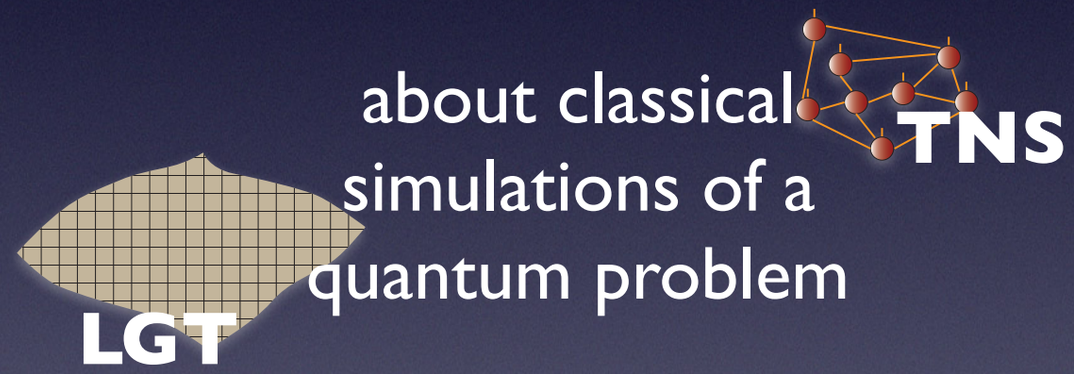
TENSOR NETWORK STATES FOR LATTICE GAUGE THEORIES



QUANTERA
QTFLAG

Mari-Carmen Bañuls
with K. Cichy (Poznan), K. Jansen, H. Saito (DESY),
J.I. Cirac (MPQ), S. Kühn (Perimeter)

MPQ
Max-Planck-Institut
für Quantenoptik
(Garching b. München)

LATTICE18

TENSOR NETWORKS

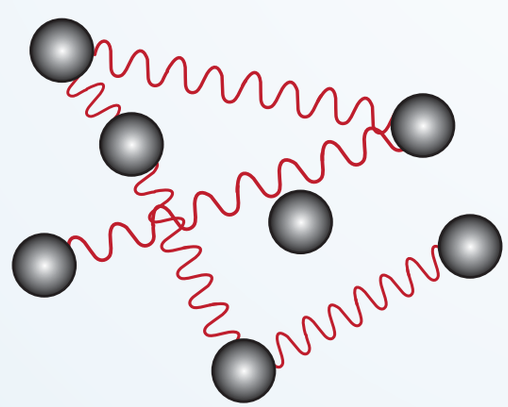
WHAT ARE TNS?

- TNS = Tensor Network States

A general state of the N-body Hilbert space has exponentially many coefficients

$$|\Psi\rangle = \sum_{i_j} c_{i_1 \ldots i_N} |i_1 \ldots i_N\rangle$$

N



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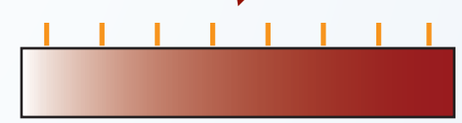

N-legged tensor

d^N

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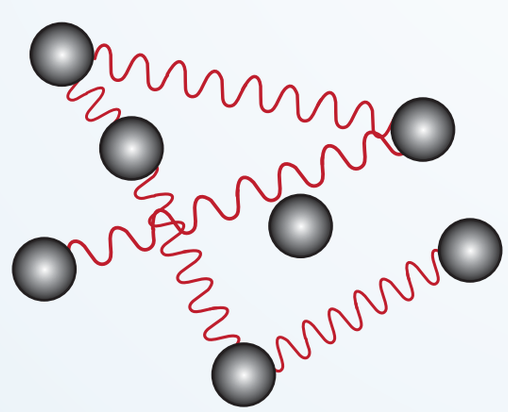

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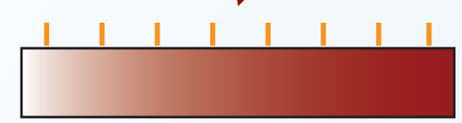

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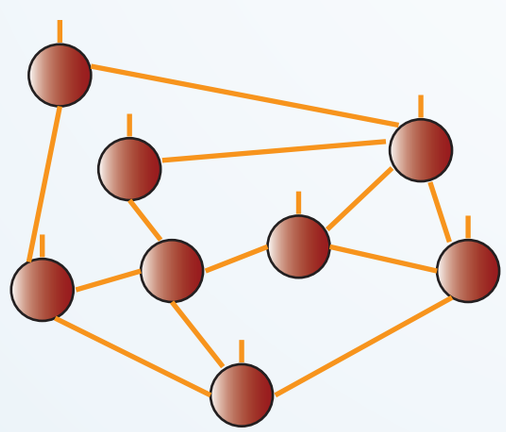


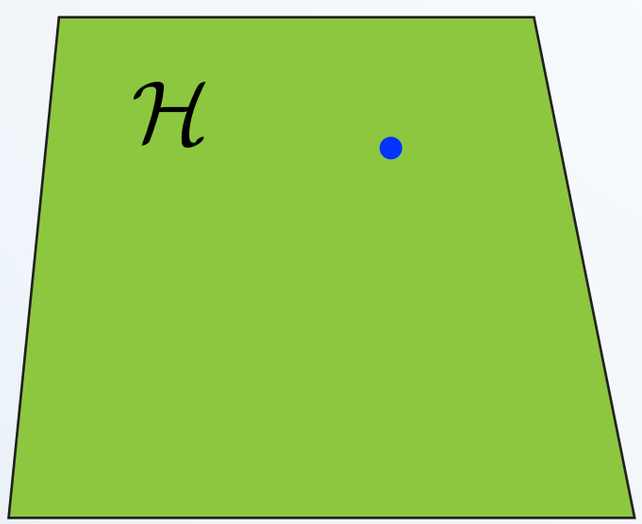
$\mathrm{poly}(N)$

A TNS has only a polynomial number of parameters

WHY SHOULD TNS BE USEFUL?

States appearing in Nature are peculiar

State at random from Hilbert space is not close to product



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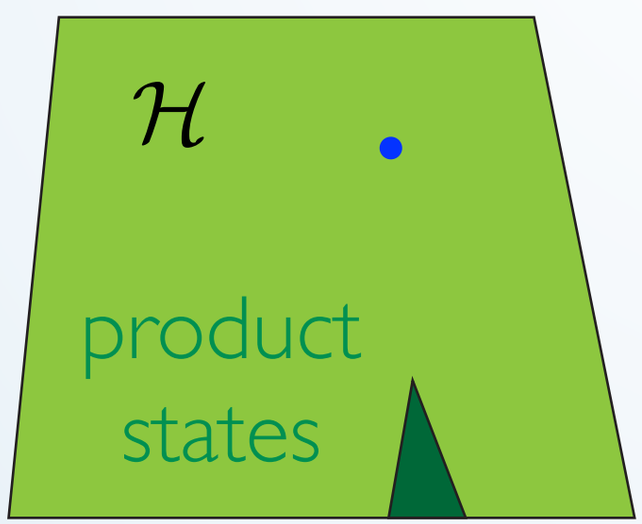

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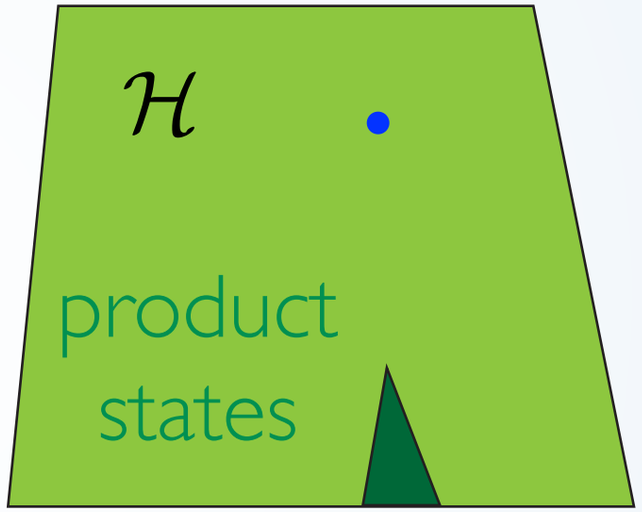

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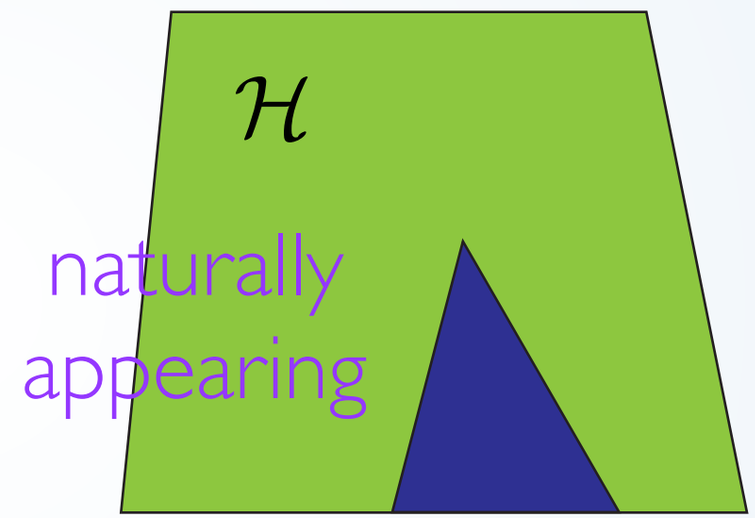

We look for the particular corner of the Hilbert space

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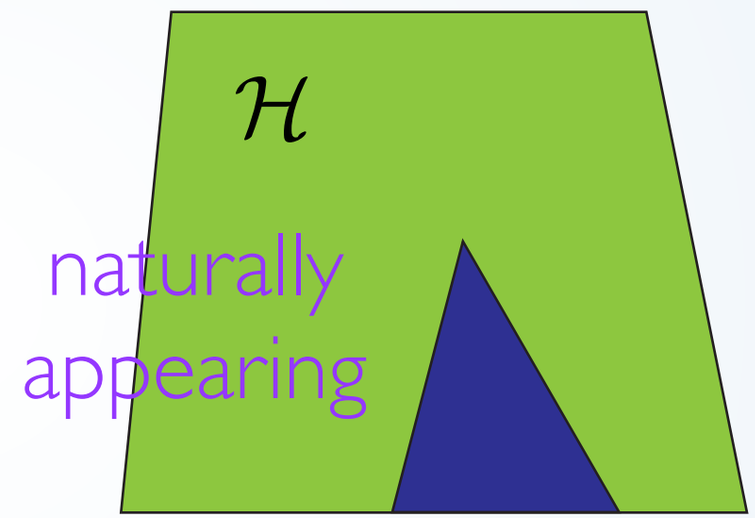

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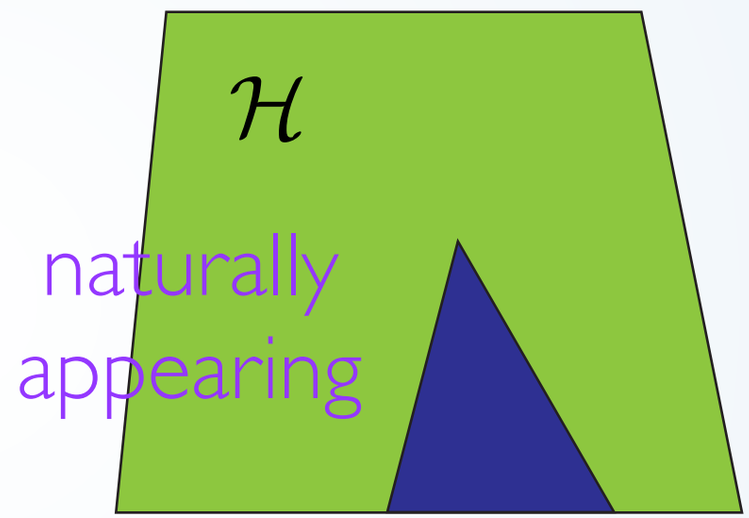
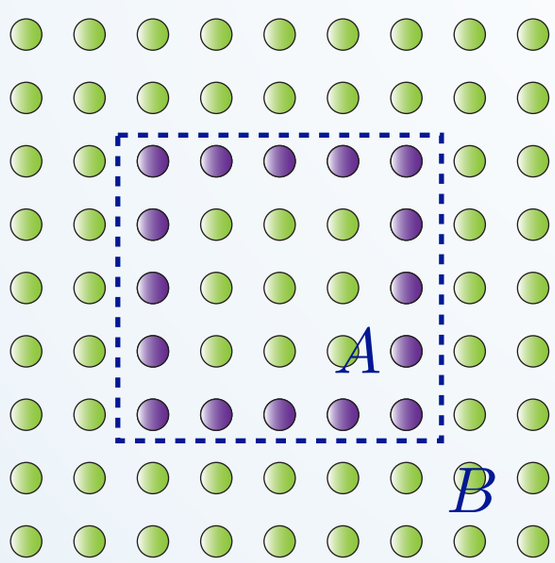


area law



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Calabrese, Cardy 2004; Wolf 2006

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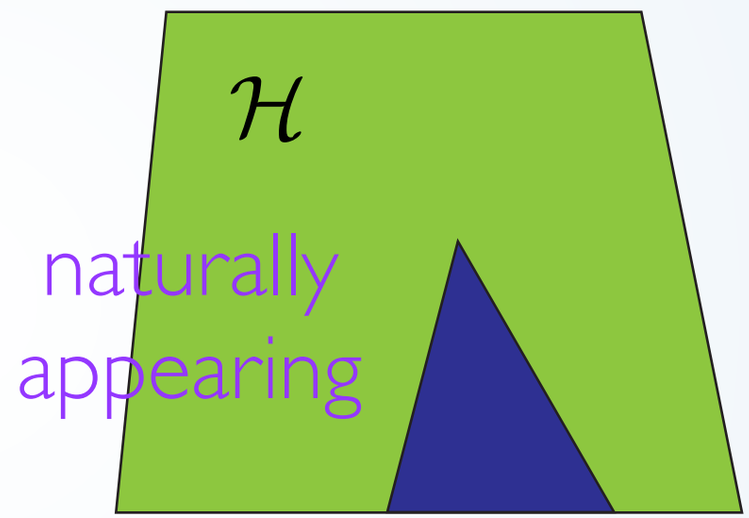

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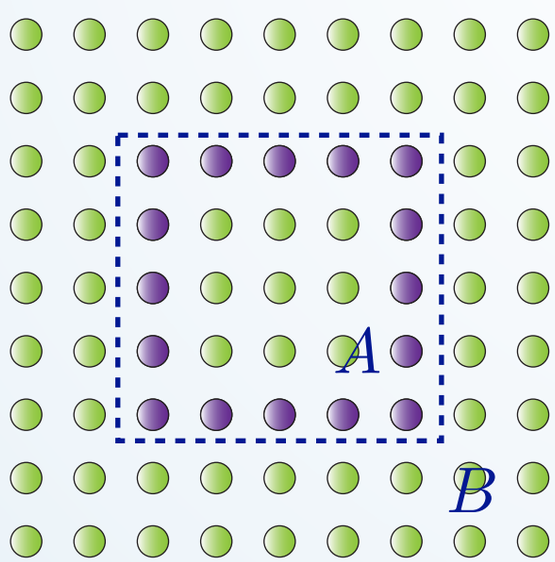

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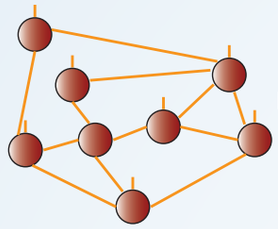
TNS = entanglement based ansatz

WHY FOR LGT?



TNS

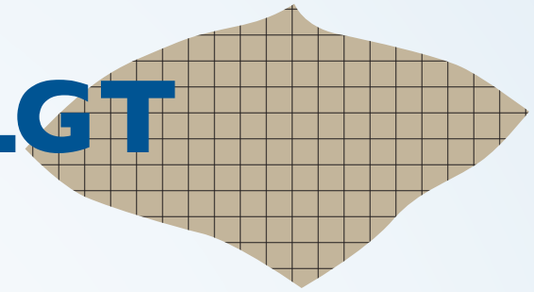
Non-perturbative for Hamiltonian systems

Extremely successful for 1D systems (MPS)

Promising improvements for higher dimensions

ground states
low-lying excitations
thermal states
time evolution

LGT



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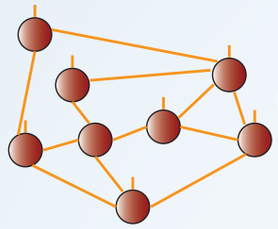

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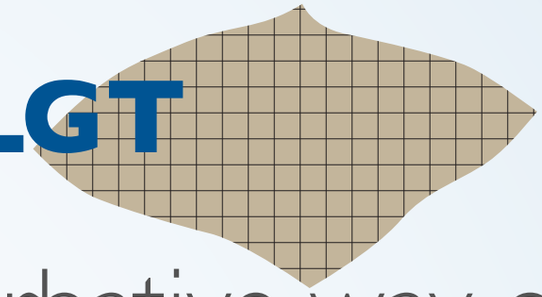

Non-perturbative way of solving QFT (QCD)

Mostly path-integral formalism & MC

4D lattice

spectrum
finite T
$64^3 \times 96$
chemical potential
time evolution

USING TNS FOR LGT

USING TNS FOR QMB

a formal approach

no sign problem

numerical algorithms

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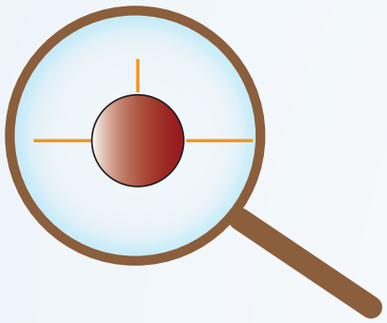

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constructing states

Chen et al PRB 2011
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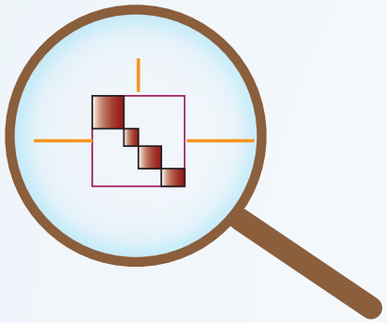

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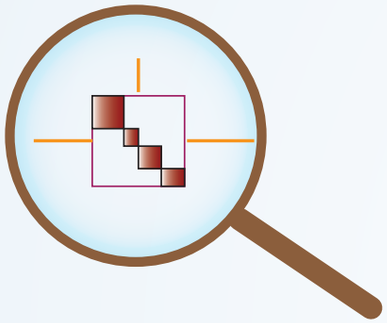

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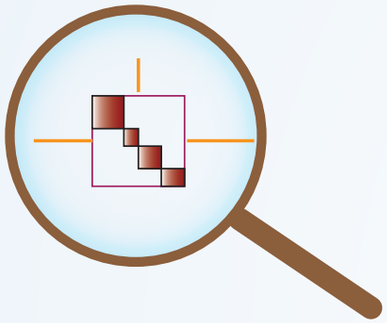

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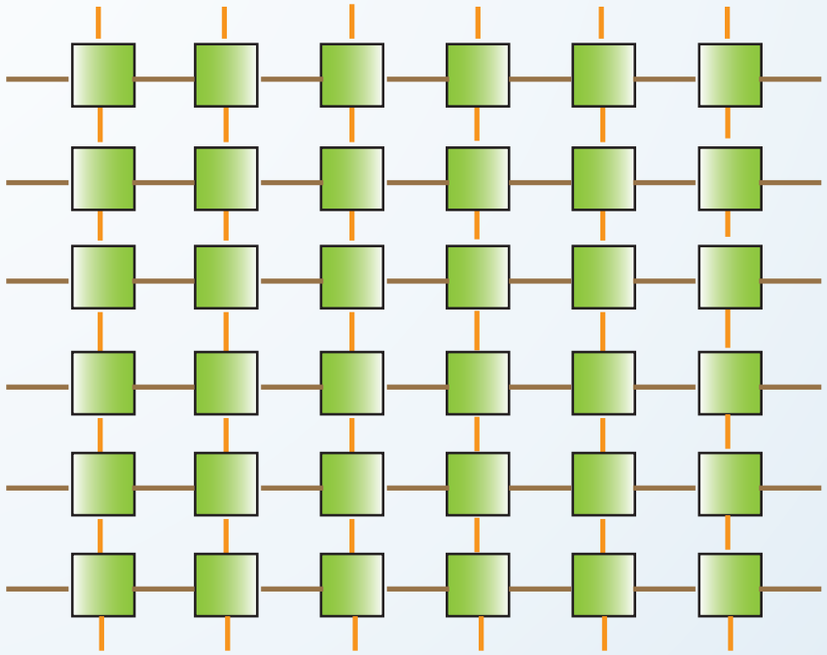

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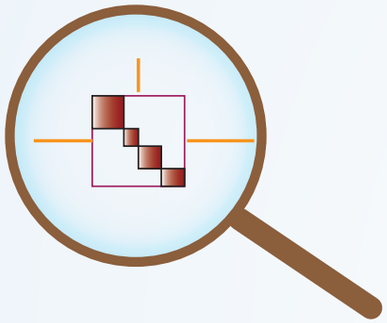

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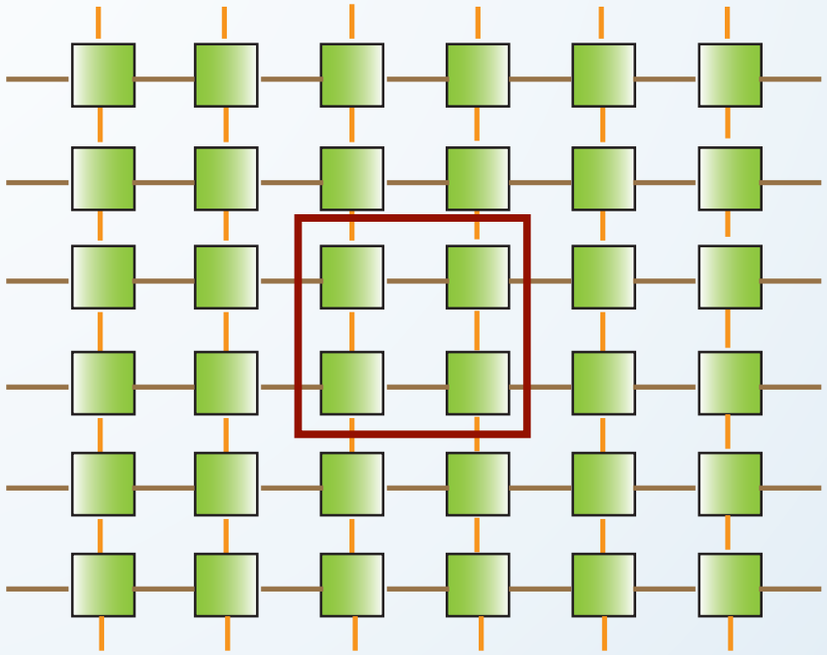

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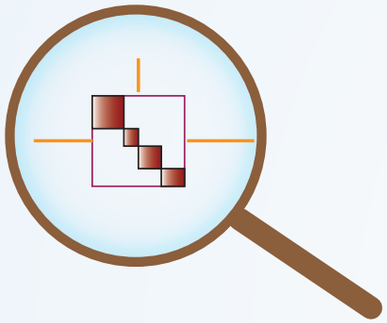

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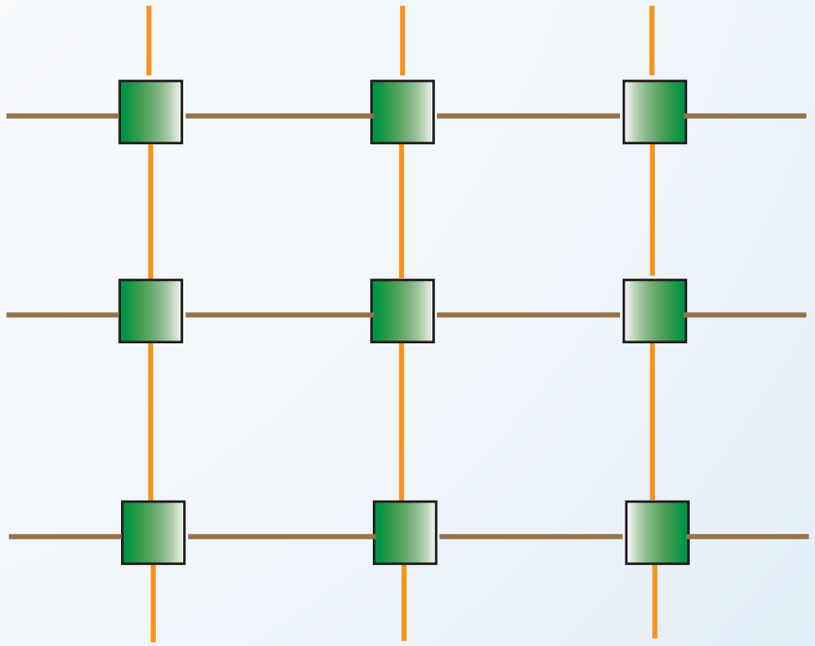

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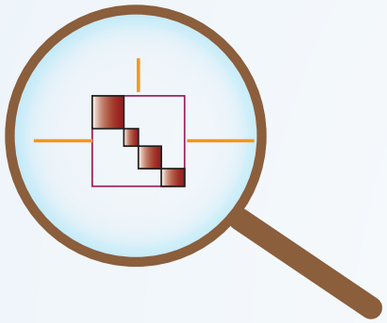

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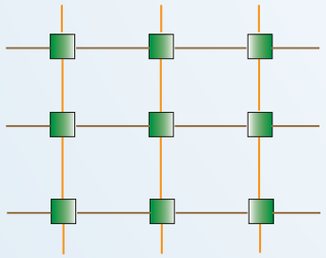

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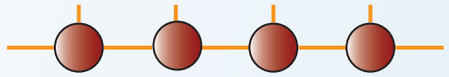

efficient algorithms for GS, low excited states, thermal, dynamics

White PRL 1992; Schollwöck RMP 2011
Vidal PRL 2003; Verstraete et al PRL 2004
Verstraete et al Adv Phys 2008; Orús Ann Phys 2014

USING TNS FOR LGT

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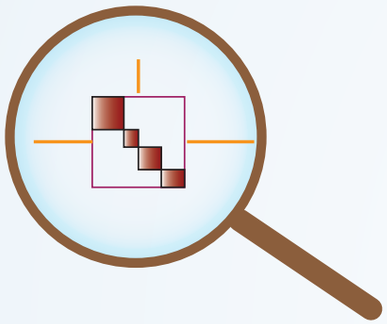

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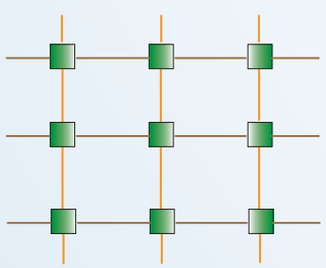

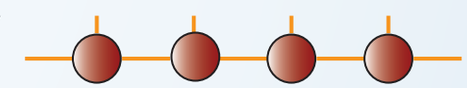

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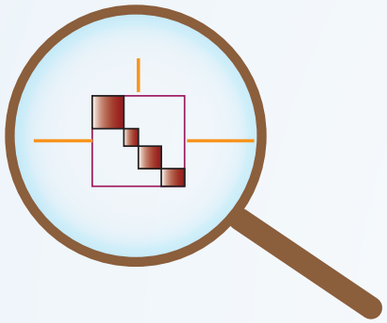

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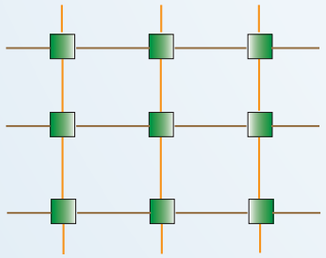

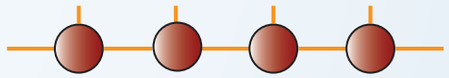

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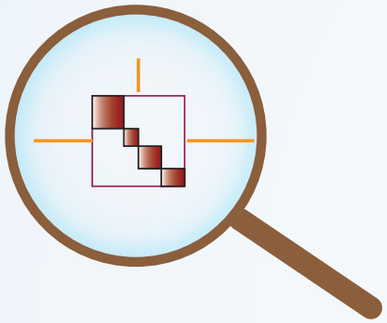

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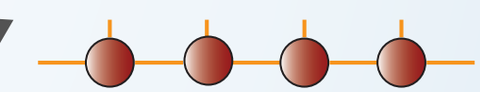

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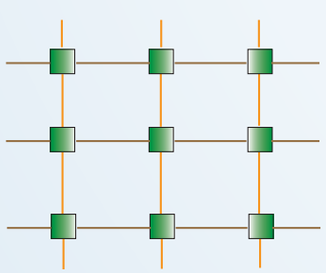

TRG approaches to classical
and quantum models

Liu et al PRD 2013
Shimizu, Kuramashi, PRD 2014,…
Kawauchi, Takeda 2015

talks by Meurice, Sakai,
Yoshimura [TheorDev]

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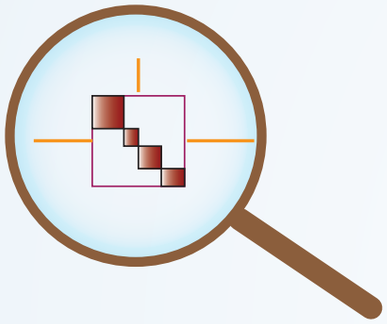

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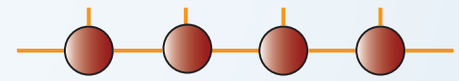
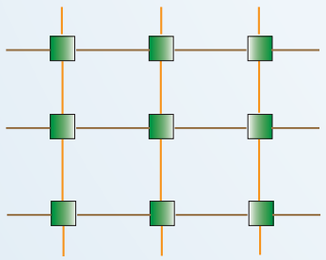


TNS as ansatz for the state



TRG approaches to classical
and quantum models

Liu et al PRD 2013
Shimizu, Kuramashi, PRD 2014,…
Kawauchi, Takeda 2015

talks by Meurice, Sakai,
Yoshimura [TheorDev]

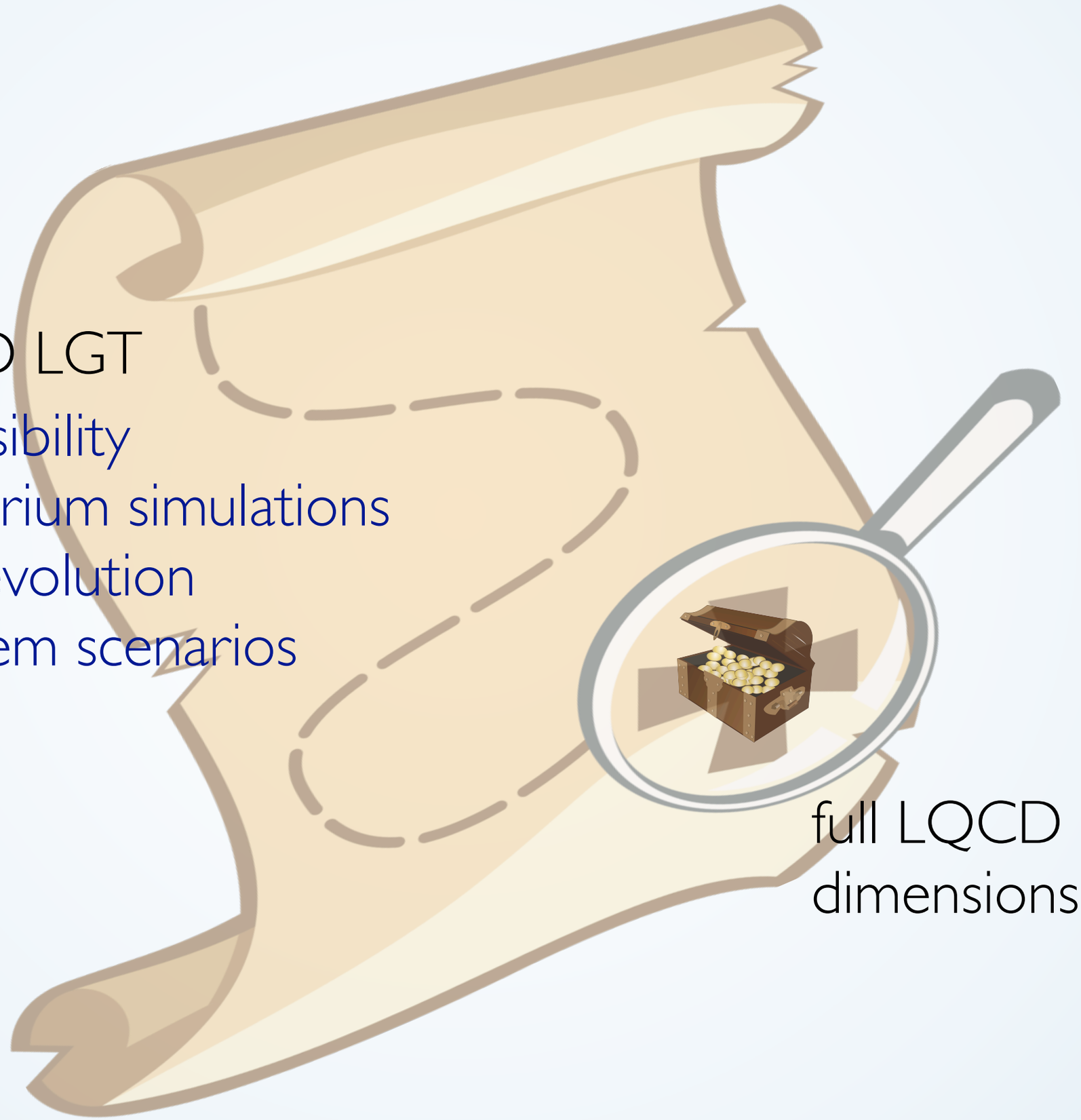
next...

a possible LGT-TNS roadmap...

a wishful LGT-TNS roadmap...

full LQCD in 3+1 dimensions

a wishful LGT-TNS roadmap...



1+1D LGT

feasibility

precise equilibrium simulations

time evolution

sign problem scenarios

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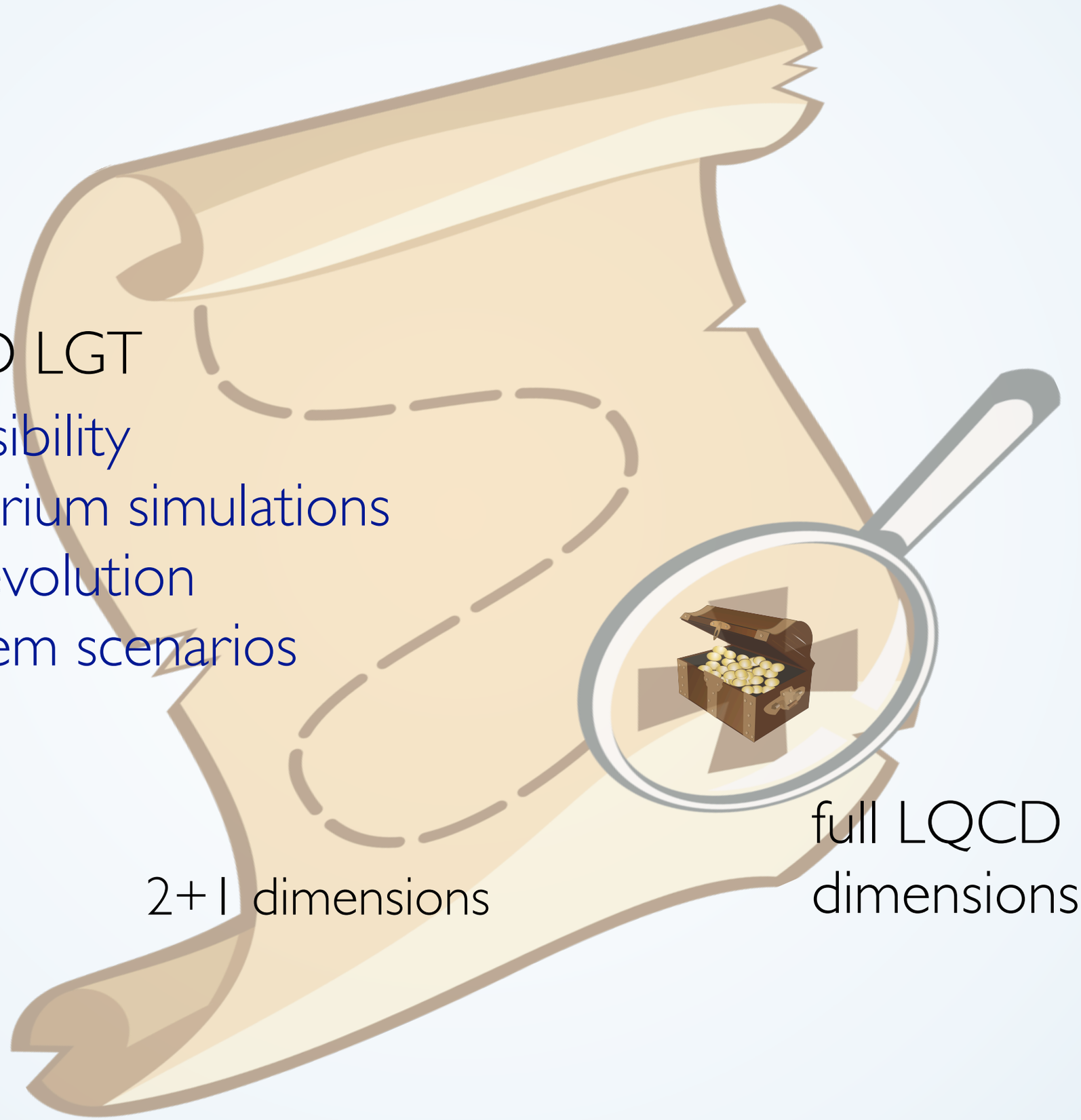

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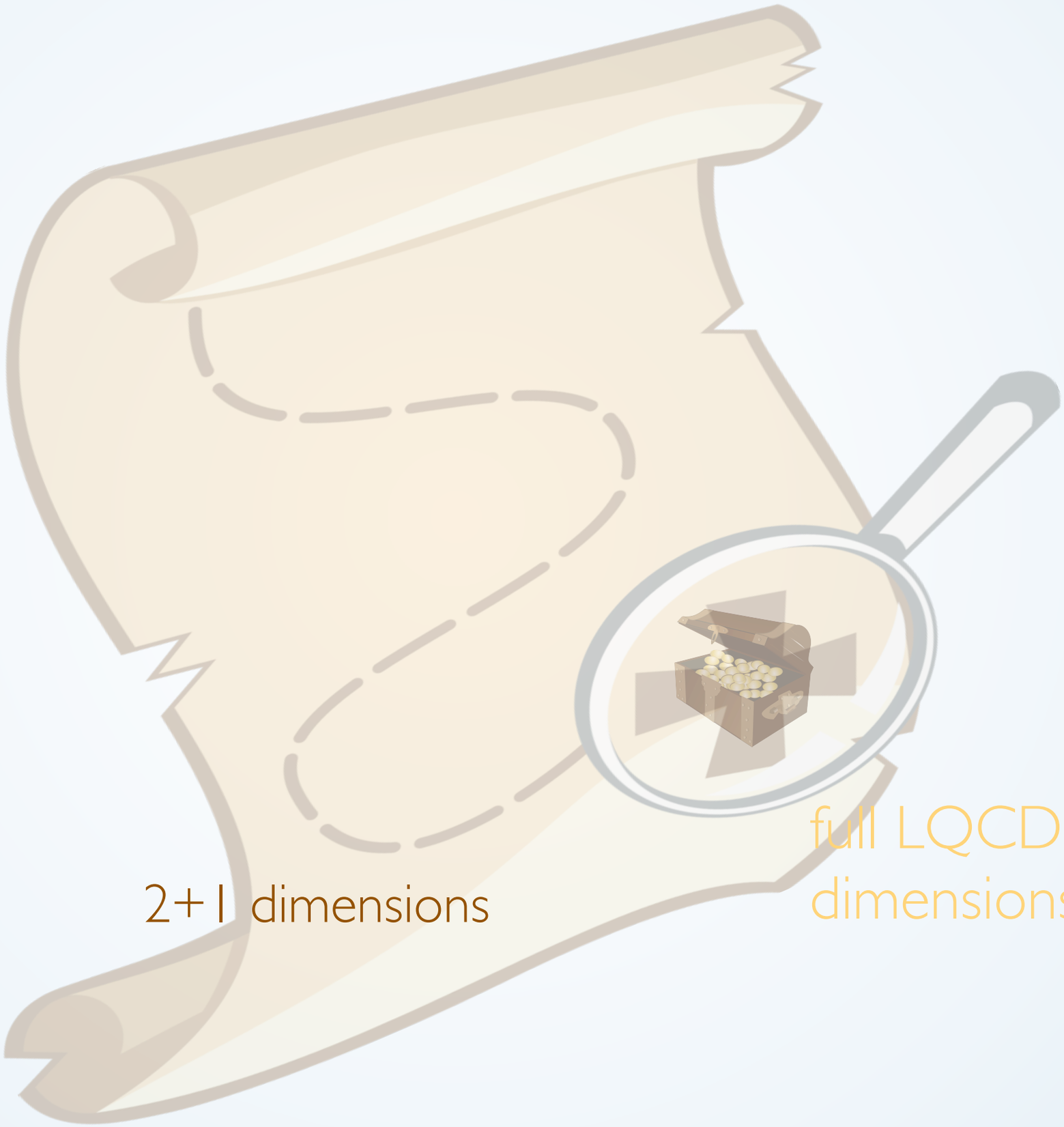

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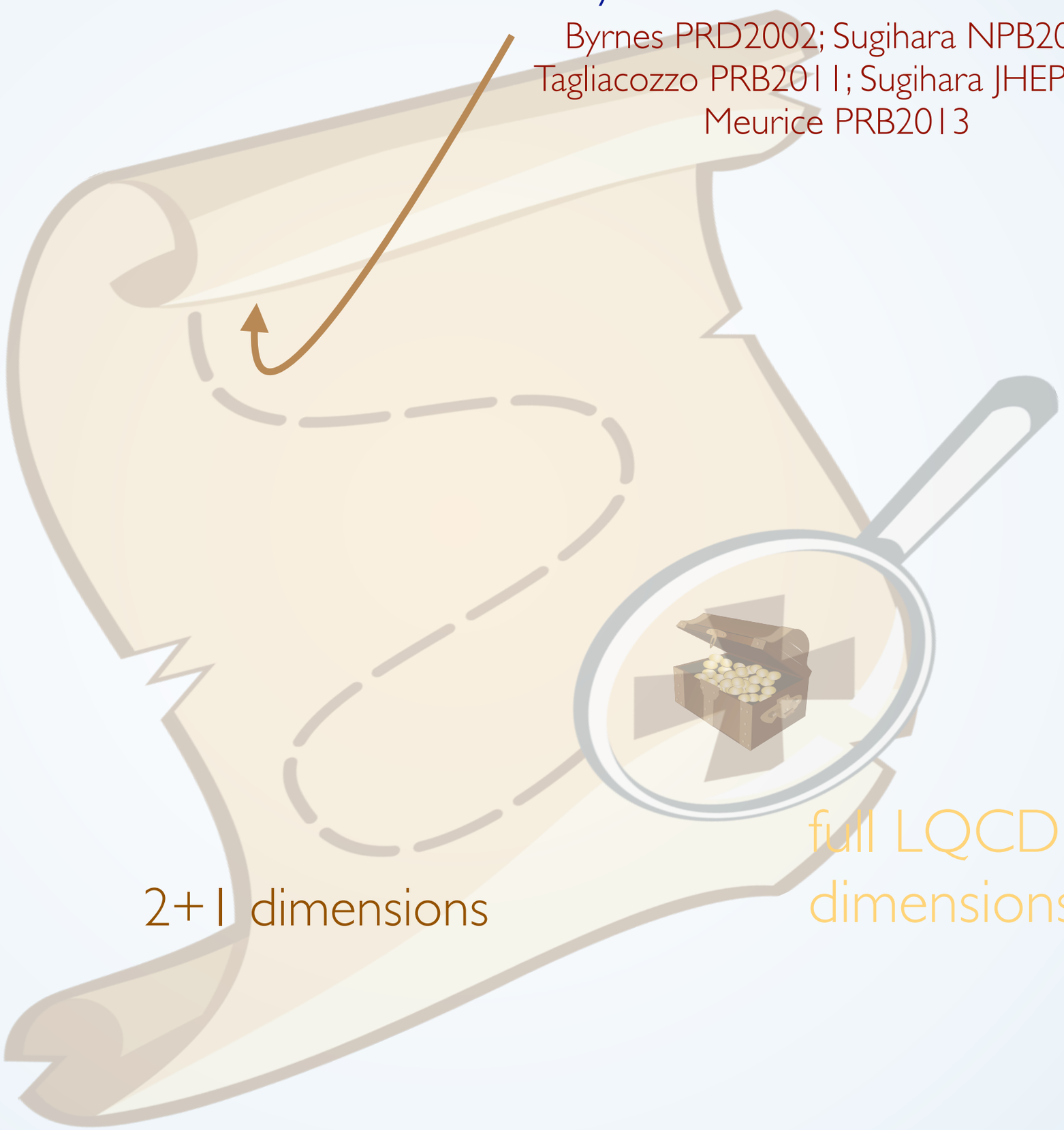

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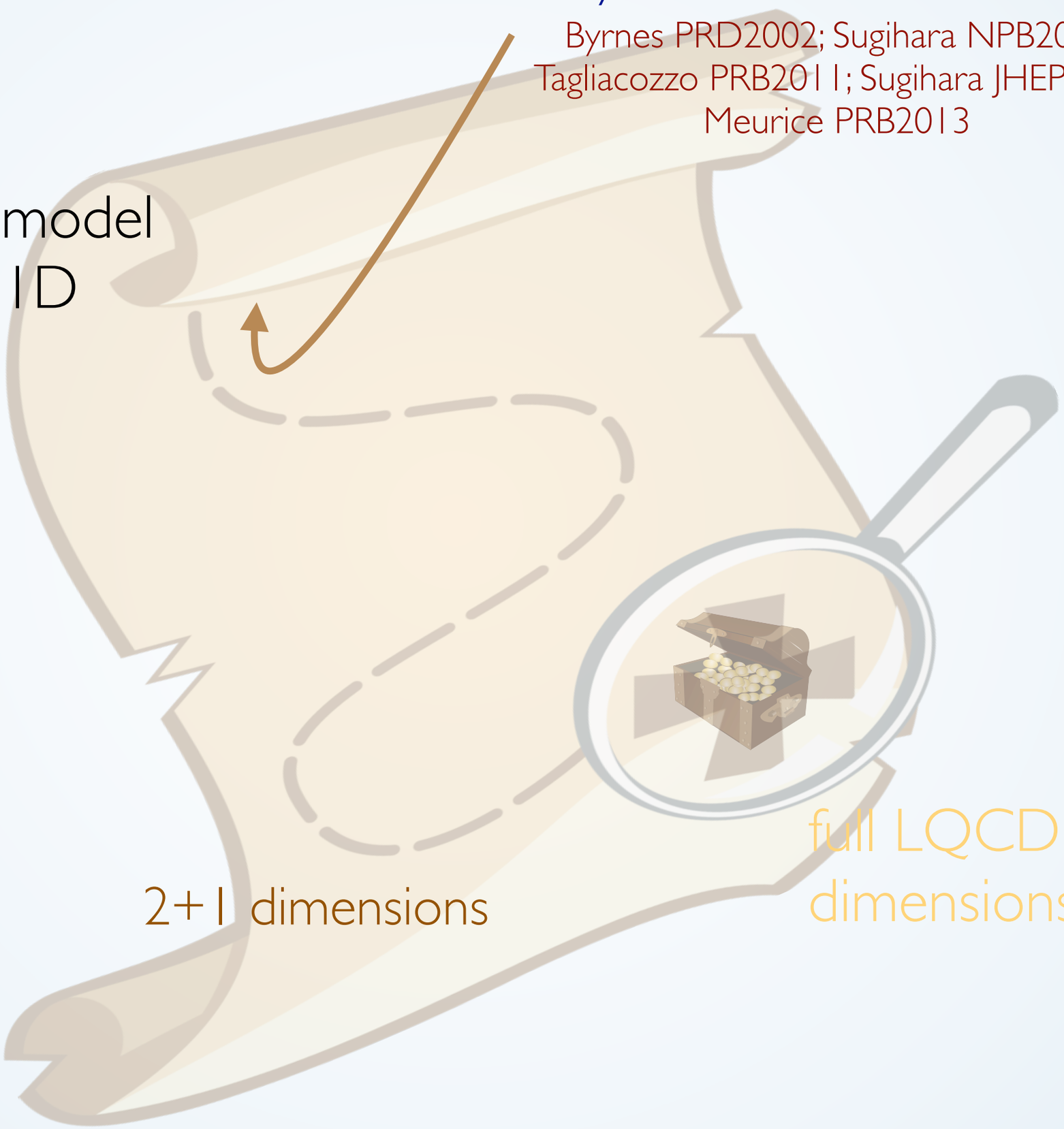

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S. Kuehn et al, PRL118 (2017) 071601

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S. Kühn et al., JHEP 07 (2015) 130;
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other 1+1 D models

talk by D. Lin [TheorD]

2+1 dimensions

full LQCD in 3+1 dimensions

SOLVING LGT WITH TNS

GENERAL STRATEGY

Hamiltonian formulation

acting on a Hilbert space

GENERAL STRATEGY

Hamiltonian formulation

acting on a Hilbert space

$\longrightarrow$ choose proper basis

GENERAL STRATEGY

Hamiltonian formulation

acting on a Hilbert space

→ choose proper basis

Finite dimensional degrees of freedom

fermions

→ ✓ no sign problem

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gauge bosons require attention

→ truncating, integrating out (also QLinks)

talk Kühn [TheorD]

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→ choose proper basis

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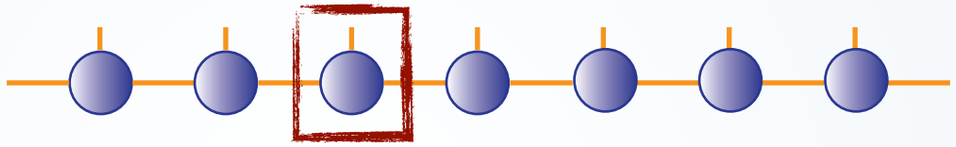
talk Kühn [TheorD]

★ Common ingredients for quantum simulation

SPECTRUM

COMPUTING THE LOW ENERGY LEVELS

Efficient algorithms to find ground state and excitations

$|E_0\rangle \simeq$ 

variational, imaginary time…

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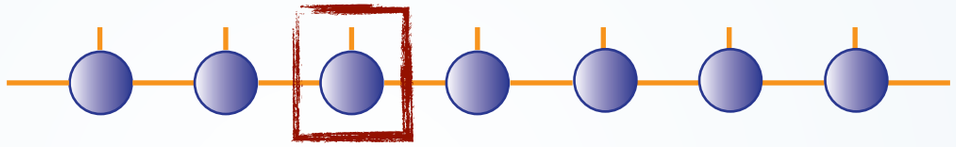
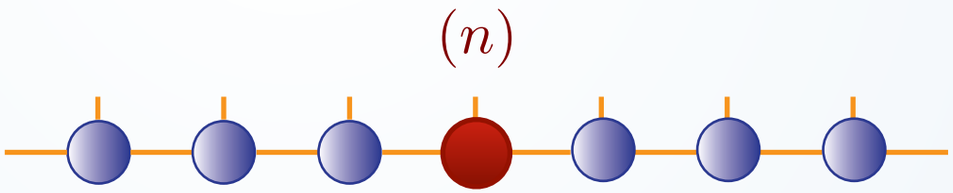
$|E_0\rangle \simeq$ 

variational, imaginary time…

work directly in the TD limit

$$\sum_n e^{ikn}$$



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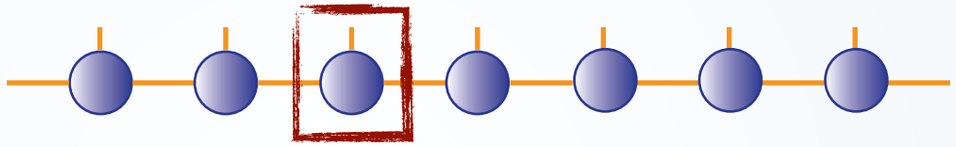
$|E_0\rangle \simeq$ 

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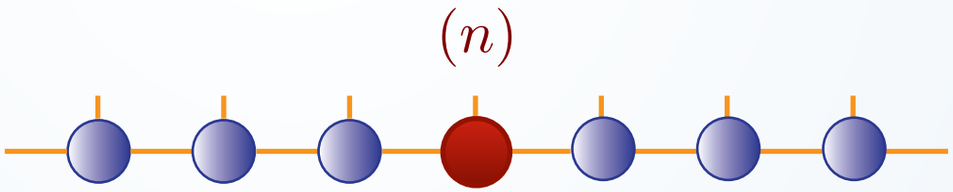
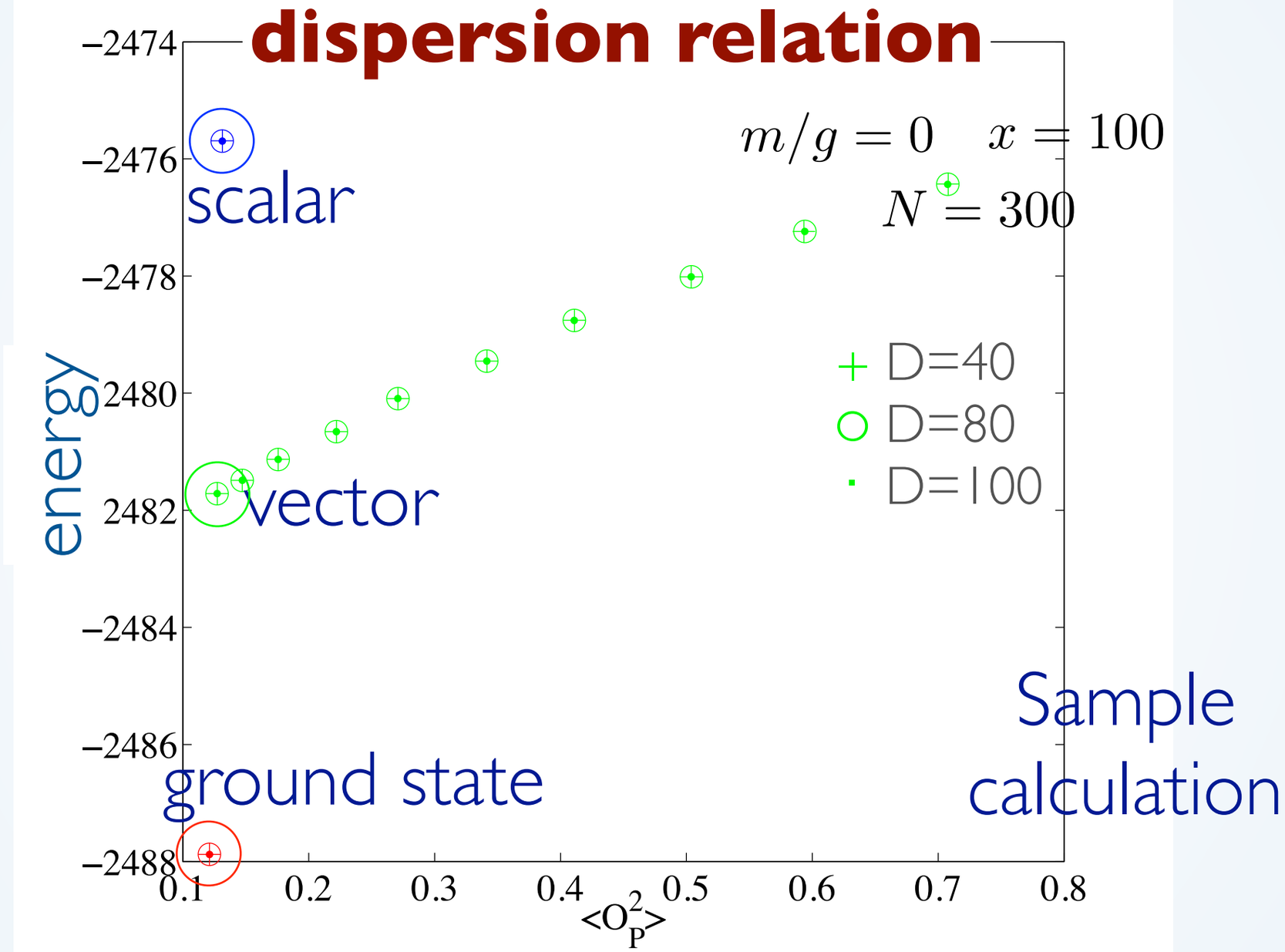
$\sum_n e^{ikn}$ 

Different strategies possible for LGT

truncate the gauge dof, integrate out

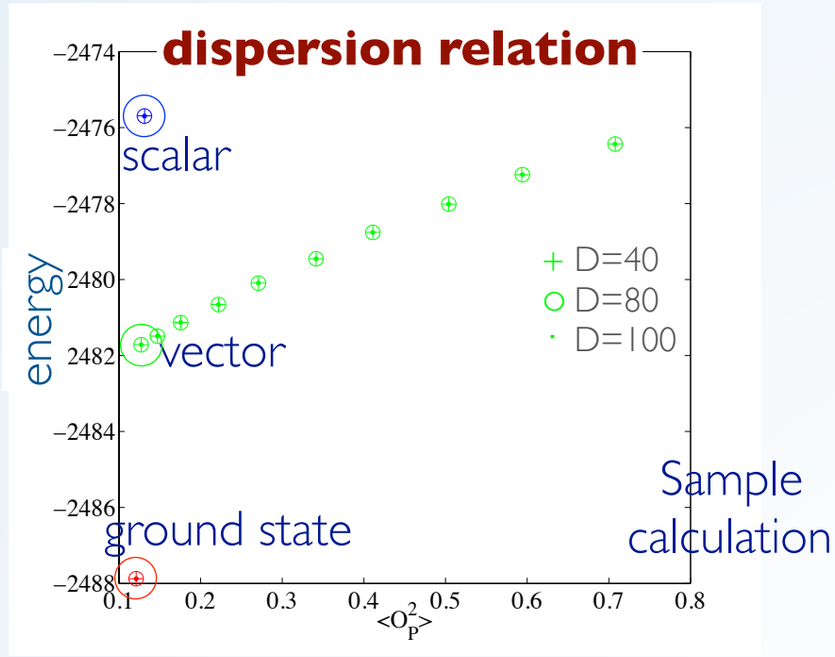
explicit symmetries in tensors

dispersion relation



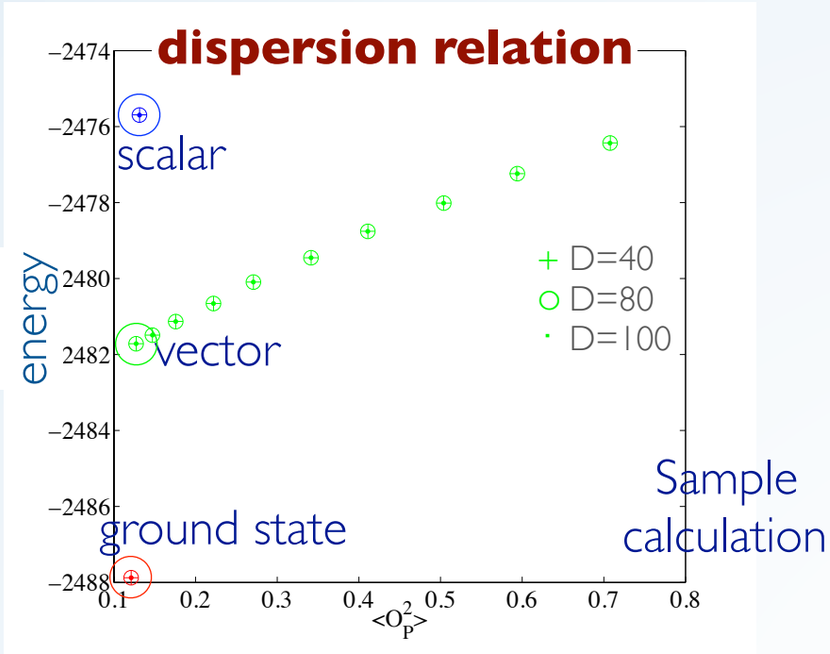
Sample calculation

MCB, Cichy, Cirac, Jansen JHEP11(2013)158
Buyens et al. PRL113 (2014) 091601

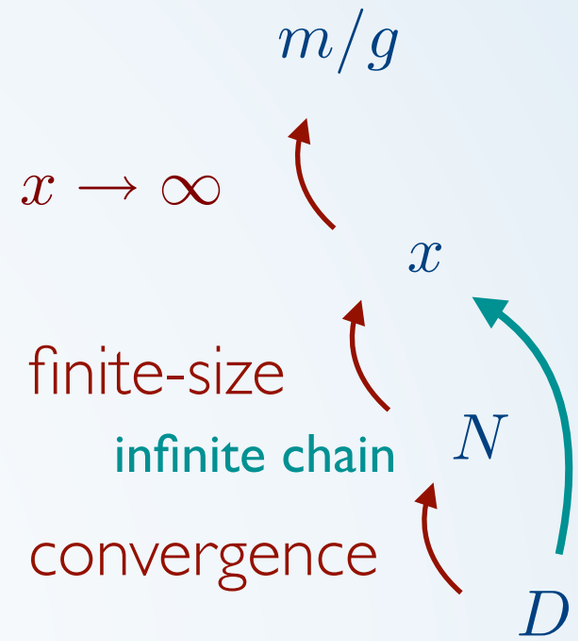


$$m/g = 0 \quad x = 100$$
$$N = 300$$

MCB, Cichy, Cirac, Jansen JHEP11(2013)158
Buyens et al. PRL113 (2014) 091601



after the continuum limit



very precise for all masses

m/g	DMRG	MPS with OBC	SCE	MPS with OBC
0	0.5641859	0.56414(26)	1,128379	1.1283(10)
125	0.53950(7)	0.53946(20)	1.22(2)	1.221(2)
0.25	0.51918(5)	0.51915(14)	1.24(3)	1.239(6)
0.5	0.48747(2)	0.48748(6)	1.20(3)	1.231(5)

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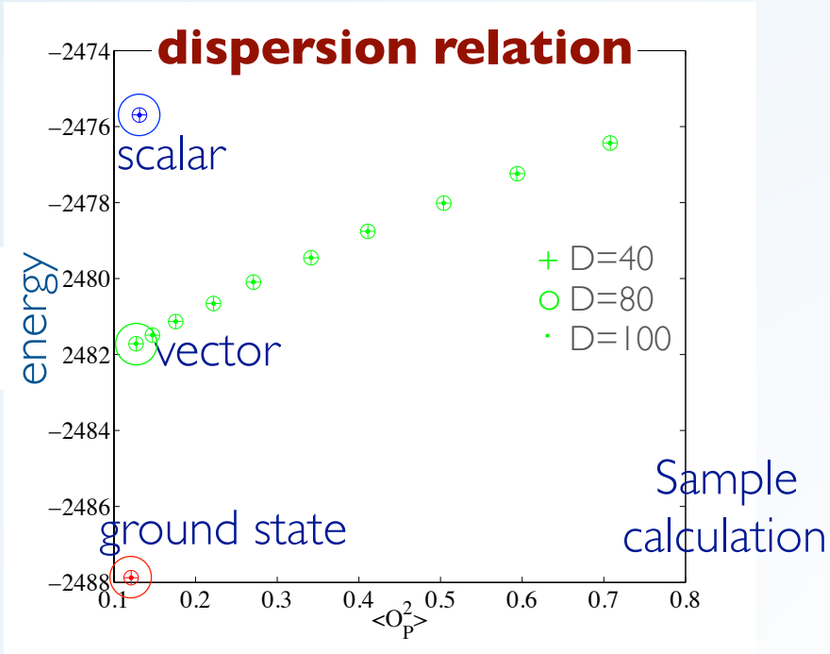

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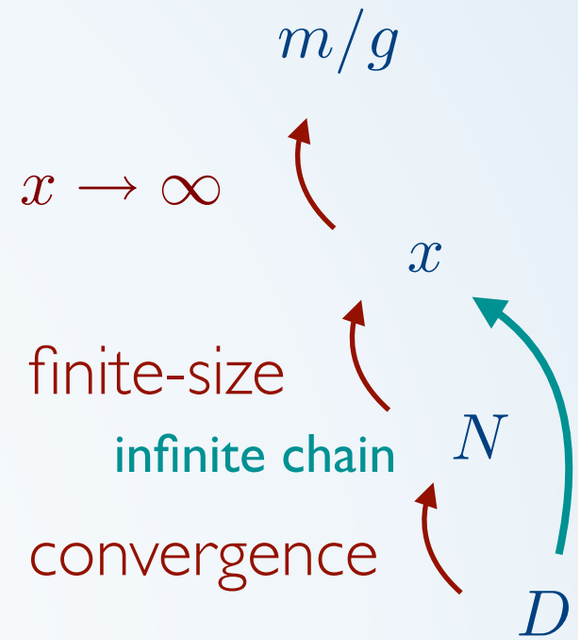

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also for SU(2) LGT

MCB, Cichy, Cirac, Jansen, Kühn PRX 7, 041046 (2017)

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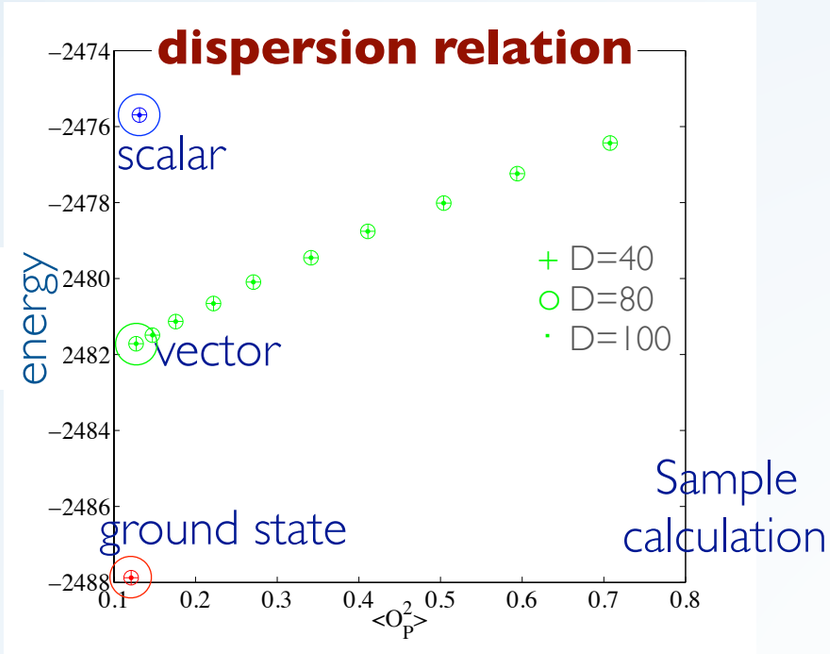

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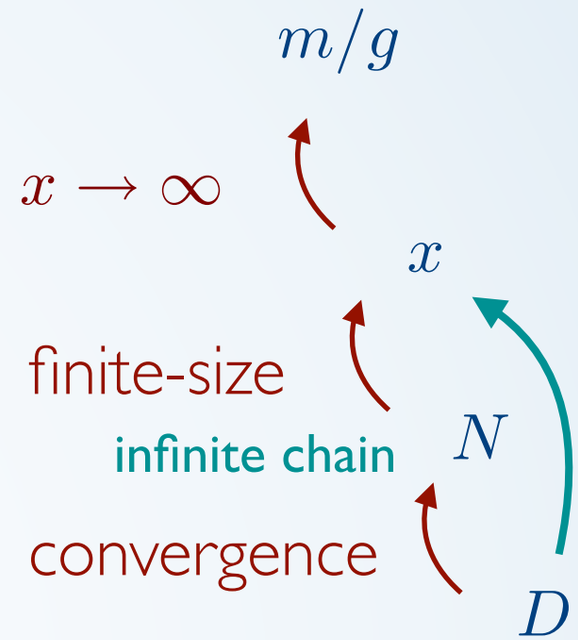

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real time, too
Buyens et al. PRL113 (2014) 091601, PRD96 (2017) 114501

MCB, Cichy, Cirac, Jansen JHEP11(2013)158
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ENTROPY

Entropy can be efficiently computed from MPS state

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gauge constraints not purely local ⇒ not all entropy physical

Casini et al 2014; Gosh et al JHEP 2015

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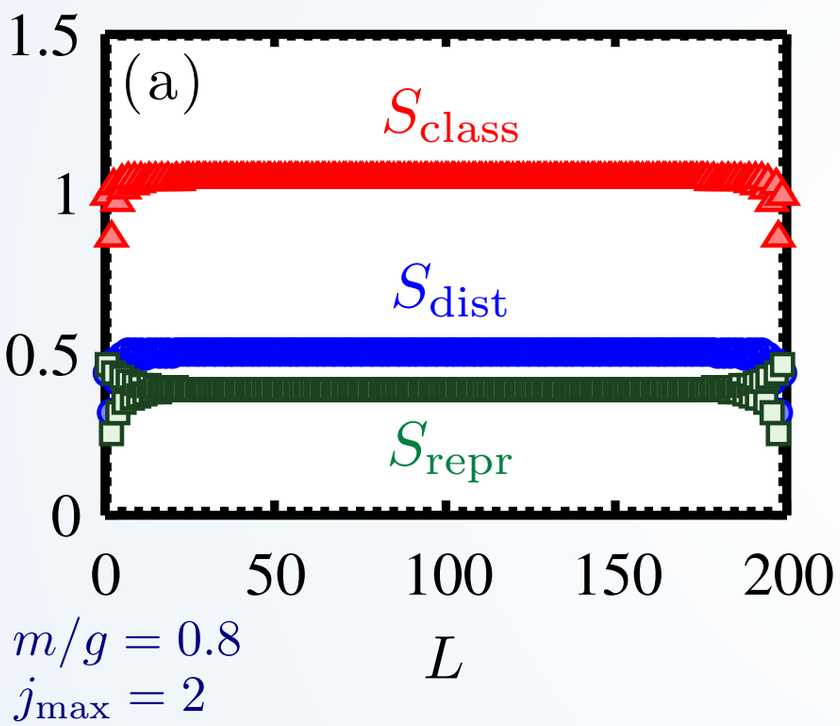

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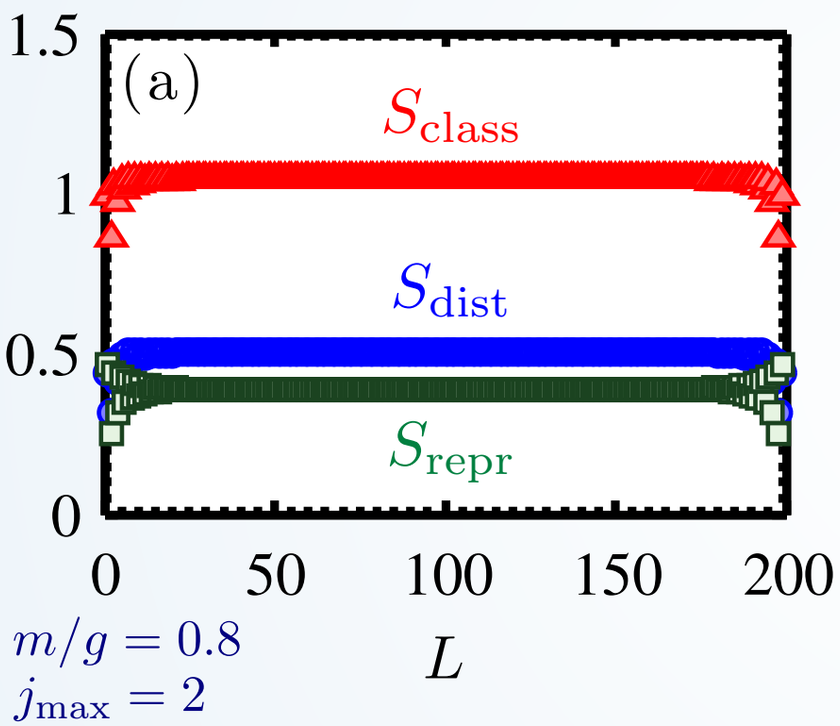

PRX7, 041046 (2017)

divergence in the continuum limit

$$S \propto \frac{c}{6} \log_2 \frac{\xi}{a}$$

Calabrese, Cardy JStatMech 2004

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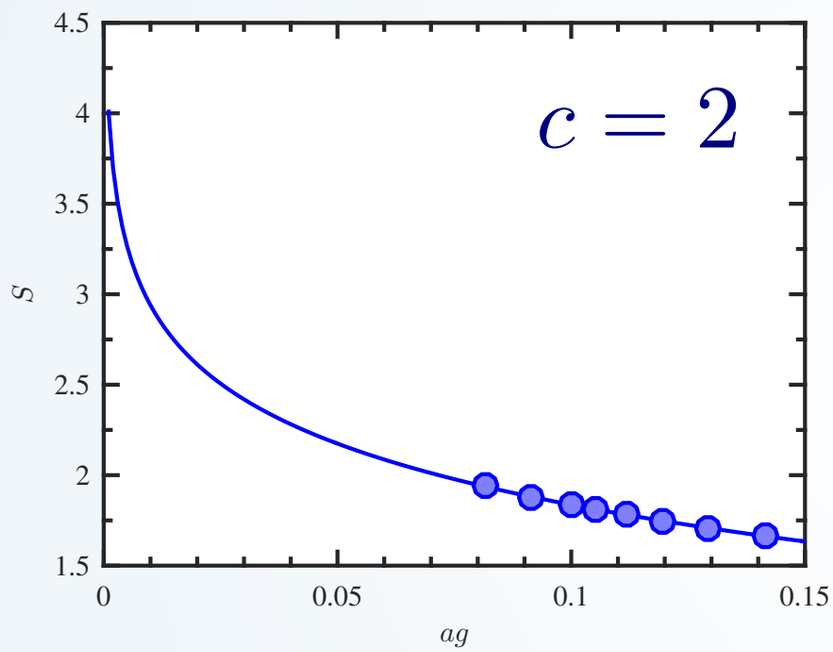

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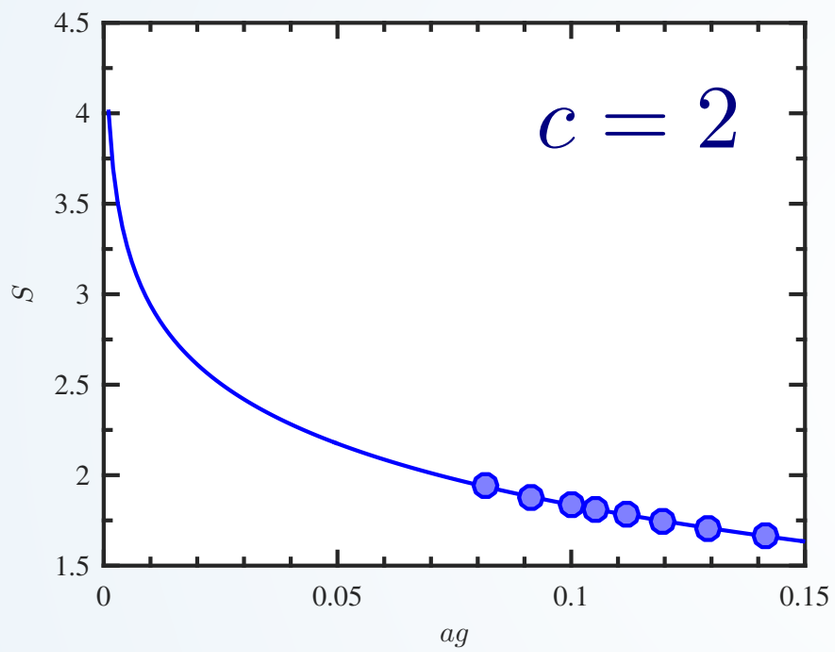
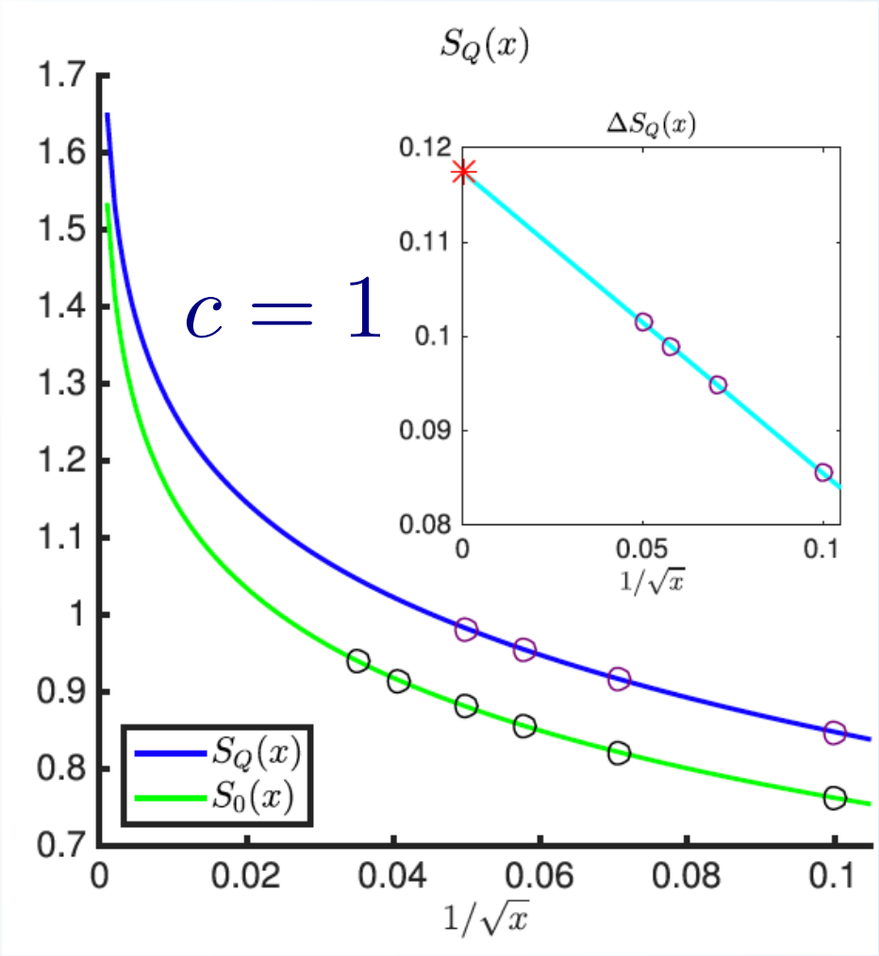


PRX7, 041046 (2017)

Schwinger



Buyens PRX6, 041040 (2016)

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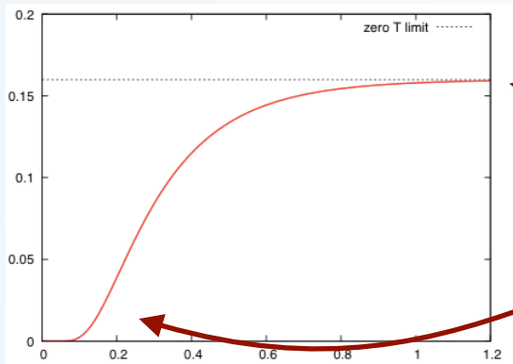
THERMAL EQUILIBRIUM

THERMAL PROPERTIES SCHWINGER

chiral condensate at finite T: analytical for m/g=0

Sachs, Wipf 92

$\frac{\Sigma}{g}$



$T \to 0$

$T \to \infty$

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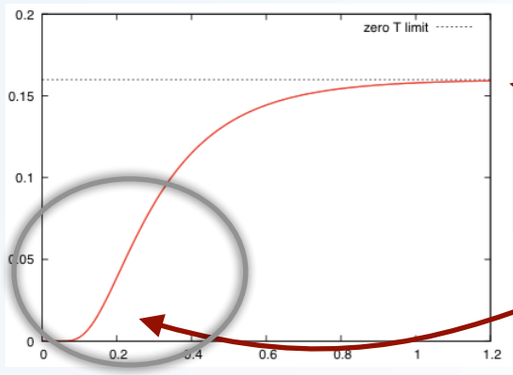

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smooth
restoration of
chiral symmetry

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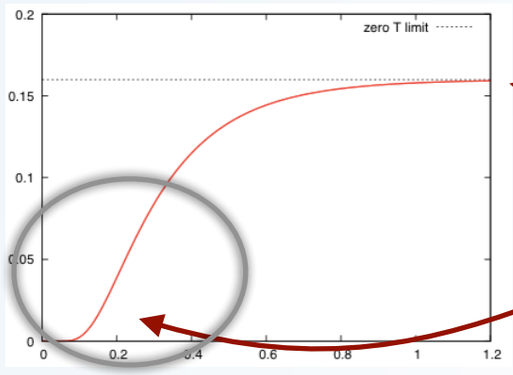

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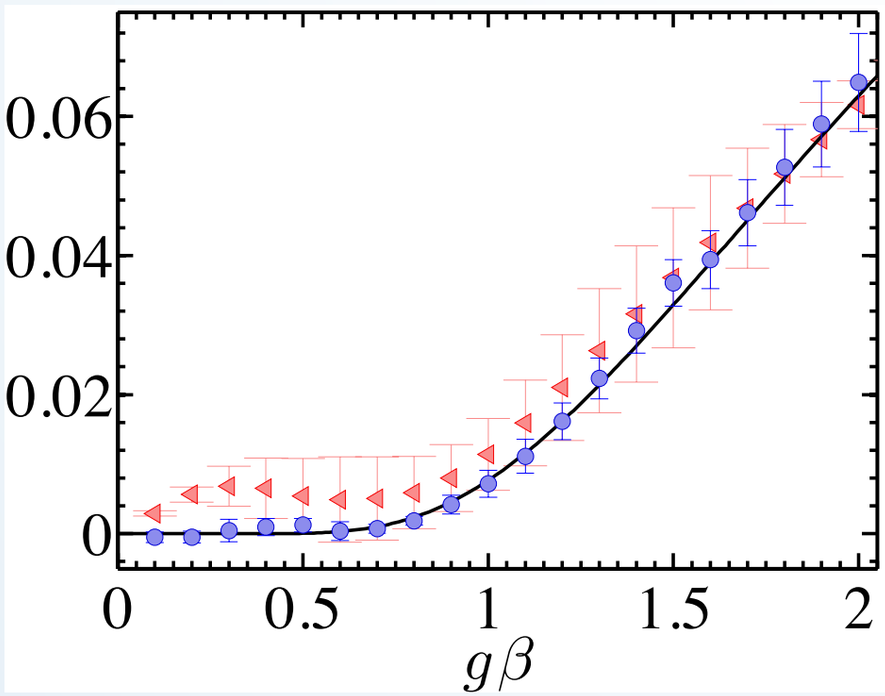
smooth restoration of chiral symmetry



PRD 92, 034519 (2015); PRD 93, 094512 (2016)

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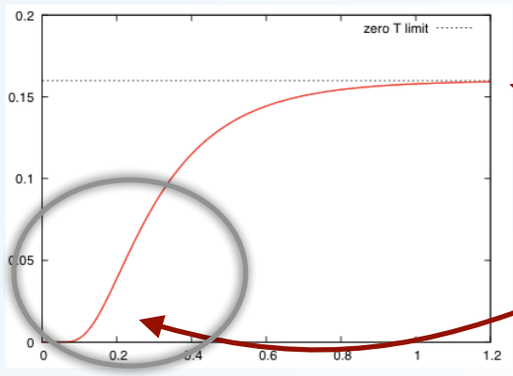

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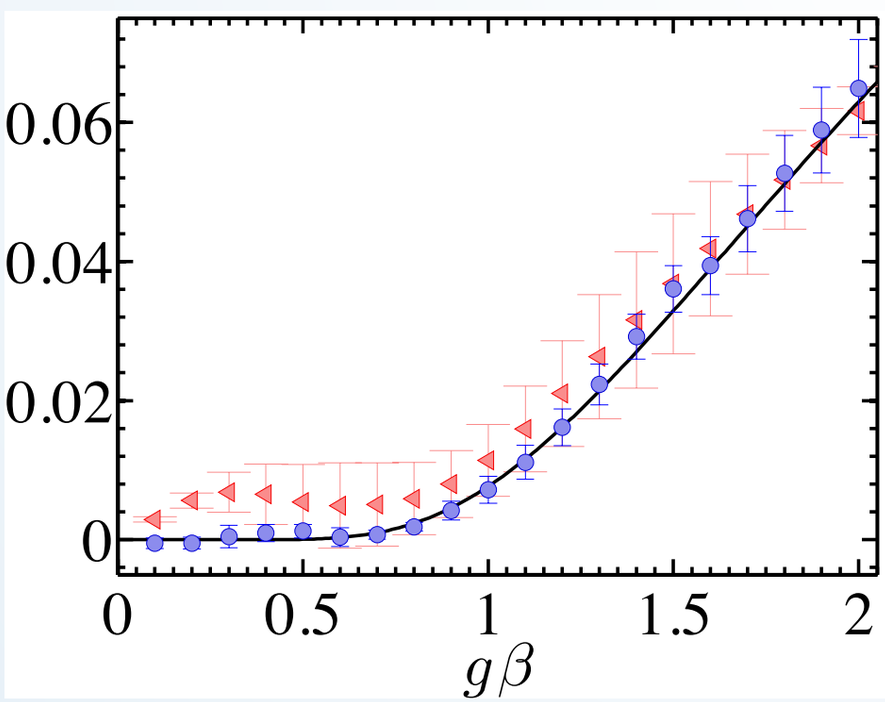
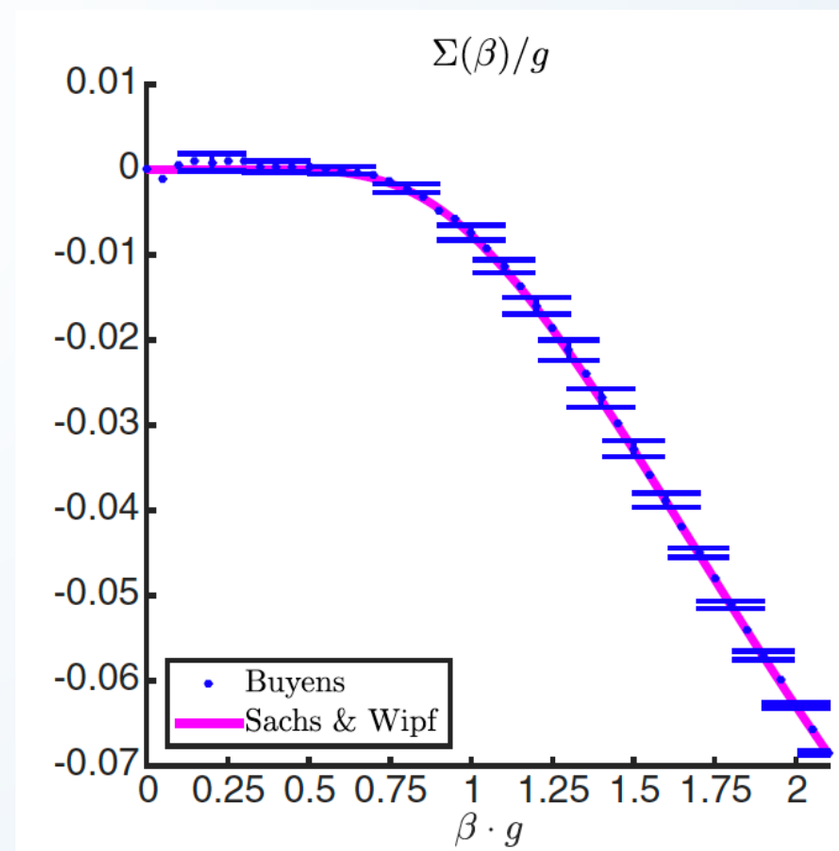


PRD 92, 034519 (2015); PRD 93, 094512 (2016)



Buyens PRD 94, 085018 (2016)

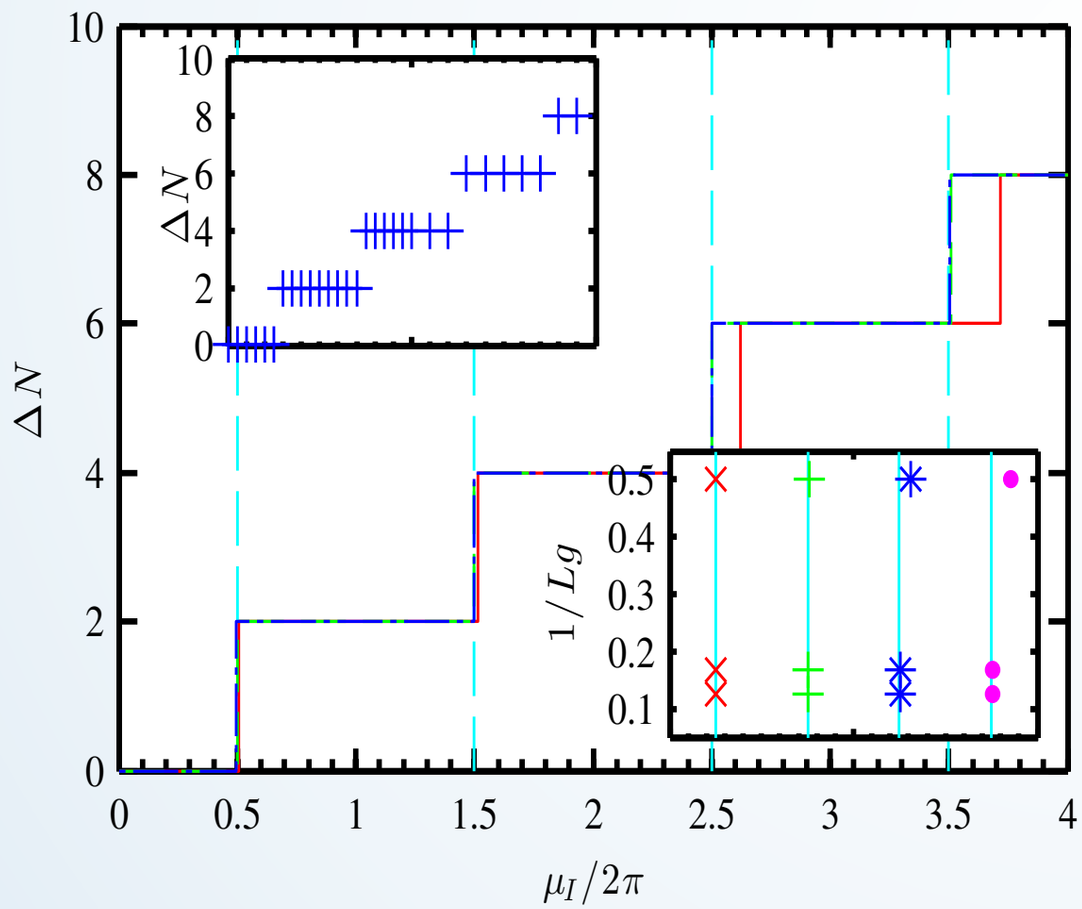
CHEMICAL POTENTIAL

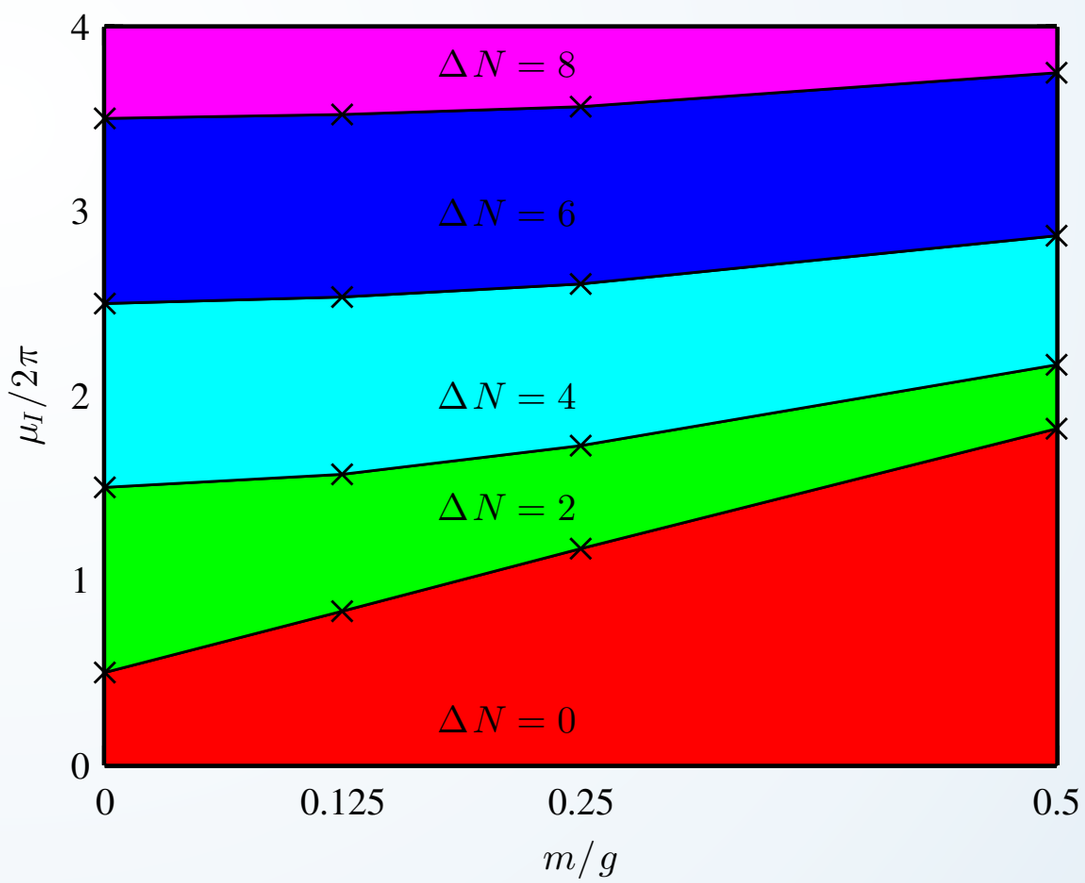
FINITE DENSITY WITH MPS

Several fermion flavors, different chemical potentials

ground state density changes (first order PT)

Montecarlo has sign problem





S. Kuehn et al, PRL118 (2017) 071601

To conclude...

TNS = entanglement based ansatz

QUANTERA
QTFLAG

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feasibility for LQFT thoroughly tested in 1+1D

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spectrum, thermal equilibrium, finite density,
(some) dynamics
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Related: proposals for quantum simulation of LGT with ultracold atoms

Zohar et al. PRL 2010, 2012 ,
Tagliacozzo et al., Nat. Comm. 2013
Banerjee et al., PRL 2012

Rico et al. PRL 2014
Pichler et al, PRX 2016
Zohar, Burrello, PRD 2015

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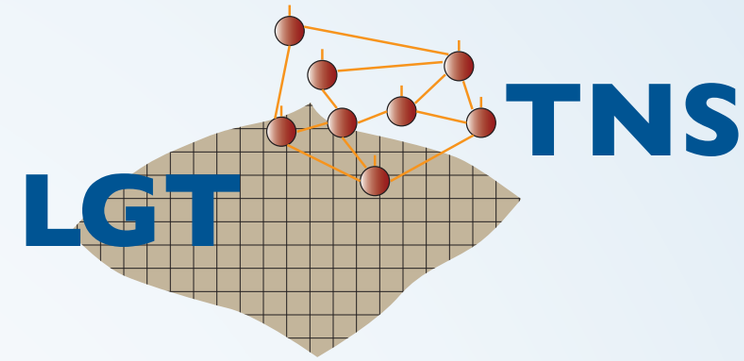
THANKS



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2D

ACTIVE RESEARCH: PEPS FOR LGT

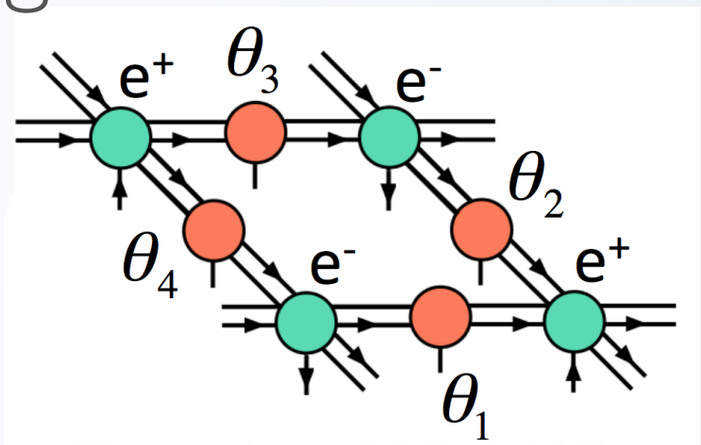
explicitly gauge invariant PEPS

Tagliacozzo et al PRX 2014
Haegeman et al PRX 2014
Zohar et al Ann Phys 2015
arXiv:1807.01294

restricted ansatz calculations

standard PEPS toolbox contains all ingredients for full variational computation

computational cost, required D



Zapp, Orús PRD 2017

restriction of the ansatz may be better strategy

e.g. fully Gaussian PEPS

Zohar, Cirac PRD 2018