Fate of a recent conformal fixed point and $\beta$-function in the $SU(3)$ BSM gauge theory with ten massless flavors

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What, why and how?

$SU(3)$ gauge theory with $N_f = 10$ flavors
IR conformal or chirally broken?

There was/is some controversy about $N_f = 12$……

$N_f = 10$ should be simpler

Relevant for model building: conformal walking: 4 + 6 model, tune masses → walking if 10 flavors conformal, if chirally broken: usual walking
What, why and how?

$N_f = 10$ interesting on its own

Last year: $N_f = 3$ sextet, $N_f = 14$ fund (both conformal, 1711.00130)

This conference: Kieran Holland: $N_f = 13$ fund (conformal)
What, why and how?

\( N_f = 10 \) interesting on its own

\[
\frac{g^2(sL) - g^2(L)}{\log(s^2)}
\]

approaching the conformal window

- fund \( N_f = 4 \) \( c = 3/10 \) \( s = 3/2 \)
- fund \( N_f = 8 \) \( c = 3/10 \) \( s = 3/2 \)
- sextet \( N_f = 2 \) \( c = 7/20 \) \( s = 3/2 \)
- fund \( N_f = 12 \) \( c = 1/5 \) \( s = 2 \)

Last year: \( N_f = 3 \) sextet, \( N_f = 14 \) fund (both conformal, 1711.00130)

This conference: Kieran Holland: \( N_f = 13 \) fund (conformal)
What, why and how?

Calculate $N_f = 10$ running coupling, $\beta$-function in continuum

Periodic finite volume gradient flow scheme

Step scaling, $L \rightarrow 2L$, discrete $\beta$-function
What, why and how?

Results in literature - domain wall

Hasenfratz, Rebbi, Witzel: 1710.11578, 8 → 16, 10 → 20, 12 → 24
Chiu: PoS LATTICE2016 (2017) 228, 8 → 16, 10 → 20, 12 → 24, 16 → 32

Discrepancy for $4.5 < g^2(L) < 6.0$
Outline

• Numerical setup
• Rooting, taste breaking, etc
• Continuum extrapolation
• Comparison with literature
• Conclusion and outlook
Numerical setup

- Tree-level improved Symanzik gauge action

- Periodic gauge field

- 4-step stout-improved rooted staggered fermions ($\rho = 0.12$)

- Anti-periodic in all directions

- $m = 0$

- $12 \rightarrow 24$, $16 \rightarrow 32$, $18 \rightarrow 36$, $20 \rightarrow 40$, $24 \rightarrow 48$
Infrared regulator $1/L$ acts similarly to $m$ in large volumes
→ stable algorithm
Rooting - taste breaking in Dirac eigenvalues

Lowest 8 eigenvalues

First (low $\beta$): doublets, then (high $\beta$): quartets
Fix $g^2(L) = 5.5$, taste breaking disappears in the continuum.
Continuum extrapolation

- Interpolate by polynomials (rather than tune)

- Larger $c$: smaller cut-off effects, larger stat errors (knew this already)

- Take $c = 1/4$, $3/10$ and $s = 2$ (also $s = 3/2$)

- Check consistency of SSC and WSC discretizations
Continuum extrapolation

\[ N_f = 10 \quad \text{SSC} \quad c = 0.25 \quad \text{interpolation} \]

\[ \beta \]

\[ g^2(L) \]

\[ 12^4 \rightarrow 24^4 \]

\[ \chi^2/\text{dof} = 0.4 \text{ and } 1.7 \]

\[ 16^4 \rightarrow 32^4 \]

\[ \chi^2/\text{dof} = 0.92 \text{ and } 0.56 \]

\[ 18^4 \rightarrow 36^4 \]

\[ \chi^2/\text{dof} = 0.97 \text{ and } 0.094 \]

\[ 20^4 \rightarrow 40^4 \]

\[ \chi^2/\text{dof} = 1.4 \text{ and } 0.69 \]
Continuum extrapolation

$N_f = 10$  SSC  $c = 0.25$  interpolation

$\chi^2$/dof = 1.7 and 0.9
Continuum extrapolation

$N_f = 10 \quad \text{beta function}$

continuum SSC: $0.45 \pm 0.046$

continuum WSC: $0.386 \pm 0.061$

$g^2 = 5, c = 0.30, s = 2$

\begin{align*}
\frac{g^2(sL) - g^2(L)}{\log(s^2)} & \approx \frac{a^2}{L^2} \times 10^{-3} \\
\end{align*}
Continuum extrapolation

\[
\frac{g^2(sL) - g^2(L)}{\log(s^2)}
\]

continuum WSC: 0.575 ± 0.043
continuum SSC: 0.646 ± 0.04

\( g^2 = 6, c = 0.30, s = 2 \)

\( N_f = 10 \) beta function
Continuum extrapolation

$N_f = 10$  beta function

continuum WSC: $0.727 \pm 0.037$
continuum SSC: $0.795 \pm 0.042$

$g^2 = 7$, $c = 0.30$, $s = 2$
Final result from $12 \rightarrow 24$, $16 \rightarrow 32$, $18 \rightarrow 36$, $20 \rightarrow 40$, $24 \rightarrow 48$

\[
\frac{g^2(sL) - g^2(L)}{\log(s^2)}
\]

\[N_f = 10 \text{ beta function}\]

\[c = 1/4 \quad s = 2\]
Final result from $12 \rightarrow 24, 16 \rightarrow 32, 18 \rightarrow 36, 20 \rightarrow 40, 24 \rightarrow 48$

$c = 3/10 \quad s = 2$
Final result

approaching the conformal window

\( \frac{g^2(sL) - g^2(L)}{\log(s^2)} \)

For:
- \( N_f = 4 \) with \( c = \frac{3}{10} \) and \( s = \frac{3}{2} \)
- \( N_f = 8 \) with \( c = \frac{3}{10} \) and \( s = \frac{3}{2} \)
- \( N_f = 12 \) with \( c = \frac{1}{5} \) and \( s = 2 \)
approaching the conformal window

- $N_f = 4$, $c = 3/10$, $s = 3/2$
- $N_f = 8$, $c = 3/10$, $s = 3/2$
- $N_f = 10$, $c = 3/10$, $s = 2$
- $N_f = 12$, $c = 1/5$, $s = 2$
approaching the conformal window

- fund $N_f = 4$, $c = \frac{3}{10}$, $s = \frac{3}{2}$
- fund $N_f = 8$, $c = \frac{3}{10}$, $s = \frac{3}{2}$
- fund $N_f = 10$, $c = \frac{3}{10}$, $s = 2$
- sextet $N_f = 2$, $c = \frac{7}{20}$, $s = \frac{3}{2}$
- fund $N_f = 12$, $c = \frac{1}{5}$, $s = 2$
Comparison with existing literature

\[
\frac{g^2(sL) - g^2(L)}{\log(s^2)}
\]

\[c = \frac{3}{10}, \quad s = 2\]

This work

5-loop

H, R, W

T-W C

Graph showing the comparison with existing literature.
Why the disagreement?

- Domain wall - too small volumes?
- Domain wall - residual mass non-zero?
- Non-universality (???)
Non-universality?

Outside conformal window: $\beta$-function positive for all $0 < g^2(L)$

Only Gaussian UV fixed point, governs continuum limit, $g_0 \rightarrow 0$

Bare perturbation theory (i.e. perturbation theory on cut-off scale) reliable close to continuum

Various discretizations can be judged to be in the right universality class by perturbation theory

Anything $= \text{continuum} + O(a)$ is okay \\
(dimension, symmetries, locality)

Staggered, Wilson, domain wall, overlap, etc. all okay
Non-universality?

Inside conformal window: $\beta$-function has simple zero at $g^2_*$ and is positive for $0 < g^2(L) < g^2_*$

Gaussian UV fixed point still there (only these 2)

Non-trivial RG flow between UV fixed point $g^2 = 0$ and IR fixed point $g^2 = g^2_*$ as $L = 0, \ldots, \infty$

In particular, running is via dimensionless quantity $\Lambda L$

Small $\Lambda L$: $g^2(L) \sim \frac{1}{\log\left(\frac{1}{\Lambda L}\right)}$

Large $\Lambda L$: $g^2(L) \sim g^2_* - \frac{\text{const}}{(\Lambda L)^\alpha}$
Non-universality?

For $0 < g^2(L) < g_*^2$ the volume $L$ is finite in physical units ($\Lambda$)

At finite $L$, i.e. $g^2(L) < g_*^2$, continuum limit is governed by Gaussian UV fixed point

Bare perturbation theory still reliable close to the continuum limit

Various discretizations can be judged to be in the right universality class by perturbation theory

For $g^2(L) < g_*^2$ exactly the same story as outside conformal window
Non-universality?

Other example: $T^3 \times R$ Hamiltonian formulation, $g^2(L) < g^2_*$

- Non-trivial finite masses $M_i = C_i/L$

- Well-defined ratios $C_{ij} = M_i/M_j = C_i/C_j$

- Lattice corrections: $O(a^2)$

- Continuum limit $g_0 \rightarrow 0$ or $\beta \rightarrow \infty$

- Same as outside conformal window
Non-universality?

Only $g^2(L = \infty) = g_*^2$ is tricky (but not needed for $g^2(L) < g_*^2$) both on $T^4$ and $T^3 \times R$

Staggered, Wilson, domain wall, overlap, etc, MUST give the same result for $g^2(L) < g_*^2$

If not, they would disagree in QCD too
Non-universality?

Proposal: fix $g^2(L) = 5.7$

What is $(g^2(sL) - g^2(L))/\log(s^2)$?

All 3 results give positive $\beta$-function

What happens for $g^2(L) > 5.7$ is irrelevant

There MUST be agreement once continuum limit is carefully/correctly done via $g_0 \to 0$ or $\beta \to \infty$

All 3 groups should agree (before going to higher $g^2(L)$)
Non-zero domain wall residual mass

Finite 5th dimension: H, W, R: mostly $L_s = 12$, T-W C: $L_s = 16$

Bare mass zero, but residual mass non-zero

Most severe: large $g^2(L) \sim 6 - 7$

Finite mass effect: $g^2(\beta, L/a, am) = g^2(\beta, L/a) + \Delta(L/a)$

Introduce: $x = g^2(L), \quad y = (g^2(sL) - g^2(L))/\log(s^2)$

Mass effect in $x$ direction: $\Delta x = \Delta(L/a)$

Mass effect in $y$ direction: $\Delta y = \frac{\Delta(sL/a) - \Delta(L/a)}{\log(s^2)}$

Volume-independent mass-dependence completely cancels, remaining effect: volume-dependent mass-dependence $\rightarrow \Delta y$ expected to be small
Is the shifted $b(g^2)_{m>0}$ curve above or below $b(g^2)_{m=0}$?

Depends on $\frac{db(g^2)}{dg^2}_{m=0}$ more or less than $\frac{\Delta y}{\Delta x} = \frac{\Delta (sL/a) - \Delta (L/a)}{\Delta (L/a) \log(s^2)}$

H,W,R: $m > 0$ is always above $m = 0$ ??
Conclusions and outlook

- No IR fixed point for $5 < g^2(L) < 8$

- Potential reasons for disagreements: too small lattice volumes and/or too large residual mass for H,W,R and T-W C

- Even with IRFP: $g^2(L) < g_\ast^2$ is universal (in usual sense)

- 3-way discrepancy should be resolved, e.g. at $g^2(L) = 5.7$

- Would be good: low energy observables in p-regime

- Hadron/glueball spectrum, chiral condensate, etc.

- Running coupling in p-regime
Thank you for your attention!
Backup slides
Results

\[ \frac{g^2(sL) - g^2(L)}{\log(s^2)} \]

\( g^2(L) \)

\( c = 1/4 \quad WSC \quad s = 3/2 \)

12->18
16->24
20->30
24->36
32->48

\[ g^2(L) \]
Results

(\(g^2(sL) - g^2(L)\)) / log(s)

c = 1/4
SSC
s = 3/2

12->18
16->24
20->30
24->36
32->48
Results

\[ \frac{g^2(sL) - g^2(L)}{\log(s^2)} \]

\( c = 1/4 \quad WSC \quad s = 2 \)

12->24
16->32
18->36
20->40
24->48


\[
\frac{g^2(sL) - g^2(L)}{\log(s^2)}
\]

c = 1/4
SSC
s = 2

12->24
16->32
18->36
20->40
24->48
Results

\[ \frac{g^2(sL) - g^2(L)}{\log(s^2)} \]

\[ g^2(L) \]

\[ c = \frac{3}{10} \quad \text{WSC} \quad s = \frac{3}{2} \]

-1
-0.5
0
0.5
1
1.5
3  4  5  6  7  8  9  10

12->18
16->24
20->30
24->36
32->48
Results

\[
\frac{(g^2(sL) - g^2(L))}{\log(s^2)}
\]

\[c = \frac{3}{10}\]

SSC

\[s = \frac{3}{2}\]
Results

\[
\frac{g^2(sL) - g^2(L)}{\log(s^2)}
\]

c = 3/10

WSC

s = 2

12->24
16->32
18->36
20->40
24->48
Results

\[ \frac{g^2(sL) - g^2(L)}{\log(s^2)} \]

\( g^2(L) \)

\( c = \frac{3}{10} \)

SSC

\( s = 2 \)

12->24
16->32
18->36
20->40
24->48