Non-perturbative renormalization of operators in near-conformal systems using gradient flow

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1) Is gradient flow a renormalization group transformation?
2) Can we use GF to calculate anomalous dimensions?
1) Is gradient flow a renormalization group transformation?
2) Can we use GF to calculate anomalous dimensions?

1) It is not, but it can be tricked:
   • normalize correctly
   • calculate appropriate quantities
→ GF acts like RG blocking with continuous scale change

2) Pilot study: $N_f=12$ flavor SU(3), determine anomalous dimension of mass and baryon operators

next talk: Andrea Carosso, $\Phi^4$ model
Wilson RG in a nutshell:

Step 1: Introduce “blocked” fields and integrate out the original ones
Step 2: rescale $\Lambda_{\text{cutoff}} \rightarrow \Lambda_{\text{cutoff}}/b$ (or $a \rightarrow b \ a$)

- The partition function is unchanged,
- The action changes $S(\phi, g_0) \rightarrow S(\phi, g')$
- The RG flow runs along the renormalized trajectory either to the $\xi=0$ trivial or $\xi=\infty$ UVFP

Credit: Wilson-Kogut 1973, Ch.11
Correlation function of $\langle \mathcal{O}(0)\mathcal{O}(x_0) \rangle_{g,m}$

An RG transformation of scale change $b$:

$$S(\phi, g) \rightarrow S(\phi, g')$$

$$\langle \mathcal{O}(0)\mathcal{O}(x_0) \rangle_{g,m} = b^{-2\Delta_0} \langle \mathcal{O}(0)\mathcal{O}(x_b = x_0 / b) \rangle_{g',m'}$$

$$\Delta_0 = d_o + \gamma_o$$ scaling dimension and $x_0 >> b$

We do not need to simulate with $S(\phi, g')$

— just use the principle of MCRG
Monte Carlo Renormalization Group

Action
\[ S(\phi, g_0) \]
\[ \rightarrow \]
\[ S(\phi, g') \]

Configuration ensemble
\[ \{ \phi \} \]
\[ \rightarrow \]
\[ \{ \Phi_b \} \]

RG transformed expectation values can be calculated without explicit knowledge of the blocked action

\[ \langle O(0)O(x_b) \rangle_{g',m'} = \langle O_b(0)O_b(x_b) \rangle_{g,m} \]

\( O_b = O(\Phi_b) \) is the operator of the blocked fields

Swendsen PhysRevLett.42.859,1979
Gradient flow could be “blocking”

GF is a continuous smoothing that removes short distance fluctuations

**Gauge flow:**  \( \frac{\partial}{\partial t} V_t = - (\partial S_W[V_t]) V_t, \quad V_0 = U \)

**Fermions** evolve on the gauge background:

\( \frac{\partial}{\partial t} \chi_t = \Delta[V_t] \chi_t, \quad \chi_0 = \psi \)

(The flow action does not have to match the model)

GF misses two important attributes of an RG transformation:

- there is no rescaling \( \Lambda_{\text{cut}} \rightarrow \Lambda_{\text{cut}} / b \) or coarse graining
- linear transformation does not have the correct normalization
  (wave function renormalization or \( Z_{\phi} = b^{-\eta/2} \))

Both issues can be solved
Gradient flow could be “blocking”  

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Both issues can be solved

GF does not flow to FP
RG transformation (b=2)

- gradient flow: $\phi_t(\phi)$
- blocked fields: $\Phi_b = Z_b \phi_t = b^{-\eta/2} \phi_t$
- Coarse grain and rescale with $b : x \rightarrow x/b$
RG transformation (b=2)

- gradient flow: \( \phi_t(\phi) \)
- blocked fields: \( \Phi_b = Z_b \phi_t = b^{-\eta/2} \phi_t \)
- Coarse grain and rescale with \( b : x \rightarrow x/b \)
GF vs RG

2-point functions do not care about decimation:

\[
\langle O_b(\Phi_b(0))O_b(\Phi_b(x_b))\rangle_{g,m} = b^{-\eta} \langle O(\phi_t(0))O(\phi_t(x_b))\rangle_{g,m}
\]

At the level of expectation values GF is a proper RG transformation
Put it together

\[
\langle O(0)O(x_{0}) \rangle_{g,m} = b^{-2\Delta_0} \langle O(0)O(x_{b} = x_{0} / b) \rangle_{g',m'}
\]

\[
\langle O(0)O(x_{b}) \rangle_{g',m'} = \langle O_b(0)O_b(x_{b}) \rangle_{g,m}
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\]

Ratio of flowed & unplowed correlators predict the anomalous dimension

\[
\frac{\langle O_t(0)O_t(x_0) \rangle}{\langle O(0)O(x_0) \rangle} = b^{2\Delta_o - 2n_o \Delta_\phi} \\
x_0 \gg b \\
\Delta_o = d_o + \gamma_o \\
\Delta_\phi = d_\phi + \eta/2
\]
Anomalous dimensions

Calculate $\eta$ by an operator that does not have an anomalous dimension: — vector or axial charge ($A(x)$)

The super-ratio

$$R(t, x_0) = \frac{\langle O_t(0)O_t(x_0) \rangle}{\langle O(0)O(x_0) \rangle} \left( \frac{\langle A(0)A(x_0) \rangle}{\langle A_t(0)A_t(x_0) \rangle} \right)^{n_0/n_A} = b^{\gamma_0}$$

independent of $x_0 >> b$ and predicts $\gamma$
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- $t$ and $b$ are still independent!
  • Natural choice: $b^2 \sim t$
- it is advantageous to flow only the source, not the sink
- $\gamma$ is universal at the FP only: set fermion mass to zero
- $t$ has to be large enough, and
Anomalous dimensions

Calculate $\eta$ by an operator that does not have an anomalous dimension:
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The super-ratio

$$R(t,x_0) = \frac{\langle O(0)O_t(x_0) \rangle}{\langle O(0)O(x_0) \rangle} \left( \frac{\langle A(0)A(x_0) \rangle}{\langle A_t(0)A_t(x_0) \rangle} \right)^{n_0/n_A} = b^{\gamma_0/2} \propto t^{\gamma_0}$$

independent of $x_0 \gg b$ and predicts $\gamma$

- $t$ and $b$ are still independent!
  - Natural choice: $b^2 \sim t$
- it is advantageous to flow only the source, not the sink
- $\gamma$ is universal at the FP only: set fermion mass to zero
- $t$ has to be large enough, and $x_0 \gg \sqrt{8t}$
Pilot study: $N_f=12$

Low statistics study with staggered fermions
- $24^3 \times 48$, $32^3 \times 64$ volumes, $m=0.0025$

- mass anomalous dimension $\gamma_m = 0.23-0.25$ from perturbation theory, FSS numerical studies, Dirac eigenmodes
- the gauge coupling walks very slow - substantial scaling violation effects are expected
- baryon and tensor anomalous dimensions would be interesting where no non-perturbative prediction exists
Ratio of ratios - pseudo scalar

\[ R_t^O(x_0) = \frac{\langle O(0)O_t(x_0) \rangle}{\langle O(0)O(x_0) \rangle} \left( \frac{\langle A(0)A(x_0) \rangle}{\langle A(0)A_t(x_0) \rangle} \right)^{n_0/n_A} = t^\gamma_0 \]

has no \( x_0 \) dependence if \( x_0 >> b \)

Oscillation is due to operator overlap
\[ \propto 2\sqrt{8t} \rightarrow \text{limits max } t \]

flow time dependence of the plateau gives anomalous dimension
Flow time dependence indicates slowly running gauge coupling
Finite volume corrections

\[ R(g',t,L) = s^{-\gamma_o} R(g,s^2t,sL) \]
\[ R(g,s^2t,s^2L) = R(g,s^2t,sL) + s^{-\gamma_o} \left( R(g,t,sL) - R(g,t,L) \right) + \text{h.o.} \]
Pseudo scalar:

\[ \gamma_m = 0.24(3), \quad t \to \infty \]

\[ \gamma_m(\beta, t) = \gamma_0 + c_\beta t^{\alpha_1} + d_\beta t^{\alpha_2} \]

extrapolate to \( t \to \infty \):

error: systematic + statistical

result consistent with other methods
Nucleon channel

**Minimal flow time dependence, but limited $x_0$ range**

Anomalous dimension is small

$\gamma_N = 0.05(5)$

(perturbative: $\gamma_N = 0.09$)
Vector channel

Oscillation pronounced but little flow time dependence

Fit as

\[ \frac{A_t e^{-m_1 x_0} + B_t e^{-m_2 x_0}}{A e^{-m_1 x_0} + B e^{-m_2 x_0}} = \frac{A_t}{A} \frac{1 + B_A}{1 + B} e^{-\Delta m x_0} \]

2 anomalous dimensions, from \(A_t/A\) and \(B_t/B\) both vanish within errors
Summary & outlook

– GF can describe an RG transformation
  • can aid our understanding of GF away from perturbation theory
  • determine anomalous dimension in conformal system (probably most promising method to get nucleon anomalous dim.)
  • determine renormalization factors in QCD (needs work)
– Finite volume effects deserve more attention
– Staggered fermions are a poor choice here (oscillations): DW is more promising
– Anyone with existing conformal configurations can try the method (but need massless or nearly massless configs)
– Beyond BSM:
  • Z factors in QCD need perturbative matching
  • 3D O(n) model: might not compete with FSS but can predict anomalous dimension of irrelevant operators
    (A. Carosso, next talk)