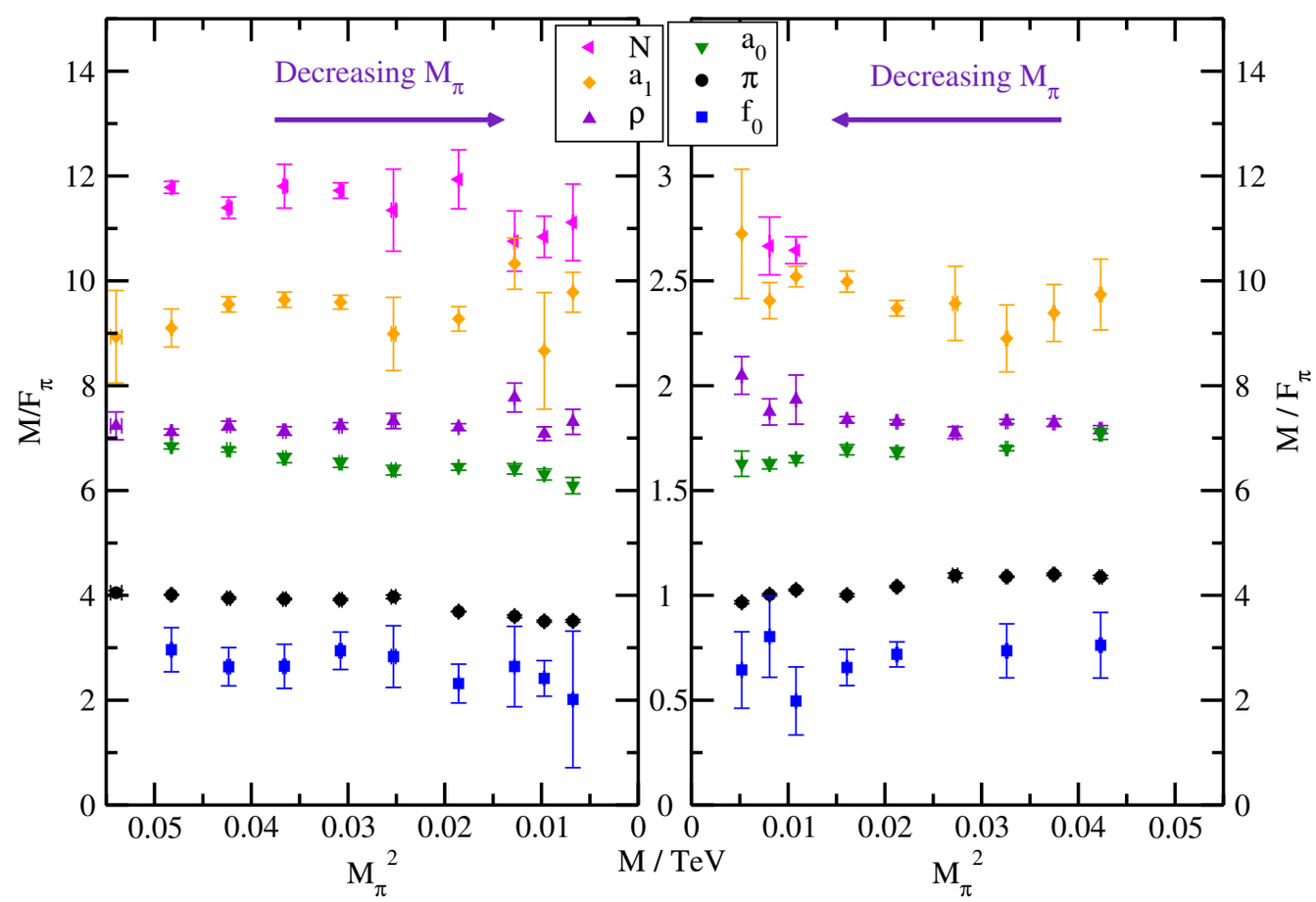
light 0++ scalar and spectrum sextet model $L_{at}HC$

July, 2013 Mainz: LatHC sextet 0++ presented

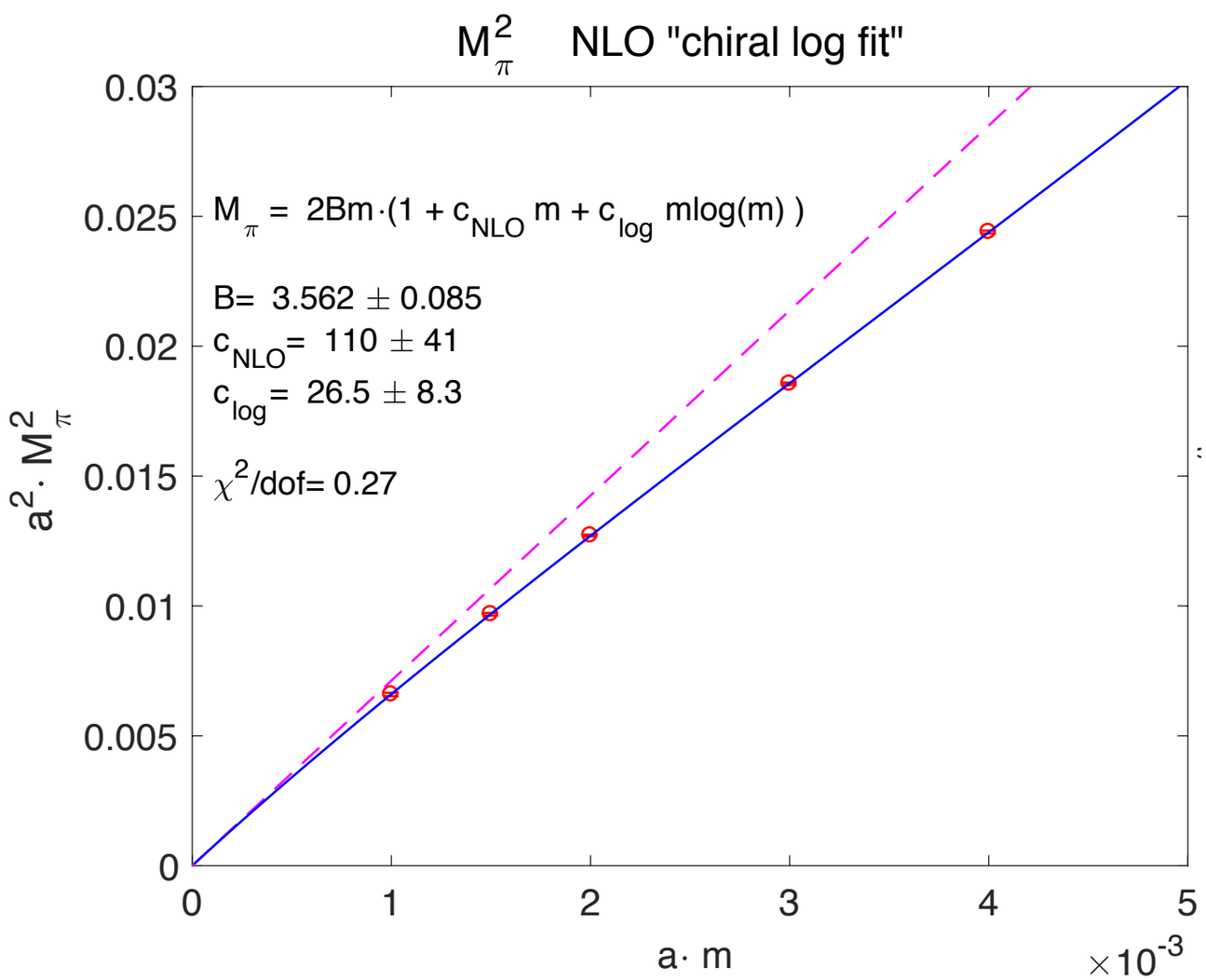


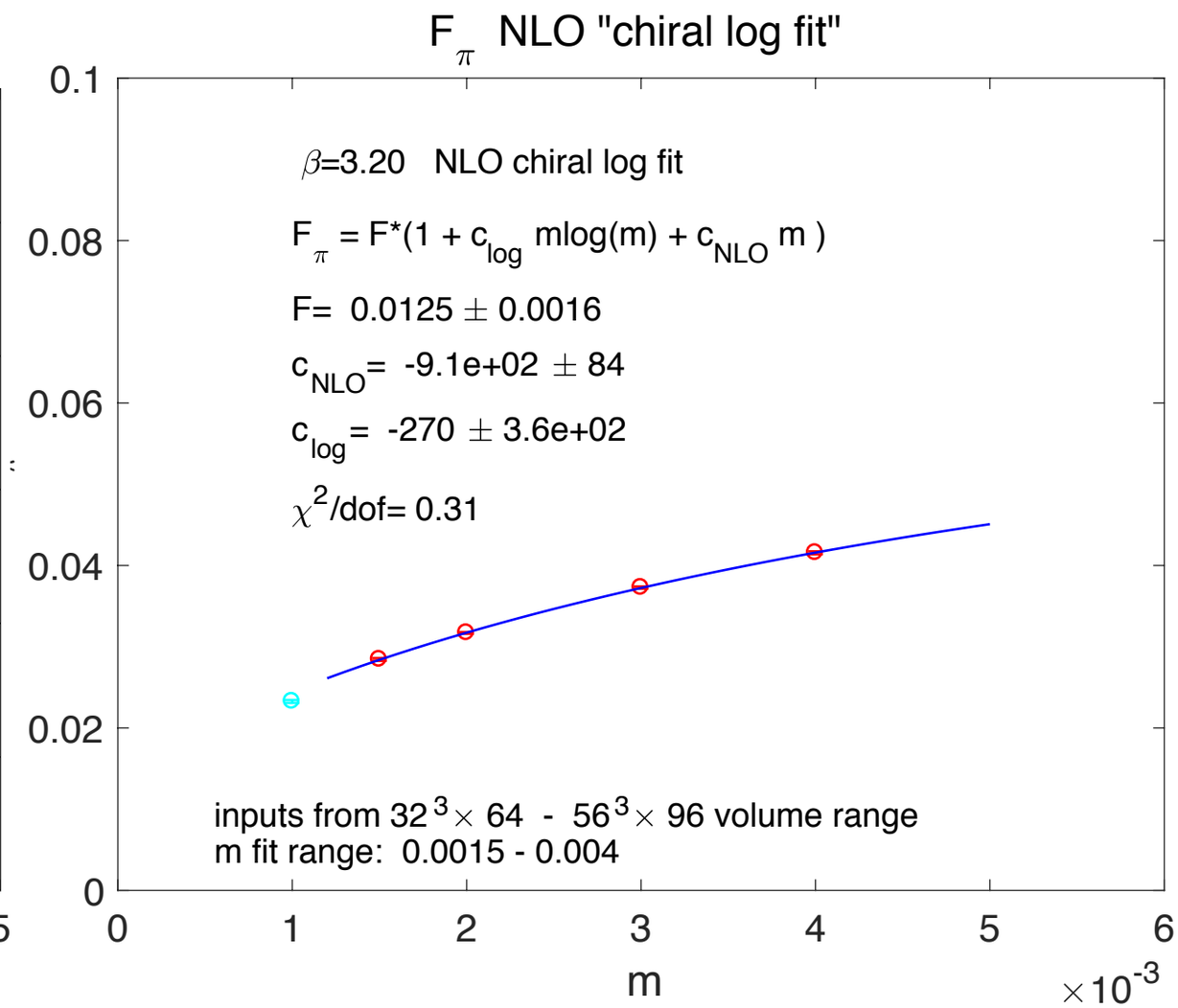
0++ is tracking the Goldstone pion $m_\pi^2 \geq m_\sigma^2$

Light scalar below mass 700 GeV to be described by linear sigma model ?



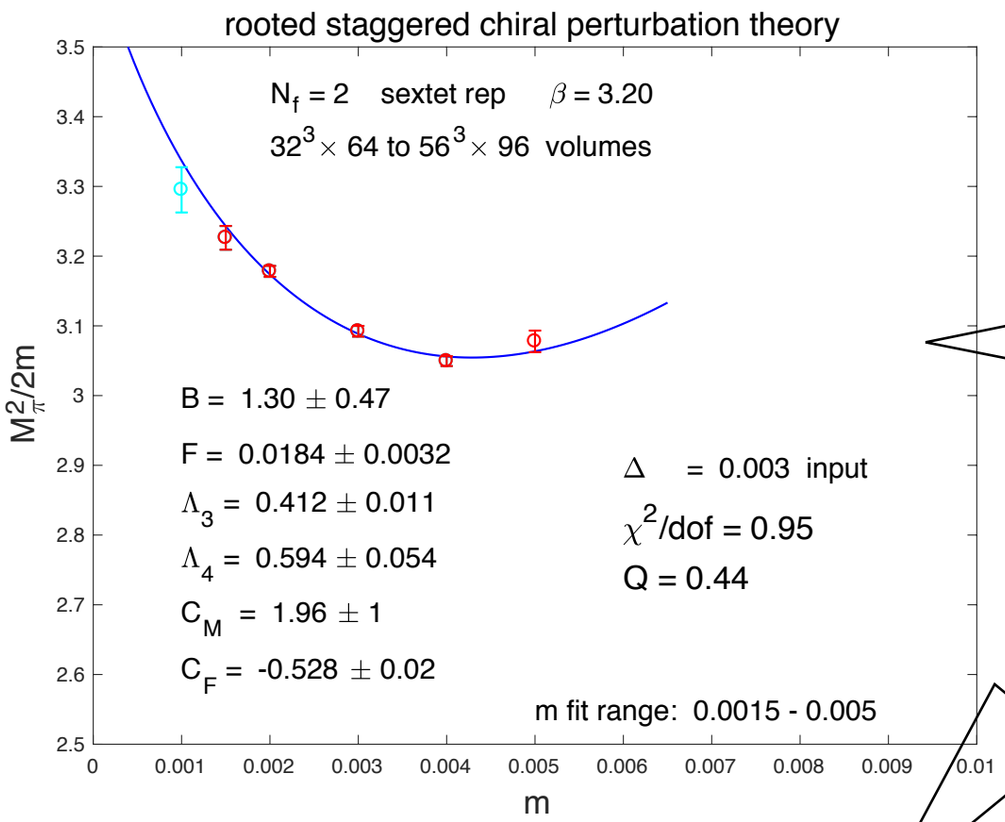
chiPT difficulties? sextet B and F inconsistency

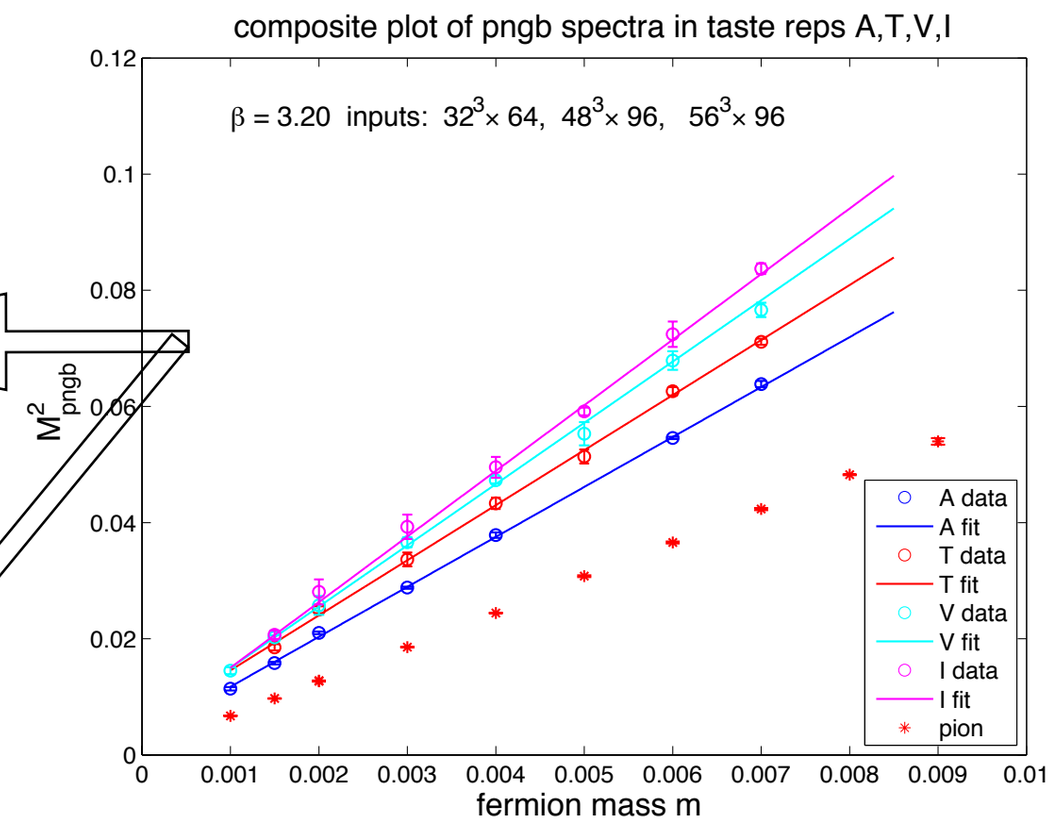


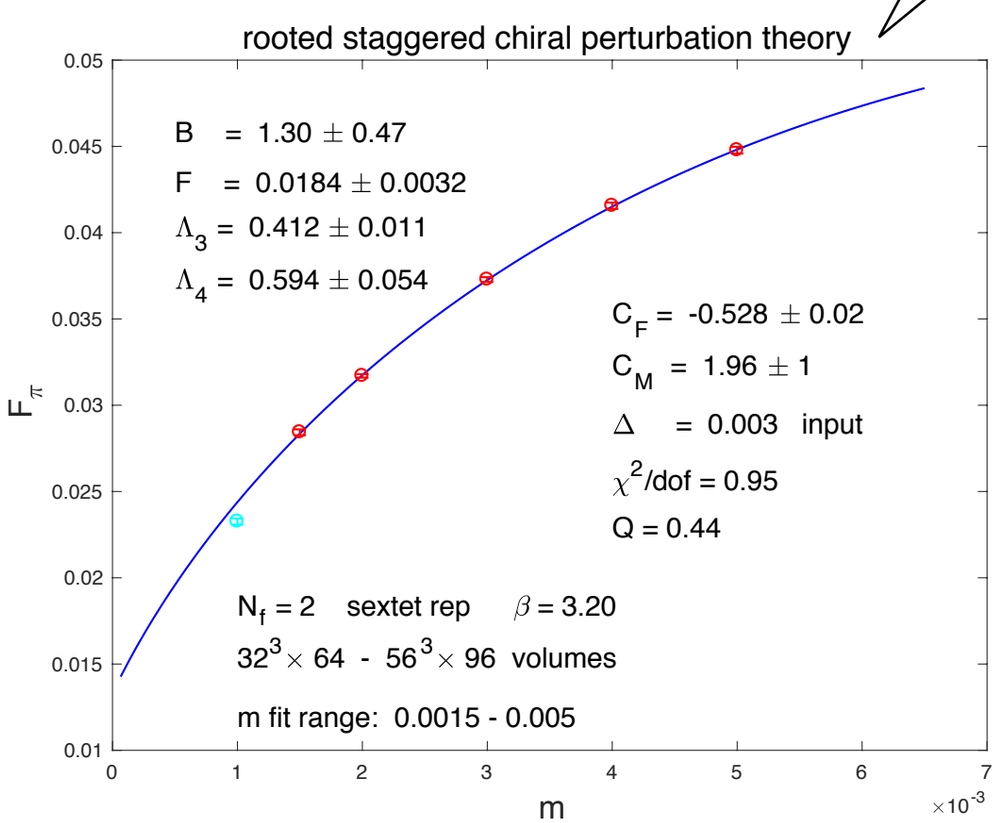


- taste broken goldstone spectrum
- 6 parameters describe 10 data
- B and F are inconsistent in the fits
- cutoff effect is “input”

chiPT difficulties? sextet B and F consistency forced but light scalar ignored







- taste broken goldstone spectrum
- 6 parameters describe 10 data
- B and F are inconsistent in the fits
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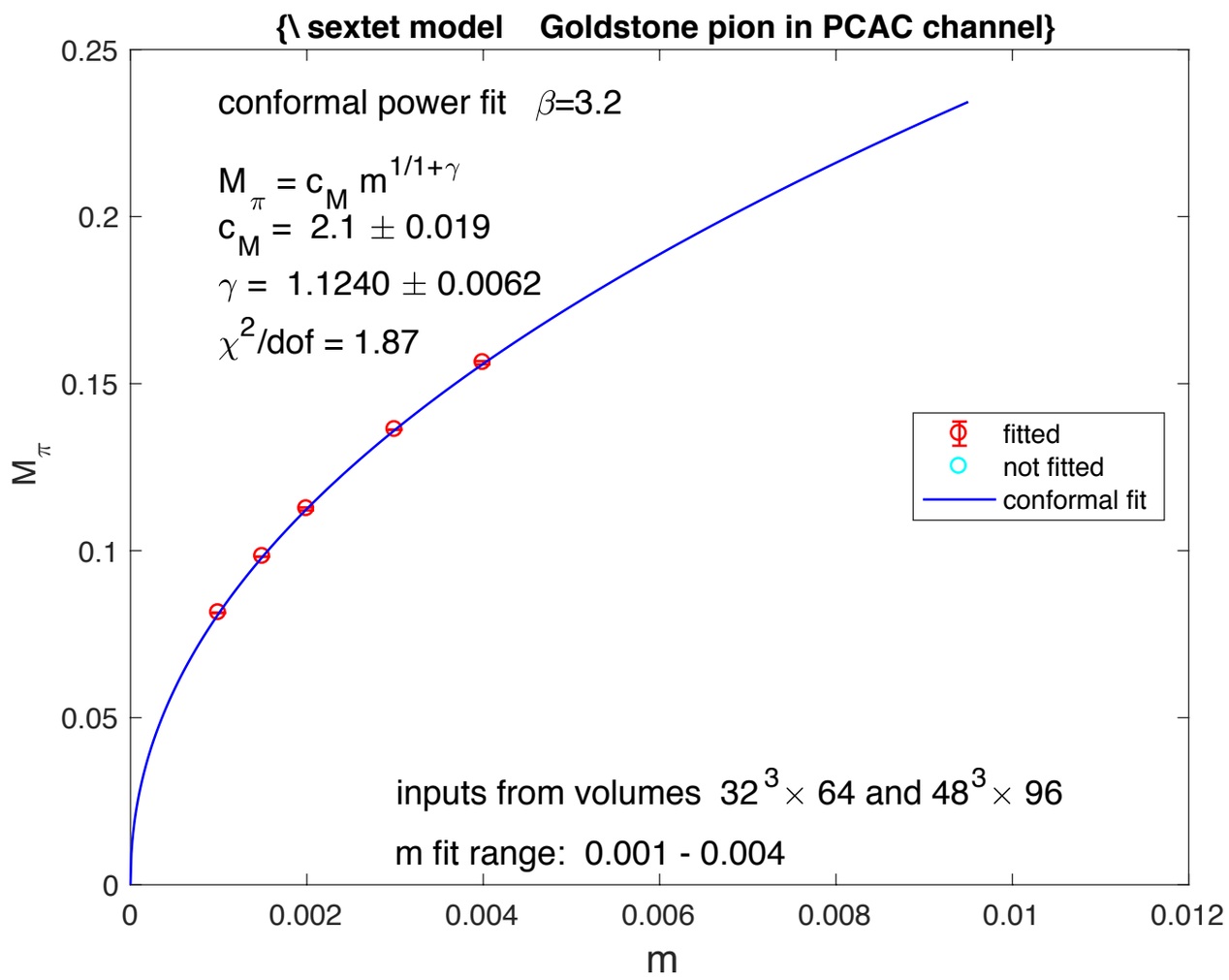
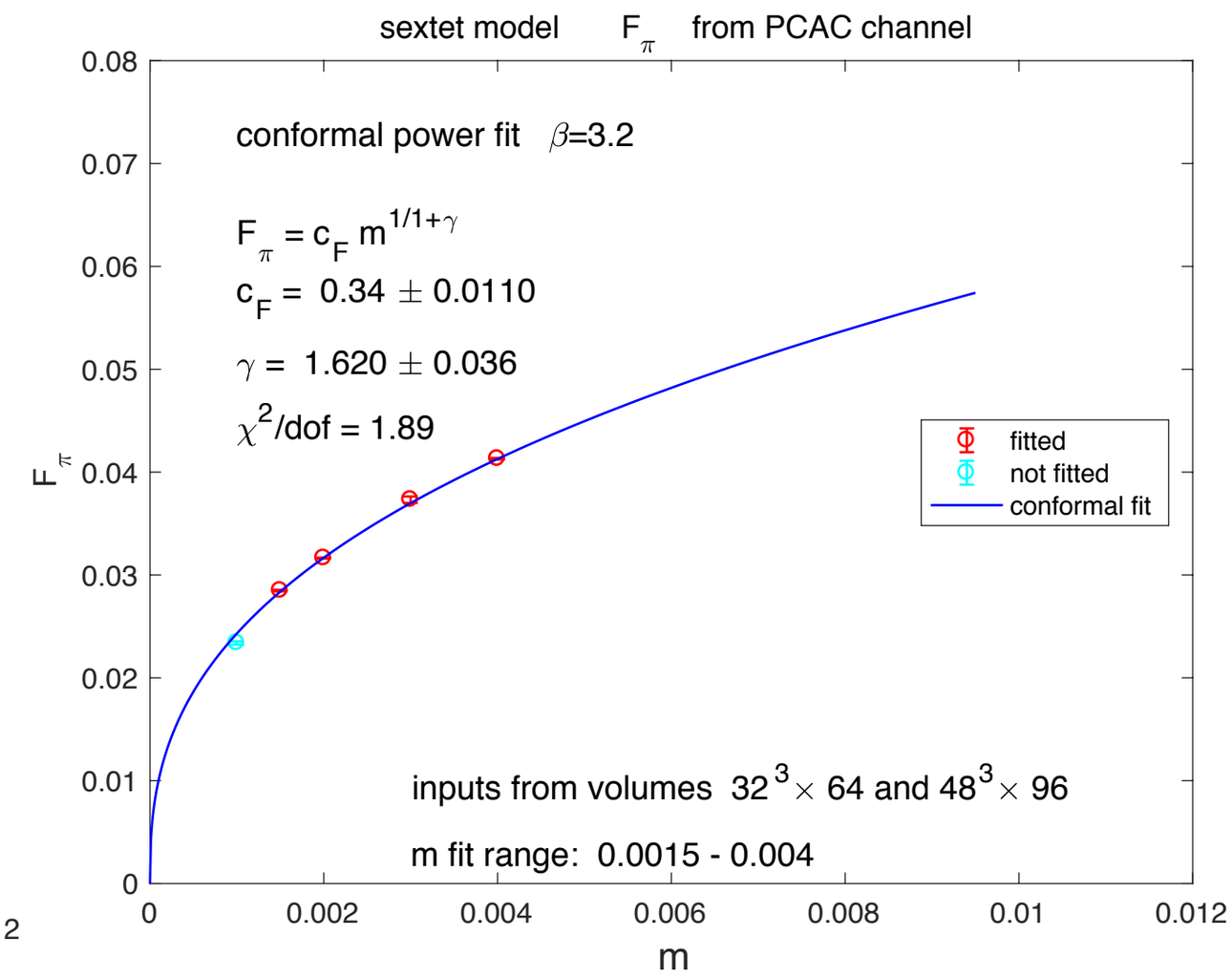
Fitted range is not “conformal mass deformation”
γ is inconsistent between M and F fits!





linear σ-model in simulations in low mass range with $m_\pi \gtrsim m_\sigma$ requires extension

$SU(2)\otimes SU(2)\sim O(4)$ for sextet model

$$L=\frac{1}{2}\left(\partial_\mu\vec{\pi}\right)^2+\frac{1}{2}\left(\partial_\mu\sigma\right)^2-\frac{1}{2}\mu^2\left(\sigma^2+\vec{\pi}^2\right)+\frac{1}{4}g\left(\sigma^2+\vec{\pi}^2\right)^2-\varepsilon\sigma$$

$$L=\frac{F^2}{4}tr\left(\partial_\mu\Sigma^\dagger\partial_\mu\Sigma\right)-\frac{F^2M^2}{4}tr\left(\Sigma+\Sigma^\dagger\right)$$

$m_\sigma^2 \geq 3m_\pi^2$ tree level relation

pion field $\Sigma=e^{i\pi_a\tau_a/F}$ with τ_a Pauli matrices,

tree level pion mass $M^2=2Bm$

$$M_\pi^2=M^2\left\{1-\frac{1}{2}\frac{M^2}{16\pi^2F^2}\bar{l}_3+O(M^4)\right\} \qquad \bar{l}_3=\frac{16\pi^2}{g_R}-\ln\frac{M_\pi^2}{M_R^2}-\frac{14}{3}$$

$$F_\pi=F\left\{1+\frac{M^2}{16\pi^2F^2}\bar{l}_4+O(M^4)\right\} \qquad \bar{l}_4=\frac{8\pi^2}{g_R}-\ln\frac{M_\pi^2}{M_R^2}-\frac{1}{3}$$

triviality analysis

(and loop expansion)

circa 1987-1988

$$\frac{1}{2}\frac{M_\pi^2}{16\pi^2F^2}\bar{l}_3<0.5\Rightarrow\frac{M_\sigma}{M}>\sqrt{2} \text{ with the condition } \frac{M_\sigma}{F}=2$$

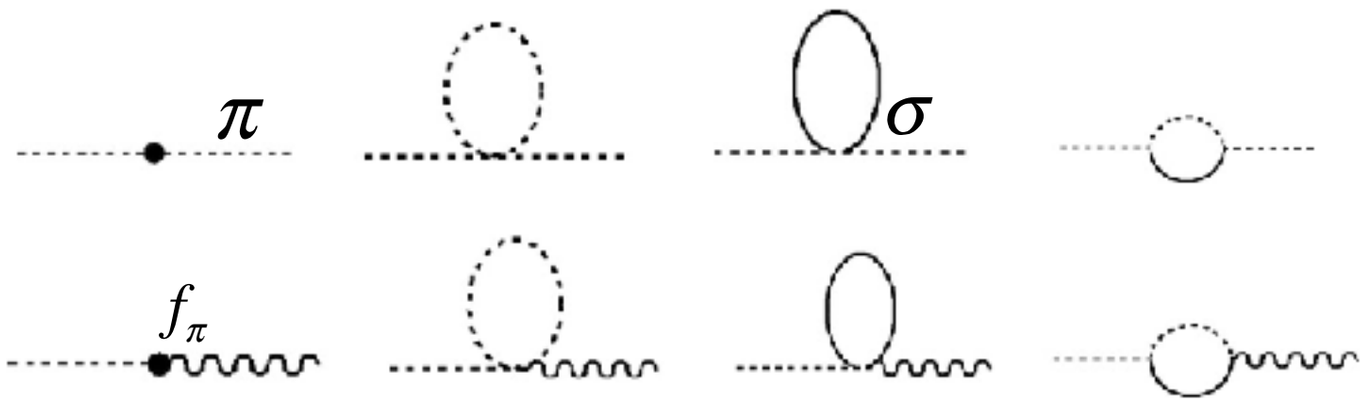
$m_\sigma^2 \geq 2m_\pi^2$ 1-loop relation

similar condition from $\bar{l}_4=\frac{8\pi^2}{g_R}-\ln\frac{M_\pi^2}{M_R^2}-\frac{1}{3}$

generalize linear O(4) σ-model in low mass $m_\pi \geq m_\sigma$ range

→ nonlinear σ-model, perhaps dilaton?

extended EFT of σ-π entanglement in the BSM Higgs sector:



Soto et al. targeting QCD

Nuclear Physics B 866 (2013) 270–292

Sanino et al. added new terms for BSM

$$L = \frac{1}{2}\partial_\mu h \partial_\mu h - V(h) + \frac{v^2}{4}\left(D_\mu \Sigma^\dagger D_\mu \Sigma\right)\cdot\left(1 + 2a\frac{h}{v} + b\frac{h^2}{v^2} + b_3\frac{h^3}{v^3} + \ldots\right)$$

$\Sigma = e^{i\pi^a\tau^a/v}$ with τ^a Pauli matrices

$$V(h) = \frac{1}{2}m_h^2 \cdot h^2 + d_3\left(\frac{m_h^2}{2v}\right)\cdot h^3 + d_4\left(\frac{m_h^2}{8v^2}\right)\cdot h^4 + \ldots$$

light 0++ scalar:

σ-particle or dilaton?

simplest dilaton Lagrangian is non-linear σ-model limit

σ-model limit (SM): $a = b = d_3 = d_4 = 1$ (or more relaxed in χSB framework)

dilaton model limit: $a = b^2,\ b_3 = 0$ scale symmetry breaking set by f_d (in far IR χSB can be triggered)

M_π, F_π, M_σ are calculated to 1-loop: extended SU(2) flavor chiral dynamics

We have been analyzing the small pion mass region in the M_π = 0.07- 0.015 range of the p-regime, also targeting the ε-regime

linear sigma model limit in of χPT p-regime simulations requires very small pion masses

$m_\pi \ll m_\sigma$ not reached in p-regime simulations

Low energy effective theory of the σ(x) dilaton field and the π^a(x) Goldstone bosons separated from the higher resonance states with SU(2) flavor in sextet model:

$$L = \frac{1}{2}\partial_\mu \chi \partial_\mu \chi - V_{\rm d}(\chi) + \frac{f_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^2 tr\left[\partial_\mu \Sigma^\dagger \partial_\mu \Sigma\right] - \frac{f_\pi^2 m_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^y tr\left(\Sigma + \Sigma^\dagger\right)$$

Golterman and Shamir
Appelquist et al.
Matsuzaki and Yamawaki
LatKMI …

$y = 3 - \gamma$ where γ is the mass anomalous dimension

$\chi(x) = f_d\, e^{\sigma(x)/f_d}$ describes the dilaton field $\sigma(x)$

pion field $\Sigma = e^{i\pi^a \tau^a / f_\pi}$ with τ^a Pauli matrices, tree level pion mass $m_\pi^2 = 2Bm$

we adapt Appelquist et al. notation for comparison

$$V_{\rm d1} = \frac{m_d^2}{2 f_d^2}\left(\frac{\chi^2}{2} - \frac{f_d^2}{2}\right)^2$$ relevant deformation of IRFP theory

$$V_{\rm d2} = \frac{m_d^2}{16 f_d^2}\chi^4\left(4\ln\frac{\chi}{f_d} - 1\right)$$ nearly marginal deformation

Golterman and Shamir
Appelquist et al.

two different dilaton potentials illustrate scope of the analysis

Appelquist et al. test Nf=8 fundamental rep and fit obsolete sextet data with paper and pencil!

test V ~ χ^p for large χ

dictionary for the effective dilaton theory coupled to Goldstone pions:

f_π	Goldstone decay constant		F_π
$m_\pi = 2mB$	Goldstone pions	chiral limit $\Rightarrow$	M_π
f_d	dilaton decay constant		F_d
m_d	dilaton mass		M_d

with fermion mass deformations m

How do we test dilaton theory? General scaling laws:

Golterman and Shamir

Appelquist et al. nf=8 tests

$$\frac{f_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^2 \cdot V_\pi(\chi)\cdot tr\left[\partial_\mu \Sigma^\dagger \partial_\mu \Sigma\right]$$

$$\frac{f_\pi^2 m_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^y \cdot V_\Sigma(\chi)\cdot tr\left(\Sigma+\Sigma^\dagger\right)$$

spoiler alert: $V_\pi(\chi)$ and $V_\Sigma(\chi)$ set to one as higher order only in tripple 1/N expansion (Golterman/Shamir) otherwise not determined

F_d minimum of dilaton potential after fermion mass m is turned on:

for V_{d1} potential: $$\left(\frac{F_d^2}{f_d^2}\right)^{2-y/2}\left(1-\frac{F_d^2}{f_d^2}\right)=\frac{2yF^2}{f_d^2}\left(\frac{m_\pi^2}{m_d^2}\right)$$

for V_{d2} potential: $$\left(\frac{F_d^2}{f_d^2}\right)^{2-y/2}\ln\left(\frac{F_d^2}{f_d^2}\right)=\frac{2yF^2}{f_d^2}\left(\frac{m_\pi^2}{m_d^2}\right)$$

F_π^2 and M_π^2 at finite fermion mass m:

$$\left.\begin{aligned}\frac{F_\pi^2}{f_\pi^2}&=\frac{F_d^2}{f_d^2}\\ \frac{M_\pi^2}{m_\pi^2}&=\left(\frac{F_d^2}{f_d^2}\right)^{y/2-1}\end{aligned}\right\}\quad\Rightarrow\quad M_\pi^2\left(F_\pi^2\right)^{1-y/2}=\underbrace{2B_\pi\left(f_\pi^2\right)^{1-y/2}}_{\substack{C\ \text{fitted}\\ y\ \text{fitted}}}\cdot m$$

scaling test I: non-chiPT M_π^2 and F_π^2

independent of dilaton potential !

now we test this in the sextet model

scaling test (independent of the shape of the dilaton potential):

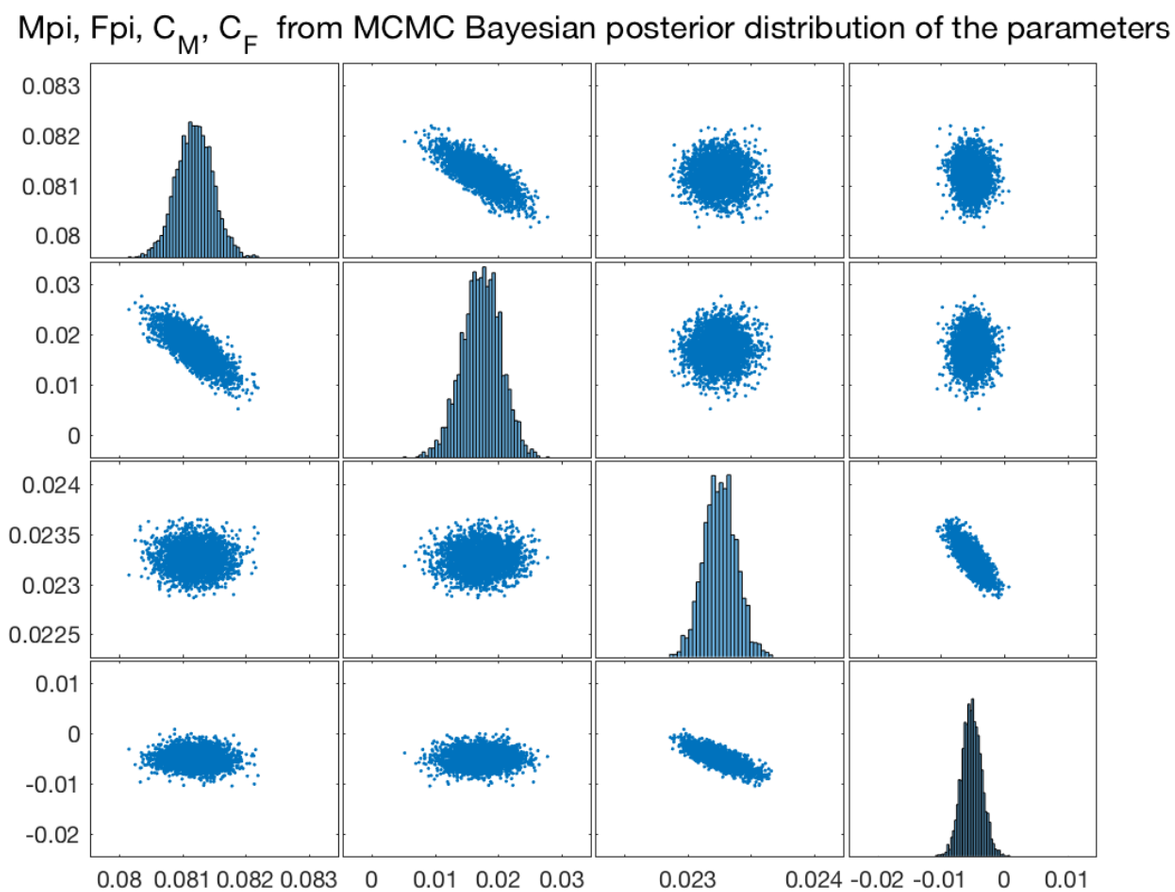
$(aM_\pi)^2 \cdot (aF_\pi)^{2-y} = C \cdot am$ with $C = 2aB_\pi (af_\pi)^{2-y}$

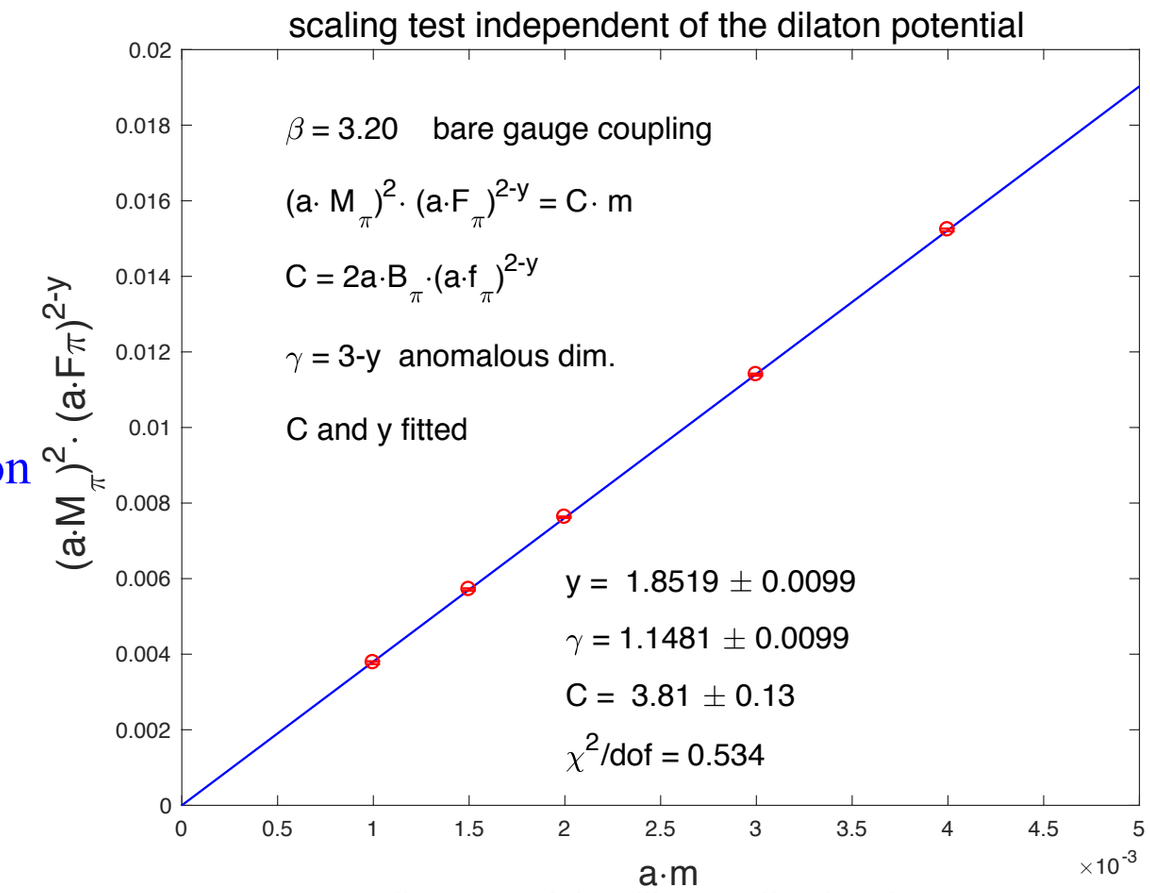
$\gamma = 3 - y$ mass anomalous dimension (what scale?)

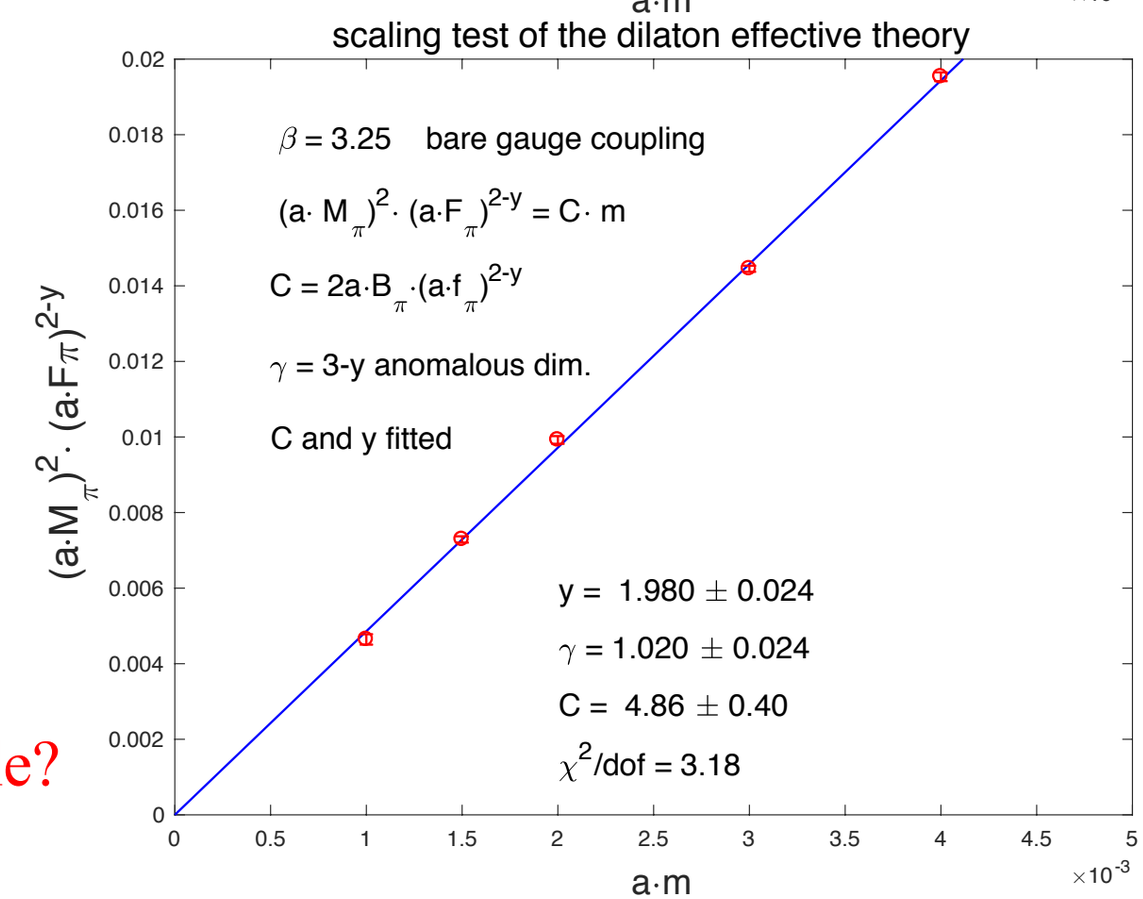
FSS and covariance matrix are used in the fits

for checks: Bayesian posterior y distribution is determined from Markov Chain Monte Carlo on the Maximum Likelihood Function

full analysis in Ricky Wong talk



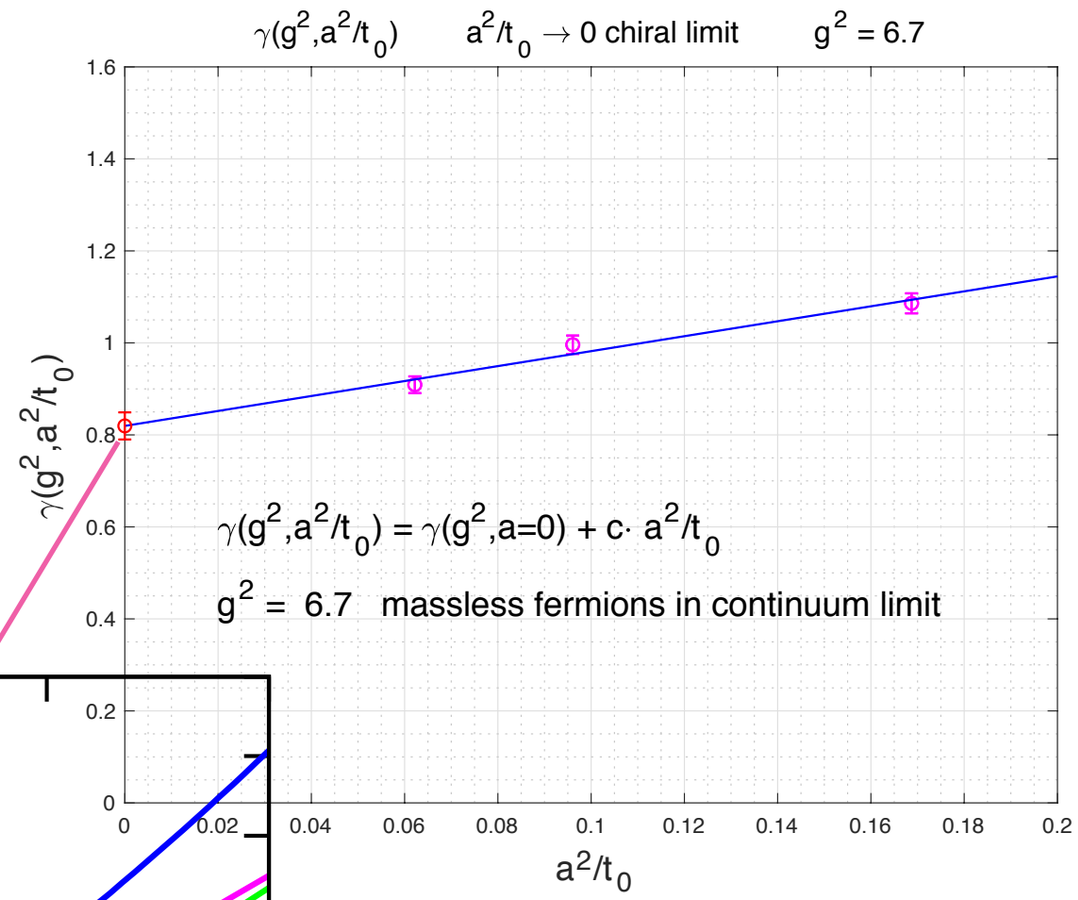


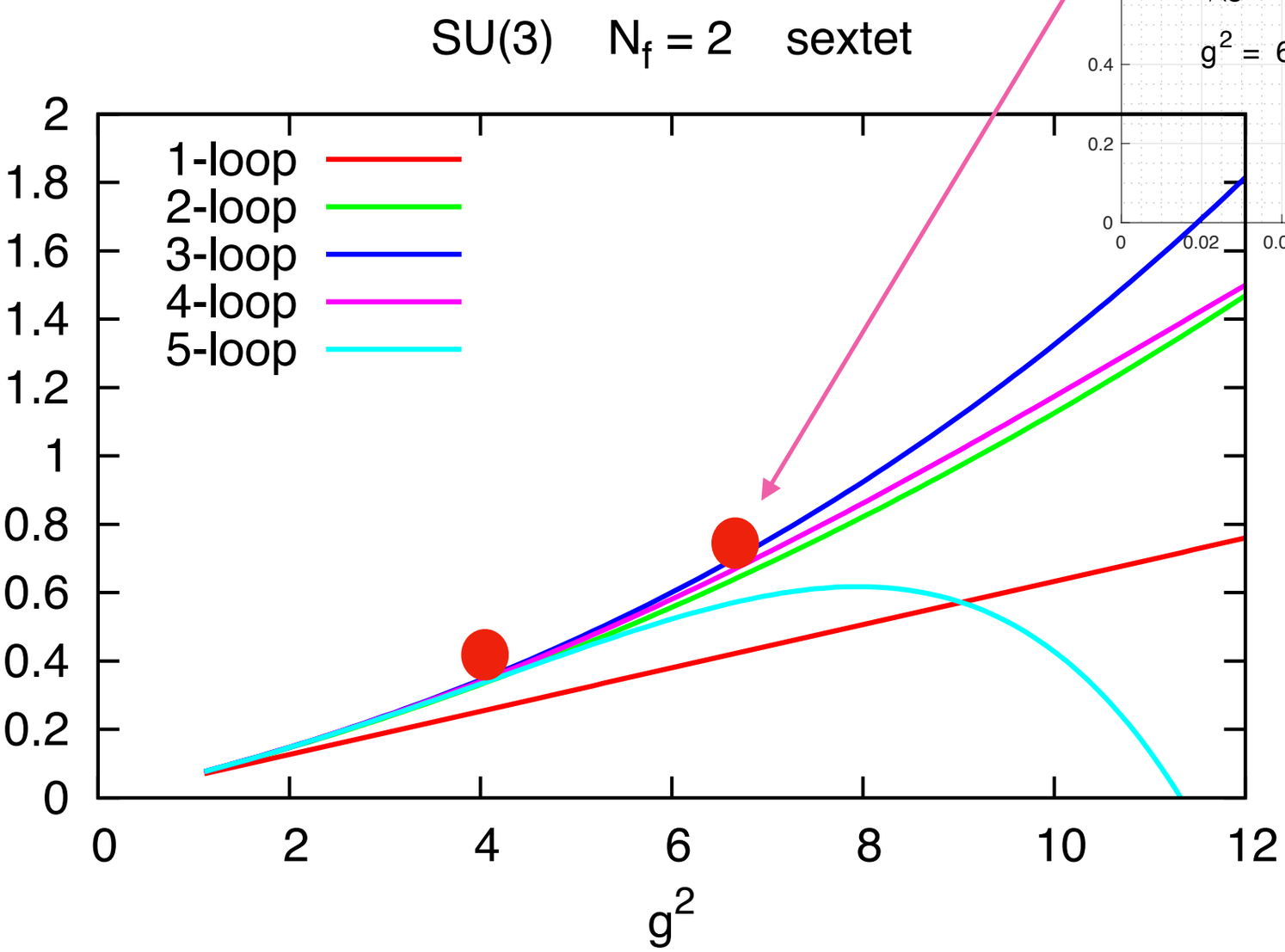


all is well? cutoff-dependent γ^*? if not, what γ scale?

mass anomalous dimension γ from Dirac spectrum: sextet data

- Chebyshev expansion of mode number
- infinite volume limit from FSS
- m -> 0 chiral limit at fixed a
- a -> 0 continuum limit





- Chebyshev expansion of mode number
- infinite volume limit from FSS
- m -> 0 chiral limit at fixed a
- a -> 0 continuum limit

scaling test II: from $V(\chi) \approx \chi^p$ large χ asymptotic shape of the dilaton potential: $(aM_\pi)^2 \cdot (aF_\pi)^{2-p} = B$

covariance matrix is used in the fits shown

cross check: p and B generated from ensembles of Fpi and Mpi at each m in Markov Chain Monte Carlo of the exact Maximum Likelihood Function without covariance matrix approx.

full analysis in Ricky Wong talk

these fits are failing (what did "the other sextet analysis" do?)

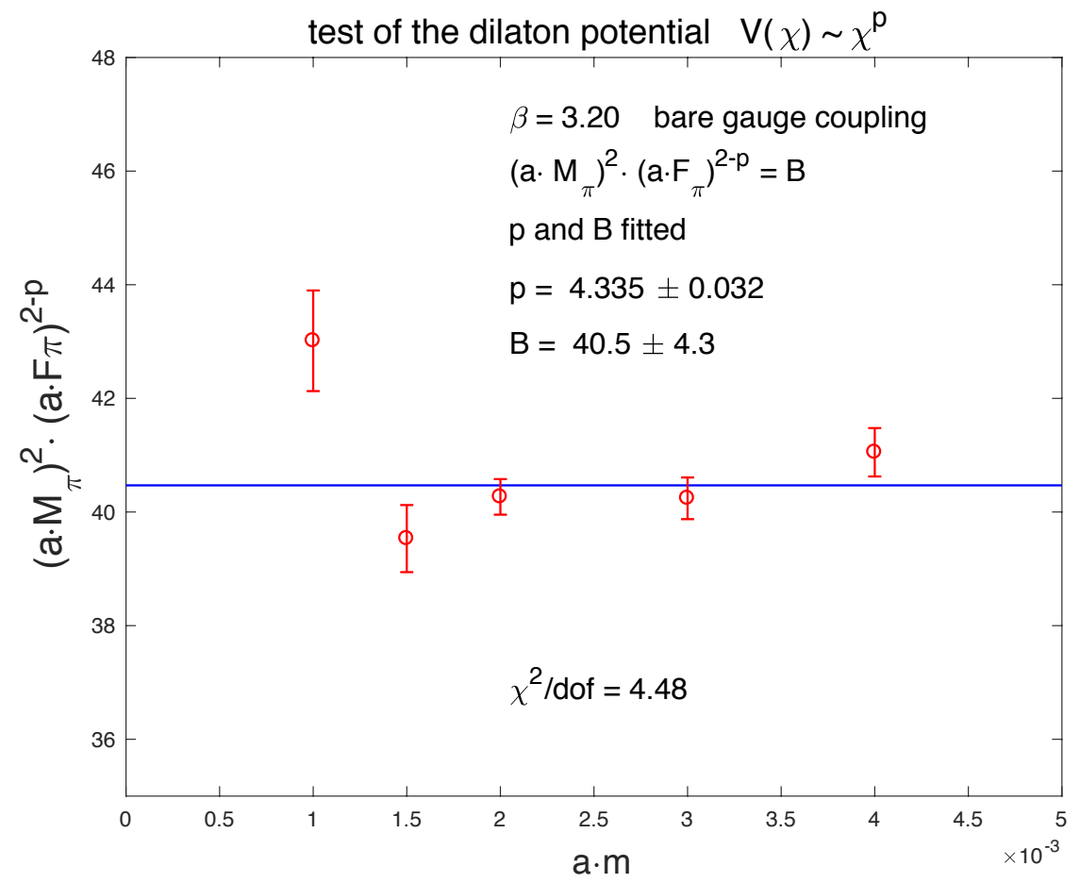
control of loop effects?

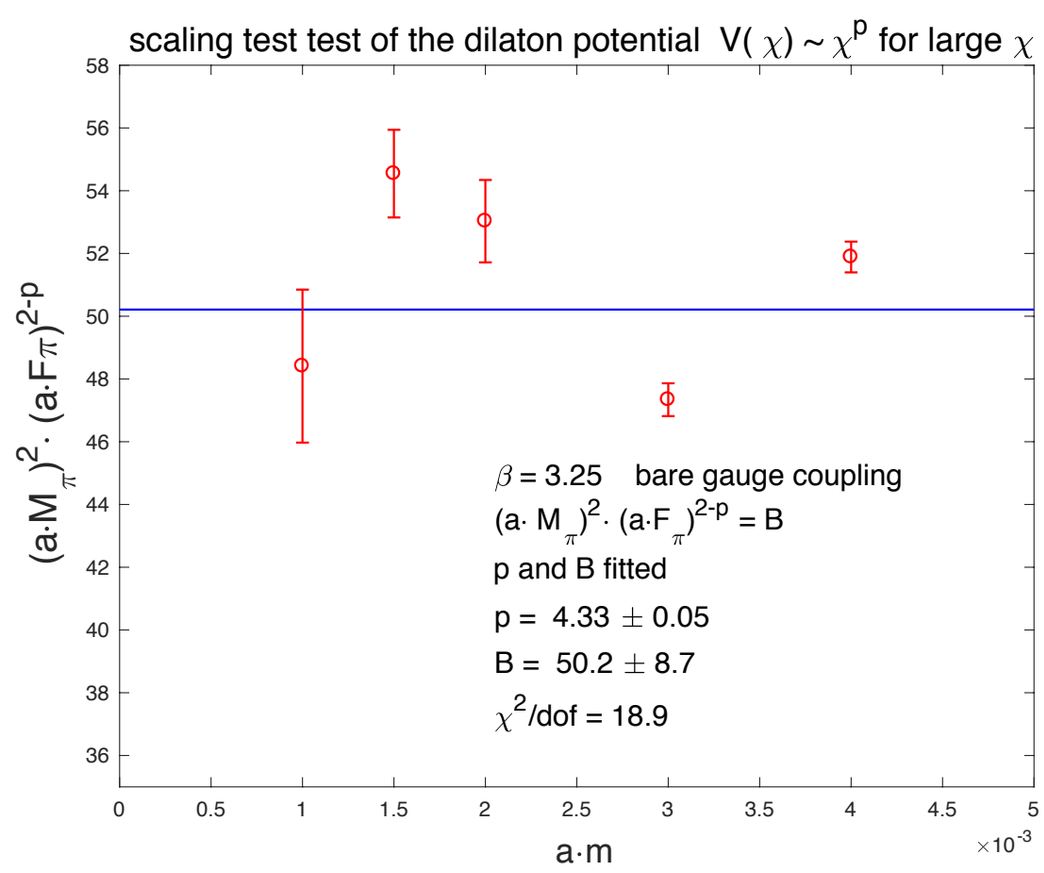
cutoff effects? not prime suspect

missing dilaton potential terms?

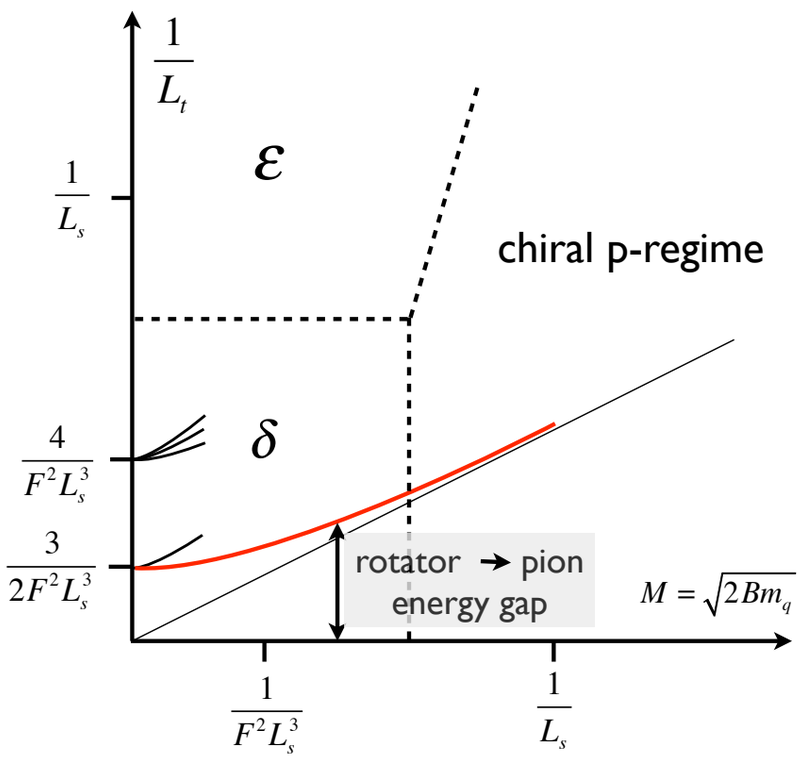
limited FSS at Q=0? not prime suspect

what is the definition and fit consistency of y and γ?



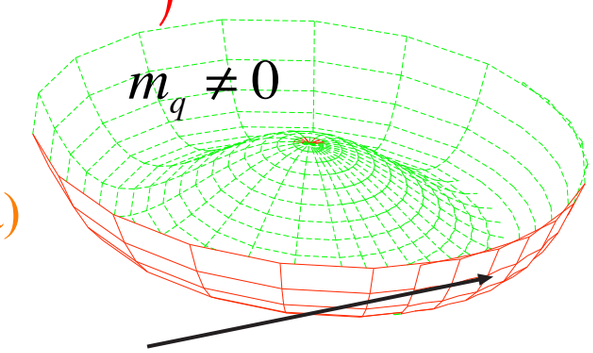


dilaton "decoupling" in the ε-regime



$$L=\frac{1}{2}\partial_\mu\chi\partial_\mu\chi-V(\chi)+\frac{f_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^2 tr\left[\partial_\mu\Sigma^\dagger\partial_\mu\Sigma\right]-\frac{f_\pi^2m_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^y tr\left(\Sigma+\Sigma^\dagger\right)$$

$$L=\frac{1}{2}\partial_\mu\chi\partial_\mu\chi-V(\chi)\ -\frac{f_\pi^2m_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^y tr\left(\Sigma+\Sigma^\dagger\right)$$



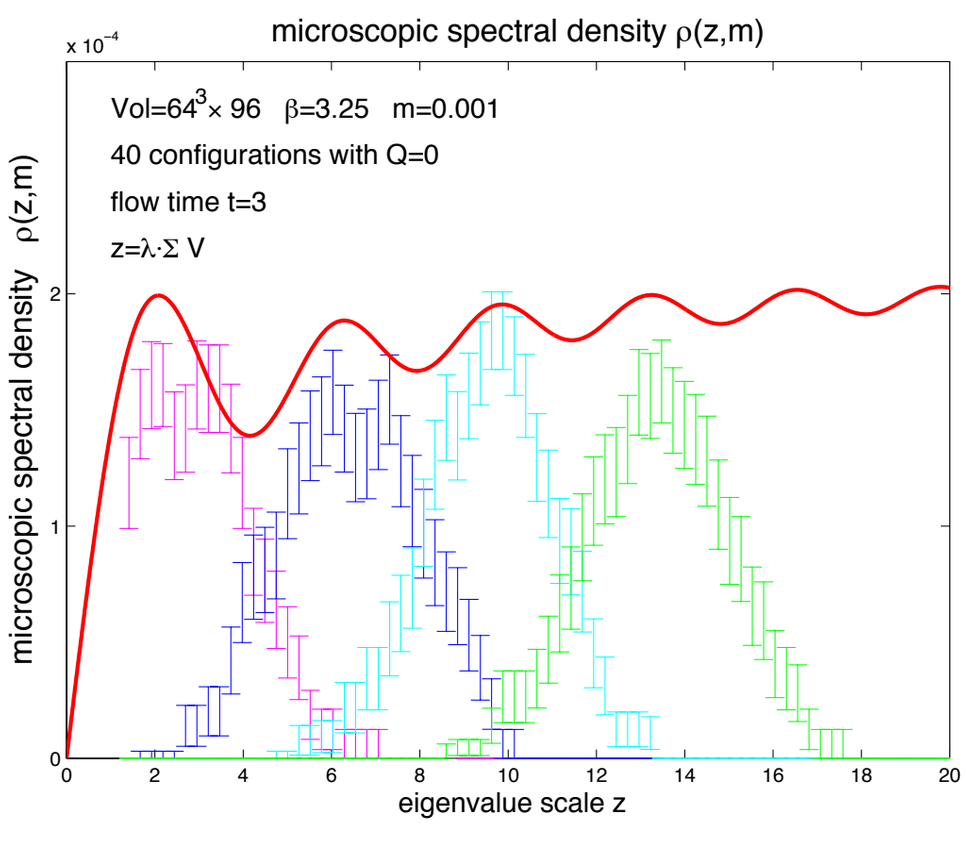
in RMT dynamics:

$m_\pi\neq 0$ mixed $\varepsilon-$regime (Damgaard-Fukaya)

pion light and dilaton "heavy"

tilted condensate

$m_\pi=0$ limit: dilaton is decoupled from pion zero mode in ε regime



- Mixed regime RMT analysis:
sea quarks earlier in p-regime this is changed now
spectrum in ϵ-regime

- taste breaking is handled in same framework
James Osborne worked out

- thanks to James for the discussions and the opportunity for checking our software on the output of his code

dilaton “decoupling” in the ε-regime

- simulations of the ε-regime are set up and running:
- staggered stout fermions at our medium fine lattice spacing
- dropping down from our lightest pion mass m*a ~ 0.07 in the p-regime
- one order of magnitude (2 orders of magnitude in the fermion mass)
- 64^4 lattice size running in the m=0.001- 0.00001 fermion mass range
- Mpi*L ~ 0.5 !!
- F*L ~ 1 is our projection - important for ϵ-regime expansion

constraint effective potential

$$\exp(-\Omega U_\Omega(\Phi)) = \prod_x \int d\phi(x)\delta\left(\Phi - \frac{1}{\Omega}\sum_x \phi(x)\right)\exp(-S[\phi])$$

scalar field $\phi(x)$ elementary, or source of composite operator

$$P(\Phi) = \frac{1}{Z}\exp(-\Omega U_\Omega(\Phi)), \qquad Z = \int d\Phi' \exp(-\Omega U_\Omega(\Phi'))$$

probability distribution of order parameter in finite volume Ω

implementation with fermion fields: jk, Lee Lin, Pietro Rossi, Yue Shen, **NPB Proc. Suppl. 9 (1989) 99-104**

$$\frac{dU_{\rm eff}}{d\Phi} = m^2\Phi + \frac{1}{6}\lambda\langle\phi^3\rangle_\Phi - N_F y\langle\bar\psi\psi\rangle_\Phi, \qquad \langle\bar\psi\psi\rangle_\Phi = \langle {\rm Tr}(D[\phi]^{-1})\rangle_\Phi$$

HMC at fixed zero momentum mode of the scalar field

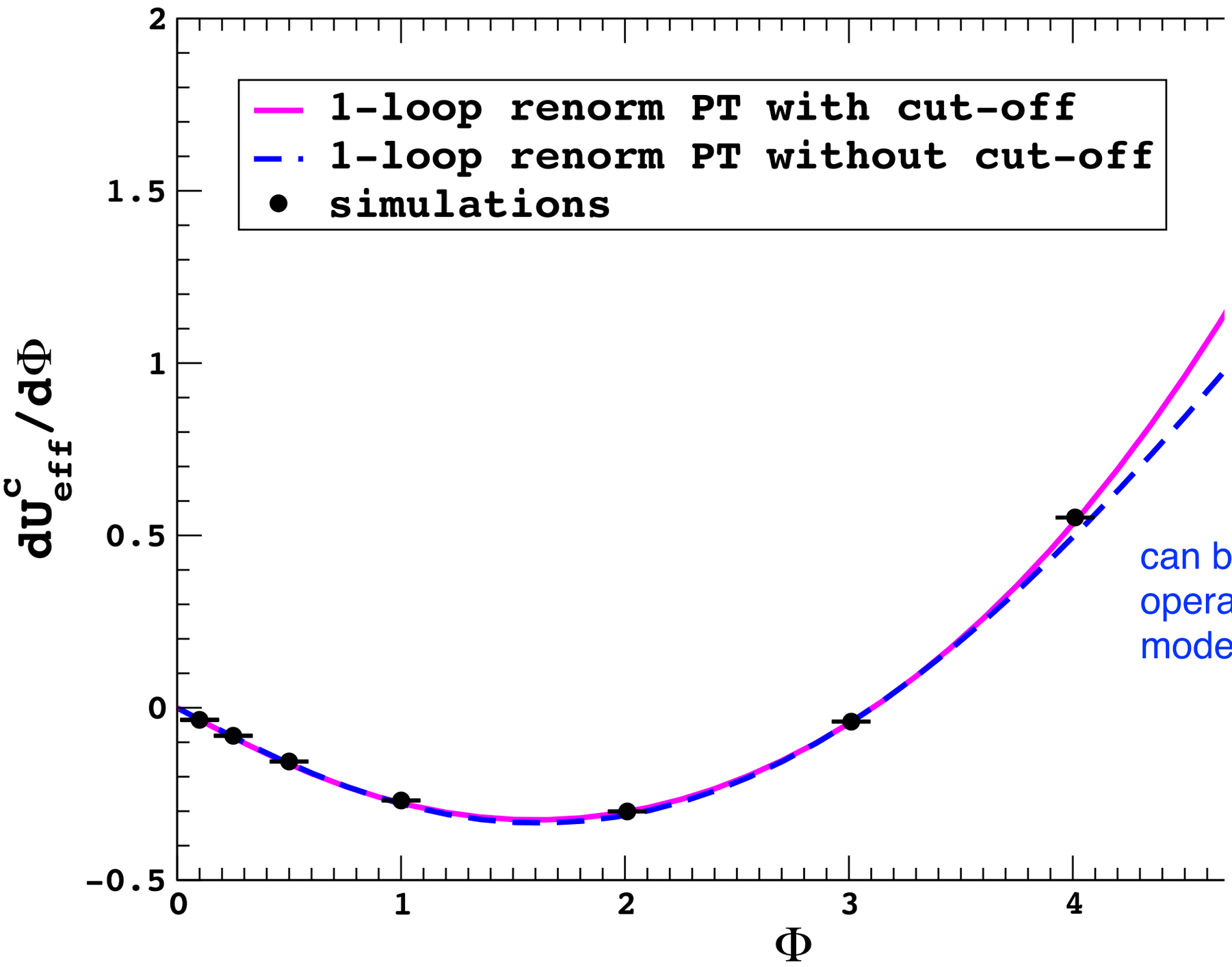
composite 0++ scalar emergent from NJL: equivalent Yukawa model

$$\dot\phi(x,t) = \pi(x,t)\ ,$$

$$\dot\pi(x,t) = -\left[\frac{\partial S_{\rm eff}}{\partial\phi(x,t)} - \frac{1}{\Omega}\sum_y \frac{\partial S_{\rm eff}}{\partial\phi(y,t)}\right]$$

$$\frac{1}{\Omega}\sum_y \phi(y,t) = \Phi, \qquad \sum_y \pi(y,t) = 0$$

constraint effective potential



jk, Kieran Holland
LatHC

can be extended to four-fermion operator and more general BSM models

can we extend the analysis to gauge theories?

Summary:

- sextet model is consistent with χSB from all angles we looked at
- general EFT approach will change the χPT analysis
- dilaton EFT is a new fresh look
- dilaton signatures are problematic in sextet model
- sources of the problem?
- missing dilaton terms? scale dependent $\gamma(\lambda)$? loop control?
- the ε-regime (RMT) is new opportunity for general EFT signatures!
- constraint effective potential method