THE 36TH INTERNATIONAL SYMPOSIUM ON LATTICE FIELD THEORY
MICHIGAN STATE UNIVERSITY, JULY 2018

THE PATH FROM FINITE TO INFINITE VOLUME

HADRONIC OBSERVABLES FROM LATTICE QCD

ZOHREH DAVOUDI

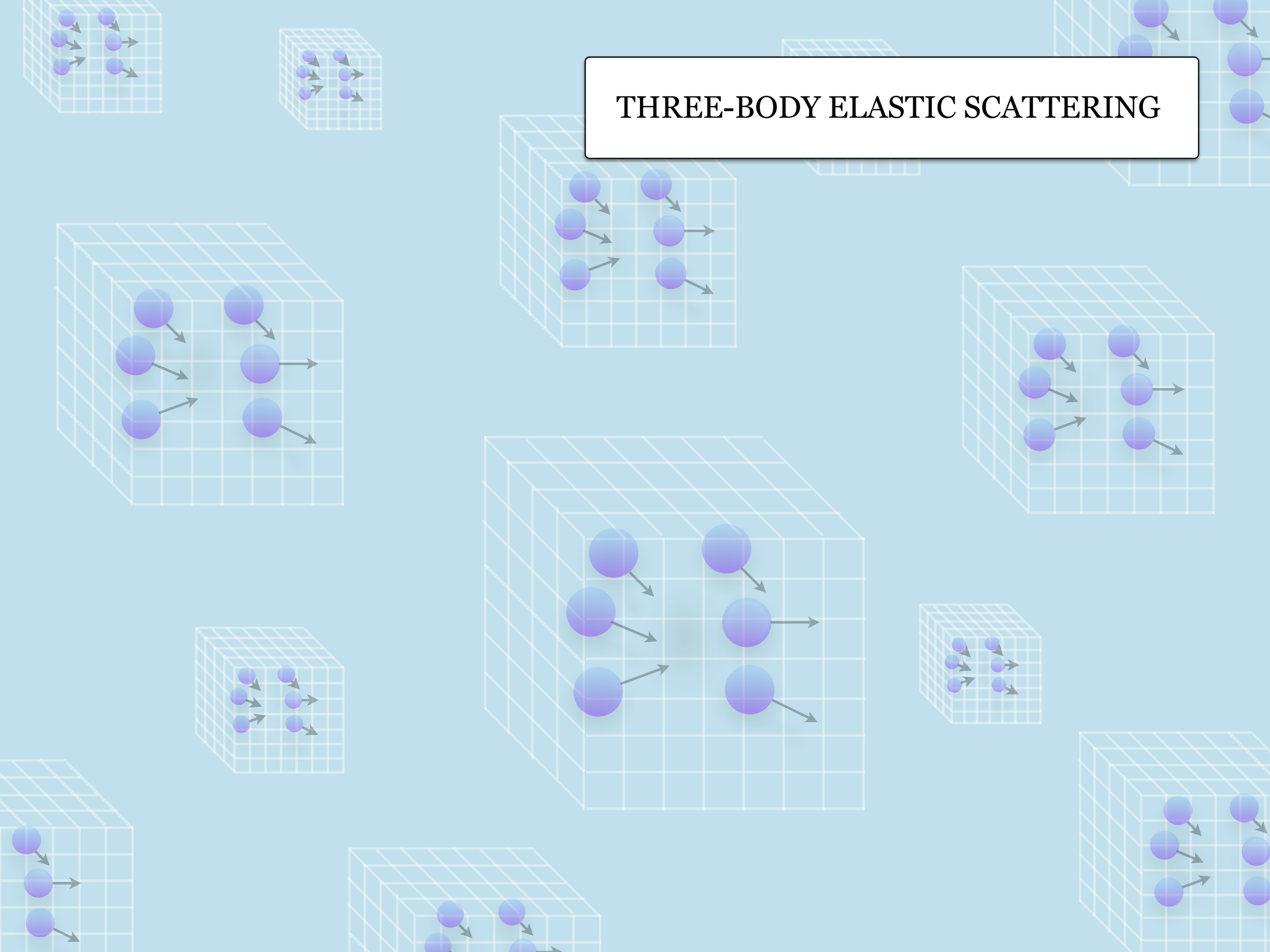
UNIVERSITY OF MARYLAND AND RIKEN FELLOW

TWO-BODY ELASTIC SCATTERING

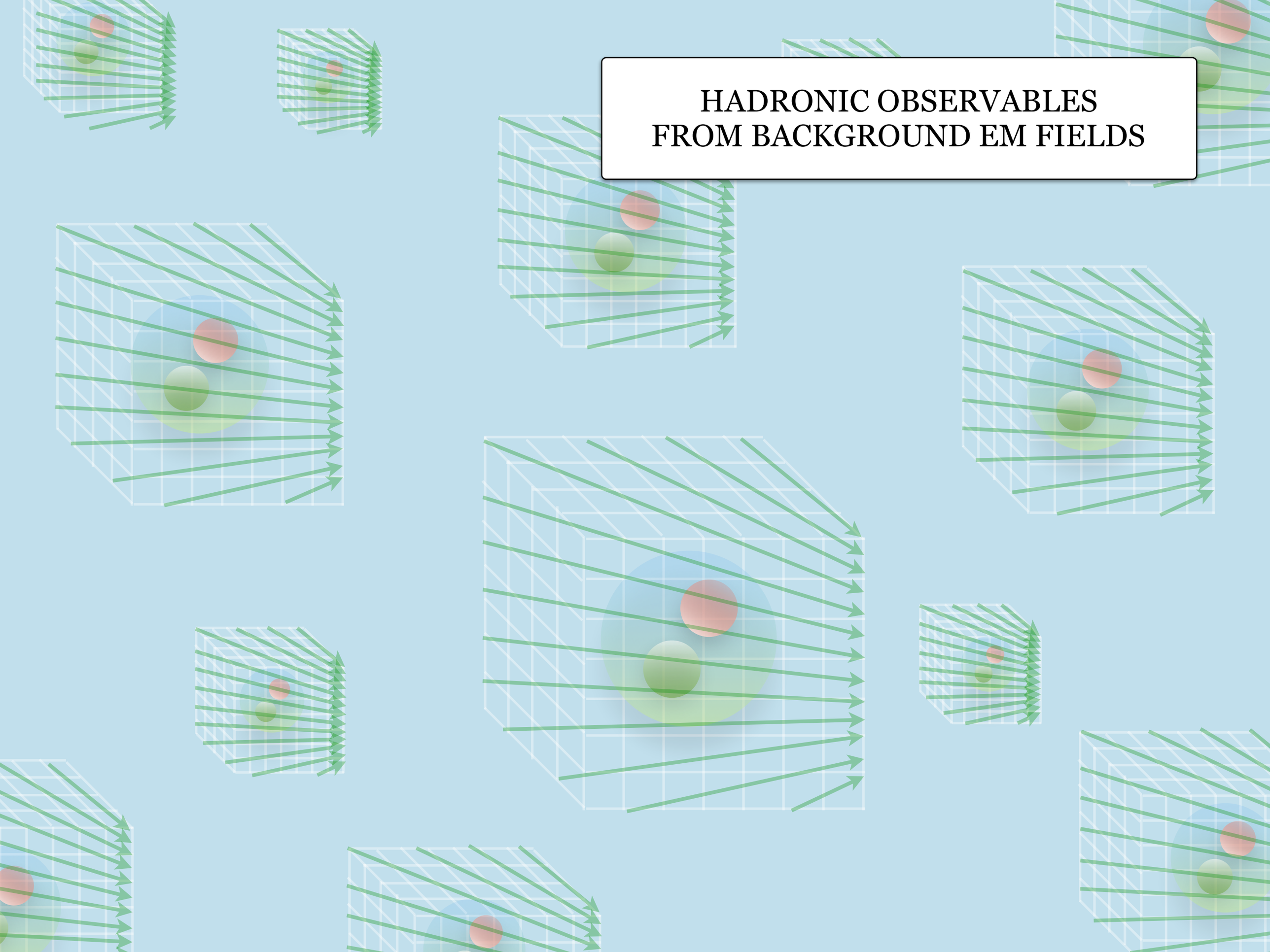
TWO-BODY INELASTIC SCATTERING

TWO-NUCLEON OBSERVABLES

THREE-BODY ELASTIC SCATTERING



HADRONIC OBSERVABLES FROM BACKGROUND EM FIELDS



HADRONIC OBSERVABLES IN PRESENCE OF QED

IT IS A GREAT HONOR TO HOLD THIS TITLE...

WHAT I HAVE ACCOMPLISHED REFLECTS ON INCREDIBLE PHYSICISTS I HAVE MET, WORKED WITH AND INSPIRED BY IN MY JOURNEY TO HERE...

A GREAT JOURNEY IN SEATTLE UNDER MARTIN'S MENTORSHIP

2010-2014

WINTER 2014, UW, SEATTLE
PHOTO BY: SILAS BEANE

FALL 2012, UW, SEATTLE
PHOTO BY: SEATTLE TIMES

MANY FUN PHYSICS CHATS IN BEAUTIFUL SEATTLE AND ELSEWHERE

SUMMER 2017, SEATTLE
PHOTO BY: ZD

FALL 2016, AACHEN
PHOTO BY: ZD

Kavli Institute for Theoretical Physics
University of California, Santa Barbara

A DELIGHTFUL COLLABORATION

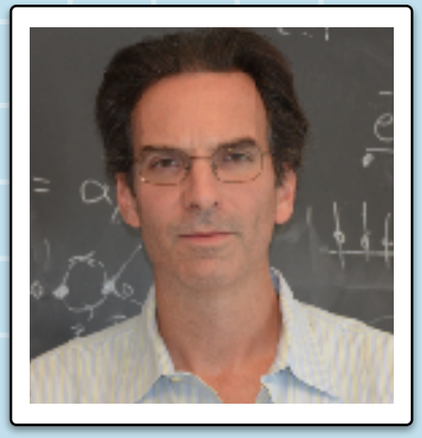
PHOTO: SUMMER 2016, SANTA BARBARA, CA

NPLQCD

PHOTO BY: ZD

AND COLLABORATORS WITHIN THE NPLQCD COLLABORATION

2016-...



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Past Programs & Workshops

2018 Programs and Workshops

INT-18-1a:

- Nuclear ab initio Theories and Neutrino Physics, February 26 - March 30, 2018

INT-18-1b:

- Multi-Scale Problems Using Effective Field Theories, May 7 - June 1, 2018

INT-18-2a:

- Fundamental Physics with Electroweak Probes of Light Nuclei, June 12 - July 13, 2018

Workshops:

- **INT-18-70W:** Multi-Hadron Systems from Lattice QCD, February 5 - 9, 2018
- **INT-18-71W:** Astro-Solids, Dense Matter, and Gravitational Waves, April 16 - 20, 2018

Rapid Reaction Workshop:

- **INT-18-72R:** INT-JINA Symposium: First multi-messenger observations of a neutron star merger and its implications for nuclear physics, March 12 - 14, 2018

2017 Programs and Workshops

INT-17-1a:

- Toward Predictive Theories of Nuclear Reactions Across the Isotopic Chart, February 27 - March 31, 2017

INT-17-1b:

- Precision Spectroscopy of QGP Properties with Jets and Heavy Quarks, May 1 - June 8, 2017

INT-17-2a:

- Neutrinoless Double-beta Decay, June 13 - July 14, 2017

INT-17-2b:

- Electromagnetic Signatures of R-process Nucleosynthesis in Neutron Star Binary Mergers, July 24 - August 18, 2017

INT-17-3:

- Spatial and Momentum Tomography of Hadrons and Nuclei, August 28 - September 29, 2017

Workshops:

- **INT-17-64W:** SIGN 2017: International Workshop on the Sign Problem in QCD and Beyond, March 20 - 24, 2017
- **INT-17-65W:** Probing QCD in Photon-Nucleus Interactions at RHIC and LHC: the Path to EIC, February 13 - 17, 2017
- **INT-17-66W:** Quantum Computing for Nuclear Physics, November 14-15, 2017
- **INT-17-67W:** Lattice QCD Input for Neutrinoless Double-β Decay, July 6 - 7, 2017
- **INT-17-68W:** The Flavor Structure of Nucleon Sea, October 2 - 13, 2017
- **INT-17-69W:** Neutron-Antineutron Oscillations: Appearance, Disappearance, and Baryogenesis, October 23 - 27, 2017

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2016 Programs and Workshops

INT-16-1:

- Nuclear Physics from Lattice QCD, March 21 - May 27, 2016

INT-16-2a:

- Bayesian Methods in Nuclear Physics, June 13 - July 8, 2016

INT-16-2b:

- The Phases of Dense Matter, July 11 - August 12, 2016

INT-16-3:

- Exploring the QCD Phase Diagram through Energy Scans, September 19 - October 14, 2016

Workshops:

- **INT-16-61W:** Flavor Observations with Supernova Neutrinos, August 15 - 19, 2016
- **INT-16-62W:** Spectrum and Structure of Excited Nucleons from Exclusive Electroproduction, November 14 - 18, 2016
- **INT-16-63W:** Theoretical Developments in Neutrino-Nucleus Scattering, December 5 - 9, 2016

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2015 Programs and Workshops

INT-15-1:

- Frontiers in Quantum Simulation with Cold Atoms, March 23 - May 8, 2015

INT-15-2a:

- Neutrino Astrophysics and Fundamental Properties, June 1 - June 26, 2015

INT-15-2b:

- Correlations and Fluctuations in p+A and A+A Collisions, July 6 - July 31, 2015

INT-15-2c:

- Equilibration Mechanisms in Weakly and Strongly Coupled Quantum Field Theory, August 3 - 28, 2015

INT-15-3:

- Intersections of BSM Phenomenology and QCD for New Physics Searches, September 14 - October 23, 2015

Workshops:

- **INT-15-58W:** Reactions and Structure of Exotic Nuclei, March 2-13, 2015
- **INT-15-59W:** TALENT/INT Lectures on "Nuclear Physics of Neutron Stars and Supernovae", June 22 - July 10, 2015
- **INT-15-60W:** Modern Exotic Hadrons, November 2 - 13, 2015

2014 Programs and Workshops

INT-14-1:

- Universality in Few-Body Systems: Theoretical Challenges and New Directions, March 10 - May 16, 2014
- Few-Body Universality in Atomic and Nuclear Physics: Recent Experimental and Theoretial Advances, May 12-16, 2014

INT-14-2a:

- Binary Neutron Star Coalescence as a Fundamental Physics Laboratory, June 30 - August 1, 2014

INT-14-2b:

- Nucleosynthesis and Chemical Evolution, July 28 - August 29, 2014

INT-14-3:

- Heavy Flavor and Electromagnetic Probes in Heavy Ion Collisions, September 15 - October 10, 2014

Workshops:

- **INT-14-55W:** Studies of 3D Structure of Nucleon, February 24-28, 2014
- **INT-14-56W:** The r-process: status and challenges, July 28 - August 1, 2014
- **INT-14-57W:** Nuclear Aspects of Dark Matter Searches, December 8-12, 2014

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2013 Programs and Workshops

INT-13-1a:

- Computational and Theoretical Advances for Exotic Isotopes in the Medium Mass Region, March 25 - April 19, 2013
- Advances in Many-Body Theory: from Nuclei to Molecules, April 3-5, 2013

INT-13-1b:

- Quantum Noise, May 6 - May 31, 2013

INT-13-2a:

- Advances in Quantum Monte Carlo Techniques for Non-Relativistic Many-Body Systems, June 24 - August 2, 2013

INT-13-2b:

- Nuclei and Fundamental Symmetries: Theory Needs of Next-Decade Experiments, August 5 - 30, 2013

INT-13-2c:

- TALENT: Training in Advanced Low Energy Nuclear Theory, July 1 - 19, 2013

INT-13-3:

- Quantitative Large Amplitude Shape Dynamics: Fission and Heavy Ion Fusion, Sept. 23 - Nov. 15, 2013
- Experimental Status and Prospects, October 14-18, 2013
- Workshop on Reactor Neutrinos, November 6-8, 2013

Workshops:

- **INT-13-52W:** Nuclear Structure and Dynamics at Short Distances, February 11 - 22, 2013

- INT Summer School on Applications of String Theory, July

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6-8, 2013

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- **INT-13-53W:** Nuclear Reactions from Lattice QCD, March 11-12, 2013
- **INT-13-54W:** Neutrino-Nucleus Interactions for Current and Next Generation Neutrino Oscillation Experiments, December 3-13, 2013

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Collisions, April 13, 2012

INT-12-2a:

ollapse Supernovae: Models and Observable
, June 25 - July 27, 2012
op on Probing the Supernova Mechanism by
ations, July 16-20, 2012

INT-12-2b:

- Lattice QCD studies of excited resonances and multi-hadron systems, July 30 - August 31, 2012

INT-12-2c:

- INT Summer School on Lattice QCD for Nuclear Physics, August 6 - 24, 2012

INT-12-3:

- Light Nuclei from First Principles, September 17 - November 16, 2012
- Structure of Light Nuclei, October 8-12, 2012
- Electroweak Properties of Light Nuclei, November 5-9, 2012

February
for
gh 2025,
April 23 - 26, 2012

- **INT-12-51W:** The "Ridge" Correlation in High-Energy Collisions at RHIC and LHC, May 7 - 11, 2012

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2011 Programs and Workshops

INT-11-1:

- Fermions from Cold Atoms to Neutron Stars: Benchmarking the Many-Body Problem, March 14-May 20, 2011

INT-11-2a:

- Extreme Computing and its Implications for the Nuclear Physics/Applied Mathematics/Computer Science Interface, June 6-July 8, 2011
- Exascale Workshop, June 27-July 1, 2011

INT-11-2b:

- Astrophysical Transients: Multi-messenger Probes of Nuclear Physics, July 11 - August 5, 2011

INT-11-2c:

- INT Summer School on Applications of String Theory, July

WORKSHOP ON

NUCLEAR REACTIONS FROM LATTICE QCD

INSTITUTE FOR NUCLEAR THEORY, SEATTLE, MARCH 2013

INSTITUTE for NUCLEAR THEORY

Back row (left to right): Emmanuel Chang, Michael Endres, Kenneth Nollett, Achim Schwenk, William Detmold, Martin Savage, Dean Lee, Saul Cohen, David Kaplan and Michael Buchoff. Front row (left to right): Huey-Wen Lin, Zohreh Davoudi, Thomas Luu, Raul Briceno, Gautam Rupak, Sinya Aoki, Brian Tiburzi and Stephen Sharpe.
Not present in the picture: Evan Berkowitz, Aurel Bulgac, Joaquin Drut, Maxwell Hansen, Gerald Miller, Sergej Moroz, Petr Navratil, Kenneth Roche and André Walker-Loud.

PHOTO: INT

INDEBTED TO GREAT MENTORS AT UW AND MIT

2010 - 2017

Back row (left to right): Emmanuel Chang, Michael Endres, Kenneth Nollett, Achim Schwenk, William Detmold, Martin Savage, Dean Lee, Saul Cohen, David Kaplan and Michael Buchoff. Front row (left to right): Huey-Wen Lin, Zohreh Davoudi, Thomas Luu, Raul Briceno, Gautam Rupak, Sinya Aoki, Brian Tiburzi and Stephen Sharpe.
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AND TO GREAT FRIENDS AND COLLABORATORS
2010 - 2017

RAUL BRICENO

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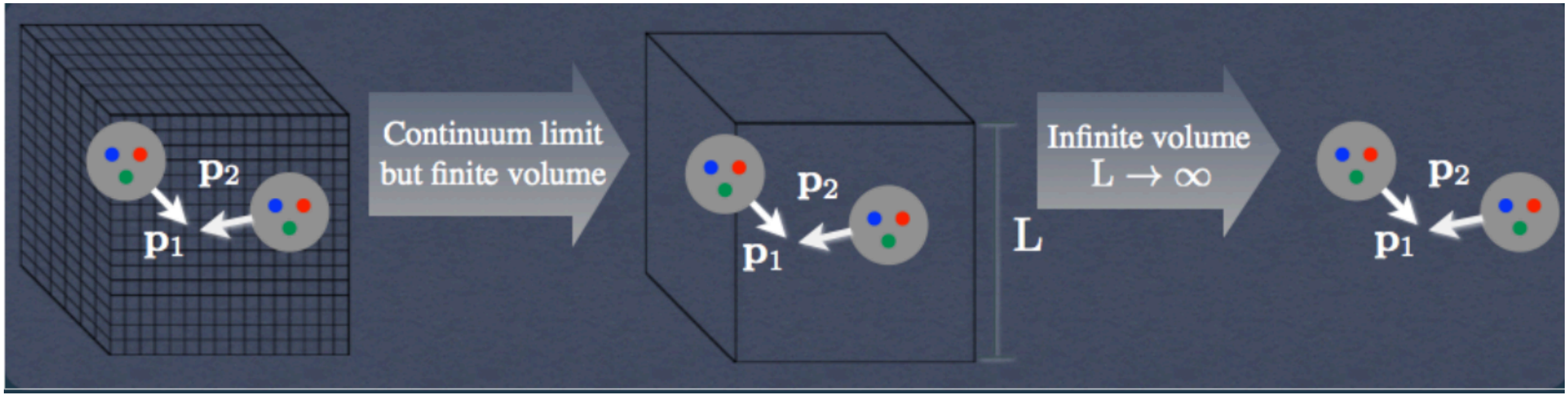
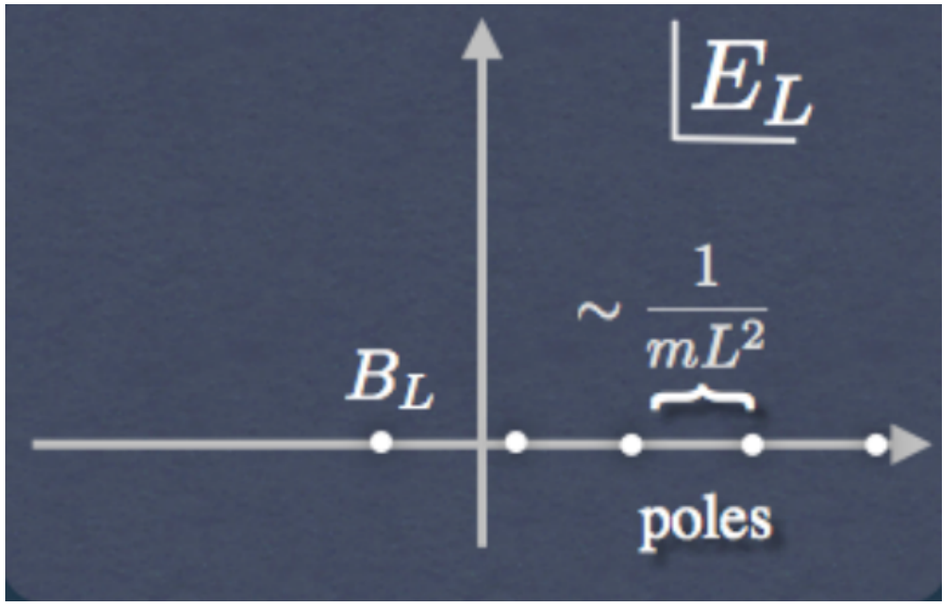
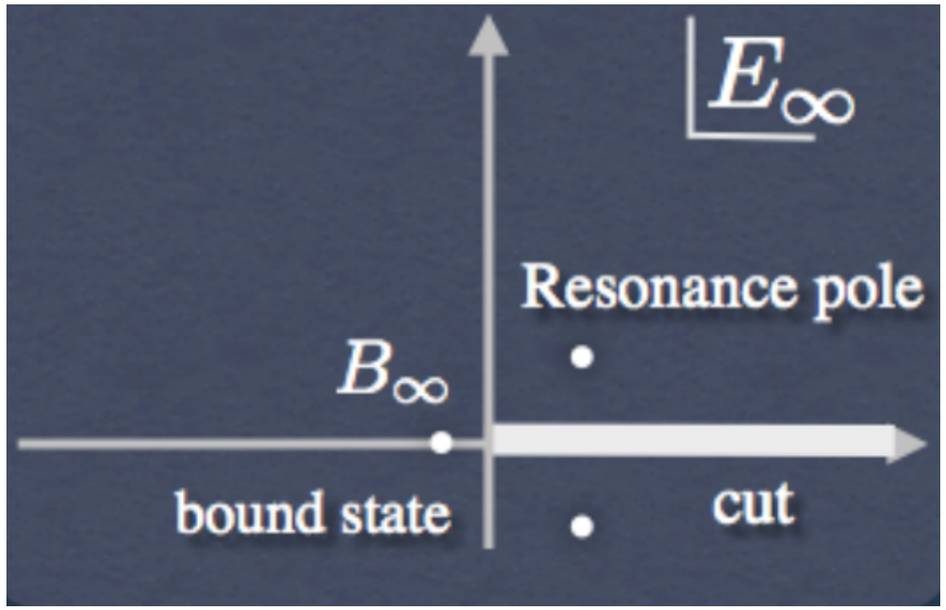
TWO-BODY ELASTIC SCATTERING

FINITE VS. INFINITE VOLUME

Maiani and Testa, Phys.Lett.B245,585(1990).
Luescher, Commun.Math.Phys.105,153(1986), Nucl.Phys.B354,531(1991).

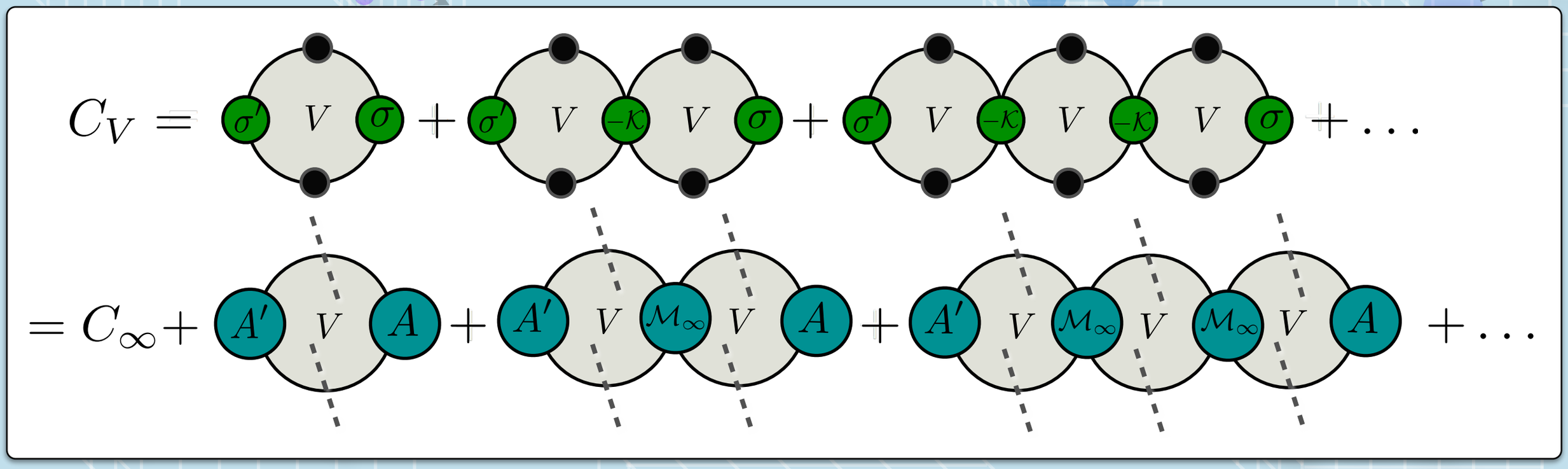


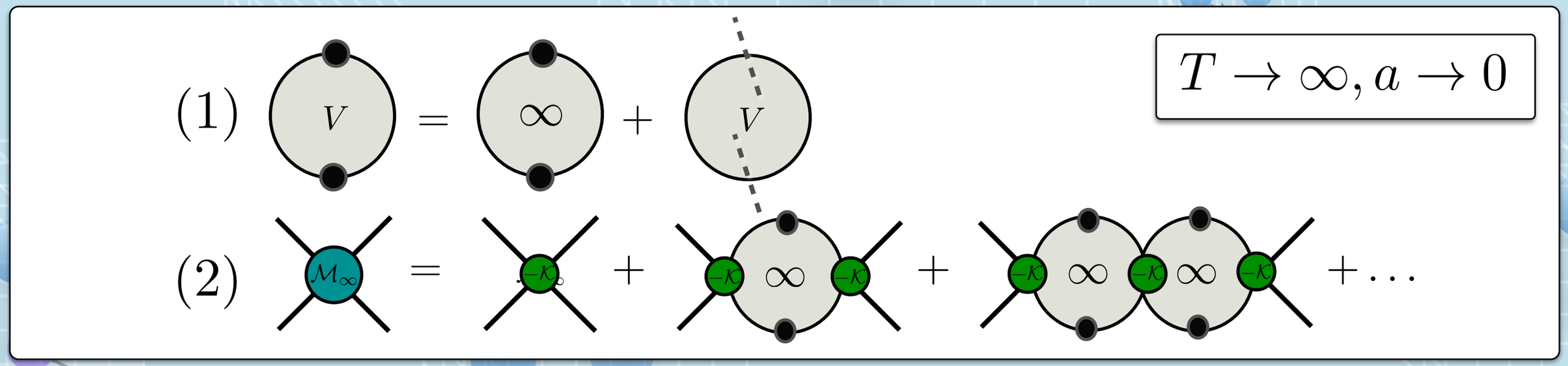




MAPPING DERIVED FROM A QFT APPROACH:

Kim, Sachrajda and Sharpe, Nucl.Phys.B727(2005)218-243.





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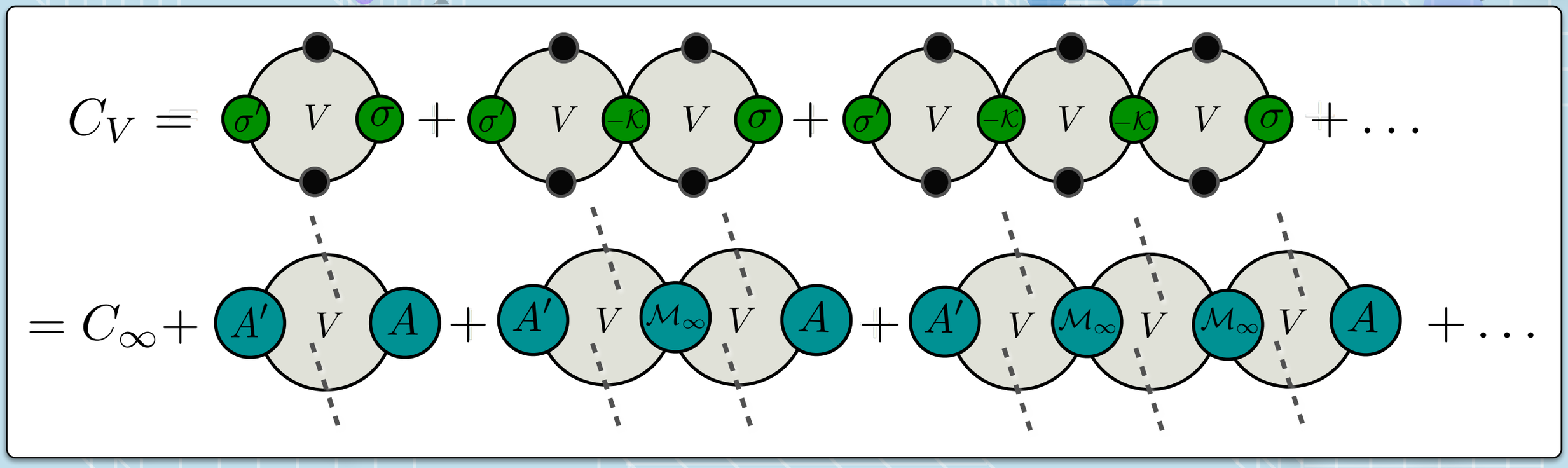
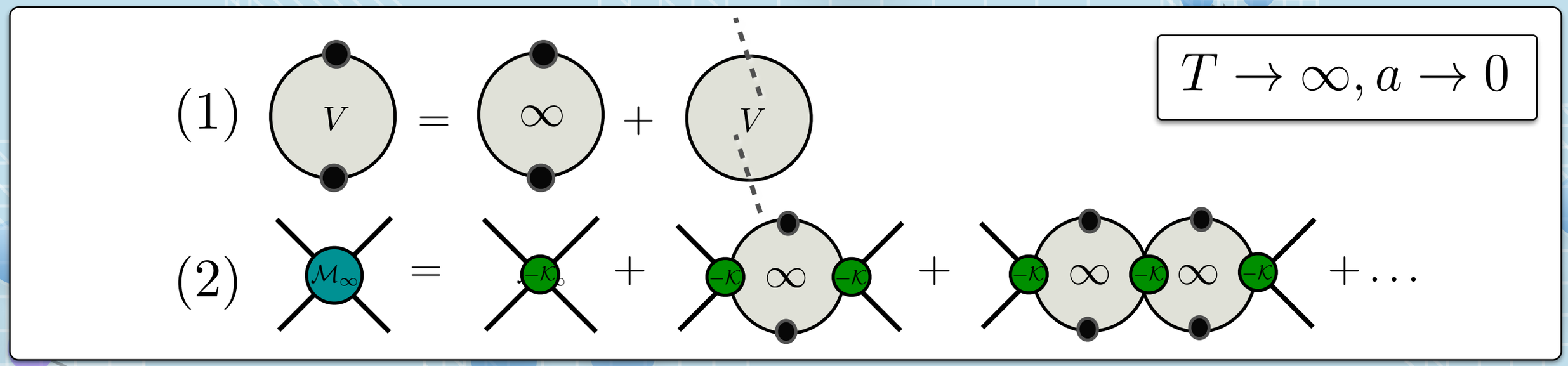


$$\det\left[\delta\mathcal{G}^V(E^*) + \mathcal{M}^{-1}(E^*)\right] = 0$$

FINITE-VOLUME FUNCTION SCATTERING AMPLITUDE



FINITE-VOLUME FUNCTION A LITTLE MORE CLOSELY…

$$(\delta\mathcal{G}^V)_{l_1,m_1;l_2,m_2} = i\frac{q^* n}{8\pi E^*}\left(\delta_{l_1,l_2}\delta_{m_1,m_2} + i\frac{4\pi}{q^*}\sum_{l,m}\frac{\sqrt{4\pi}}{q^{*l}}c^{\mathbf{P}}_{lm}(q^{*2})\int d\Omega^* Y^*_{l_1m_1}Y^*_{lm}Y_{l_2m_2}\right)$$

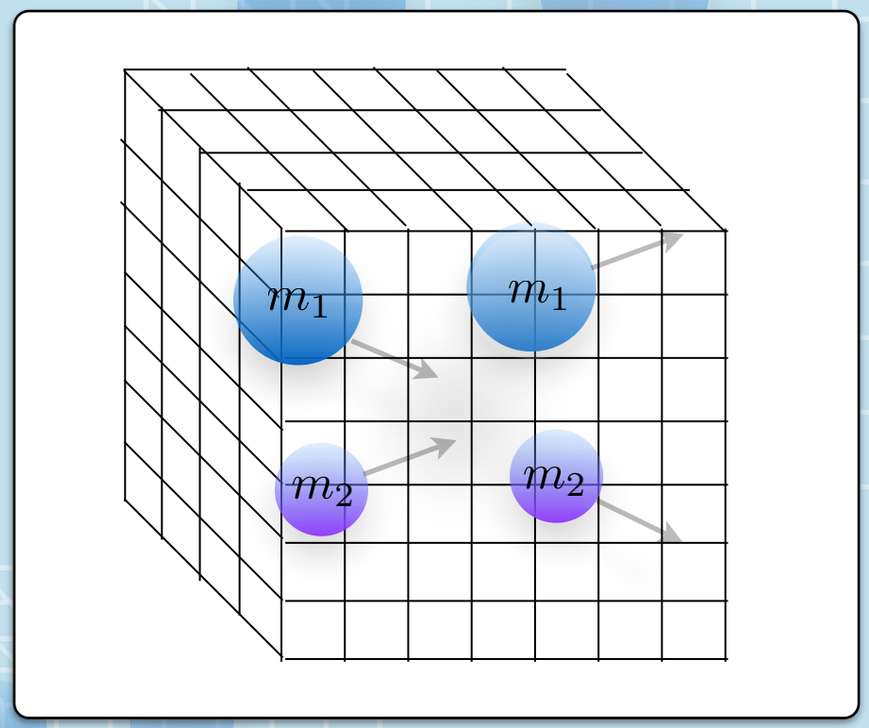
$$c^{\mathbf{P}}_{lm}(x) = \frac{1}{\gamma}\left[\frac{1}{L^3}\sum_{\mathbf{k}} - \mathcal{P}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\right]\frac{\sqrt{4\pi}Y_{lm}(\hat{k}^*)\ k^{*l}}{k^{*2} - x}$$

$$\mathbf{k}^* = \gamma^{-1}\left[\mathbf{k}_{\parallel} - \frac{1}{2}(1 + \frac{m_1^2 - m_2^2}{E^{*2}})\mathbf{P}\right] + \mathbf{k}_{\perp}$$

ZD and Savage, Phys. Rev.D84,114502(2011).

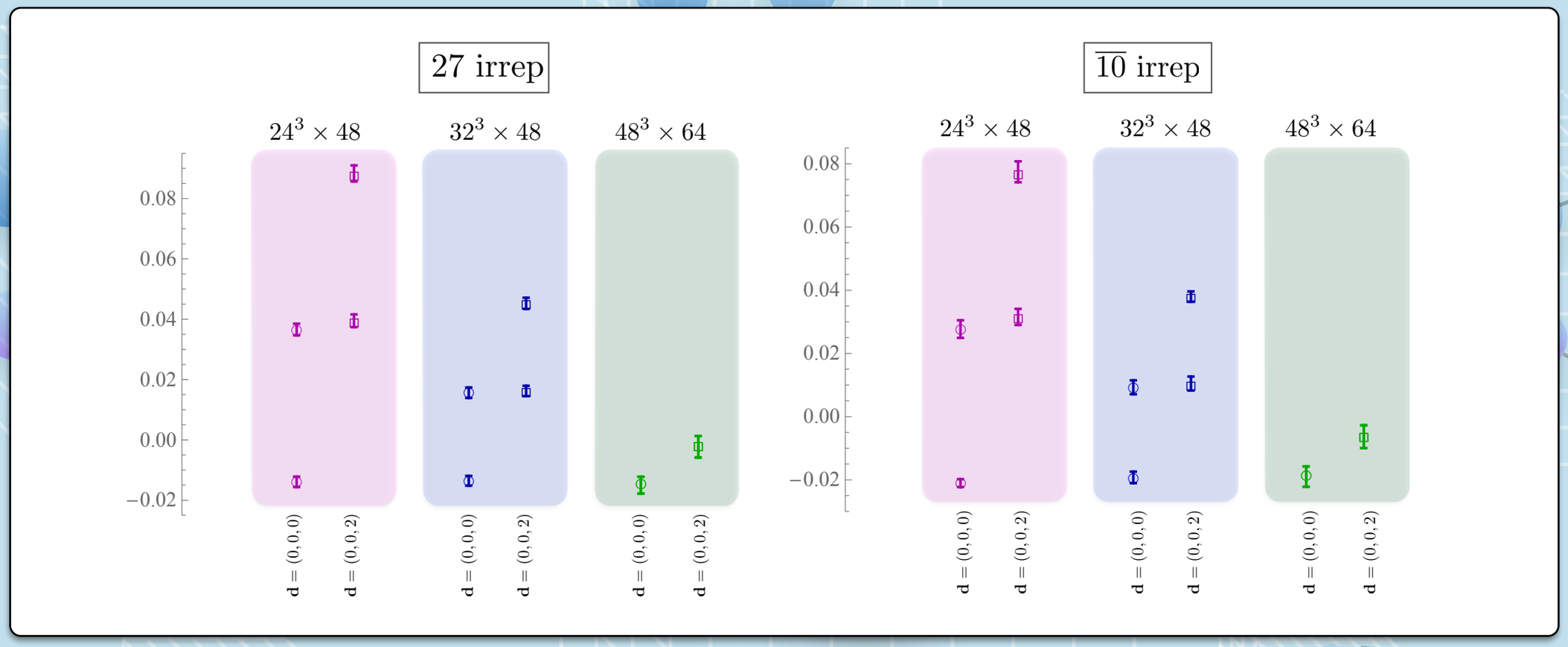
Fu, Phys.Rev.D85,014506 (2012)
Leskovec and Prelovsek, Phys.Rev.D85,114507(2012)

$$\det\left[\delta\mathcal{G}^V(E^*) + \mathcal{M}^{-1}(E^*)\right] = 0$$



LOWEST-LYING SPECTRA

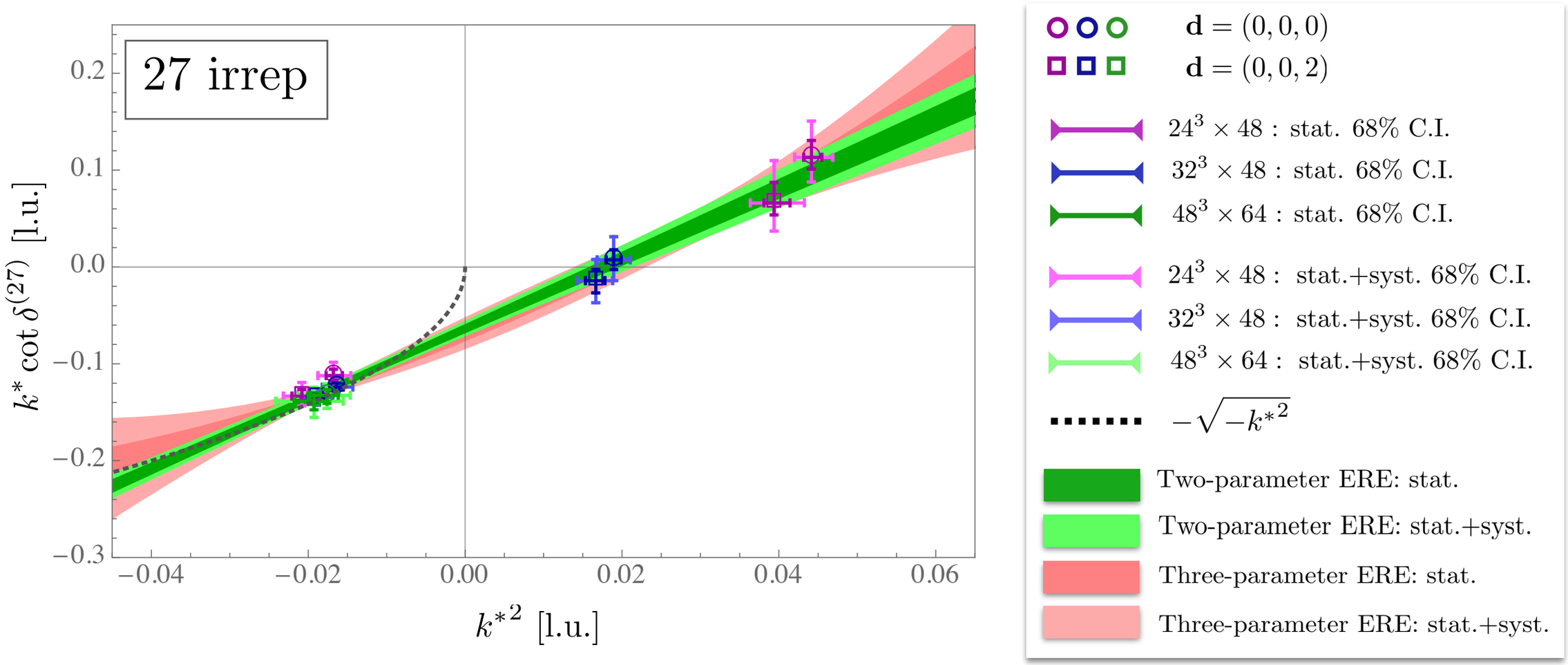
Wagman et al.(NPLQCD), Phys.Rev.D 96,114510(2017).



LOW-ENERGY SCATTERING PHASE SHIFT IN S-WAVE

$N_f = 3,\ m_\pi = 0.806\ \text{GeV},\ a = 0.145(2)\ \text{fm}$

Wagman et al.(NPLQCD), Phys.Rev.D 96,114510(2017).



$$B = 20.6^{(+1.8)(+2.8)}_{(-2.4)(-1.6)}\ \text{MeV}$$

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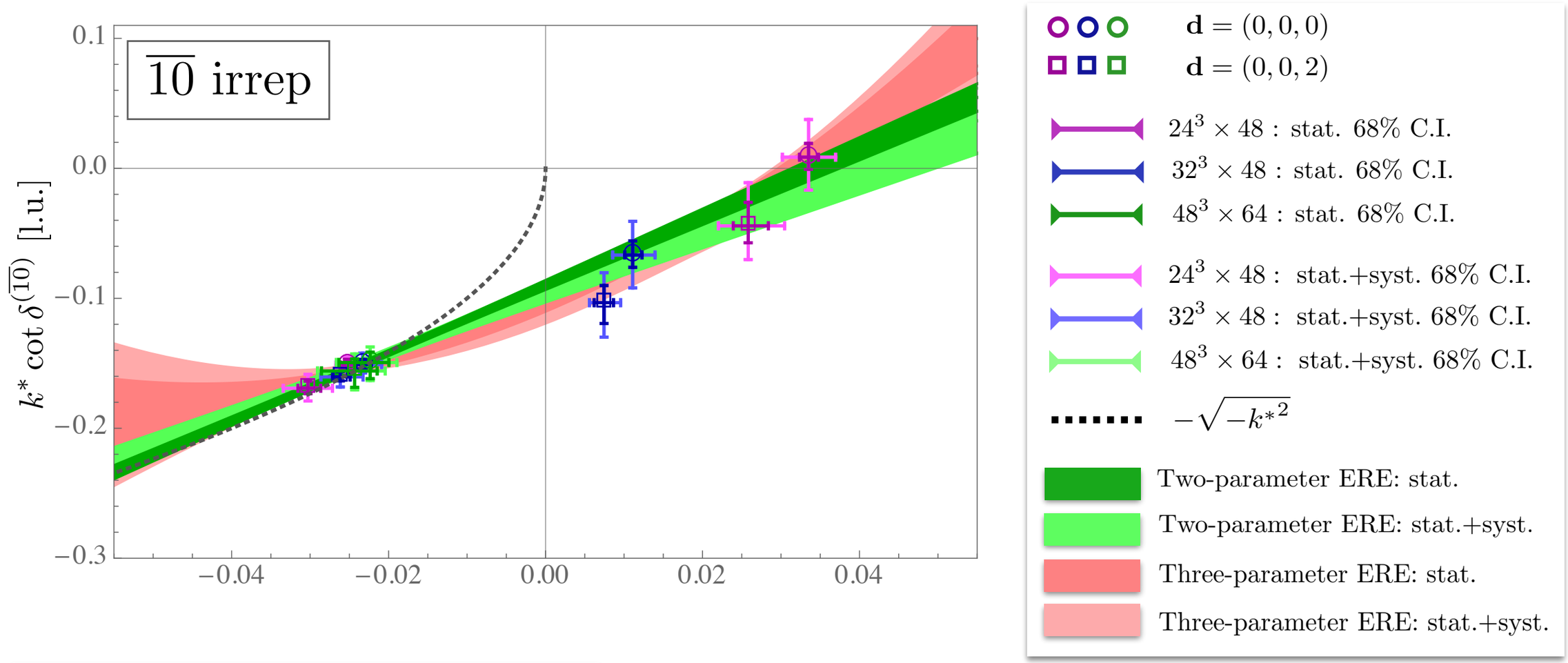

$$B = 27.9^{(+3.1)(+2.2)}_{(-2.3)(-1.4)}\ \text{MeV}$$

EVIDENCE FOR SU(6) SPIN-FLAVOR SYMMETRY PREDICTED AT LARGE Nc

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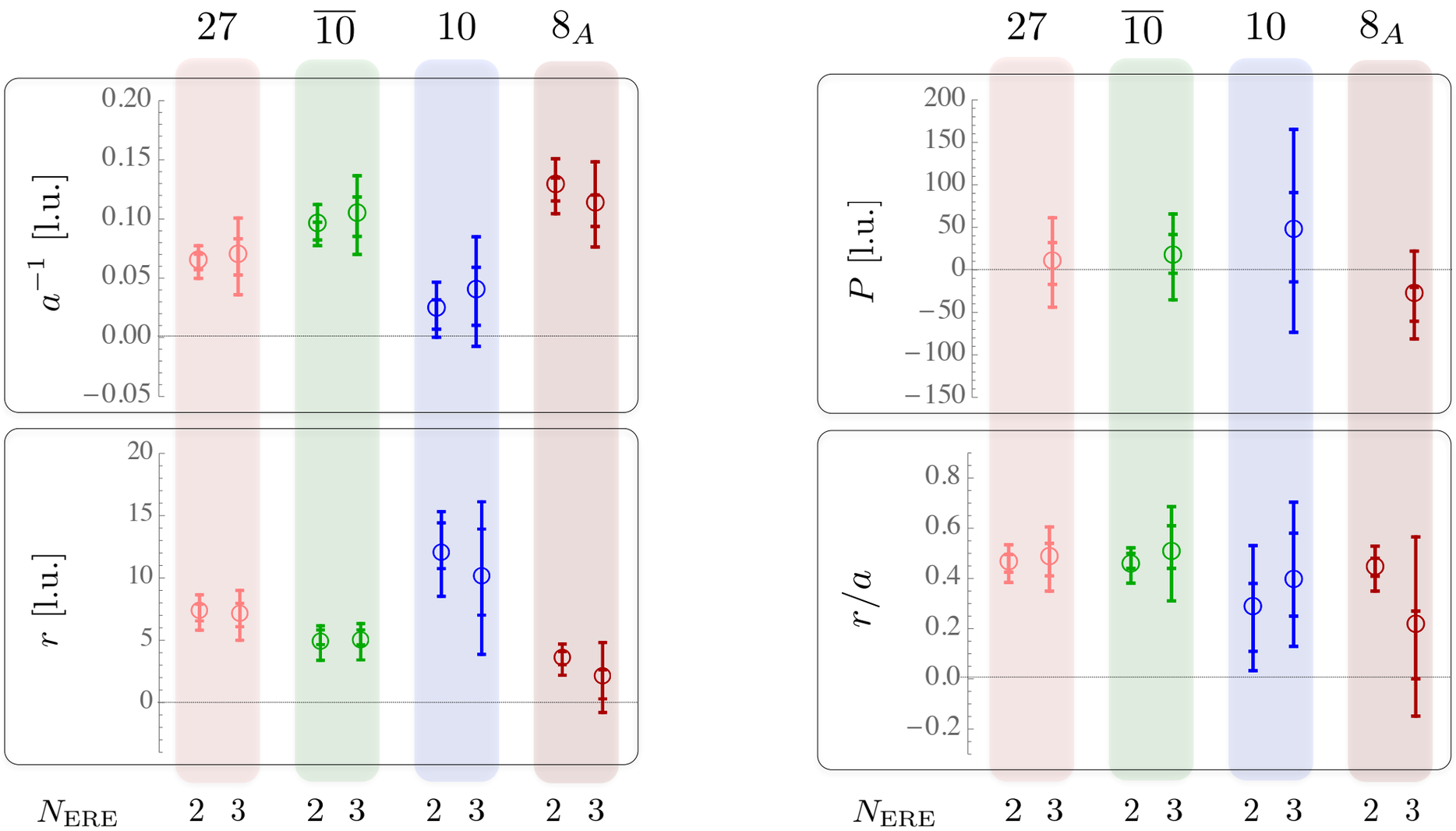


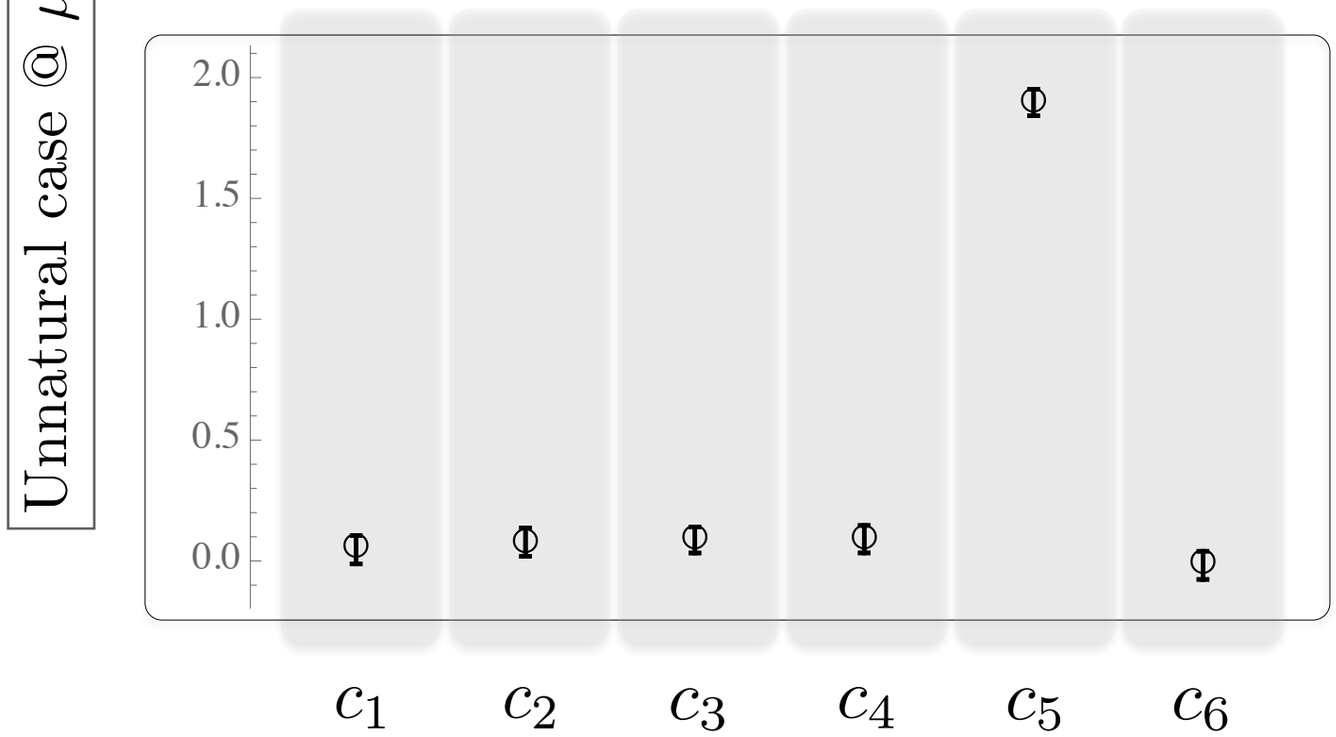
LOW-ENERGY EFFECTIVE INTERACTIONS FROM QCD

$N_f = 3,\ m_\pi = 0.806\ \text{GeV},\ a = 0.145(2)\ \text{fm}$

Wagman et al.(NPLQCD), Phys.Rev.D 96,114510(2017).

$$\mathcal{L}^{(0)}_{BB} = -c_1 \mathrm{Tr}(B_i^\dagger B_i B_j^\dagger B_j) - c_2 \mathrm{Tr}(B_i^\dagger B_j B_j^\dagger B_i) - c_3 \mathrm{Tr}(B_i^\dagger B_j^\dagger B_i B_j)$$
$$- c_4 \mathrm{Tr}(B_i^\dagger B_j^\dagger B_j B_i) - c_5 \mathrm{Tr}(B_i^\dagger B_i)\mathrm{Tr}(B_j^\dagger B_j) - c_6 \mathrm{Tr}(B_i^\dagger B_j)\mathrm{Tr}(B_j^\dagger B_i).$$

Savage and Wise, Phys.Rev. D53 (1996) 349.

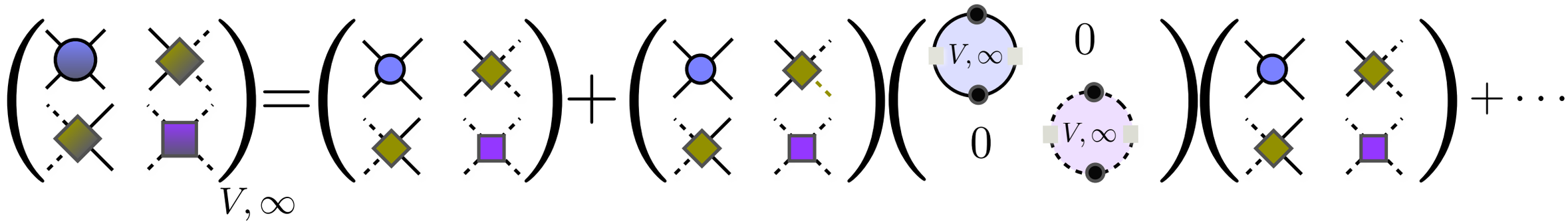


$SU(3) \longrightarrow SU(6) \longrightarrow SU(16)$

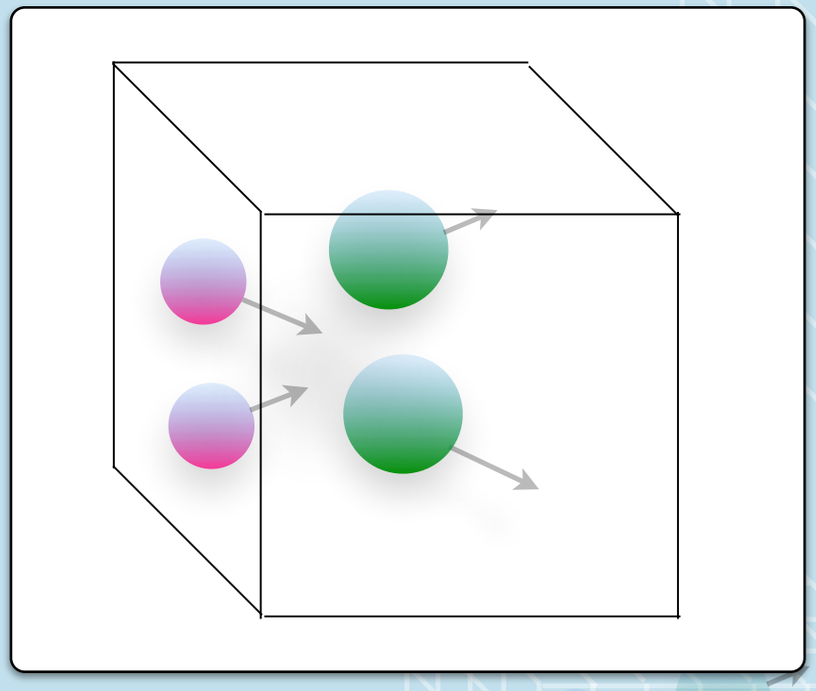
Kaplan and Savage, Phys.Lett.B365:244-251,1996.

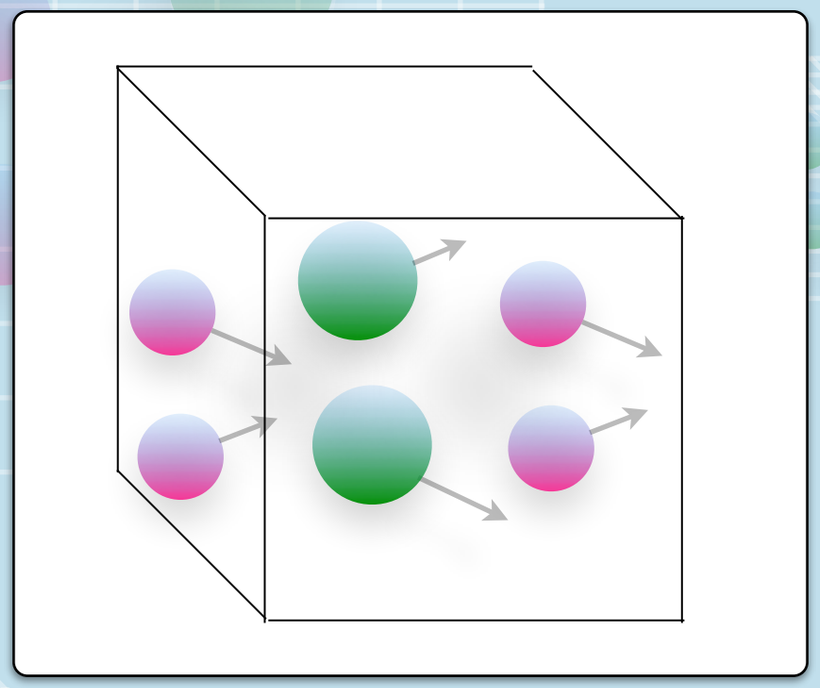
TWO-BODY INELASTIC SCATTERING

COUPLED-CHANNEL GENERALIZATION IS STRAIGHTFORWARD:



Briceno and ZD, Phys. Rev.D88,094507(2013).





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$$(\delta\mathcal{G}_i^V)_{l_1,m_1;l_2,m_2} = i\frac{q_i^* n_i}{8\pi E^*}\left(\delta_{l_1,l_2}\delta_{m_1,m_2} + i\frac{4\pi}{q_i^*}\sum_{l,m}\frac{\sqrt{4\pi}}{q_i^{*l}}c_{lm}^{\mathbf{P}}(q_i^{*2})\int d\Omega^* Y_{l_1m_1}^* Y_{lm}^* Y_{l_2m_2}\right)$$

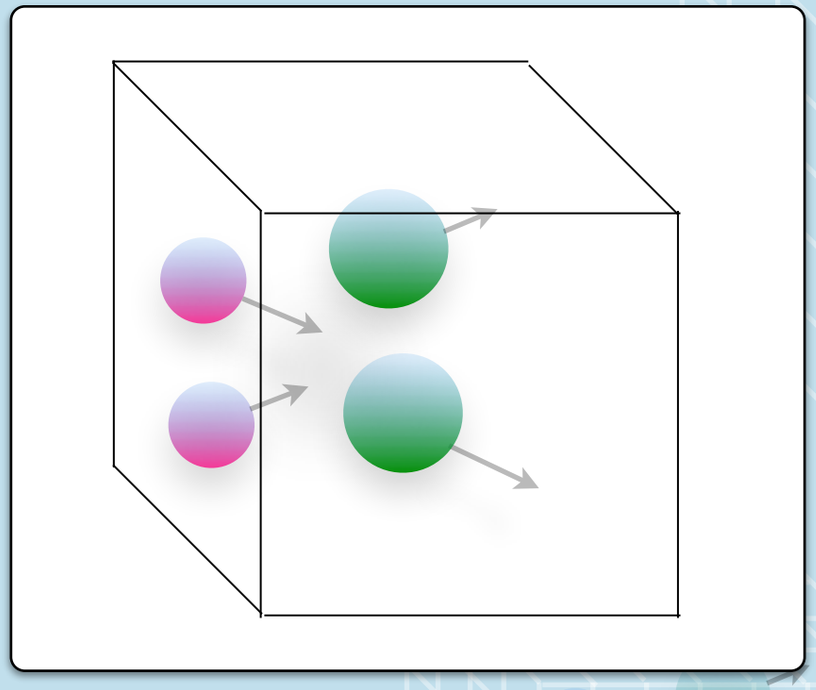
$$c_{lm}^{\mathbf{P}}(x) = \frac{1}{\gamma}\left[\frac{1}{L^3}\sum_{\mathbf{k}} - \mathcal{P}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\right]\frac{\sqrt{4\pi}Y_{lm}(\hat{k}^*)\ k^{*l}}{k^{*2} - x}$$

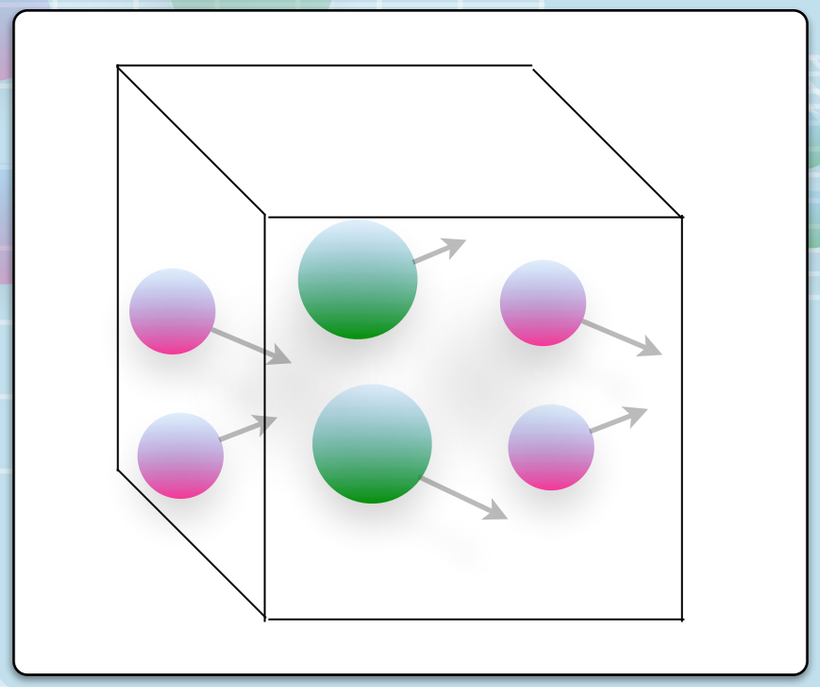
$$\mathbf{k}^* = \gamma^{-1}\left[\mathbf{k}_{\parallel} - \frac{1}{2}(1 + \frac{m_1^2 - m_2^2}{E^{*2}})\mathbf{P}\right] + \mathbf{k}_{\perp}$$

Briceno and ZD, Phys. Rev.D88,094507(2013).

He, Feng and Liu, JHEP 0507,011 (2005)
Hansen and Sharpe, Phys.Rev.D.86.016007(2012)

$$\mathrm{Det}\left[\delta\mathcal{G}^V(E^*) + \mathcal{M}^{-1}(E^*)\right] = 0$$





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LELLOUCH-LEUSCHER FORMULA FOR *pp* FUSION

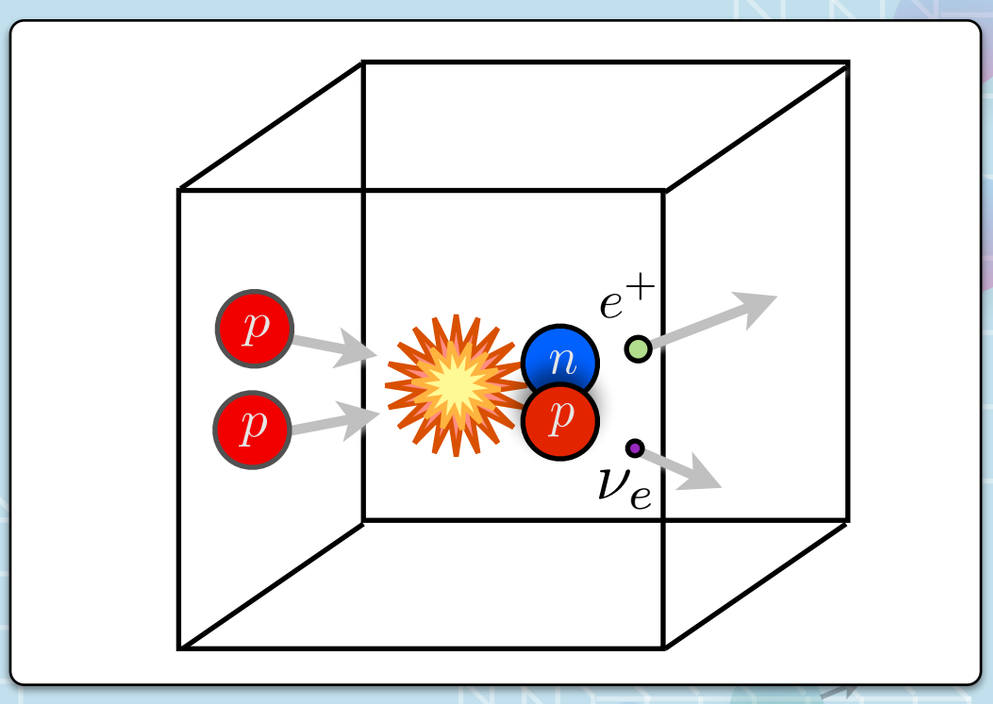
$$\left(\frac{MV}{2}\right)^2 \csc^2\delta^{(^1S_0)} \csc^2\delta^{(^3S_1)} \left(\phi' + \delta^{(^1S_0)\prime}\right)\left(\phi' + \delta^{(^3S_1)\prime}\right) |\mathcal{M}_W^V|^2 = \left(\frac{4\pi}{M}\widetilde{L}_{1,A} + \frac{4\pi}{M} g_A G_1^V\right)^2$$

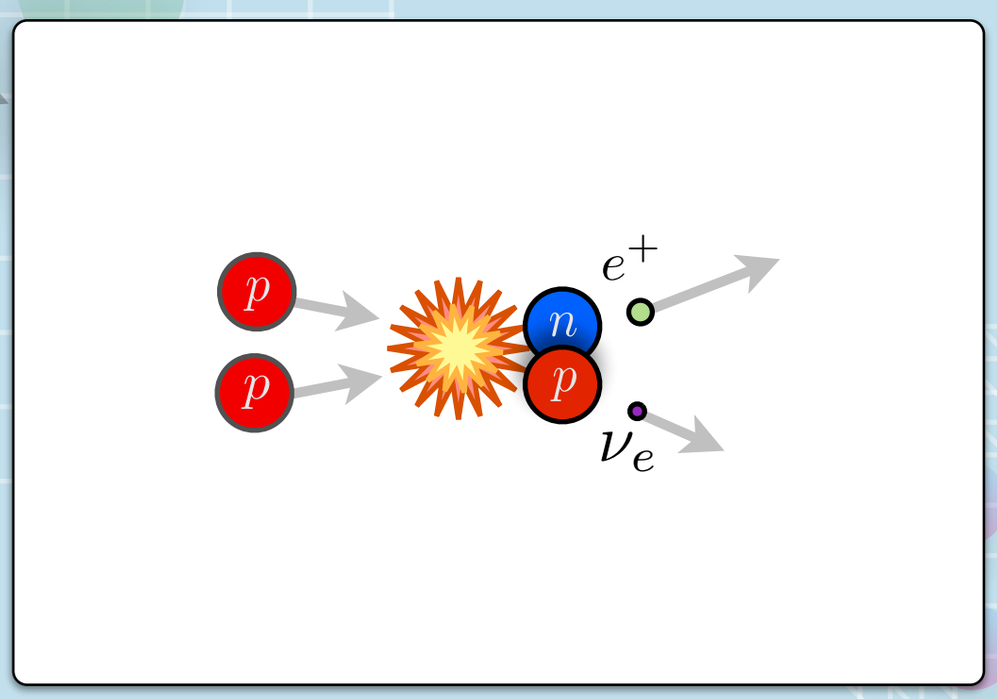
Briceno and ZD, Phys. Rev.D88,094507(2013).

BACKGROUND FIELD APPROACH:
Detmold and Savage, Nucl.Phys.A743(2004)170.

GENERAL 2 TO 2 PROCESS:
Briceno and Hansen, Phys.Rev.D94,013008(2016).

$$\mathrm{Det}\left[\delta\mathcal{G}^V(E^*) + \mathcal{M}^{-1}(E^*)\right] = 0$$

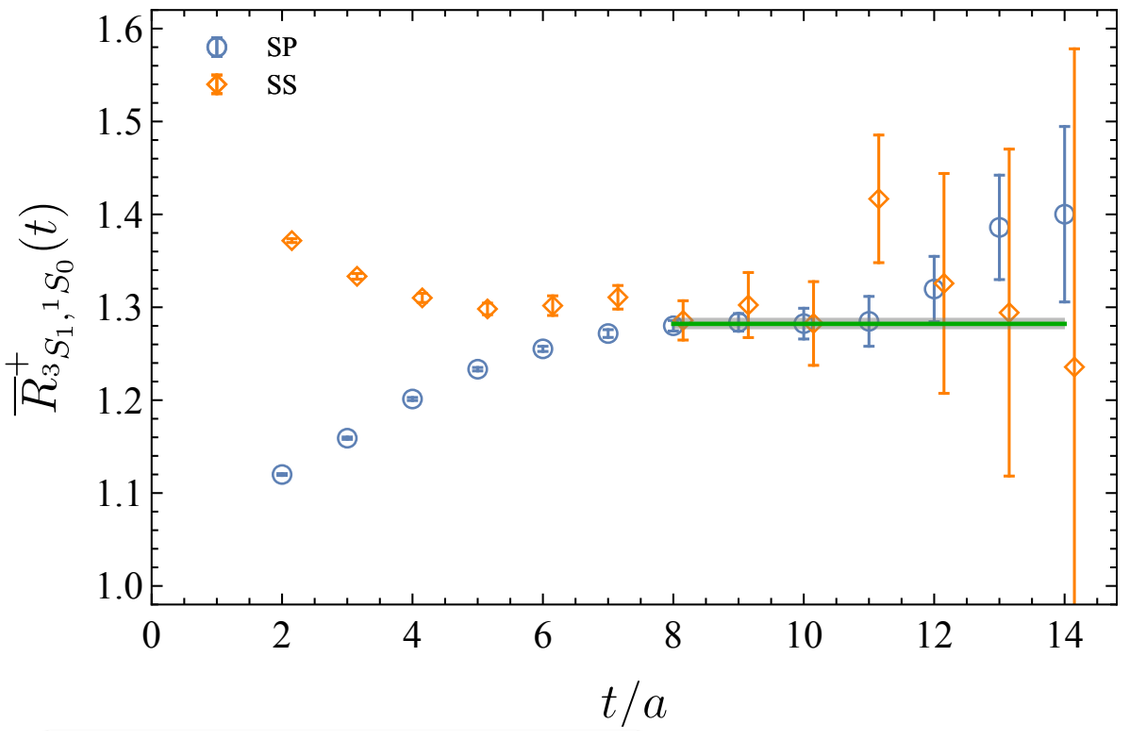


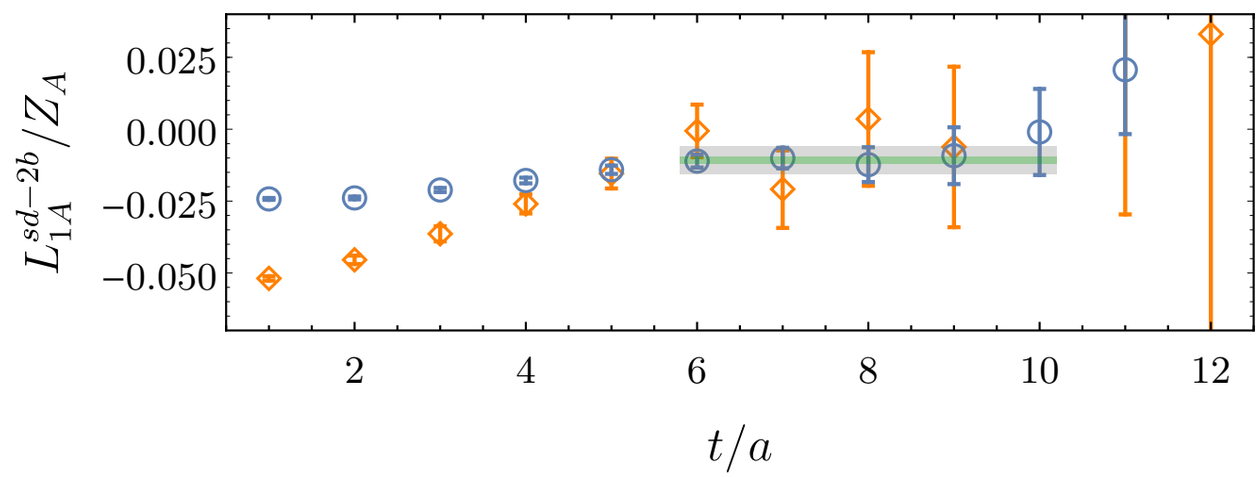


pp FUSION FROM LQCD AND EFT

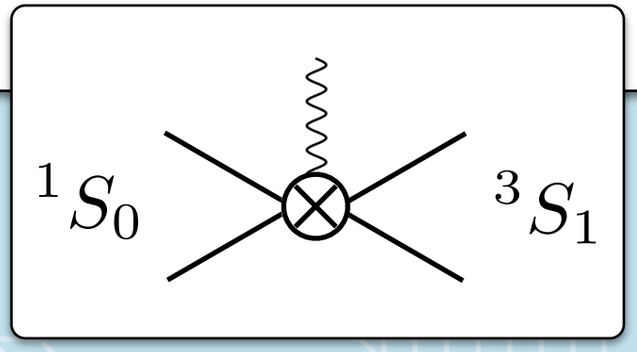
$N_f = 3,\ m_\pi = 0.806\ \text{GeV},\ a = 0.145(2)\ \text{fm}$

Savage et al.(NPLQCD), Phys. Rev.Lett.119,062002(2017).





$$L_{1,A}^{sd-2b} \equiv \frac{|\langle pp|A_3^+|d\rangle| - g_A}{Z_A} = -0.011(01)(15)$$



$$L_{1,A} = 3.9(0.1)(1.0)(0.3)(0.9)\ \text{fm}^3 \quad @\ \mu = m_\pi^{\text{phys.}} = 140\ \text{MeV}$$

TWO-NUCLEON OBSERVABLES

TWO NUCLEON SYSTEMS ARE COUPLED CHANNELS

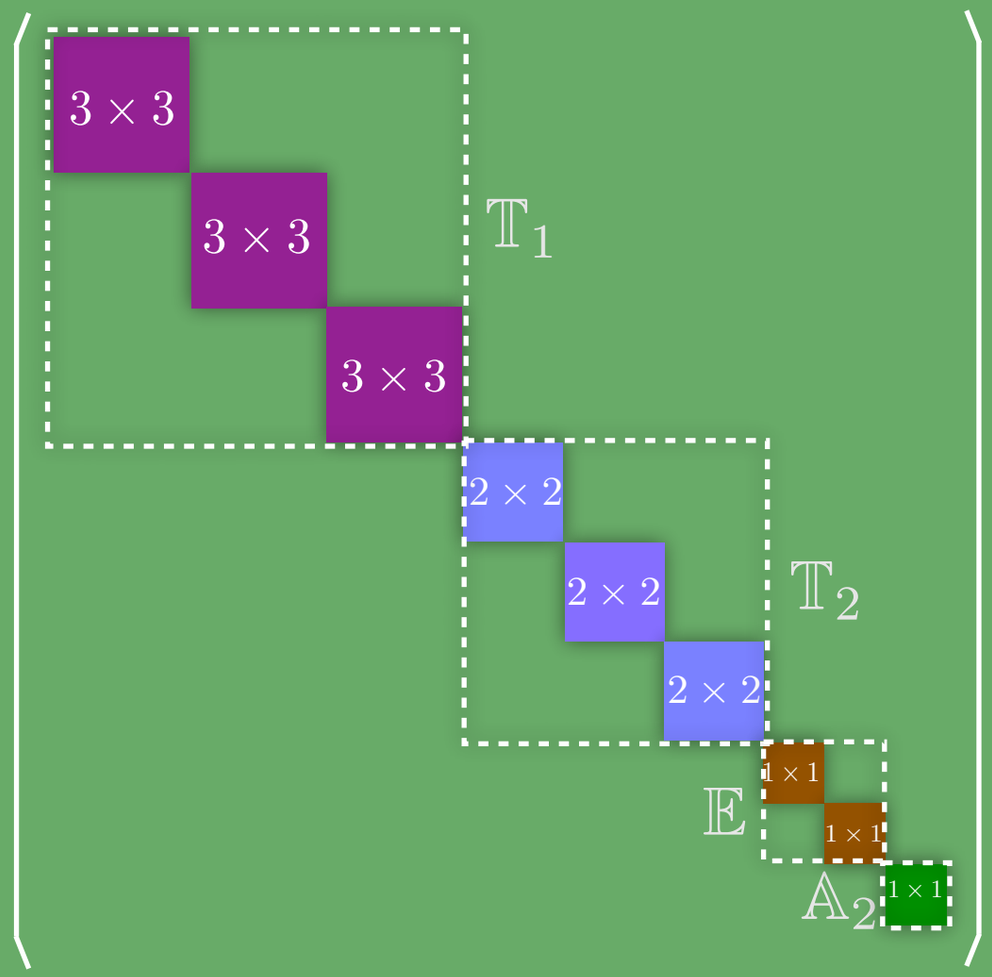
Briceno, ZD and Luu, Phys.Rev.D88, 034502(2013).

$$[\delta\mathcal{G}^V]_{JM_J,IM_I,LS;J'M'_J,I'M'_I,L'S'} = \frac{iMk^*}{4\pi}\delta_{II'}\delta_{M_IM'_I}\delta_{SS'}\left[\delta_{JJ'}\delta_{M_JM'_J}\delta_{LL'} + i\sum_{l,m}\frac{(4\pi)^{3/2}}{k^{*l+1}}c^{\mathbf{P}}_{lm}(k^{*2})\right.$$

$$\left.\times\sum_{M_L,M'_L,M_S}\langle JM_J|LM_L,SM_S\rangle\langle L'M'_L,SM_S|J'M'_J\rangle\int d\Omega\, Y^*_{L,M_L}Y^*_{l,m}Y_{L',M'_L}\right]$$

$$L \leq 3, \mathcal{P} = +$$

$$\mathcal{M} = \begin{pmatrix} \mathcal{M}_{1,S} & \mathcal{M}_{1,SD} & 0 & 0 \\ \mathcal{M}_{1,SD} & \mathcal{M}_{1,D} & 0 & 0 \\ 0 & 0 & \mathcal{M}_{2,D} & 0 \\ 0 & 0 & 0 & \mathcal{M}_{3,D} \end{pmatrix}, \text{ QC} \rightarrow$$

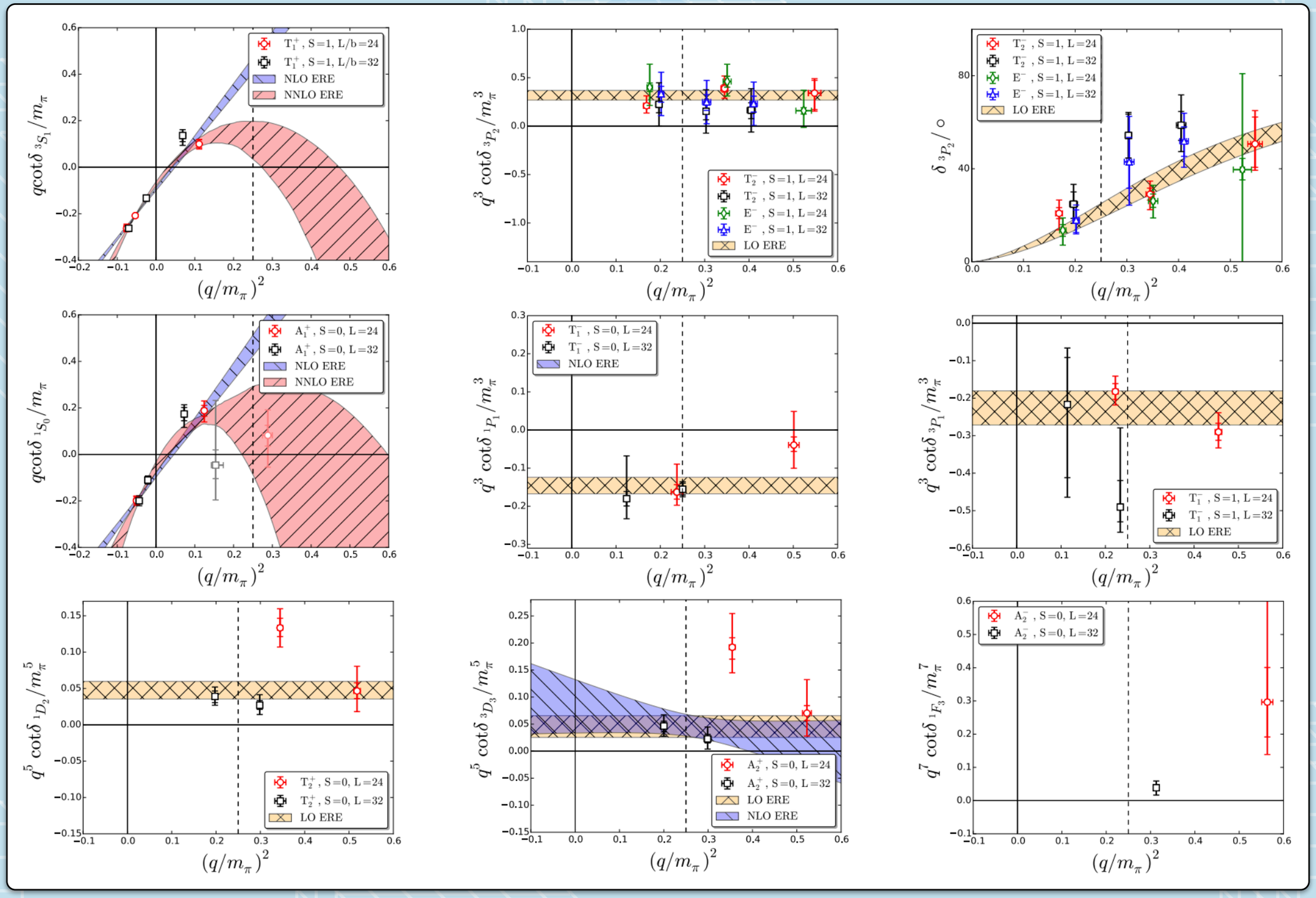


IDENTIFIED 49 QCS FOR 16 SCATTERING PARAMETERS

THE APPLICATION OF THE NN QCs

$N_f = 3,\ m_\pi = 0.806\ \text{GeV},\ a = 0.145(2)\ \text{fm}$

Berkowitz et al, Phys.Let.B,765,285-292(2017).



BEATING SEPARATION OF SCALES WITH NEW IDEAS

DEUTERON IS A SHALLOW BOUND STATE OF PROTON AND NEUTRON.

$$B_d = 2.224644(24)\ [\mathtt{MeV}]$$

$$\eta = -\tan\epsilon_1|_{B_d} \approx 0.02713(6)$$

IT'S WAVEFUNCTION HAS A TINY D-WAVE ADMIXTURE.

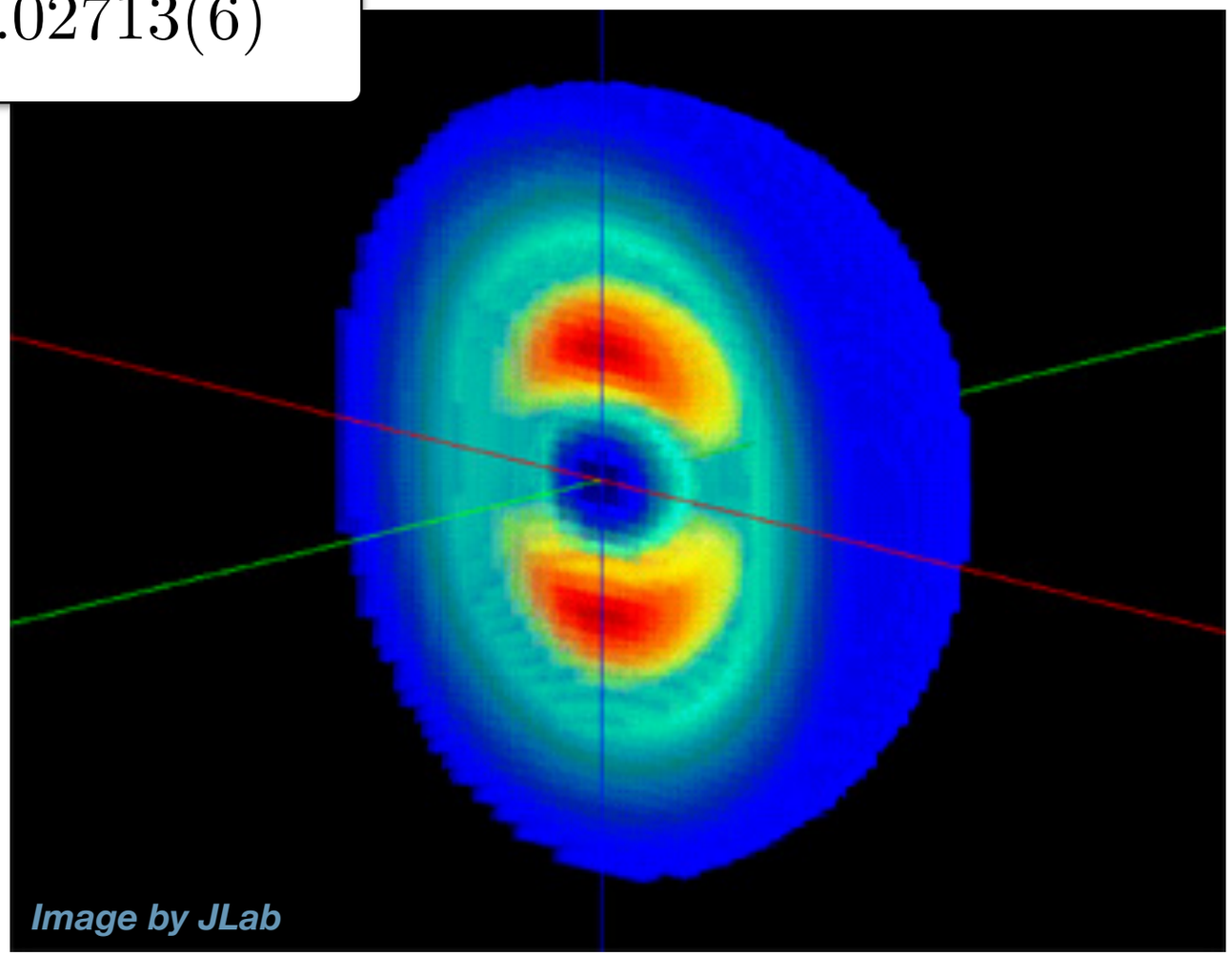
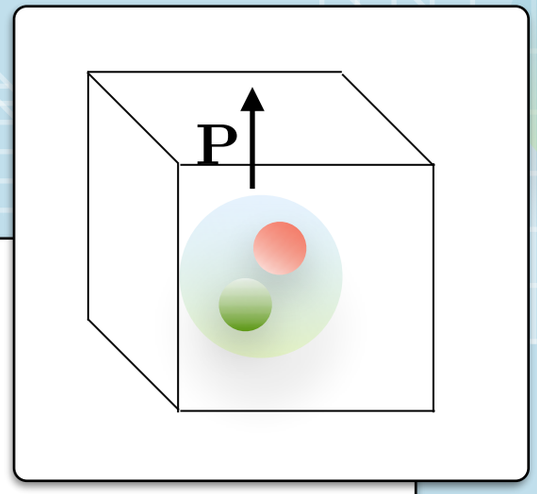


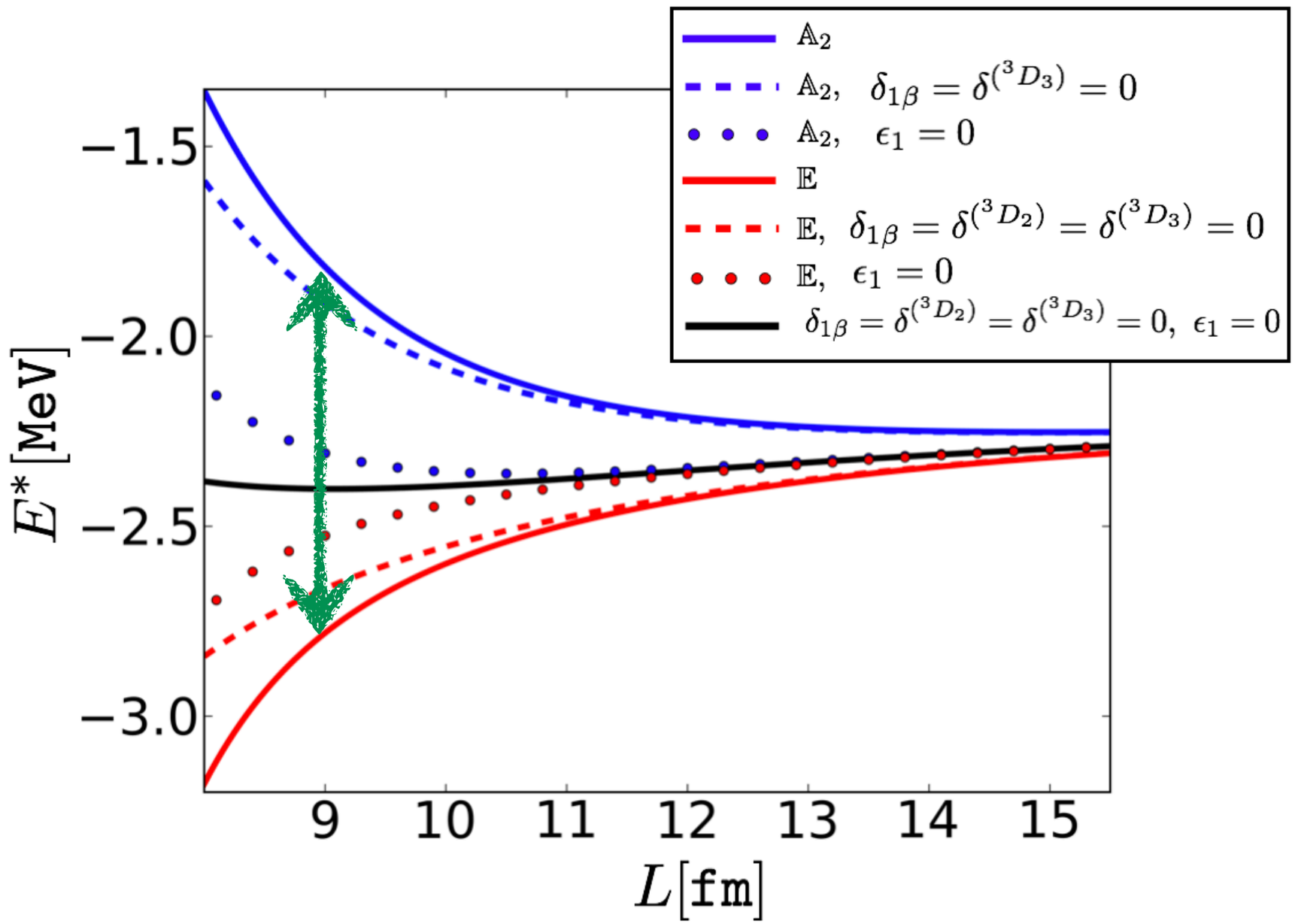
Image by JLab

DISCOVERING SENSITIVITY TO S-D MIXING IN A FINITE VOLUME IN BOSSTED SYSTEMS



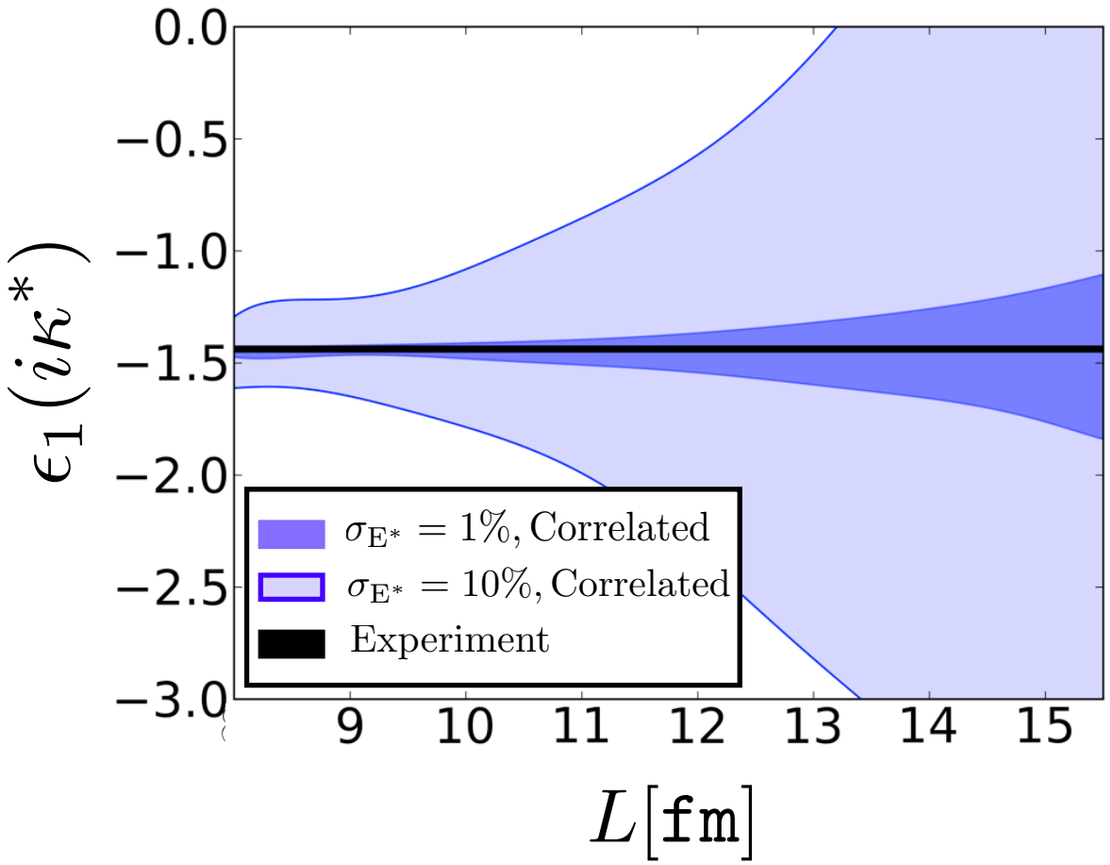
$$\mathbf{P} = \frac{2\pi}{L}(0,0,1)$$

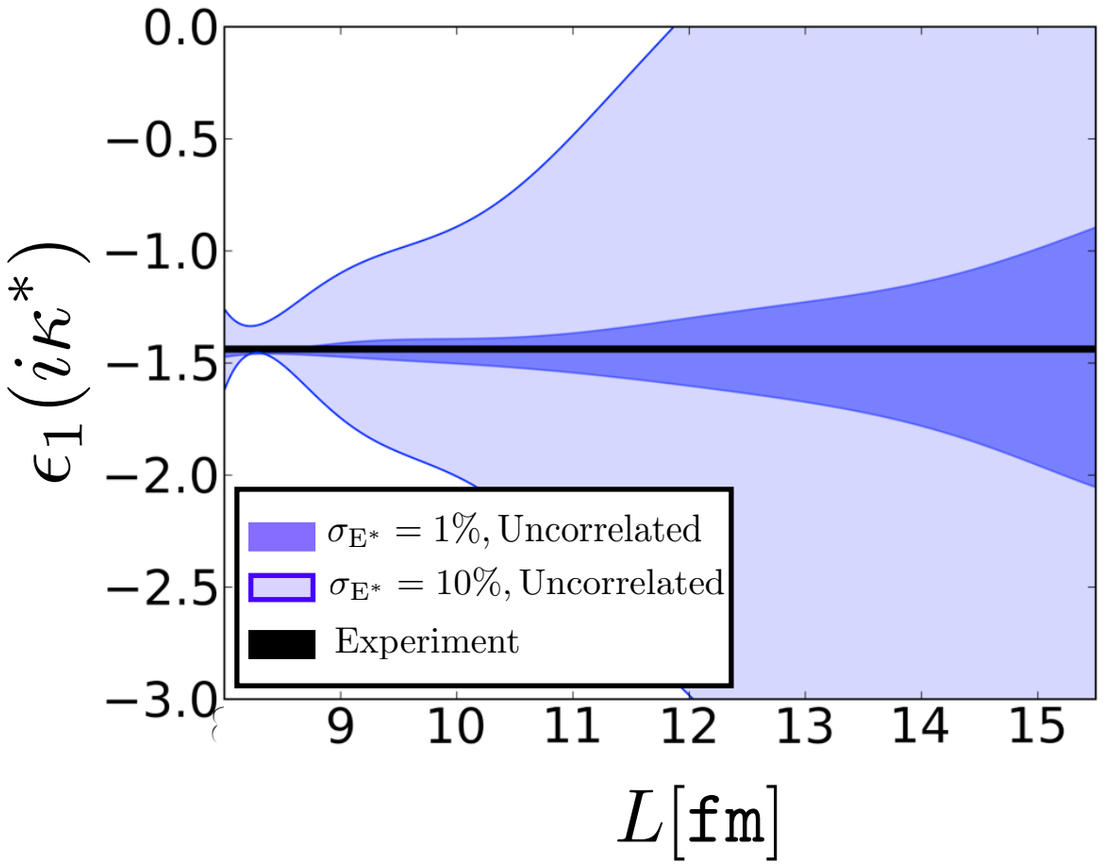
Briceno, ZD, Luu and Savage, Phys.Rev.D88, 114507(2013).



REQUIRED PRECISION ON ENERGIES AT THE PHYSICAL POINT FOR AN EXTRACTION OF THE MIXING ANGLE

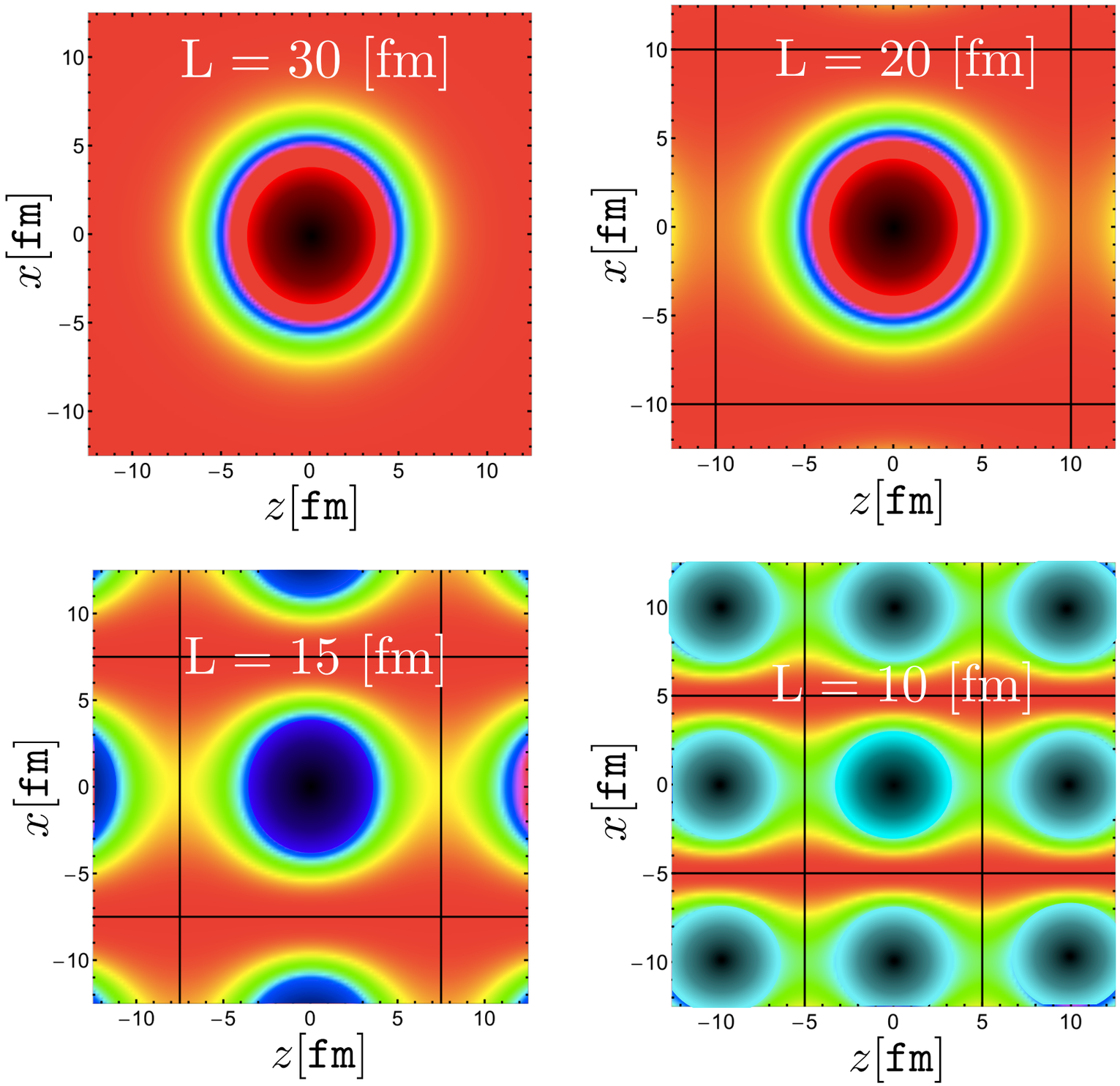
Briceno, ZD, Luu and Savage, Phys.Rev.D88, 114507(2013).





DEUTERON ASIMPTOTIC WAVEFUNCTION IN A FINITE VOLUME (E IRREP)

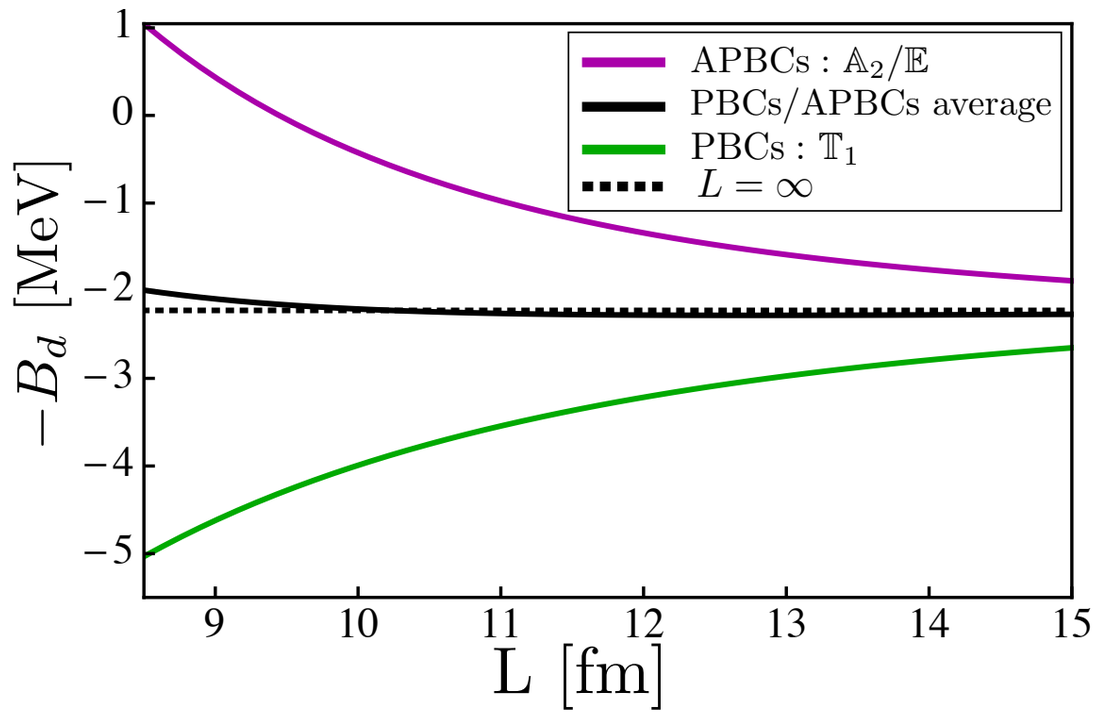
$$\mathbf{P} = \frac{2\pi}{L}(0,0,1)$$

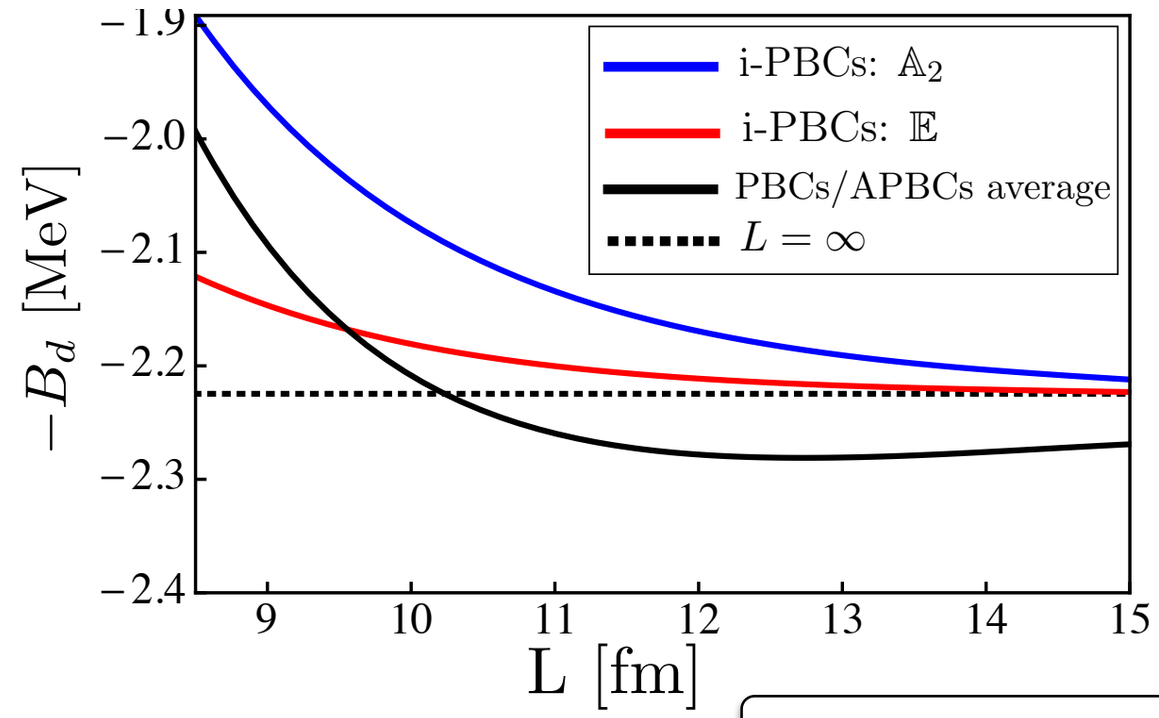


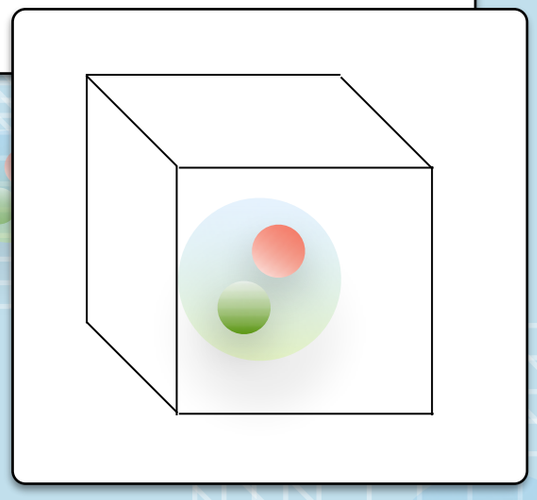
Briceno, ZD, Luu and Savage, Phys.Rev.D88, 114507(2013).

SMART CHOICES OF BOUNDARY CONDITIONS SUPPRESS VOLUME EFFECTS

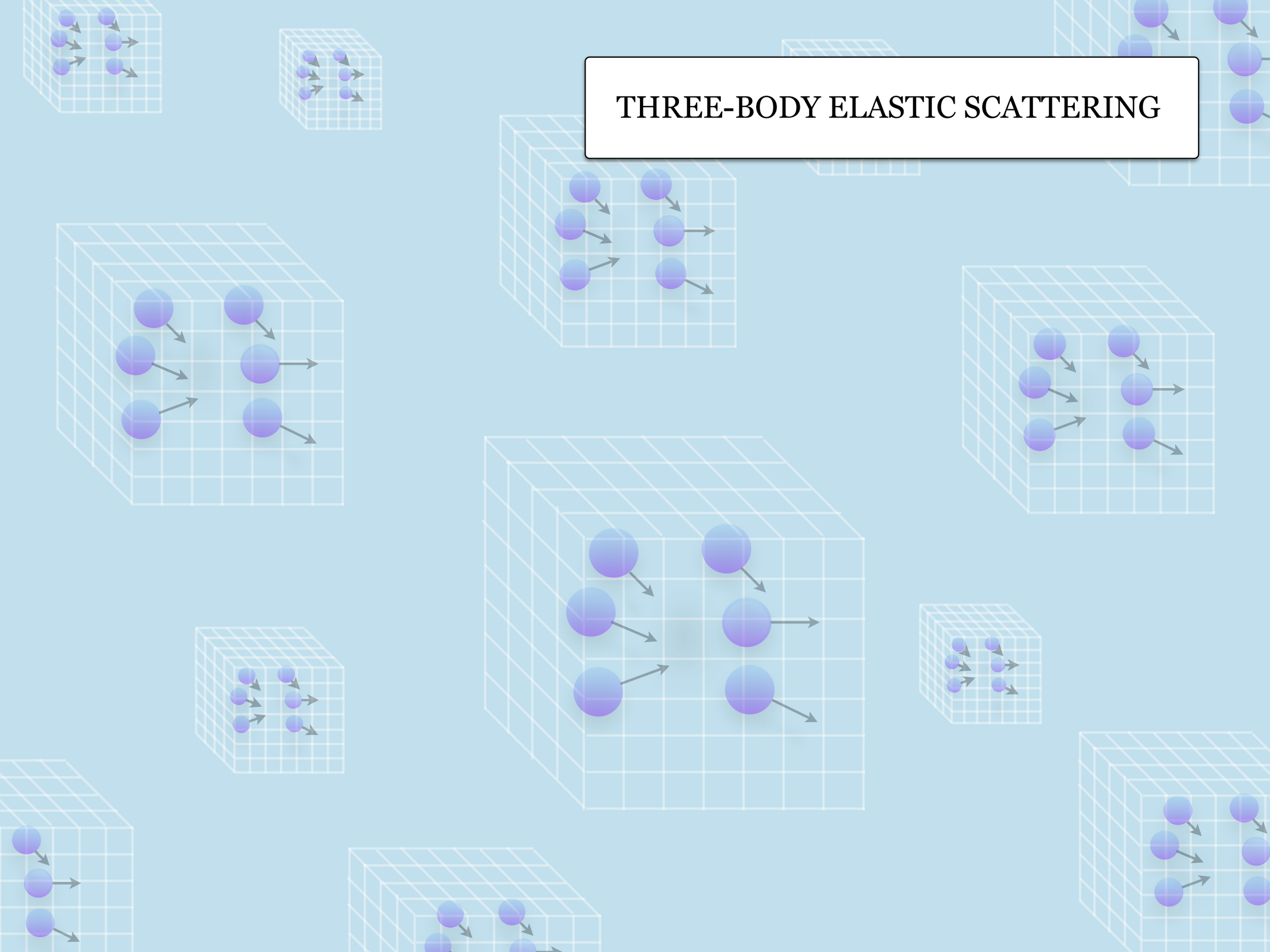
Briceno, ZD, Luu and Savage, Phys.Rev.D89,074509 (2014).





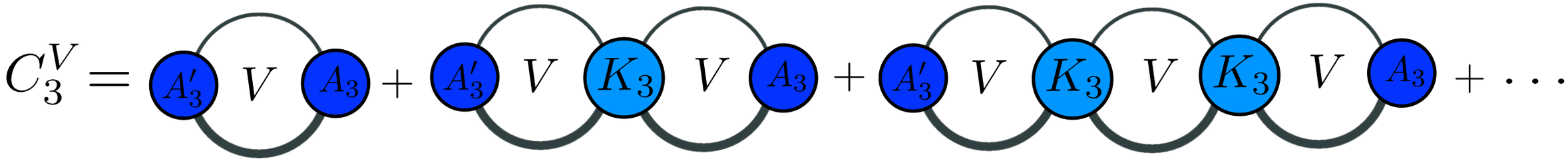


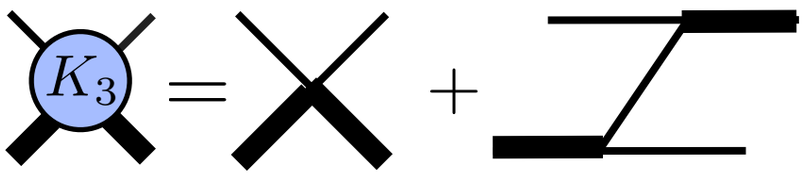
THREE-BODY ELASTIC SCATTERING



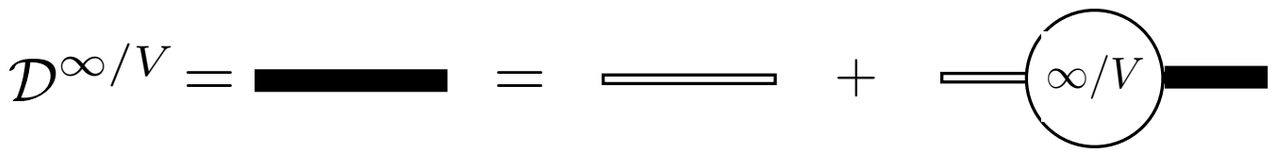
THREE-HADRON QUANTIZATION CONDITION

Briceno and ZD, Phys.Rev. D87,094507(2013).





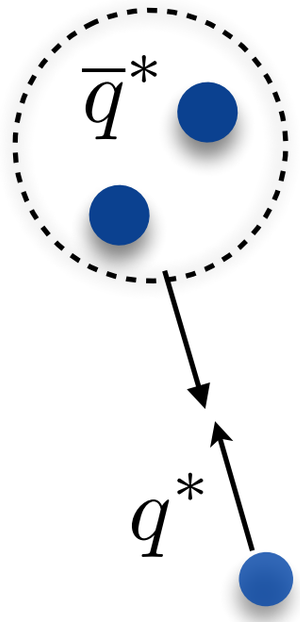
THREE-BODY KERNEL AT LO



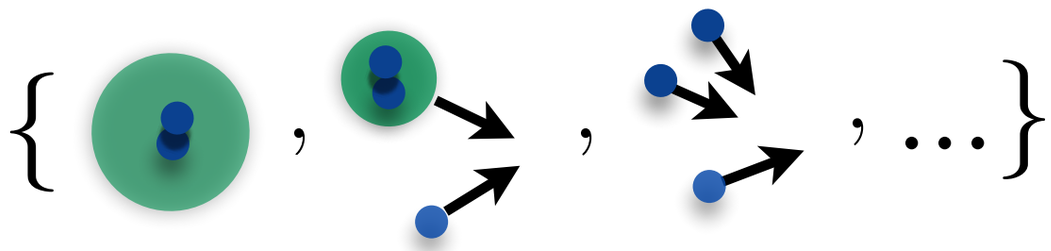
FULL DIMER PROPAGATOR

LUESCHER TWO-BODY QC FOR THE DIMER SYSTEM

$$\boxed{\bar{q}^*_\kappa \cot \delta_d = 4\pi \; c_{00}^{q^*}(\bar{q}^{*2}_\kappa)} \qquad (I)$$



THREE-PARTICLE STATES



THREE-HADRON QUANTIZATION CONDITION

Briceno and ZD, Phys.Rev. D87,094507(2013).

$$\mathrm{Det}(1+\tilde{\mathcal{M}}_V^{\infty}\delta\tilde{\mathcal{G}}^V) \equiv \det_{\mathrm{oc}}[\det_{\mathrm{pw}}(1+\tilde{\mathcal{M}}_V^{\infty}\delta\tilde{\mathcal{G}}^V)] = 0 \quad (II)$$

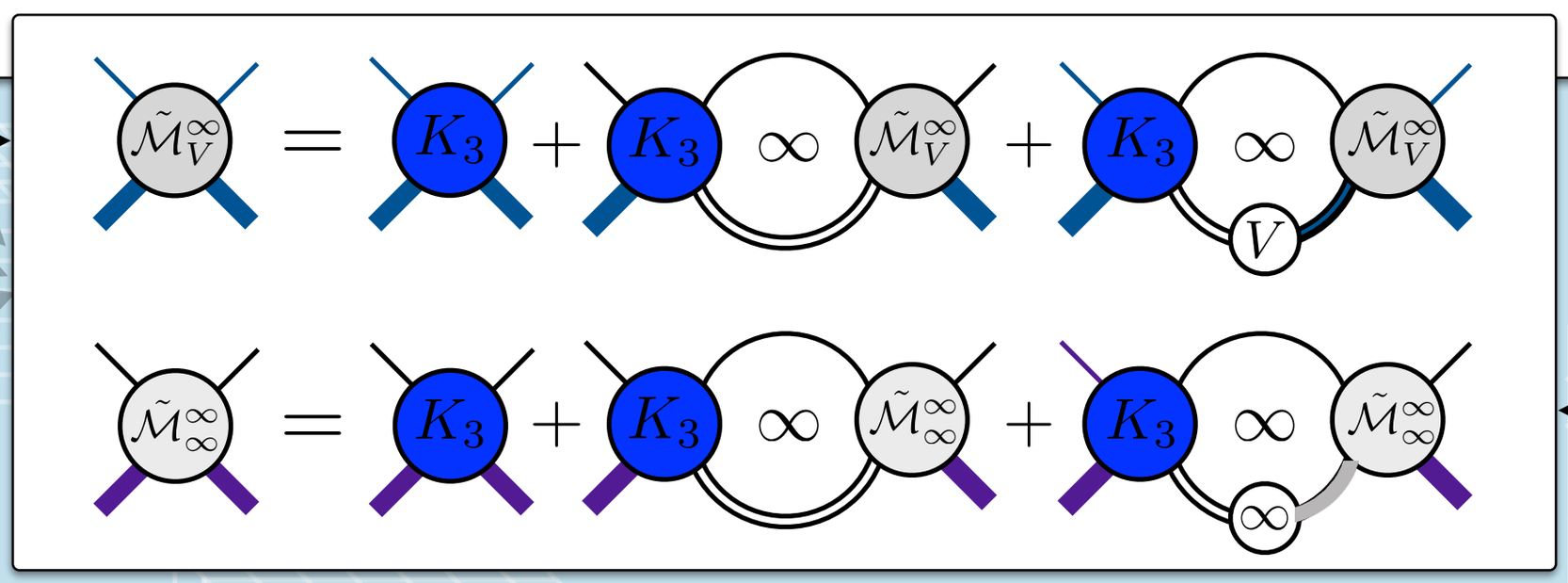
OVER OPEN KINEMATIC CHANNELS

OVER PARTIAL-WAVE CHANNELS OF BOSON-PARTICLE STATES

$$\tilde{\mathcal{M}}_V^{\infty}(\mathbf{p},\mathbf{k};\mathbf{P},E) = \tilde{\mathcal{M}}_{\infty}^{\infty}(\mathbf{p},\mathbf{k};\mathbf{P},E) - \int \frac{d^3q}{(2\pi)^3}\tilde{\mathcal{M}}_{\infty}^{\infty}\mathbf{p},\mathbf{q};\mathbf{P},E)\delta\mathcal{D}^V(E-\frac{q^2}{2m},|\mathbf{P}-\mathbf{q}|)\tilde{\mathcal{M}}_V^{\infty}(\mathbf{q},\mathbf{k};\mathbf{P},E)$$

DIAGONAL IN ANGULAR MOMENTUM

MIXES THE DIMER-PARTICLE STATES



THREE-HADRON QUANTIZATION CONDITION

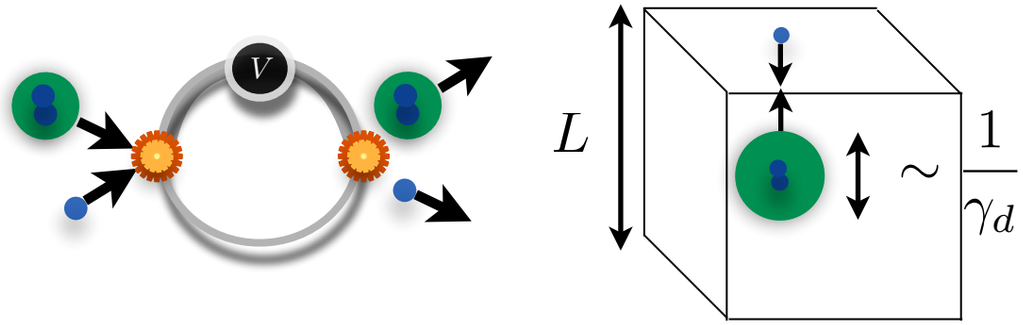
Briceno and ZD, Phys.Rev. D87,094507(2013).

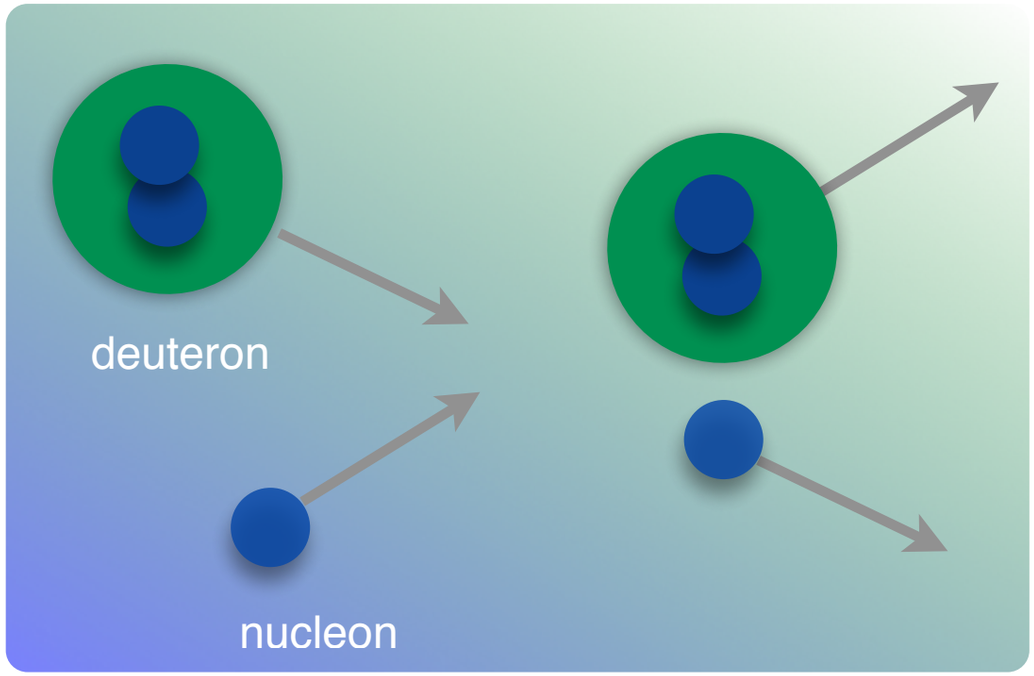
$$\mathrm{Det}(1+\tilde{\mathcal{M}}_V^{\infty}\delta\tilde{\mathcal{G}}^V) \equiv \det_{\mathrm{oc}}[\det_{\mathrm{pw}}(1+\tilde{\mathcal{M}}_V^{\infty}\delta\tilde{\mathcal{G}}^V)] = 0 \quad (II)$$

OVER OPEN KINEMATIC CHANNELS

OVER PARTIAL-WAVE CHANNELS OF BOSON-PARTICLE STATES

$$q_0^* \cot\delta_{Bd} = 4\pi\, c_{00}^P(q_0^*) + \eta\frac{e^{-\gamma_d L}}{L}$$





Polejaeva and Rusetsky, Eur.Phys.J.A48,67(2012).
Hansen and Sharpe, Phys.Rev.D90,116003(2014).
Hammer, Pang and Rusetsky, JHEP09,109(2017).
Briceno, Hansen and Sharpe, Phys.Rev.D95,074510(2017).
Mai and Doring, Eur.Phys.J.A,12440-1(2017).
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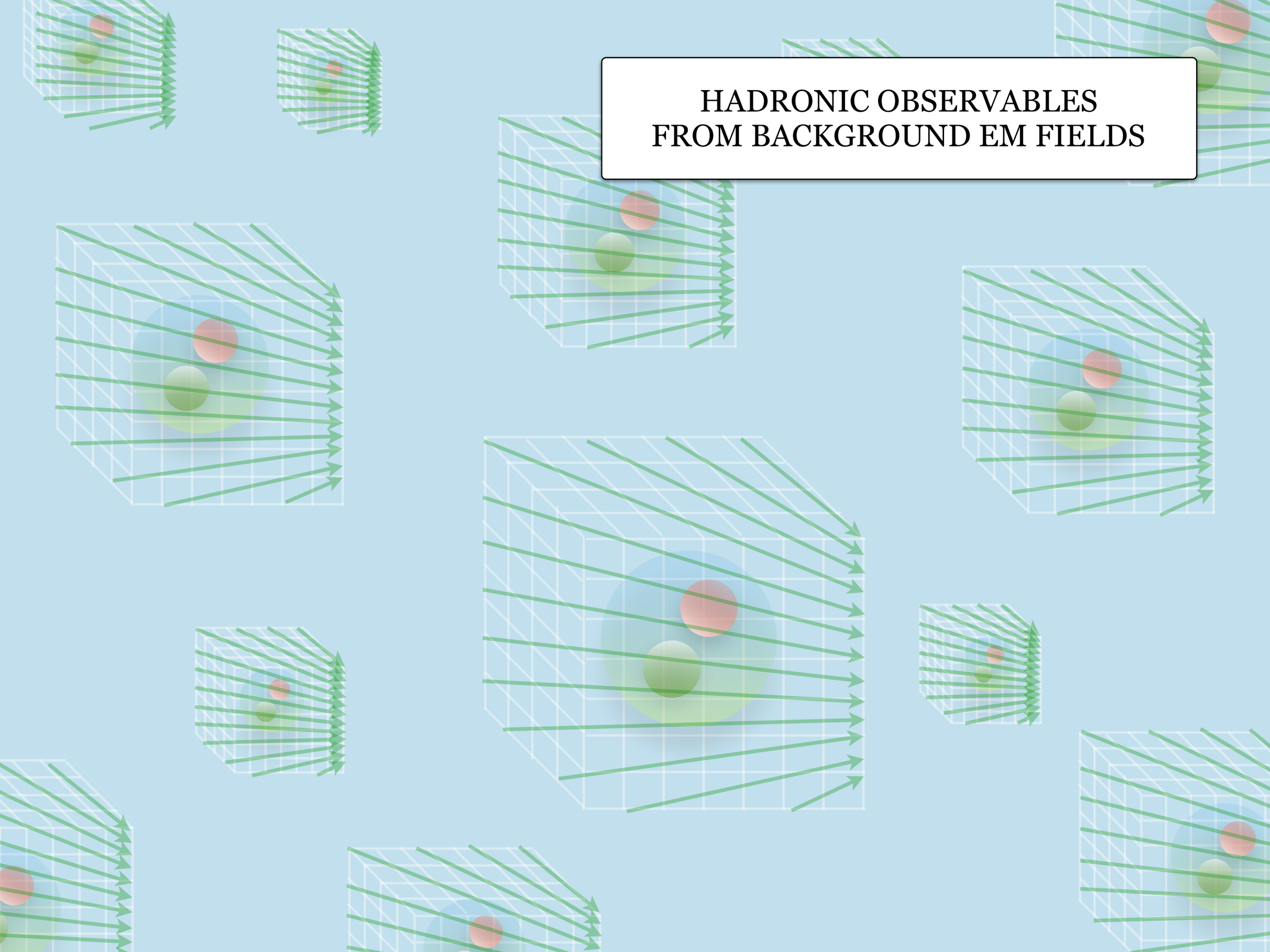
HADRONIC OBSERVABLES FROM BACKGROUND EM FIELDS



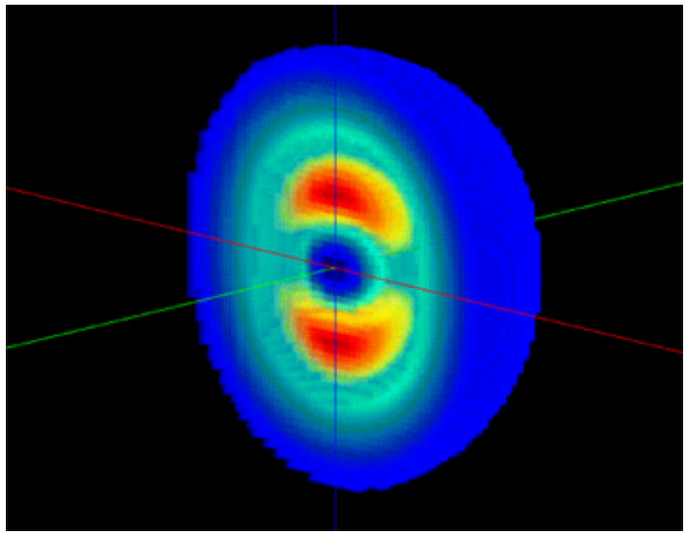
BACKGROUND EM FIELDS AND FAR MORE APPLICATIONS

1) EM CHARGE RADIUS

ZD and Detmold, Phys.Rev.D93, 014509(2016).

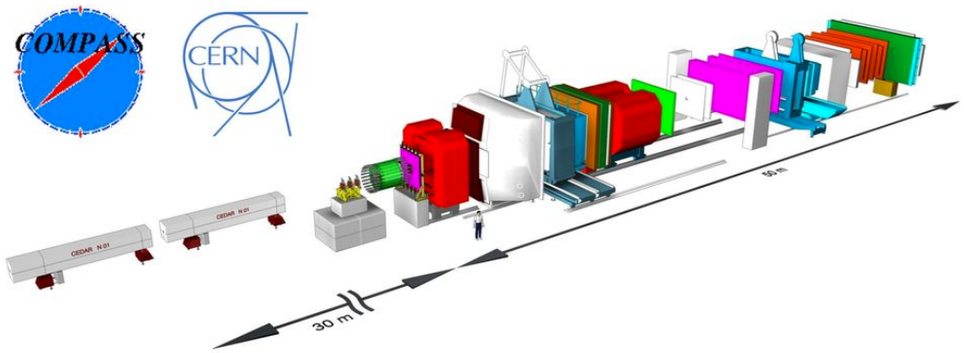
2) ELECTRIC QUADRUPOLE MOMENT

ZD and Detmold, Phys.Rev.D93,014509(2016).



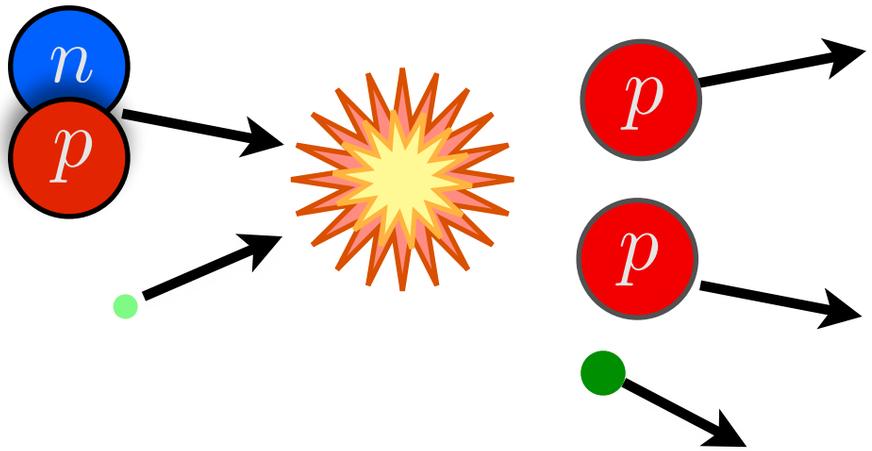
3) FORM FACTORS

Detmold, Phys.Rev.D71,054506(2005).



4) AXIAL BACKGROUND FIELDS

Beane at al, Phys.Rev.Lett.115,132001(2015).



A LINEAR ELECTRIC FIELD IN A PERIODIC LATTICE

$$A_\mu = \left(-\frac{E_0}{2}(\mathbf{x}_3 - \left[\frac{\mathbf{x}_3}{L}\right] L)^2, \mathbf{0}\right) \quad \rightarrow \quad \mathbf{E} = E_0 \mathbf{x}_3 \hat{\mathbf{x}}_3$$

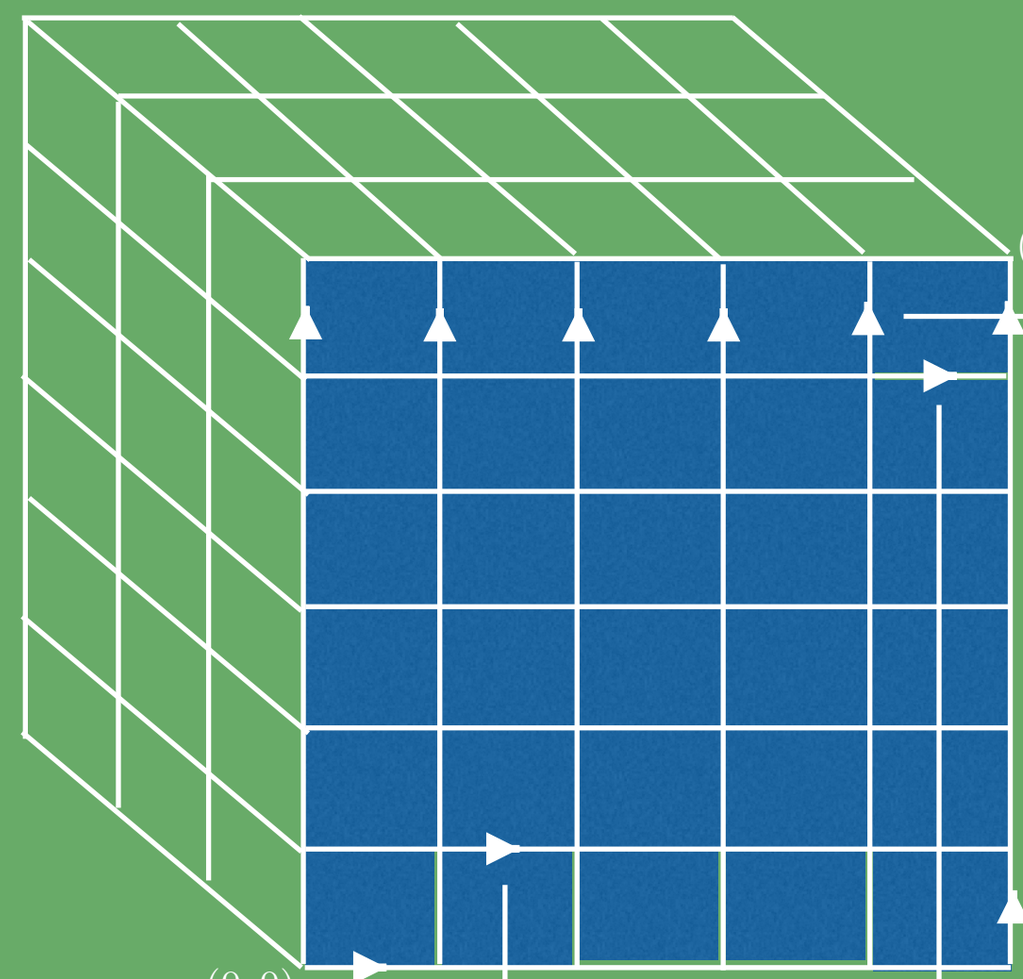
PERIODIC BC

$$e^{ie\hat{Q}E_0 L^2 T/2} = 1$$

$$E_0 = \frac{4\pi n}{e\hat{Q}L^2 T}$$

QUANTIZATION CONDITION
FOR THE SLOPE OF THE FIELD

$$= e^{ie\hat{Q}E_0 a_t a_s/2}$$



MODIFIED LINK

$$1 \times e^{-ie\hat{Q}E_0 L^2 t/2}$$

$$1$$

$$1$$

$$e^{-ie\hat{Q}E_0 a_t/2}$$

$$e^{-(L-a_s)^2 ie\hat{Q}E_0 a_t/2}$$

ZD and Detmold, Phys.Rev.D 92,074506(2015).

AN OSCILLATORY ELECTRIC FIELD IN A PERIODIC LATTICE

$$A_\mu = (A_0, -\mathbf{A}) = \left(\frac{ia}{q_3} e^{iq_3 \mathbf{x}_3}, 0, 0, 0\right) \;\rightarrow\; \mathbf{E} = a e^{iq_3 \mathbf{x}_3} \hat{\mathbf{x}}_3$$

PERIODIC BC

$$e^{-\frac{e\hat{Q}a}{q_3}(1-e^{iq_3 L})T} = 1$$

$$q_3 = \frac{2\pi n}{L}$$

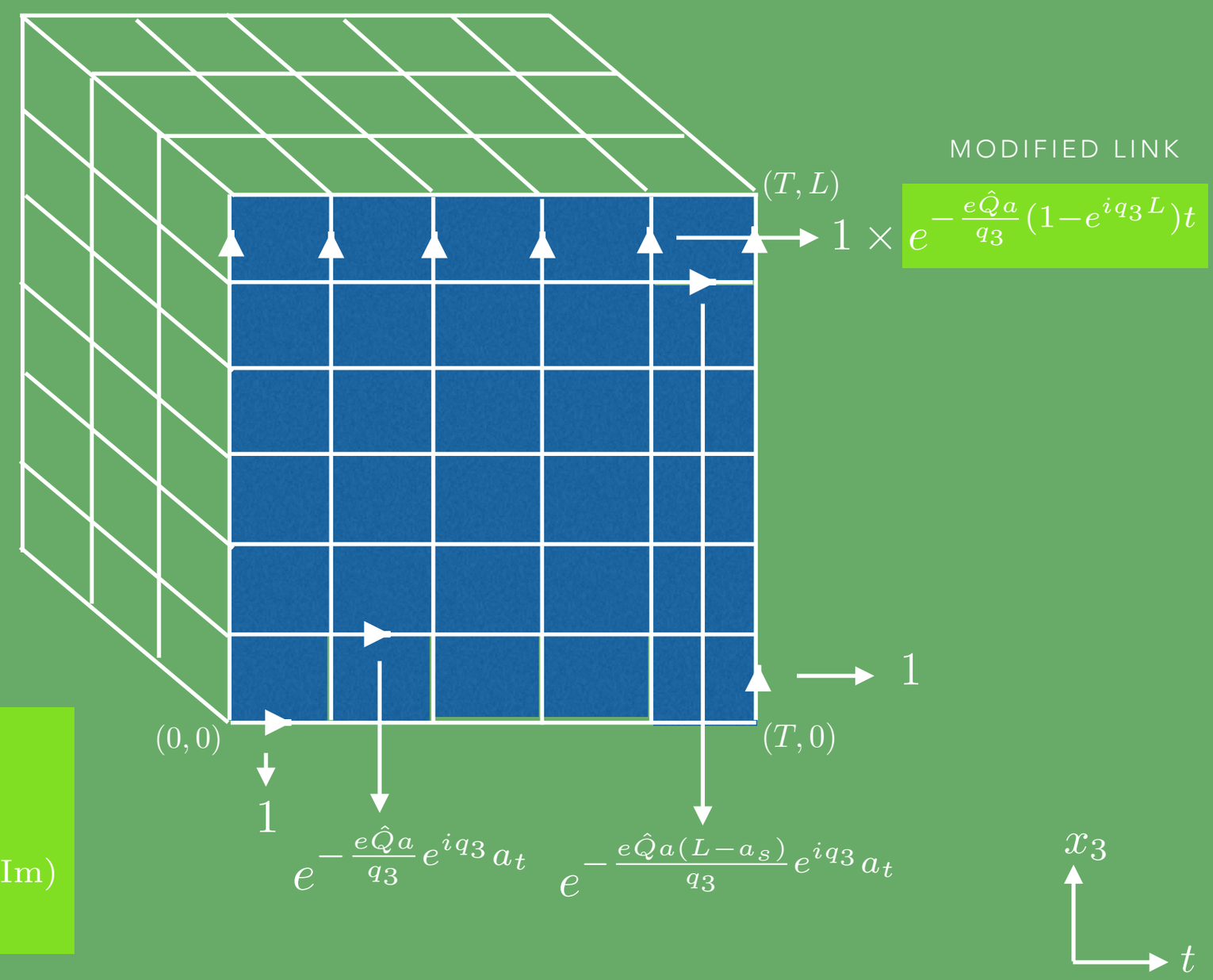
QUANTIZATION CONDITION FOR THE FREQUENCY OF THE FIELD

OR

QUANTIZATION CONDITION FOR THE AMPLITUDE OF THE FIELD

$$a^{(\mathrm{Im})} = \frac{\pi q_3 n'}{e\hat{Q}T},$$

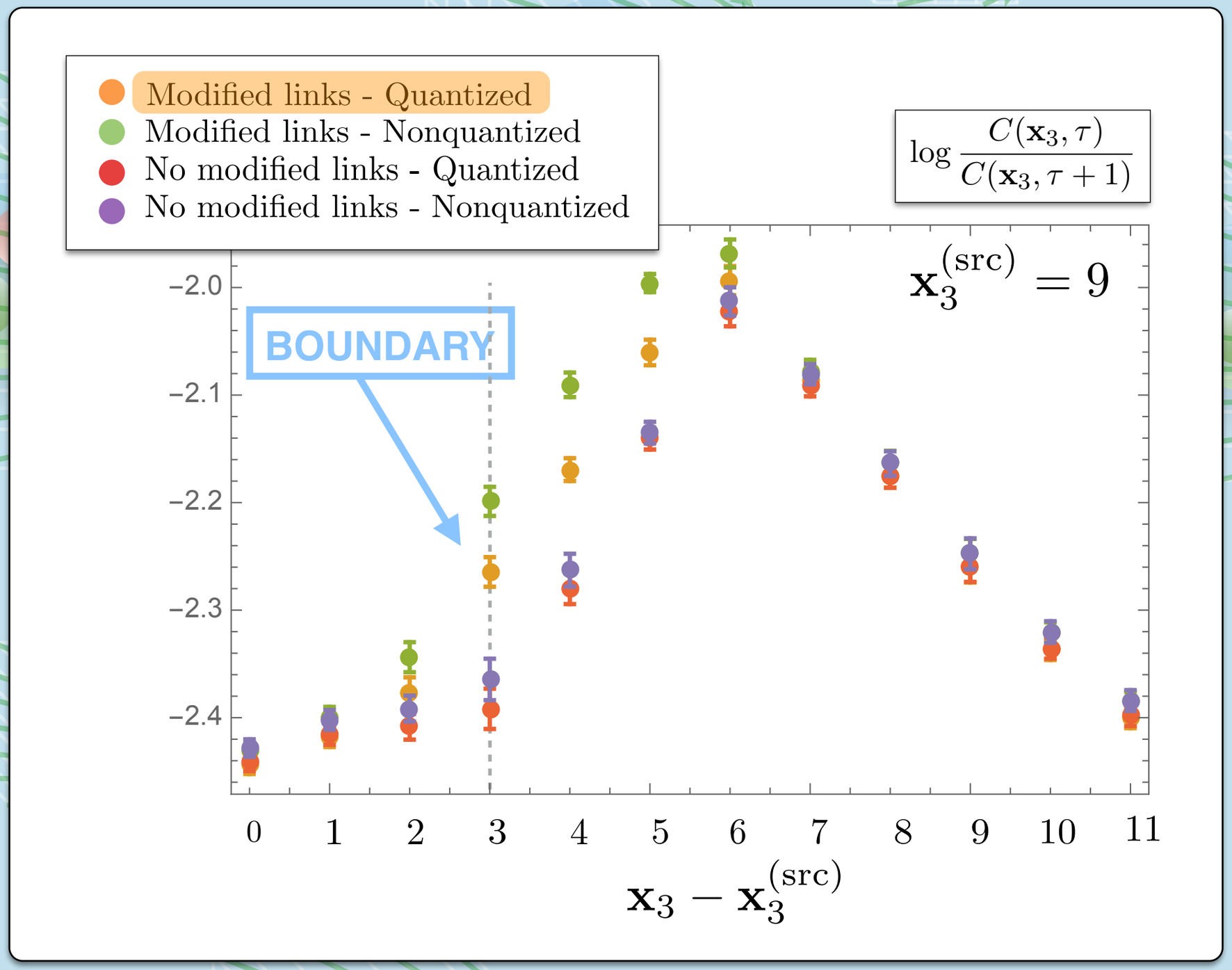
$$a^{(\mathrm{Re})} = -\frac{\sin(q_3 L)}{1-\cos(q_3 L)} a^{(\mathrm{Im})}$$



ZD and Detmold, Phys.Rev.D 92,074506(2015).

NEUTRAL PION CORRELATION FUNCTION WITH $\mathbf{E} = E_0 x_3 \mathbf{x}_3$

ZD and Detmold, Phys.Rev.D 92,074506(2015).

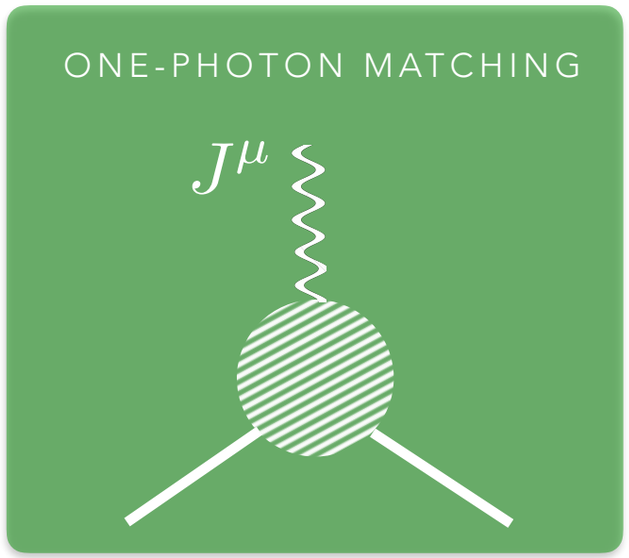


MATCHING LQCD AND EFT FOR CHARGE RADIUS AND ELECTRIC QUADRUPOLE MOMENT

A SPIN-1 THEORY

ZD and Detmold, Phys.Rev.D92,074506(2015).

$$\mathcal{L} = \frac{1}{2}W^{\dagger\mu\nu}W_{\mu\nu} + M^2 V^{\dagger\alpha}V_\alpha - \frac{1}{2}W^{\dagger\mu\nu}(D_\mu V_\nu - D_\nu V_\mu) - \frac{1}{2}((D_\mu V_\nu)^\dagger - D_\nu V_\mu^\dagger)W^{\mu\nu} +$$
$$ieC^{(0)}\, F_{\mu\nu}\, V^{\dagger\mu}V^\nu + \frac{ieC_1^{(2)}}{M^2}\, \partial_\mu F^{\mu\nu}((D_\nu V^\alpha)^\dagger V_\alpha - V^{\dagger\alpha}D_\nu V_\alpha) +$$
$$\frac{ieC_2^{(2)}}{M^2}\, \partial^\alpha F^{\mu\nu}((D_\alpha V_\mu)^\dagger V_\nu - V_\nu^\dagger D_\alpha V_\mu) + \frac{ieC_3^{(2)}}{M^2}\, \partial^\nu F^{\mu\alpha}((D_\mu V_\alpha)^\dagger V_\nu - V_\nu^\dagger D_\mu V_\alpha) + \mathcal{O}\left(\frac{1}{M^4}, F^2\right)$$

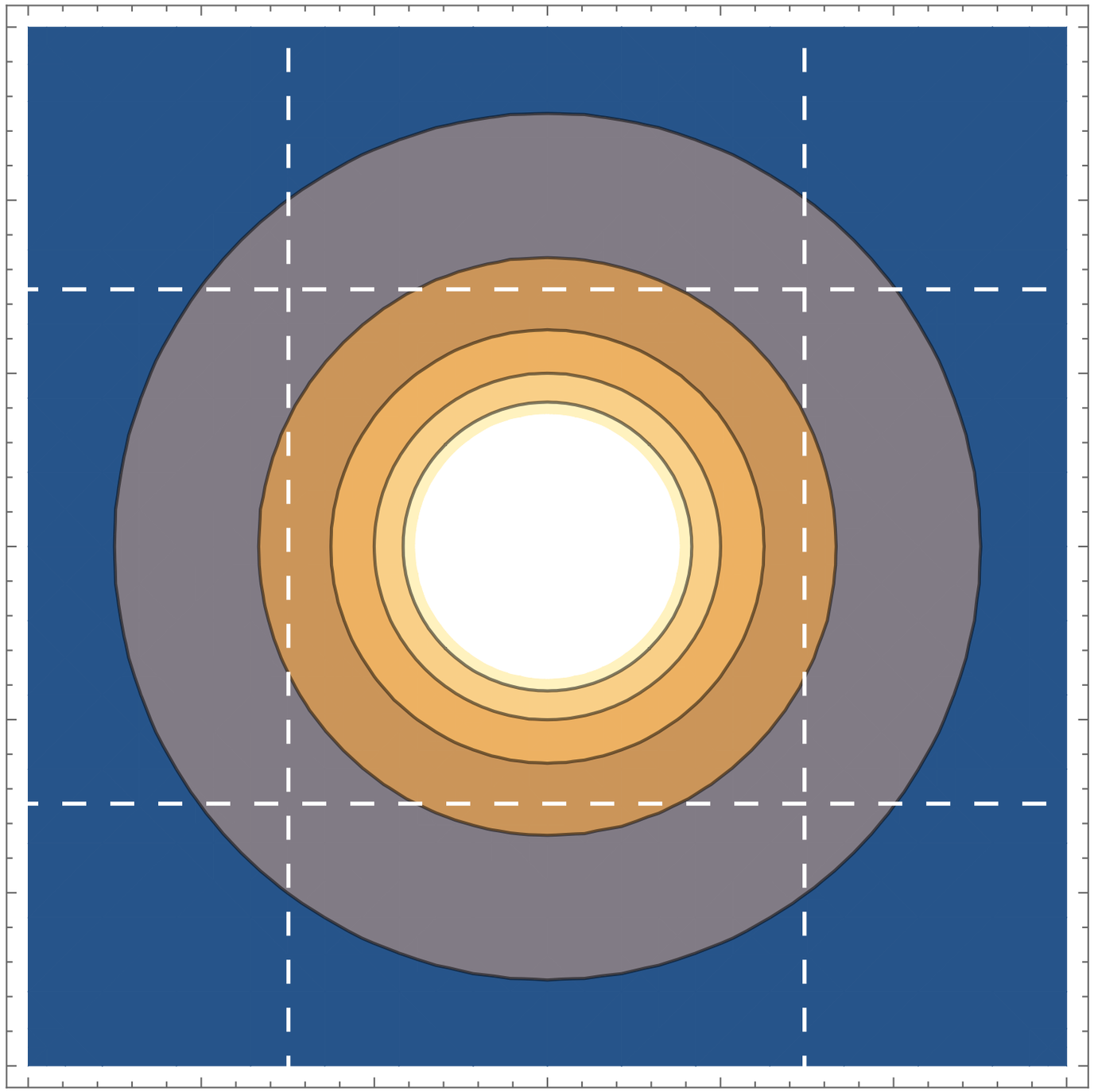


$$C^{(0)} = \overline{\mu}_1 - Q_0,$$
$$C_1^{(2)} = \frac{1}{6e}\left(M^2 \langle r^2\rangle_E + e\overline{Q}_2\right),$$
$$C_2^{(2)} = \frac{1}{4}(-Q_0 + \overline{Q}_2 + \overline{\mu}_1) + \frac{1}{6e}M^2\langle r^2\rangle_M,$$
$$C_3^{(2)} = \frac{1}{2}(-Q_0 + \overline{Q}_2 + \overline{\mu}_1).$$

NOW MATCH TO EITHER CORRELATION FUNCTIONS OR ENERGY EIGENVALUES OF LQCD IN A LINEAR ELECTRIC FIELD.

HADRONIC OBSERVABLES IN PRESENCE OF QED

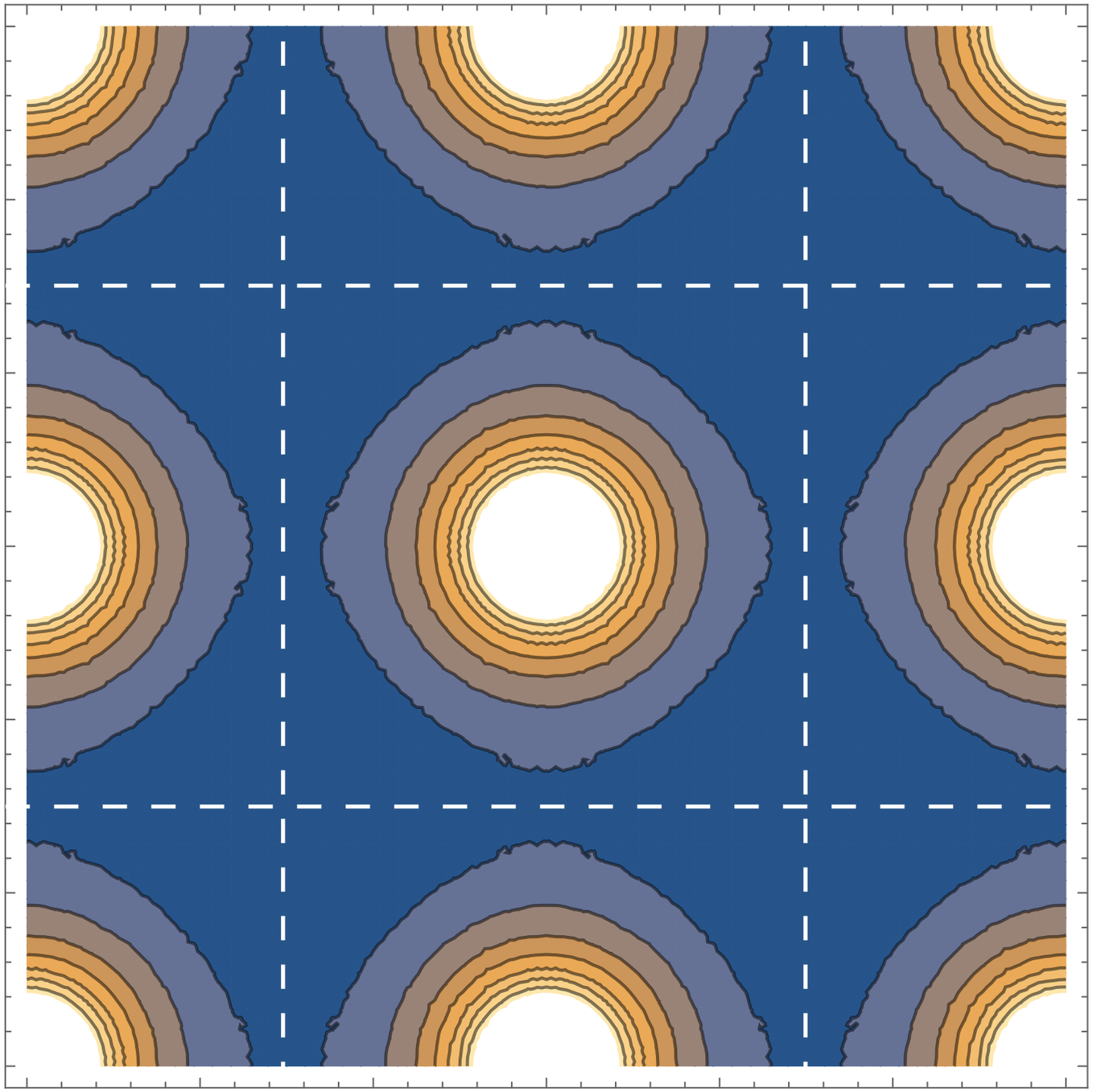
COULOMB POTENTIAL OF A POINT CHARGE **WITHOUT** THE UNIFORM CHARGE DENSITY



x-y plane

ZD and Savage, Phys. Rev. D 90, 054503 (2014).

COULOMB POTENTIAL OF A POINT CHARGE **WITH** THE UNIFORM CHARGE DENSITY



x-y plane

ZD and Savage, Phys. Rev. D 90, 054503 (2014).

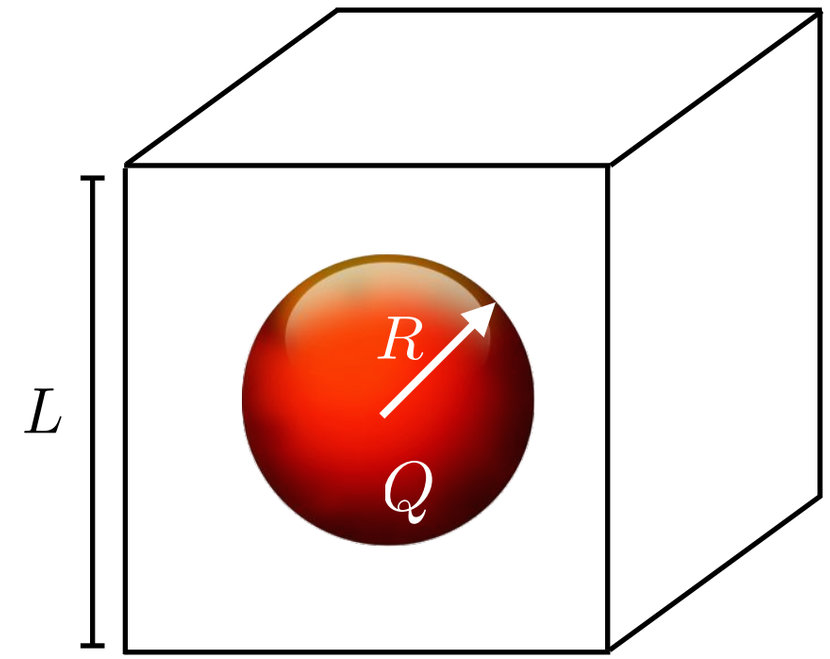
A CLASSICAL EXAMPLE: CHARGED SPHERE IN A FINITE VOLUME

SELF ENERGY

$$U(R,L) = \frac{1}{2}\int d^3r d^3r' \rho \, V(\mathbf{r}-\mathbf{r}')$$

$$\rho = \frac{Q}{\frac{4}{3}\pi R^3}$$

$$V(\mathbf{r}-\mathbf{r}') = \frac{Q}{4\pi|\mathbf{r}-\mathbf{r}'|}$$



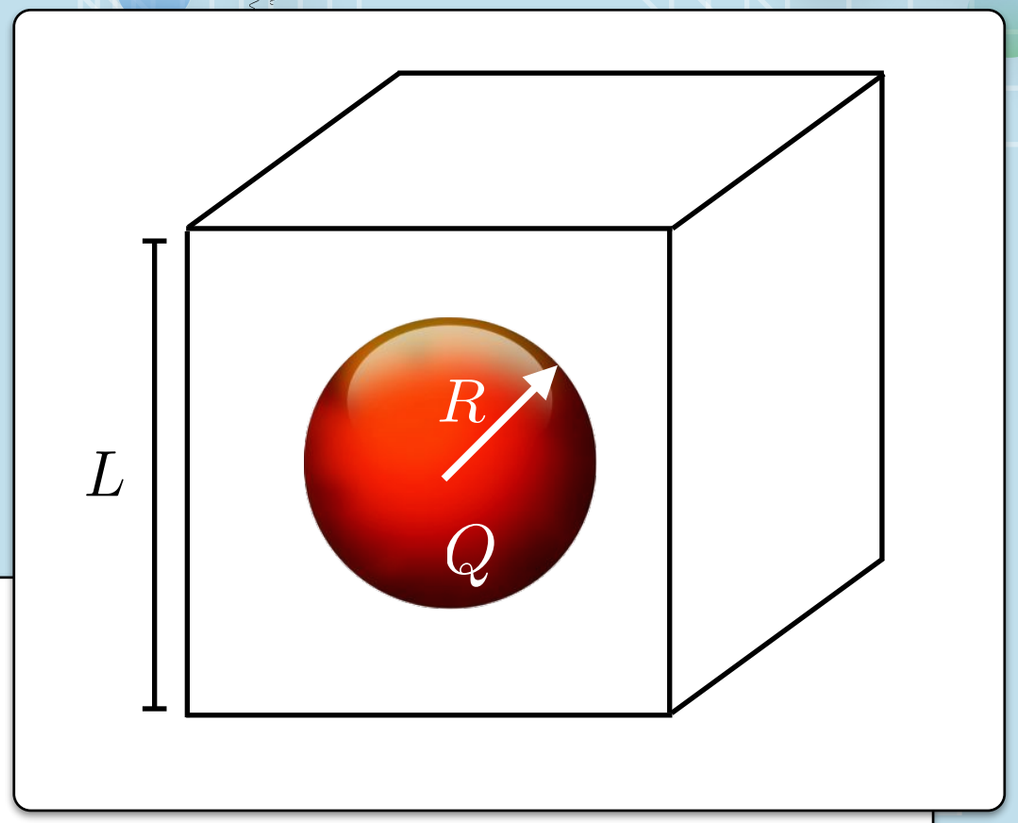
ZERO MODE REMOVED

ZD and Savage, Phys. Rev. D 90, 054503 (2014).

$$V(\mathbf{r}-\mathbf{r}') = \frac{1}{L^3}\sum_{\mathbf{k}\in\frac{2\pi\mathbf{n}}{L},n\neq\mathbf{0}} \frac{e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}}{\mathbf{k}^2} = V^{(\infty)}(\mathbf{r}-\mathbf{r}') + \sum_{m\ \neq\mathbf{0}}\int \frac{d^3k}{(2\pi^3)}\frac{e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}}{\mathbf{k}^2}e^{i\mathbf{k}\cdot\mathbf{m}L}$$

POISSON RE-SUMMATION
(NOT EXACTLY: INCLUDE THE ZERO MODE BUT INTRODUCE AN IR REGULATOR. SUBTRACT OFF THE IR DIVERGENCE AT THE END.)

A CLASSICAL EXAMPLE: CHARGED SPHERE IN A FINITE VOLUME



ZD and Savage, Phys. Rev. D 90, 054503 (2014).

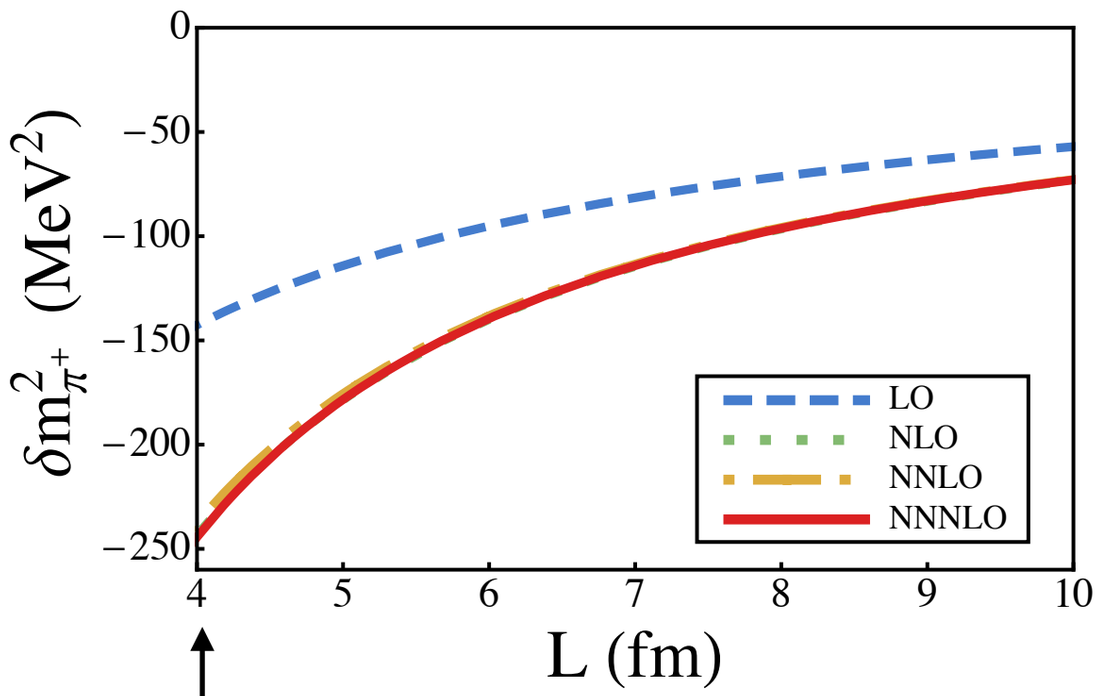
$$U(R,L) = \frac{3}{5}\frac{Q^2}{4\pi R} + \frac{Q^2}{8\pi L}c_1 + \frac{Q^2}{10L}\left(\frac{R}{L}\right)^2 + \ldots$$

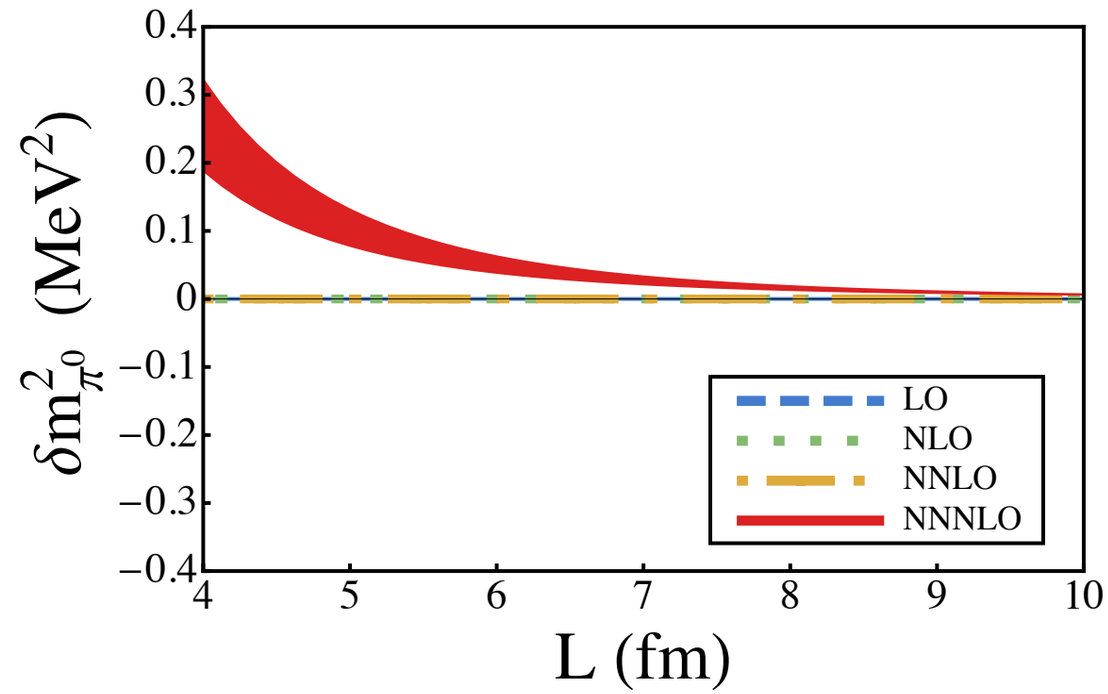
INDEPENDENT OF R

$\frac{Q^2}{6L^3}\langle r^2\rangle$ WITH $\langle r^2\rangle = \frac{3}{5}R^2$

$$c_1 = -2.83729$$

FINITE-VOLUME QED CORRECTIONS TO THE MASS OF THE PION





$\sim 10\%$ at $L = 4$ [fm]

ZD and Savage, Phys. Rev. D 90, 054503 (2014).

Hilf and Polley, Phys.Lett.B131,412(1983).
Duncan et al, Phys.Rev.Lett.76,3894(1996).
Hayakawa, Uno, Prog.Theor.Phys.120,413(2008).
Borsanyi et al, Science 347:1452-1455(2015).
Lucini et al, JHEP02(2016)076.
Endres et al, Phys.Rev.Lett.117,072002(2016).
Carrasco et al, PhysRevD.91.074506(2015).

LOW-ENERGY EFFECTIVE FIELD THEORY OF HADRONS COUPLED TO QED_L NEEDS CAUTION

$\mathrm{QED}_L \neq$ HEAVY FIELD EFFECTIVE THEORY AT $\mathcal{O}(1/L^3)$

$$S_{\mathrm{L,int.}}[A_\mu, J_\mu] = \int_{\mathbb{T}^4} \mathrm{d}^4x\, A_\mu(x) \left[J_\mu(x) - \tfrac{1}{L^3} \int_{\mathbb{T}^3} \mathrm{d}^3\mathbf{y}\, J_\mu(t, \mathbf{y})\right]$$

A RESIDUAL MASS ADDED TO THE EFFECTIVE LAGRANGIAN FIXES THE DISCREPANCY:

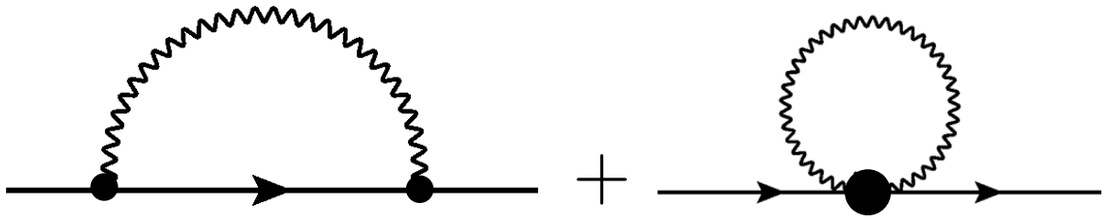
$$\delta m_{\phi,u} = -\frac{q^2|\mathbf{v}|^2}{8m_\phi \omega(\mathbf{p}) L^3}$$

$$\delta m_{\psi,u} = q^2 \frac{2\omega(\mathbf{p})^2 + m_\psi^2}{8m_\psi \omega(\mathbf{p})^3 L^3}$$

Boyle, ZD, Harrison, Juttner, Portelli and Savage, in preparation (2018).

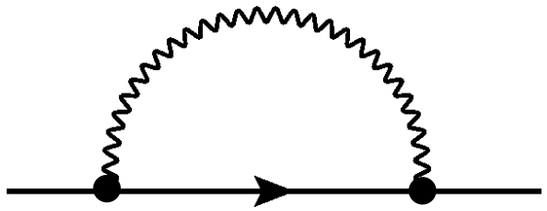
FINITE-VOLUME QED CORRECTIONS TO THE SELF ENERGY OF CHARGED STRUCTURELESS PARTICLES [JAMES HARRISON'S TALK ON WEDNESDAY]

SPIN-0



$$\Delta\omega_0^2 = M^2 Q^2 \left\{ \frac{1}{\gamma}\frac{c_{2,1}(\mathbf{v})}{4\pi^2 M L} + \frac{c_1}{2\pi M^2 L^2} - \left(\frac{1}{\gamma} - \frac{1}{\gamma^3}\right)\frac{1}{2M^3 L^3} + \cdots \right\}$$

SPIN-1/2



$$\Delta\omega_{\frac{1}{2}} = M Q^2 \left\{ \frac{1}{\gamma}\frac{c_{2,1}(\mathbf{v})}{8\pi^2 M L} + \frac{c_1}{4\pi M^2 L^2} + \left[\frac{2}{\gamma} + \frac{1}{\gamma^3}\right]\frac{1}{4M^3 L^3} + \cdots \right\}$$

Boyle, ZD, Harrison, Juttner, Portelli and Savage, in preparation (2018).

SEE YIMING CAI'S TALK ON THURSDAY FOR OUR APPROACH TO $1 \to 2$ DECAY PROCESSES WITH STRONG AND QED INTERACTIONS

THE PATH FROM FINITE TO INFINITE VOLUME:

A LOT MORE TO DO AND LOTS OF EXCITING APPLICATIONS!

THANK YOU