Towards models with an unified dynamical mechanism for elementary particle masses

Roberto Frezzotti & Giancarlo Rossi

Physics Department and INFN of Roma Tor Vergata
Centro Fermi – Roma – Italy

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Talk based on

  and work in preparation

& on the results of recent lattice work (talk by M. Garofalo) in collaboration with

- P. Dimopoulos, G.C. Rossi (Univ. of Roma Tor Vergata and Centro Fermi)
  and M. Garofalo (INFN of Roma Tor Vergata)
- B. Kostrzewa, F. Pittler, C. Urbach (HISKP - University of Bonn)

Outline:

- Elementary fermion mass mechanism as a non-perturbative “anomaly”
- Mass mechanism in a Toy Model with weak interactions \( (g_W > 0) \)
- Towards a realistic BSM model: mechanism & all interactions ?
- Outlook & Conclusions
Fermion mass as a non-perturbative (NP) “anomaly”

Results in the talk by M. Garofalo \(\iff\) unnoticed NP feature in QFT

1. Consider a renormalizable UV regulated gauge model (RGI scale \(\Lambda_S\)) with fermionic chiral symmetries \((\tilde{\chi}_{L,R})\) broken in hard way at the UV cutoff scale, \(\Lambda_{UV}\)

2. Include fundamental scalar doublet \((\phi)\) in order to have an exact chiral symmetry involving fermions and scalars: prerequisite for EW theory & no \(\Lambda_{UV} \bar{Q}Q\) term

3. Interplay of strong gauge interactions and \(\tilde{\chi}_{L,R}\) breaking \(\Rightarrow\) NP “anomaly” in the Nambu-Goldstone phase \((\langle \phi \rangle = v)\): seen in \(\tilde{\chi}_{L,R}\) Schwinger Dyson Eq.s (SDE)

4. In the critical model, \(\eta = \eta_{cr}(\rho)\), where \(\tilde{\chi}_{L,R}\) breaking is minimized we have

   bare \(\tilde{\chi}_L\) SDE:
   \[
   \partial \tilde{J}^{Li}(x) = \eta \tilde{\delta}^{Li} O_4^Y(x) + \Lambda_{UV}^{-2} \rho \tilde{\delta}^{Li} O_6^{\text{Wil}}(x)
   \]

   but for insertion in correlators at momenta \(p\) s.t. \(\Lambda_S \ll p \ll \Lambda_{UV}\)

   a) renormalized \(\tilde{\chi}_L\) SDE in Wigner phase:
   \[
   Z_J \partial \tilde{J}^{Li}(x) = \Lambda_{UV}^{-2} O_{n \geq 6}^{Li}(x)
   \]

   b) renormalized \(\tilde{\chi}_L\) SDE in NG phase:
   \[
   Z_J \partial \tilde{J}^{Li}(x) = O_4^{Li}(x) + \Lambda_{UV}^{-2} O_{n \geq 6}^{Li}(x)
   \]
NP fermion mass “anomaly” and LE effective action

What’s $O_{4}^{Li}(x)$? In what context is it defined? Is it UV finite & RG invariant?

1. If non-zero, it should be finite & RG invariant: @ $\eta = \eta_{cr}(\rho)$ the current $Z_{j} \tilde{J}^{Li}$ is conserved up to $O(\Lambda_{UV}^{-2})$ in Wigner phase $\Rightarrow$ SDE’s r.h.s. RG invariant in NG phase

2. No exact WTI for $\tilde{\chi}_{L,R}$ fermionic transformations (even @ $\eta_{cr}$) $\Rightarrow$ at NP level the renormalized form of $\tilde{\chi}_{L,R}$ SDE at low energy may differ in Wigner and NG phases

[NP renormalization ultimately based on "common wisdom" + numerical evidence]

3. NP term in r.h.s. of the renormalized $\tilde{\chi}_{L,R}$ SDE $\iff$ NP $\tilde{\chi}_{L,R}$ breaking terms in the LE effective action, $\Gamma_{LE}$, for correlators at momenta $\rho$ s.t. $\Lambda_{S} \ll \rho \ll \Lambda_{UV}$

4. @ $\eta = \eta_{cr}(\rho)$ no $\tilde{\chi}_{L,R}$ breaking term is allowed in $\Gamma_{Wigner}^{LE}$ as $\Lambda_{UV}^{-1} \rightarrow 0$; but $\Gamma_{NG}^{LE}$ may include $\Lambda_{S} C_{1}(\bar{Q}_{L} U Q_{R} + \text{h.c.})$, $\ldots \frac{\Lambda_{S}^{2}}{\nu^{4}} C_{-2}((\bar{Q}_{L} U Q_{R})(\bar{Q}_{L} U Q_{R}) + \text{h.c.}), \ldots$

$\rho$ generic, effective fields in $\Gamma_{NG}^{LE}$: $U = \exp(\frac{i}{\nu} \sum_{j=1,2,3} \tau^{i} \zeta^{j})$, $R = \nu + \zeta^{0}$

$O_{4}^{Li}(x) = \tilde{\delta}^{Li}[\Lambda_{S} C_{1}(\bar{Q}_{L} U Q_{R} + \text{h.c.}) + \ldots]$ describes NP $\tilde{\chi}_{L,R}$ breaking in NG phase
NP mass in LE eff. action: universality, naturalness

Implications of RG invar. \( \Lambda_S C_1 (\bar{Q}_L U Q_R + \text{h.c.}) + \ldots \) in \( \Gamma_{LE}^{NG} \) for predictive power?

1. The whole \( \tilde{\chi}_{L,R} \) breaking NP term in \( \Gamma_{LE}^{NG} \), or in the r.h.s. of renormalized SDEs
   a) is irreducibly different as an operator from Yukawa’s \( R(\bar{Q}_L U Q_R + \text{h.c.}) \)
   b) cannot & needs not be renormalized by adding any Lagrangian counterterm
   c) involves coefficients (\( C_1 \)) depending on the \( \tilde{\chi}_{L,R} \) breaking UV parameters (\( \rho \))

2. LE physics determined by gauge coupling & \( \tilde{\chi}_{L,R} \) parameters at the UV-scale:
   “universality” \( \iff \) equivalence classes of different UV complete models with
   identical \( \tilde{\chi}_{L,R} \) effects at LE \( \iff \) take the representative with simplest \( \tilde{\chi}_{L,R} \) term;

3. \( m_{Q}^{\text{eff}} = C_1'' \Lambda_S'' \) with \( C_1 = O(\alpha_S^2) \) and \( ''\Lambda_S'' \sim \frac{\Lambda_S v^2}{\Lambda_S^2 + v^2} \) or so, vanishing as \( v \rightarrow 0 \)
   is the elementary fermion mass in \( \Gamma_{LE}^{NG} \), normalized at same scale as \( \bar{Q}_L U Q_R \)
   is natural à la ’t Hooft: RGI scale \( \Lambda_S \), no Yukawa mass due to minimization of \( \tilde{\chi}_{L,R} \)

4. Massless Goldstones \( \zeta^{1,2,3} : \ldots \) can be eaten up by massive \( W \) gauge bosons
Mechanism in Toy Model with weak interactions \((g_W > 0)\)

UV regularization left unspecified: hard UV cutoff \(\Lambda_{UV} \sim \frac{1}{b}\)
Simplest toy model with weak interactions

Introducing weak gauge interactions for L-handed fermions, what about

★ definition and restoring of $\tilde{\chi}_{L,R}$ (fermionic chiral) symmetries?

★ $W$-boson mass: from strong interactions & same mechanism as for fermions?

Extended toy model with minimal fermion content (avoiding global SU(2) anomaly):

gauge: $\text{SU}(3)_s \times \text{SU}(2)_L$ ($g_Y = 0$) & Dirac fermions: $Q \in 3$ & $N \in 1$ of $\text{SU}(3)_s$

$$
\mathcal{L}_{\text{toy}}(Q, N, A, \Phi, W) = \mathcal{L}_{\text{kin}}(Q, N, A, \Phi, W) + \mathcal{V}(\Phi) + \mathcal{L}_{\text{Wil}}(Q, N, A, \Phi, W) + \mathcal{L}_{\text{Yuk}}(Q, N, \Phi)
$$

$$
\mathcal{L}_{\text{kin}} = \frac{1}{4} F_{\mu\nu}^A A^a_{\mu\nu} + \frac{1}{4} F_{\mu\nu}^W F_{\mu\nu}^W + \bar{Q}_L \gamma_\mu D_{\mu}^A W Q_L + \bar{Q}_R \gamma_\mu D_{\mu}^A W Q_R
$$

$$
+ \bar{N}_L \gamma_\mu D_{\mu}^W N_L + \bar{N}_R \gamma_\mu \partial_\mu N_R + \frac{1}{2} \text{Tr}[\Phi^\dagger \tilde{D}_{\mu}^W D_{\mu}^W \Phi]
$$

$$
\mathcal{L}_{\text{Wil}} = \frac{b^2}{2} \rho \left( \bar{Q}_L \tilde{D}_{\mu}^A W \Phi D_{\mu}^A Q_R + \bar{Q}_R \tilde{D}_{\mu}^A \Phi^\dagger D_{\mu}^A W Q_L + N_L \tilde{D}_{\mu}^W \Phi \partial_\mu N_R + \bar{N}_R \tilde{D}_{\mu} \Phi^\dagger D_{\mu}^W N_L \right)
$$

$\phi$: $\text{SU}(2)_L$ doublet, $\text{SU}(3)_s$ singlet

matrix notation: $\Phi = [\tilde{\varphi}|\varphi]$ , $\tilde{\varphi} = -i \tau^2 \varphi^*$
Toy model with weak interactions: symmetries

Usual quartic scalar potential \((\hat{m}_\Phi^2 = m_0^2 - m_{cr}^2)\) and Yukawa term (coupling \(\eta\)):

\[
\mathcal{V}(\Phi) = \frac{m_0^2}{2} \text{Tr}[\Phi^\dagger \Phi] + \frac{\lambda_0}{4} (\text{Tr}[\Phi^\dagger \Phi])^2 \quad \mathcal{L}_{Yuk}(Q, \Phi) = \eta (\bar{Q}_L \Phi Q_R + \bar{Q}_R \Phi^\dagger Q_L)
\]

SU(2)_L gauge symmetry: \(W^1,2,3\) bosons & covariant derivatives on \(f = Q, N\), e.g.

\[
D^A_W f_L = (\partial_\mu - i \delta_{f,Q} g_s \lambda^a A^a_\mu - ig \tau^i \frac{\tau^i}{2} W^i_\mu) f_L \quad \bar{f}_L D^A_W = \bar{f}_L (\partial_\mu + i \delta_{f,Q} g_s \lambda^a A^a_\mu + ig \tau^i \frac{\tau^i}{2} W^i_\mu)
\]

Global SU(2)_L \times SU(2)_R invariance, if \(W\)'s transform (as \(\in su(2)_L\)) under \(\tilde{\chi}_L\):

\[
\chi_L \equiv \tilde{\chi}_L \otimes \chi^\phi_L \quad \text{and} \quad \chi_R \equiv \tilde{\chi}_R \otimes \chi^\phi_R \quad \text{with}
\]

\[
\tilde{\chi}_L : Q[N]_L \to \Omega_L Q[N]_L, \quad \bar{Q}[N]_L \to \bar{Q}[N]_L \Omega_L^\dagger, \quad \chi^\phi_L : \Phi \to \Omega_L \Phi, \quad \Omega_L \in SU(2)_L,
\]

\[
W_\mu \to \Omega_L W_\mu \Omega_L^\dagger,
\]

\[
\tilde{\chi}_R : Q[N]_R \to \Omega_R Q[N]_R, \quad \bar{Q}[N]_R \to \bar{Q}[N]_R \Omega_R^\dagger, \quad \chi^\phi_R : \Phi \to \Phi \Omega_R^\dagger, \quad \Omega_R \in SU(2)_R,
\]

\(\tilde{\chi}_{L,R}\) and \(\chi^\phi_{L,R}\) transf.s are no symmetries ... term \(\text{Tr}[\Phi^\dagger D^W_\mu D^W_\mu \Phi]\) breaks \(\tilde{\chi}_L\), too
g_W > 0 : maximal $\tilde{\chi}$–symmetry restoring at $(\rho_{cr}, \eta_{cr})$

\[ \partial_\mu \langle \tilde{J}^{L; i}_\mu (x) \hat{O}(0) \rangle = \langle \tilde{D}^i_L \hat{O}(0) \rangle \delta(x) - \eta \langle \sum_{f=Q,N} (\bar{f}_L \tau^i_2 \Phi_R - \bar{f}_R \Phi^\dagger \tau^i_2 f_L) (x) \hat{O}(0) \rangle + \]

\[ - \frac{b^2}{2} \rho \langle \sum_{f=Q,N} (\bar{f}_L \delta^A_{\mu} W^i_2 \Phi^A_{\mu} f_R - \bar{f}_R \delta^A_{\mu} \Phi^\dagger \tau^i_2 D^A_{\mu} f_L) (x) \hat{O}(0) \rangle + \]

\[ + \frac{i}{2} g_W \langle \text{Tr} \left( \Phi^\dagger [\tau^i_2, W_\mu] D^W_{\mu} \Phi + \Phi^\dagger \delta^W_{\mu} [W_\mu, \tau^i_2] \Phi \right) (x) \hat{O}(0) \rangle \]

\[ \tilde{J}^{L; i}_\mu = \sum_{f=Q,N} \{ \bar{f}_L \gamma_\mu \frac{\tau^i_2}{2} f_L - \frac{b^2}{2} \rho \left( \bar{f}_L \frac{\tau^i_2}{2} \Phi D^A_{\mu} f_R - \bar{f}_R \delta^A_{\mu} \Phi^\dagger \frac{\tau^i_2}{2} f_L \right) \} + g_W \text{Tr} \left( [W_\nu, F^W_{\mu \nu}] \frac{\tau^i_2}{2} \right) \]

upon renormalization one expects the $\tilde{\chi}_L$–SDE’s of the LE effective theory to read

\[ Z_J \partial_\mu \langle \tilde{J}^{L; i}_\mu (x) \hat{O}(0) \rangle = \langle \tilde{D}^i_L \hat{O}(0) \rangle \delta(x) - (\eta - \bar{\eta}_L) \langle \sum_{f=Q,N} (\bar{f}_L \frac{\tau^i_2}{2} \Phi_R - \text{h.c.}) (x) \hat{O}(0) \rangle + \]

\[ + (1 - \bar{\gamma}) \frac{i}{2} g_W \langle \text{Tr} \left( \Phi^\dagger [\frac{\tau^i_2}{2}, W_\mu] D^W_{\mu} \Phi + \Phi^\dagger \delta^W_{\mu} [W_\mu, \frac{\tau^i_2}{2}] \Phi \right) (x) \hat{O}(0) \rangle + \cdots + \mathcal{O}(b^2) \]

Critical model: \[ \eta_{cr} = \bar{\eta}_L (g_s, g_W; \mu_0, \lambda_0; \eta_{cr}, \rho_{cr}) , \quad 1 = \bar{\gamma}(g_s, g_W; \mu_0, \lambda_0; \eta_{cr}, \rho_{cr}) \]

here $\tilde{\chi}_L$ and $\tilde{\chi}_R$ restored at low momenta, up to possible NP terms $\sim \Lambda_s^n$, $n = 2, 1, \ldots$
\[ \hat{m}_\Phi^2 > 0 : \text{full restoring of } \tilde{\chi}\text{–symmetry } \oplus (\rho_{cr}, \eta_{cr}) \]

Wigner phase at \((\rho_{cr}, \eta_{cr})\): \(\mathcal{V}(\Phi)\) has a single minimum \((\hat{m}_\Phi^2 > 0) \Rightarrow \text{renormalized SDEs of } \tilde{\chi}_{L(R)} : \)

\[ Z_J \partial_\mu \left\langle \tilde{j}_{\mu}^{L(R), i}(x) \hat{O}(0) \right\rangle = \left\langle \hat{\Delta}_{L(R)}^{i}(0) \right\rangle \delta(x) + O(b^2) \]

\(\star\) requiring full \(\Phi\) decoupling at low energy determines \((\rho_{cr}, \eta_{cr}) : \) at LO see figure

\(\star\) no NP operators \(\sim \Lambda_s\) occur in r.h.s. \(\iff\) due to \(\nu_\Phi = 0\) here \(U\)-field is undefined

\[ \Phi \text{ is fully decoupled & auxiliary in the effective action valid for } \Lambda_{UV} \gg p \gg \Lambda_s \]

\[ \Gamma_{\text{loc}}^{\text{Wig}} \equiv \Gamma_{\text{loc}}^{\hat{m}_\Phi^2 > 0} = \frac{1}{4} [(F^A F^A) + (F^W F^W)] + \sum_{f=Q, N} [\tilde{f}_L \mathcal{D}^A f_L + \tilde{f}_R \mathcal{D}^A f_R] + \mathcal{V}_{\text{eff}}^{\hat{m}_\Phi^2 > 0}[\Phi] \]

\[ \rho_{cr} \sim O(1/\sqrt{N_F^{\text{tot}}}) \]
\( \hat{m}_\Phi^2 < 0 \) : dynamical \( \tilde{\chi} \) SB & NP masses \( @ (\rho_{cr}, \eta_{cr}) \)

NG phase at \( (\rho_{cr}, \eta_{cr}) \): \( \mathcal{V}(\Phi) \) has many degenerate minima \( (\hat{m}_\Phi^2 < 0) \) \( \Rightarrow \)

\( \star \) by def. of \( (\rho_{cr}, \eta_{cr}) \) no \( O(\nu) \) \( W \)-mass and \( O(\nu) \) \( Q(N) \)-mass terms: e.g. at LO

\[ v^2 [\begin{array}{c} \tilde{\chi} \cr \tilde{\chi} \end{array}] \left[ \begin{array}{cc} m_\phi & 0 \\ 0 & m_\phi \end{array} \right] [\begin{array}{c} \tilde{\chi} \cr \tilde{\chi} \end{array}] = 0 \]

\( \star \) under dynamical \( \tilde{\chi} \) SB vacuum polarized by residual \( O(b^2 \nu) \) \( \tilde{\chi} \)-breaking terms

\( \star \) interplay of \( O(b^2) \) \( \tilde{\chi}_{L,R} \) and \( \tilde{\chi} \) SB dynamics \( \Rightarrow \) \( O(b^0) \) \( \tilde{\chi}_{L,R} \) terms in

\[ \Gamma^{NG}_{loc} = \Gamma^{\hat{m}_\Phi^2 < 0}_{loc} + C_{1,Q} \Lambda_s [\bar{Q}_L U Q_R + \bar{Q}_R U^\dagger Q_L] + C_{2} \Lambda_s^2 \text{Tr} [U^\dagger \overleftrightarrow{D}_\mu D_\mu U] \]

\( \Leftrightarrow (M_W^{eff})^2 = g_W^2 C_2 \Lambda_s^2 \) and \( m_{eff}^Q = C_{1,Q} \Lambda_s \) from common mechanism; \( m_{eff}^N = 0 \)

short distance NP vertex corrections

NP mass \( @ \) low energy
NP masses in $\Gamma_{\text{loc}}^{NG} @ (\rho_{cr}, \eta_{cr})$: remarks

1. Effective NP masses modulated by gauge couplings & loop suppression factors:
   
   \[ C_2 = O(g_s^4 \rho_{cr}^6 N_f; \sim 3 \text{ loop suppr.}), \quad C_{1,Q} = O(g_s^4 \rho_{cr}^3 N_f; \sim 2 \text{ loop suppr.}) \]

   \[ C_{1,N} = 0 \] owing to a symmetry of $L_{\text{toy}}$ valid in case of sterile R-handed fermions $N_R(x) \rightarrow N_R(x) + c$, \[ \tilde{N}_R(x) \rightarrow \tilde{N}_R(x) + \bar{c}, \quad c, \bar{c} \text{ constant} \] [Goltermann & Petcher, 1990]

2. Absence in $\Gamma_{\text{loc}}^{NG}$ of term $\tilde{C} \Lambda_s R \text{Tr}\left[U^\dagger \overset{\leftarrow}{D}_\mu W D_\mu U\right]$ is peculiar of the critical model:

   as $\rho_{cr}^2 - \rho^2 \rightarrow 0^+$ & $\eta \rightarrow \eta_{cr}$ we have $1 - \bar{\gamma} \rightarrow 0^+$, $m_\zeta^2 \sim |\hat{m}_\Phi|/(1 - \gamma) \rightarrow +\infty$

   \[ \Rightarrow \text{decoupling of } \zeta^0. \] For $\Gamma_{\text{loc}}^{NG}$ to describe the decoupling of $\zeta^0$, e.g. in $WW \rightarrow WW$ amplitudes, \[ |\tilde{C}| \leq O((1 - \bar{\gamma})^{1/2}) \xrightarrow{\rho \rightarrow \rho_{cr}, \eta \rightarrow \eta_{cr}} 0 \] is necessary

3. NP kinetic term for GB’s: canonical normalization $\Rightarrow \ U = \exp(i\zeta^2/\sqrt{C_2 \Lambda_s})$

   basic GBs NP-ly coupled so as to provide longitudinal d.o.f.s for massive $W$s

4. Further $\tilde{\chi}$-breaking terms in $\Gamma^{NG}$:

   \[ \frac{1}{\Lambda_s^2} \left[ (\bar{Q}_L U Q_R)(\bar{Q}_R U^\dagger Q_L), \ [\text{Tr}(D^W_\mu U^\dagger D^W_\mu U)]^2, \ ... \right. \]
Towards a realistic BSM model: mechanism & all interactions?

A possible explanation of why nothing new is seen close to the EW scale.

Hints at $\Lambda_T \sim 5 \text{ TeV}$: calling for experimental search in that energy range.
Hypothesis: effective mass of elementary particles stemming from our mechanism

Consistency of hypothesis with experimentally observed masses implies (hints at)

- new Tera-strong SU($N_T$) interaction with RGI scale $\Lambda_T > M_W \gg \Lambda_{QCD}$

- new set of Tera-fermions subjected to the new force (besides to SM interactions)
  - $Q_L \in (N_T, 3, 2; Y_Q)$, $L_L \in (N_T, 1, 2; Y_L)$
  - $Q^u_R \in (N_T, 3, 1; Y^u_Q)$, $L^u_R \in (N_T, 1, 1; Y^u_L)$
  - $Q^d_R \in (N_T, 3, 1; Y^d_Q)$, $L^d_R \in (N_T, 1, 1; Y^d_L)$

with (irrep. of SU($N_T$), SU(3)$_c$, SU(2)$_L$; $Y = Q_{em} - T_3$) ; besides SM fermions, e.g.
  - $q_L \in (1, 3, 2; 1/6)$, $\ell_L \in (1, 3, 2; -1/2)$
  - $t_R \in (1, 3, 1; 2/3)$, $\nu_R \in (1, 1, 1; 0)$
  - $b_R \in (1, 3, 1; -1/3)$, $\tau_R \in (1, 1, 1; -1)$

- composite higgs: a bound state in $WW$, $ZZ$, $Q\bar{Q}$, $L\bar{L}$ ... channel; Tera-fermions & -strong force crucial for binding; needed for unitarity of $WW \rightarrow WW$ scattering at LE
UV complete Lagrangian: towards a realistic model

\[ \mathcal{L}^{BSMM} = \frac{1}{4} \left( F^B F^B + F^W F^W + F^A F^A + F^G F^G \right) + \frac{1}{2} \text{Tr} \left( \Phi^\dagger \overleftrightarrow{D^B, W}_\mu \overleftrightarrow{D^B, W}_\mu \Phi \right) + \mathcal{V}(\Phi) + \]

+ \sum_{3 \text{ families}} \left[ \bar{q}_L D^{BWA} q_L + \bar{q}^u_R D^{BA} q^u_R + \bar{q}^d_R D^{BA} q^d_R + \ell_L D^{BW} \ell_L + \bar{\ell}^u_R D^B \ell^u_R + \bar{\ell}^d_R D^B \ell^d_R \right] + \\
+ \left[ \bar{Q}_L D^{BWAG} Q_L + \bar{Q}^u_R D^{BAG} Q^u_R + \bar{Q}^d_R D^{BAG} Q^d_R + \bar{L}_L D^{BWG} L_L + \bar{L}^u_R D^{BG} L^u_R + \bar{L}^d_R D^{BG} L^d_R \right] + \\
+ \sum_{f=q, \ell, Q, L} \left[ \eta_{f, cr} \left( \bar{f}_L \phi f_R + \text{h.c.} \right) + \frac{1}{2} b^2 \rho_f O^{u}_{i, \tilde{\chi}} + \eta_{f, cr} \left( \bar{f}_L \phi f_R + \text{h.c.} \right) + \frac{1}{2} b^2 \rho_f O^{d}_{i, \tilde{\chi}} \right]

\text{where } f \text{ runs over doublets } q^{(3)}, \ell^{(3)}, q^{(2)}, \ell^{(2)}, q^{(1)}, \ell^{(1)}, Q \text{ and } L: \text{ in principle different } \rho_f \text{'s, but}

\tilde{\chi}\text{-symmetry restoring } \Rightarrow \sum_{f=1}^{N_{\text{ferm}}^{\text{tot}}} \rho_f^2 (1 + O(\rho_f^2)) = O(1) \quad \text{and} \quad \eta_f = \eta_{f, cr}(\{\rho\})

• In NG phase: dynamical mechanism yields \( m_{Q, L}^{\text{eff}} \sim \Lambda_T \) (as \( \alpha_T(\Lambda_T) = O(1) \) ) \( \implies m^{\text{eff}} \) for \( W^\pm, Z, q \) and \( \ell \) (not for \( \nu \)'s) \( [\sim \text{Bardeen, Hill & Lindner, 1990 + “naturalness”}] \)

• Assuming \( \rho_{Q, L} \approx \rho_{q^{(3)}} \approx \rho_{\ell^{(3)}} \): \( m_W^{\text{eff}} \) and \( m_t^{\text{eff}} \) may be \( (0.10 \div 0.01)m_{Q, L}^{\text{eff}} \) due to loop suppression of \( m^{\text{eff}} \)'s (lattice test) & gauge coupling dependence: \( g^4_T|_{\Lambda_T} \gg g^4_3|_{\Lambda_T} \)
Masses of EW bosons, Tera-fermions and top quark

- \((M_Z^{\text{eff}})^2 = [(g_W^2 + g_Y^2)/(g_W^2)](M_W^{\text{eff}})^2\), while \(M_{\gamma}^{\text{eff}} = 0\)

\[
\Gamma_{LE}^{\text{NG}} \supset C_2 \Lambda_T^2 \frac{1}{2} \text{Tr} [D_{\mu}^{W,B} U^\dagger D_{\mu}^{W,B} U] \supset C_2 \Lambda_T^2 \left[ g_W^2 \sum_{j=1}^{3} (W^j \cdot W^j) + g_Y^2 B \cdot B + 2 g_W g_Y W^3 \cdot B \right]
\]

\(\Rightarrow\) diagonalization in \(W^3 - B\) sector gives massless \(\gamma\) and \(M_2^2 = (g_W^2 + g_Y^2) C_2 \Lambda_T^2\)

owing to the custodial \(\text{SU}(2)_L \times \text{SU}(2)_R\) symmetry of \(\mathcal{L}^{BSMM}\) in the \(g_Y \to 0\) limit

- Tera-hadron resonances with \(E_{\text{CoM}} \simeq (2 \div 3) m_{Q(L)}^{\text{eff}} \sim 5 \div 10\) TeV to be observed

\[
m_{Q(L)}^{\text{eff}} = C_{1,Q(L)} \Lambda_T
\]
\[
m_t = C_{1,t} \Lambda_T
\]
\[
(M_W^{\text{eff}})^2 = g_W^2 C_2 \Lambda_T^2
\]

\[
m_{Q(L)}^{\text{eff}} = C_{1,Q(L)} \Lambda_T \quad C_{1,Q(L)} |\bar{\mu}| = O(g_T^4 |\bar{\mu}| \rho_{Q,L}^3 N_{Q+L}; \sim 2 \text{ loop suppr.}) \simeq O(1)
\]
\[
m_t = C_{1,t} \Lambda_T \quad C_{1,t} |\bar{\mu}| = O(2g_3^4 |\bar{\mu}| \rho_t^3 N_{Q+L}; \sim 2 \text{ loop suppr.}) = O(0.05)
\]
\[
(M_W^{\text{eff}})^2 = g_W^2 C_2 \Lambda_T^2 \quad C_2 |\bar{\mu}| = O(g_T^4 |\bar{\mu}| \rho_{Q,L}^6 N_{Q+L}; \sim 3 \text{ loop suppr.}) \simeq O((0.03)^2)
\]
3rd family SM fermions – unitarity & $h$ boson

- Ratio $M_{T-\text{meson}}/m_t^\text{pole}$: key info for experiment, computable with controlled errors via lattice simulations with T-strong & QCD interactions, $q$ and (unquenched) $Q$

- Order of magnitude of $m_t^\text{eff}/m_t = O(\alpha_Y^2/\alpha_3^2)$, $m_b^\text{eff}/m_t = O(\alpha_W^2/\alpha_3^2)$

and $m_{\nu_\tau}^\text{eff} = 0$ (due to a shift symmetry of sterile $\nu_R$) can be understood

\[
O^t_{q,\tilde{\chi}} = \bar{q}_L \overset{\text{BWA}}{\leftrightarrow} D^B_{\mu} \phi D^A_{\mu} t_R \quad O^b_{q,\tilde{\chi}} = \bar{q}_L (D D^{BW} \phi) b_R \quad O^\tau_{\ell,\tilde{\chi}} = \bar{\ell}_L \overset{\text{BWA}}{\leftrightarrow} D^B_{\mu} \phi D^B_{\mu} \tau_R \quad [+\text{h.c.}]
\]

more accuracy & other families require info/assumptions on $\tilde{\chi}_{L,R}$ terms in $\mathcal{L}^{BSMM}$

- Model $\mathcal{L}^{BSMM}$ unitary with $M_W^\text{eff} \ll M_{T-\text{meson}}, \alpha_W \ll 1$, no fundamental higgs: interactions must produce (among other bound states) one scalar state with SM-like couplings to $W/Z$ or several scalars with tuned couplings for $W_L W_L \rightarrow W_L W_L$ to be unitary at energies $\ll \Lambda_T$ [Cornwall, Levin & Tiktopoulos 1974, Lee, Quigg & Thacker 1977]
125 GeV Higgs boson \((h)\) as \(WW + ZZ\) bound state

**WW channel:** assume binding force from T-hadron exchange to form \(h\), \(M_h < 2M_W\)

\[
G(p) = \int d^4x e^{-i\vec{p}\cdot\vec{x} - ip_0x_0} V^{-1}_3 \int d^3z \left\langle W(\vec{x}, x_0) W(\vec{z} + \vec{x}, x_0) W^\dagger(\vec{0}, 0) W^\dagger(\vec{0}, 0) \right\rangle
\]

in free theory has a cut starting at \(p^2 = -4M_W^2\); in interaction due to weak and T-strong forces

\[
G(p) = \frac{g_{\text{analyt}}(E_p^2, \vec{p}^2)}{p^2 + 4M_W^2} \left\{ 1 + \frac{\Delta_0^2}{p^2 + 4M_W^2} + \ldots \right\} = \frac{g_{\text{analyt}}(E_p^2, \vec{p}^2)}{p^2 + 4M_W^2 - \Delta_0^2}
\]

with

\[
\Delta_0^2 = O(1) g_W^4 4M_W^2 \quad \text{LE quantity, computable} \quad \Rightarrow \quad \text{a pole is expected to appear at}
\]

\[
p^2 = -M_h^2 = -4M_W^2 + \Delta_0^2 \quad \leftrightarrow \quad M_h = 2M_W \left( 1 - O(1) g_W^4 \right)^{1/2}
\]

Likely just one bound state: in non-relativistic approximation binding would be given by a potential well of height \(V_0 \sim M_W\) and width \(a \sim \Lambda_T^{-1}\), with \(8m_{\text{red}} V_0 a^2 \ll 1\)
Effective Lagrangian for $E < 1$ TeV, $h$-couplings, . . .

A model like $\mathcal{L}^{BSMM}$ should be described for $E \ll \Lambda_T$ by the effective Lagrangian

$$\Gamma_{LE} = \frac{1}{4} F_A F_A + \frac{1}{4} (F_W F_W + F_B F_B) + \sum_f (\bar{f}_R D_f^{A,B} f_R + \bar{f}_L D_f^{A,W,B} f_L) +$$

$$+ \frac{1}{2} \partial_\mu h \partial^\mu h + V_{\text{eff}}(h) + \left[ C_2 \Lambda_T^2 + C'_2 \Lambda_T h + C''_2 h^2 \right] \frac{1}{2} \text{Tr} \left[ D_{\mu}^{W,B} U^\dagger D_{\mu}^{W,B} U \right] +$$

$$+ \sum_{f \neq \nu} [m_f + y_f h](\bar{f}_L u_Y f_R + \bar{f}_R u_Y^\dagger f_L) + \mathcal{O}\left(\frac{1}{\Lambda_T}\right), \quad \Lambda_T = \text{a few TeV}$$

where $U = \exp\left[ \frac{i \vec{\zeta} \cdot \vec{\tau}}{\sqrt{C_2} \Lambda_T} \right] = \left[ \tilde{u}_{Y=\frac{1}{2}} \left| u_{Y=\frac{1}{2}} \right. \right] \equiv \left[ \tilde{u} = -i \tau_2 u^* \mid u \right]$ and

- $m_h < \Lambda_T$ and $h$-couplings as (to be) measured in experiments, for instance
  $$2 C_2 \Lambda_T / C'_2 \approx m_f / y_f \quad \text{(unitarity)} \quad \text{and} \quad m_f \approx y_f \sqrt{C_2} \Lambda_T \quad \text{(plausible, non-trivial)}$$

- flavour changing currents much like in the SM: here no tree level FCNC

- loop effects consistent with precise EW data: contribution of non-SM $T$-fermions to $S$-parameter
  $$\sim \frac{1}{M_T^{2\text{-meson}}} < \frac{1}{100 v_{SM}^2}, \text{ likely } > 12 \ T\text{-fermion doublets allowed in } \mathcal{L}^{BSMM}$$
Conclusions

A mass mechanism for fermions and EW gauge bosons: based on strong interactions, fermion chirality ($\tilde{\chi}$) broken at the UV cutoff scale & an exact symmetry ($\chi$) that once gauged describes EW interactions; changing “universality” paradigm: irrelevant terms control LE physics.

Once combined with experimental info it has interesting implications:

- **new strong interaction** with RGI scale $\Lambda_T > v_{SM}$ and $\sim$ a few TeV
- **new fermions** with mass $O(\Lambda_T)$ confined in detectable resonances
- solving the **naturalness problem**: EW & top mass scale derived from $\Lambda_T$
- **composite Higgs boson**: a bound state in the WW+ZZ channel
- **a low energy** ($p < 1$ TeV) effective action quite similar to the SM (composite Higgs couplings may show small deviations from SM)
- **insights on fermion mass hierarchy pattern**: $\frac{m_{\tau}}{m_t}$, $\frac{m_b}{m_t}$, $m_\nu \approx 0$, ...
Backup slides
Predictivity in models with $\tilde{\chi}$-breaking & NP mass

Renormalizable model: low energy $\tilde{\chi}_L$ SDE at the point of maximal $\tilde{\chi}$ symmetry:

$$\partial_\mu \tilde{J}^{L,i}_\mu = 0 \quad \text{(Wigner phase)}$$

$$\partial_\mu \tilde{J}^{L,i}_\mu = \sum_f C_{1,f} \Lambda_T D_f^{L,i} + \frac{i g_W}{2} C_2 \Lambda_T^2 \text{tr} \left( U^\dagger \left[ \frac{\tau^i}{2}, W_\mu \right] D_\mu^{WB} U - \text{h.c.} \right) \quad \text{(NG phase)}$$

\* RGI of l.h.s. $\Rightarrow$ RGI (& UV-finite) NP $\tilde{\chi}$-breaking terms on the r.h.s.

with $D_f^{L,i} = [\bar{f}_L \tau^i U f_R - \text{h.c.}]$ and $C_{1,f} = O(\rho_{f,cr}^2) \alpha_{coup(f)}^{n(f)} [1 + O(\alpha\ldots)]$

\* effective masses: $C_{1,f} \Lambda_T \leftrightarrow m^{\text{eff}}_f$, $C_2 g^2_W \Lambda_T^2 \leftrightarrow (m^{\text{eff}}_W)^2$

UV cutoff $b^{-1} \to \infty$ at fixed $M_{\text{glueball}}, M_{\text{proton}}, G_F, \sin^2 \theta_W \leftrightarrow \hat{\alpha}_{T,S,W,Y}$

$\tilde{\chi}$-symmetry $\Rightarrow \sum_{f=1}^{N_{\text{ferm}}^{\text{tot}}} \rho_{f,cr}^2 (1 + O(\rho_{f,cr}^2)) = O(1)$ entails bounds for the $\rho_{f,cr}$’s

$\rightarrow \rho_{Q,cr}, \rho_{L,cr}$ control $m^{\text{eff}}_Q, m^{\text{eff}}_L$, as well as $m^{\text{eff}}_W, m^{\text{eff}}_Z$

$\rightarrow \rho_{t,cr}$ controls $m^{\text{eff}}_t$, ... $\rho_{\tau,cr}$ controls $m^{\text{eff}}_\tau$, ...

Are the $\rho_{f,cr}$’s of the most massive fermions ($Q, L, 3rd$ SM family) of similar size ?

Plausible, if all fermions $\in$ some GUT multiplet & peculiar $\tilde{\chi}$ terms for 2nd/1st family