N=4 3d lattice SYM

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Motivations (3d)

- Mirror symmetry
- Dimensionful coupling, yet CFTs
- Holographic cosmology (Skenderis et al.)
    - Has exactly the quartic adjoint scalar potential terms that are needed
    - Can do a better job than $\Lambda CDM$ at modeling small angle $\ell < 30$ CMB
    - Lattice simulations can potentially predict large angle statistics and CMB anomalies
- Holography and $AdS_4 \times X_6$
Parent theory

- Dimensional reduction of $N=1$ 6d SYM (all in adjoint).

$$\mathcal{L} = \frac{1}{2g^2} \mathrm{Tr} F_{\mu\nu} F_{\mu\nu} + \frac{i}{g} \mathrm{Tr} \Psi^T C \Gamma_\mu D_\mu \Psi$$

$$A_\mu \rightarrow A_i, \quad \phi_\alpha, \quad i = 0, 1, 2; \quad \alpha = 1, 2, 3$$

$$\Psi_p, \quad p = 1, \ldots, 8 \rightarrow \psi_\alpha^I, \quad \alpha = 1, 2; \quad I = 1, 2, 3, 4$$

- Hence “$N=4$.”
Continuum (twisted)

- Spacetime group
  \[ SO(4) \simeq SU(2)_L \times SU(2)_R \]

- R-symmetry group
  \[ SU(2)_R \times U(1)_R \]

- Twisted rotation group
  \[ SU(2)' = \text{diag}(SU(2)_r \times SU(2)_R) \]
Continuum (twisted, cont.)

- Donaldson-Witten twist

\[ g^2 L_{4d}^{\mathcal{N}=2} = \text{Tr} \left( \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} D_\mu \bar{\phi} D^\mu \phi - \alpha [\phi, \bar{\phi}]^2 \right. \]

\[ \left. - \frac{i}{2} \eta D_\mu \psi^\mu + i \alpha \phi \{\eta, \eta\} - \frac{i}{2} \bar{\phi} \{\psi_\mu, \psi^\mu\} + L_x \right), \]

\[ L_x = \text{Tr} \left( \frac{i}{8} \phi \{\chi_{\mu\nu}, \chi^{\mu\nu}\} - i \chi^{\mu\nu} D_\mu \psi_\nu \right) \]

- Dimensionally reduce to 3d. \( D_2 \to [\phi_3, \cdot] \)
Previous work

- Anosh Joseph [1307.3281]
- Blau-Thompson twist [hep-th/9612143]

\[ SU(2)' = \text{diag}(SU(2)_E \times SU(2)_N) \]

- From the dimensional reduction

\[ SO(6) \rightarrow SO(3) \times SO(3) \cong SU(2)_E \times SU(2)_N \]

- Earlier Q=8 formulation by orbifold method [Cohen, Kaplan, Katz & Unsal 2003]
Our twisted lattice

- Must complexify everything.
- Dynamical lattice spacing, as usual in these twisted/orbifold lattices.
- Lift additional fields with generic mass terms.

\[
\mathcal{L} = \text{Tr} \left( \frac{1}{4} \tilde{F}_{\mu\nu}(n) F_{\mu\nu}(n) + \frac{1}{2} \tilde{D}^+_{\mu} \phi(n) \tilde{D}^+_{\mu} \phi(n) - \alpha [\phi(n), \bar{\phi}(n)]^2 \\
+ \frac{\mathcal{i}}{2} \tilde{D}^+_{\mu} \eta(n) \psi_{\mu}(n) + \mathcal{i} \alpha \phi(n) \{ \eta(n), \eta(n) \} - \frac{\mathcal{i}}{2} \bar{\phi}(n) (\psi_{\mu}(n) \bar{\psi}_{\mu}(n) + \bar{\psi}_{\mu}(n - e_{\mu}) \psi_{\mu}(n - e_{\mu})) \right) + \mathcal{L}_\chi,
\]

\[
\mathcal{L}_\chi = \text{tr} \left[ \frac{\mathcal{i}}{8} (\phi(n) \bar{\chi}_{\mu\nu}(n) \chi_{\mu\nu}(n) + \phi(n + e_{\mu} + e_{\nu}) \chi_{\mu\nu}(n) \bar{\chi}_{\mu\nu}(n)) \\
- \frac{\mathcal{i}}{2} (\bar{\chi}_{\mu\nu}(n) \tilde{D}^+_{\mu} \bar{\psi}_{\nu}(n) + \chi_{\mu\nu}(n) \tilde{D}^+_{\mu} \psi_{\nu}(n)) \right].
\]
Lattice gauge invariance and Q invariance of

\[ \chi_{\mu\nu}(n) = \frac{1}{2} \epsilon_{\mu\nu\rho\lambda} \bar{\chi}_{\rho\lambda}(n + e_{\mu} + e_{\nu}). \]

\[ \sum_{\mu=1}^{4} e_{\mu} = 0. \]

So, the theory must be 3d.
After using EOM,

\[ \mathcal{L} = Q \text{Tr} \left( \frac{1}{4} \chi_{\mu\nu}(n) F_{\mu\nu}(n) + \frac{1}{2} \bar{D}_{\mu} \phi(n) \psi_{\mu}(n) + \alpha \eta(n) \left[ \phi(n), \bar{\phi}(n) \right] \right) \]

\[ -\frac{1}{8} \epsilon_{\mu\nu\rho\lambda} \text{tr} (F_{\mu\nu}(n) F_{\rho\lambda}(n + e_{\mu} + e_{\nu}) ) , \]

Last term is Q invariant using lattice Bianchi identity.
Where $Q$, which is nilpotent, acts as:

\[
\begin{align*}
Q \phi(n) &= 0, \quad Q \bar{\phi}(n) = i\eta(n), \\
Q \eta(n) &= [\bar{\phi}(n), \phi(n)], \\
Q U_\mu(n) &= i\psi_\mu(n), \quad Q \bar{U}_\mu(n) = -i\bar{\psi}_\mu(n) \\
Q \psi_\mu(n) &= D_\mu^+ \phi(n), \quad Q \bar{\psi}_\mu(n) = \bar{D}_\mu^+ \phi(n) \\
Q \chi_{\mu\nu}(n) &= \bar{F}_{\mu\nu}(n) + \frac{1}{2} \epsilon_{\mu\nu\rho\lambda} F_{\rho\lambda}(n + e_\mu + e_\nu).
\end{align*}
\]
Counterterms

- Fine-tuning is calculable because the theory is super-renormalizable.
- As will see in arguments below, all counterterms are one-loop.
Power of super-renormalizability

- So, finite number of counterterms that can all be calculated at one loop in lattice perturbation theory.
- Includes the effects of the mass terms and scalar Q breaking.
Example from SUSY QM

- Finite CT in SUSY QM with naïve discretization [JG et al., 2004]
- Doublers appearing in this diagram (cancelled by scalar loop in the Q-exact case)
- Also shows power of Symanzik improvement
Prospects

- Currently coding up the Anosh Joseph discretization b/c it is much cleaner.
- Seems to have only one CT due to exact Q, point group, lattice gauge invariance.

\[ \mathcal{L}(n) = \frac{1}{g^2} Q \text{Tr} \left( \chi_{ab}(n) \mathcal{D}_a^+ U_b(n) + \eta(n) \bar{\mathcal{D}}_a U_a(n) + \frac{1}{2} \eta(n) d(n) + B_{abc}(n) \bar{\mathcal{D}}_a \chi_{bc}(n) \right) \]

- For dimension counting it is best to have canonical normalization (otherwise everything is very confusing)

\[ \Phi \rightarrow g\Phi, \quad \Psi \rightarrow g\Psi, \quad U_m = \frac{1}{ag} e^{agA_m} \]
Renormalization in the Blau-Thompson twist

- \([\text{boson}] = \frac{1}{2}, \ [\text{fermion}] = 1, \ [d]=3/2, \ [F]=3/2, \ [Q] = \frac{1}{2}\)
- So since loop is multiplied by \([g^2] = 1\)
- Must have no more than dim=2 to be unsuppressed by lattice spacing.
- But also be fermionic if under \(Q\).
- So restricting to \(Q\)-exact operators, must be of form
  \[Q\text{Tr}\Psi, \ Q\text{Tr}\Psi\Phi, \ Q(\text{Tr}\Psi\text{Tr}\Phi)\]
- Point group, lattice gauge invariance, limit to [shifts d EOM by constant]
  \[Q\text{Tr}\eta = \text{Tr}d\]
- Perturbative calculations underway, including Symanzik improvement.
- Must write down all dim=3 Q invariant fermionic operators.
Conclusions

- The holographic cosmology doesn’t really require SUSY, but it will be interesting to see how it impacts large angle predictions.
- Concrete realization of Symanzik improvement vs. SUSY in 3d should be very enlightening.
- In the Blau-Thompson twist we only have one 1-loop CT to determine in order to get full N=4 SUSY. Symanzik improvement will require more.