Strongly interacting rotating bosons via complex stochastic quantization

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1949: Onsager predicts rotating superfluids will form vortices

1979: First observation of vortices in rotating $^4$He

1990s-2000s: rotating BECs in $^4$He and dilute atomic gases

Advances in theory

*Science* 292 5516 (2001)

Rotating Bose-Einstein condensates

Theoretical advancements in study of rotating superfluids since 1950s
Theoretical progress

- Why are we stuck?
- Many-body quantum systems $\rightarrow$ Quantum Monte Carlo

\[ Z = \int D\phi e^{-S[\phi]} \]

\[ \langle \mathcal{O} \rangle = \frac{1}{Z} \int D\phi \ e^{-S[\phi]} \mathcal{O}[\phi] = \int D\phi \mathcal{P}[\phi] \mathcal{O}[\phi] \]

- Evaluate stochastically, with \( \mathcal{P}[\phi] = \frac{e^{-S[\phi]}}{Z} \)
The sign problem

• Action for non-relativistic rotating bosons:

\[ S = \int \phi^* (\mathcal{H} - \mu - \omega_z L_z) \phi + \lambda \int (\phi^* \phi)^2 \]

\[ \omega_z L_z = i \omega_z (x \partial_y - y \partial_x) \]

• Complex action
• Usual Quantum Monte Carlo methods do not work
• Proposed solution: Complex Langevin Method
• Generalization of stochastic quantization to complex dynamical variables

\[ \phi \rightarrow \phi^R + i\phi^I \]

\[ S[\phi^R + i\phi^I] = u[\phi^R + i\phi^I] + iv[\phi^R + i\phi^I] \]

• Leads to two coupled SDEs:

\[ d\phi^R = \text{Re}[K]dt + \eta \sqrt{dt} \]

\[ d\phi^I = \text{Im}[K]dt \]

\[ K = -\nabla S[\phi^R + i\phi^I] \]
• Relativistic Bose gas at finite chemical potential

\[ S = \int d^4x \left[ |\partial_\nu \phi|^2 + (m^2 - \mu^2)|\phi|^2 + \mu(\phi^* \partial_t \phi - \partial_t \phi^* \phi) + \lambda (\phi^* \phi)^2 \right] \]

\[ S[\mu]^* = S[-\mu] \]

• Lattice action

\[ S = \sum_x \left[ (2d + m^2)\phi_x^* \phi_x + \lambda(\phi_x^* \phi_x)^2 - \sum_{\nu=1}^{4} (\phi_x^* e^{-\mu \delta_{\nu,4}} \phi_{x+\hat{\nu}} + \phi_{x+\hat{\nu}}^* e^{\mu \delta_{\nu,4}} \phi_x) \right] \]

• Use CL to compute density, field modulus squared

$\mu = 0.0$
$\mu = 0.7$
$\mu = 1.125$
μ = 1.5
Relativistic Bose gas at finite chemical potential

Density EOS of spin polarized unitary Fermi gas

- $\beta\hbar$ from 0 to 2.0 (bottom to top)
- Dashed lines: 3\textsuperscript{rd} order virial expansion
- 3+1 dimensional lattice
  - $N_x = 11$, $N_t = 160$

CL results show good agreement with the virial expansion in the virial region

• CL is not always successful
  • The Excursion Problem
    • The probability distribution is not suppressed enough for large values of the complexified variables
    • Causes the imaginary drift term to “run away”
  • The Singular Drift Problem
    • The probability distribution is not suppressed enough near singularities in the drift term

• We don’t yet know how to prove when CL will work
  • Important to have checks to ensure validity
  • Comparisons with existing theoretical benchmarks, experimental measurements
Action for our system

\[ S = \int \phi^* (\mathcal{H} - \mu - \omega_z L_z) \phi + \lambda \int (\phi^* \phi)^2 \]

\[ \omega_z L_z = i \omega_z (x \partial_y - y \partial_x) \]
• Preliminary results for rotating, 2+1D system:
  • Average Angular Momentum dependence on rotation frequency
  • $N_x = 12, N\tau = 20, \tau = 0.2$
CL in non-relativistic rotating bosons

- Preliminary results for rotating, 2+1D system:
  - Moment of Inertia dependence on rotation frequency
  - \( Nx = 12, \ N\tau = 20, \ \tau = 0.2 \)
Future directions

- Decrease $|\beta \mu|$ to study superfluid regime
- Density should show triangular vortex lattice structure
- We expect to see discontinuities in the circulation observable

\[
\Gamma[l] = \frac{1}{2\pi} \int_{l \times l} dx \left( \theta_{t,x+j} - \theta_{t,x} \right)
\]

\[
\theta_{t,x} = \tan^{-1} \left( \frac{\text{Im}[\phi_{t,x}]}{\text{Re}[\phi_{t,x}]} \right)
\]
Summary and Conclusions

- Many systems of interest inaccessible to QMC due to sign problem
- CL allows us to circumvent the sign problem
- Under some circumstances, CL fails
- Preliminary results for rotating non-relativistic bosons are promising
- More work still to come
Thank you!

Funding sources:

Prof. Joaquín Drut
Andrew Loheac
Chris Shill
Josh McKenney
Yaqi Hou