$B_s \to D_s^{(*)} l \bar{\nu}$ with Heavy HISQ Quarks

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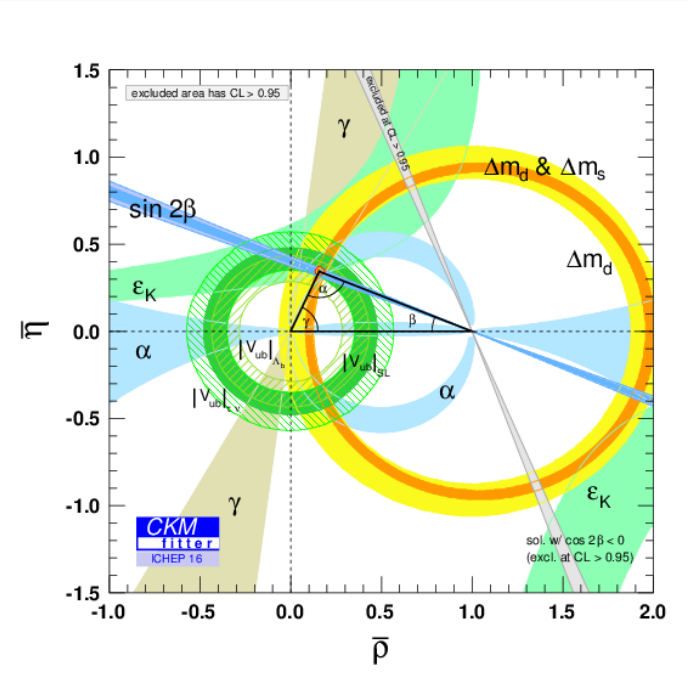
Lattice 2018, Michigan State University

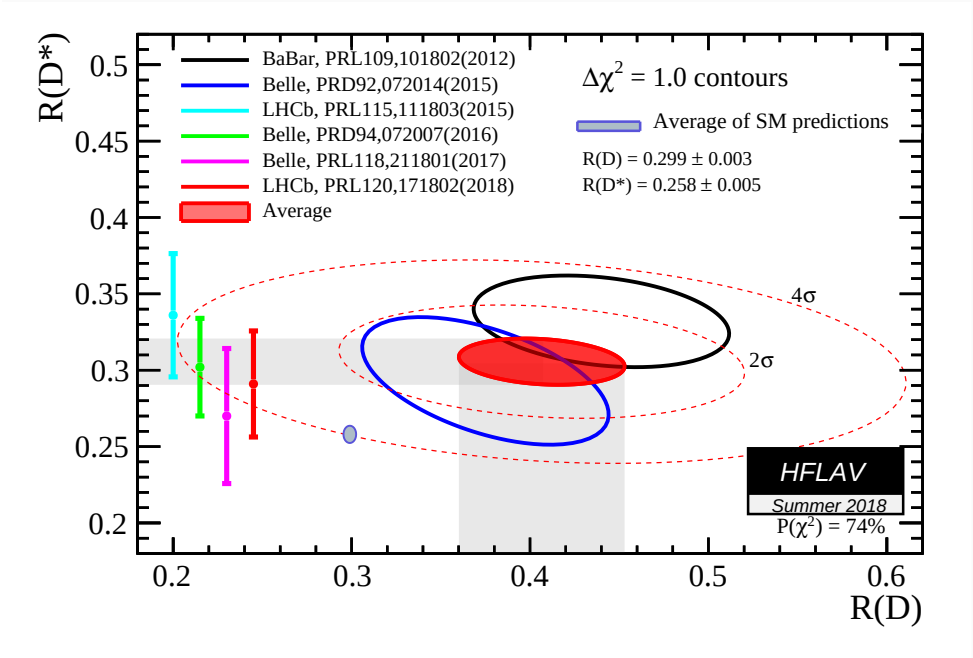
Outline

Background

Motivation

- Semileptonic decays are an important input to determinations of **CKM matrix elements**.

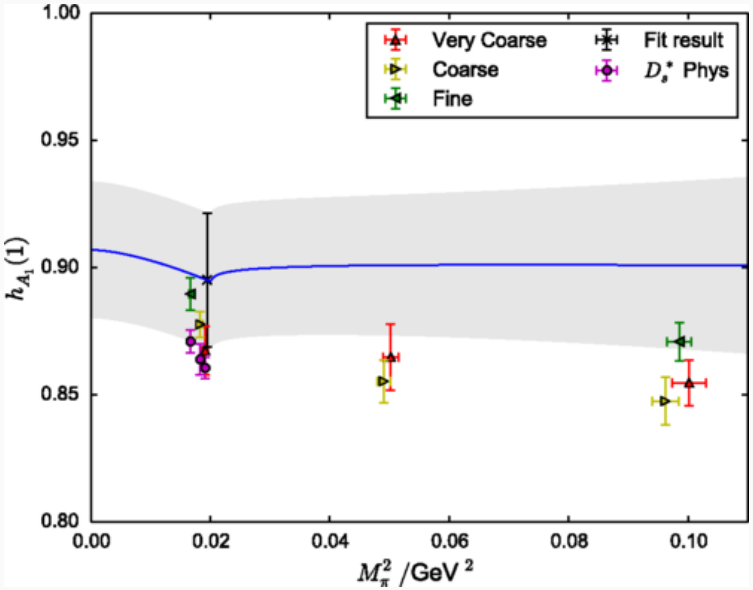


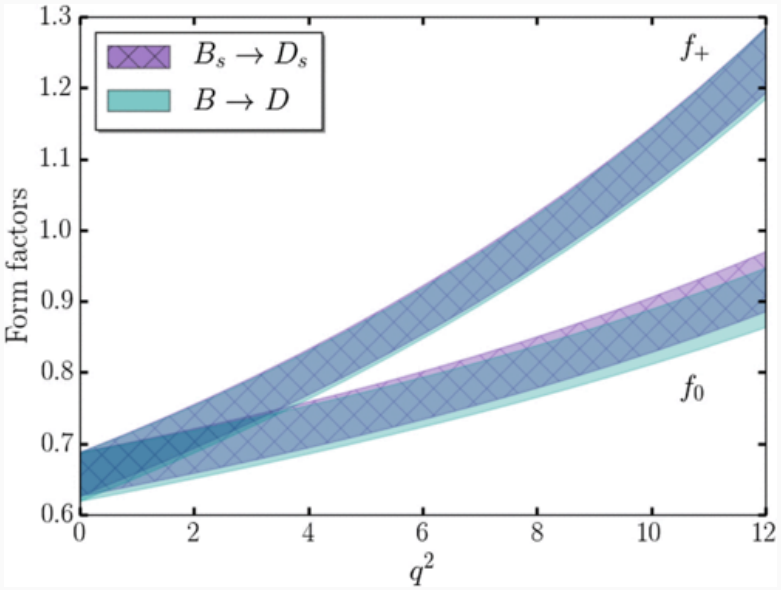


- $\exists$ number of **tensions** in B decays that point to e.g. lepton flavour violation, for example in $B \to D^*$ and $B \to D$.

Connection to $B \to D^{(*)}$

- $B_s \to D_s^{(*)}$ more feasible than $B \to D^{(*)}$ on the lattice, **avoids valence light quarks**.





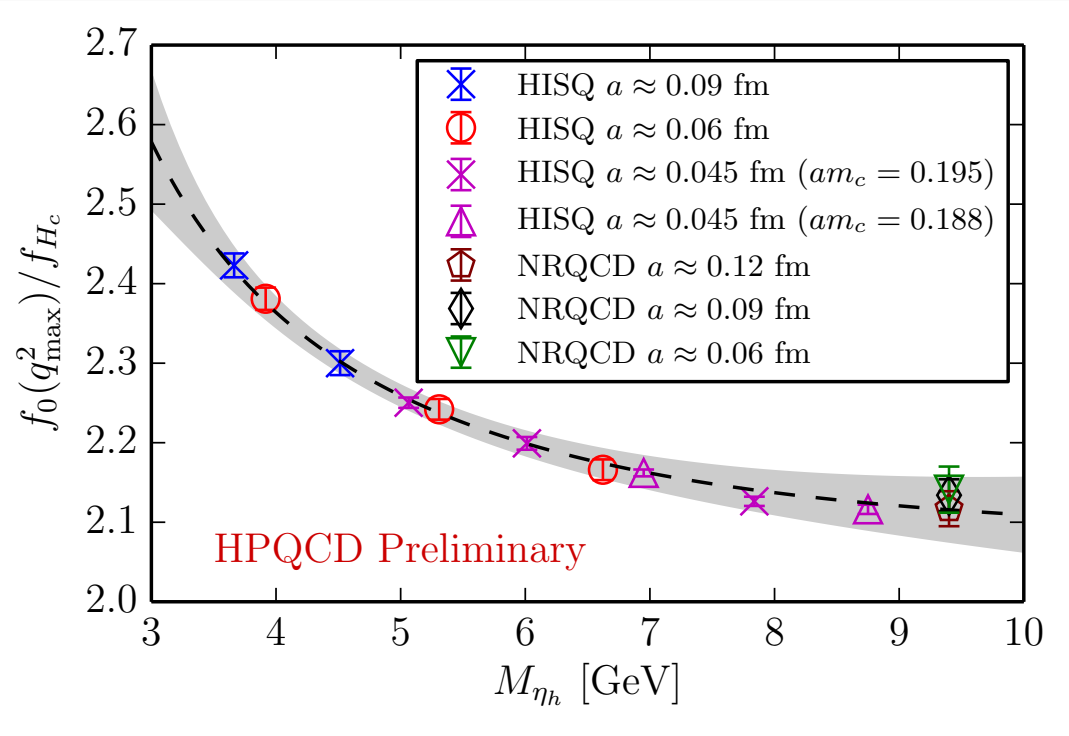
- $b \to c$ transitions **insensitive to spectator quark mass** $\implies$ $B_s \to D_s^{(*)} \simeq B \to D^{(*)}$.

$$h^s_{A_1}(q^2_{\max})/h_{A_1}(q^2_{\max}) = 1.013(22) \quad \text{[Harrison et al. 1711.11013]}$$

$$f^s_0(m_\pi^2)/f_0(m_\pi^2) = 1.006(62) \quad \text{[Monahan et al. 1703.09728]}$$

Heavy HISQ: A Recent History

- HISQ = **H**ighly **I**mproved **S**taggered **Q**uarks [0610092]
- Successful at simulating light,strange & charm quarks.
- b quarks more difficult due to $(am_b)^2$ discretization effects. Recent calculations have used *heavy-HISQ* approach; simulate with light b's and extrapolate to physical b mass.



- example - $B_c \to \eta_c$ form factors [Lytle et al. 1605.05645]

Benefits of Heavy HISQ

$$S_{\text{HISQ}} = \sum_x \bar{\psi}(x) \left(\gamma \cdot \left(\nabla_\mu(W) - \frac{a^2}{6}(1+\epsilon)\nabla^3_\mu(X) \right) + m \right) \psi(x)$$

- All normalization are done **non-perturbatively** - self-contained calculations - no matching errors.
  - NRQCD: $\mathcal{O}(\alpha_s^2, \alpha_s/m_b, 1/m_b^2)$ matching errors.
  - Fermilab: $\mathcal{O}(\alpha_s^2)$ matching errors.
  - Heavy HISQ: **No matching errors**
- Fully relativistic
- Well controlled extrapolations to physical m_b using **HQET**

Calculation Details

Calculation Details

- Goal: Deduce $B_s \to D_s$ form factors $f_{0,+}$ @ all q^2, and $B_s \to D_s^*$ form factor h_{A_1} @ zero recoil.
- We use the CSD3 cluster @ Cambridge, part of STFC's DiRAC II facility.
- 2nd Generation MILC Gluon Ensembles [1212.4768]:
  - 2+1+1 flavours in the sea, using HISQ action
  - Lüscher-Weisz Gauge action, improved up to $\mathcal{O}(N_f \alpha_s a^2)$.

a/fm	$N_x^3 \times N_t$	am_l	am_s	am_c	am_s^{val}	am_c^{val}	am_h^{val}
0.0884(6)	$32^3 \times 96$	0.0074	0.037	0.440	0.0376	0.45	0.5, 0.65, 0.8
0.05922(12)	$48^3 \times 144$	0.0048	0.0048	0.286	0.0234	0.274	0.427, 0.525, 0.65, 0.8
0.04406(23)	$64^3 \times 192$	0.00316	0.0158	0.188	0.0165	0.194	0.5, 0.65, 0.8

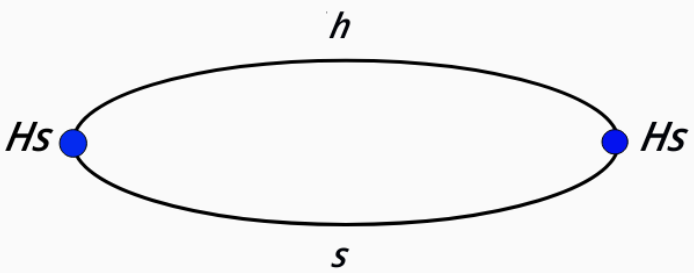
Masses tuned in Chakraborty et. al. [1408.4169].

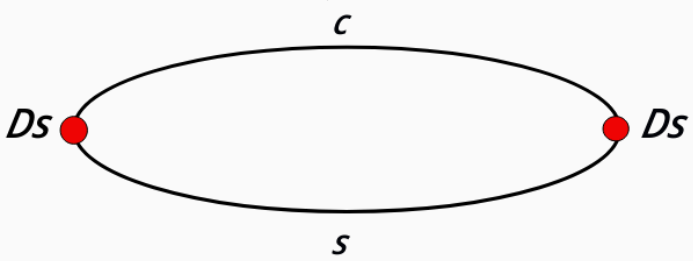
Correlation Functions

- Use **random wall sources**.
- Calculate 2- and 3-point correlation functions: $\sum_{\underline{x},\underline{y}} \langle \Phi^\dagger_{D_s^{(*)}}(\underline{x},0)\Phi_{D_s^{(*)}}(\underline{y},t)\rangle$, $\sum_{\underline{x},\underline{y}} \langle \Phi^\dagger_{H_s}(\underline{x},0)\Phi_{H_s}(\underline{y},t)\rangle$, $\sum_{\underline{x},\underline{y},\underline{z}} \langle \Phi^\dagger_{H_s}(\underline{x},0)J_\mu(\underline{y},t)\Phi_{D_s^{(*)}}(\underline{z},T)\rangle$.
- Extract $\langle D_s^{(*)}|J_\mu|H_s\rangle$ from a simultaneous **Bayesian fit**.
- 2-point fit form:

$$C_H(t) = \sum_{k=0}^{N_{\rm exp}} |a_k^H|^2 (e^{-E_k^H t} + e^{-E_k^H (N_t - t)})$$

$$+(-1)^t \sum_{k=0}^{N_{\rm exp}} |a_k^{H_o}|^2 (e^{-E_k^{H_o} t} + e^{-E_k (N_t - t)})$$





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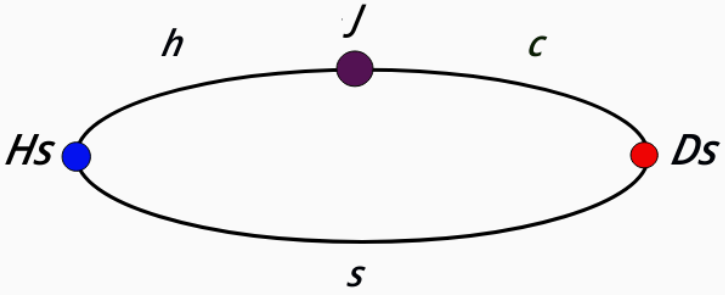
  $\sum_{\underline{x},\underline{y},\underline{z}} \langle \Phi^\dagger_{H_s}(\underline{x},0)J_\mu(\underline{y},t)\Phi_{D_s^{(*)}}(\underline{z},T)\rangle$.
- Extract $\langle D_s^{(*)}|J_\mu|H_s\rangle$ from a simultaneous **Bayesian fit**.
- 3-point fit form:

$$C_{J_\mu}(t,T) = \sum_{k,j=0}^{N_{\rm exp}} (A_{jk} e^{-E_k^{H_s} t} e^{-E_j^{D_s^{(*)}}(T-t)} + B_{jk}(-1)^t e^{-E_k^{B_{so}} t} e^{-E_j^{D_s^{(*)}}(T-t)}$$

$$+ C_{jk}(-1)^{T-t} e^{-E_k^{H_s} t} e^{-E_j^{D_{so}^{(*)}}(T-t)} + D_{jk}(-1)^T e^{-E_k^{B_{so}} t} e^{-E_j^{D_{so}^{(*)}}(T-t)})$$



$$\langle D_s^{(*)}|J_\mu|H_s\rangle = 2A_{00}\sqrt{E_0^{H_s} E_0^{D_s^{(*)}}}/a_0^{H_s} a_0^{D_s^{(*)}}$$

Currents & Normalizations

- Use only **local** HISQ currents.
- $B_s \to D_s$: Use scalar S and temporal vector V_0 currents.

$$S = \bar{\psi}_c \psi_h \quad , \quad V_0 = \bar{\psi}_c \gamma_0 \psi_h$$

  S is **absolutely normalized** in HISQ. Z_{V_0} fixed via Ward identity between V_0 and S @ zero recoil:

$$(M_{H_s} - M_{D_s}) Z_{V_0} \langle D_s | V_0 | H_s \rangle = (m_h - m_c) \langle D_s | S | H_s \rangle$$

- $B_s \to D_s^*$: Use an axial vector current $A_k = \bar{\psi}_c \gamma_5 \gamma_k \psi_h$. Z_{A_k} fixed via Ward identity between axial vector and (absolutely normalized) pseudoscalar [1311.6669].

$$M_{H_s} Z_{A_k} \langle 0 | A_k | H_s \rangle = (m_h + m_c) \langle 0 | P | H_s \rangle$$

Form Factors

- From currents $\langle D_s^{(*)}|J_\mu|H_s\rangle \equiv \langle J_\mu\rangle$, can deduce form factors.
- $B_s \to D_s$:

$$f_0(q^2) = \frac{m_b - m_c}{M_{H_s}^2 - M_{D_s}^2}\langle S\rangle$$

$$f_+(q^2) = \frac{1}{2M_{H_s}}\frac{\delta^M\langle S\rangle - q^2 Z_{V_0}\langle V_0\rangle}{\underline{p}_{D_s}^2}$$

$$(\delta^M = (m_h - m_c)(M_{H_s} - M_{D_s}))$$

- $B_s \to D_s^*$:

$$h_{A_1}(q^2_{\max}) = 2\sqrt{M_{H_s}M_{D_s^*}}Z_{A_k}\langle A_k\rangle$$

Continuum & Heavy Mass Extrapolation

$$h_{A_1}(q^2_{\max}) = \left(1 + p\log\left(\frac{M_{\eta_h}}{M_{\eta_c}}\right)\right) \sum_{i,j,k=0}^{2,2,2} b_{ijk} \left(\frac{\Lambda_{\mathrm{QCD}}}{M_{\eta_h}}\right)^i \left(\frac{am_h}{\pi}\right)^{2j} \left(\frac{am_c}{\pi}\right)^{2k}$$

$$+ \left(c_0 + c_1 \frac{\Lambda_{\mathrm{QCD}}}{M_{\eta_h}} + c_2 \left(\left(\frac{am_c}{\pi}\right)^2 + \left(\frac{am_h}{\pi}\right)^2\right)\right) (M_{\eta_c} - M_{\eta_c}^{\mathrm{phys}})$$

$$+ \left(s_0 + s_1 \frac{\Lambda_{\mathrm{QCD}}}{M_{\eta_h}} + s_2 \left(\left(\frac{am_c}{\pi}\right)^2 + \left(\frac{am_h}{\pi}\right)^2\right)\right) (M_{\eta_s}^2 - M_{\eta_s}^{\mathrm{phys2}})$$

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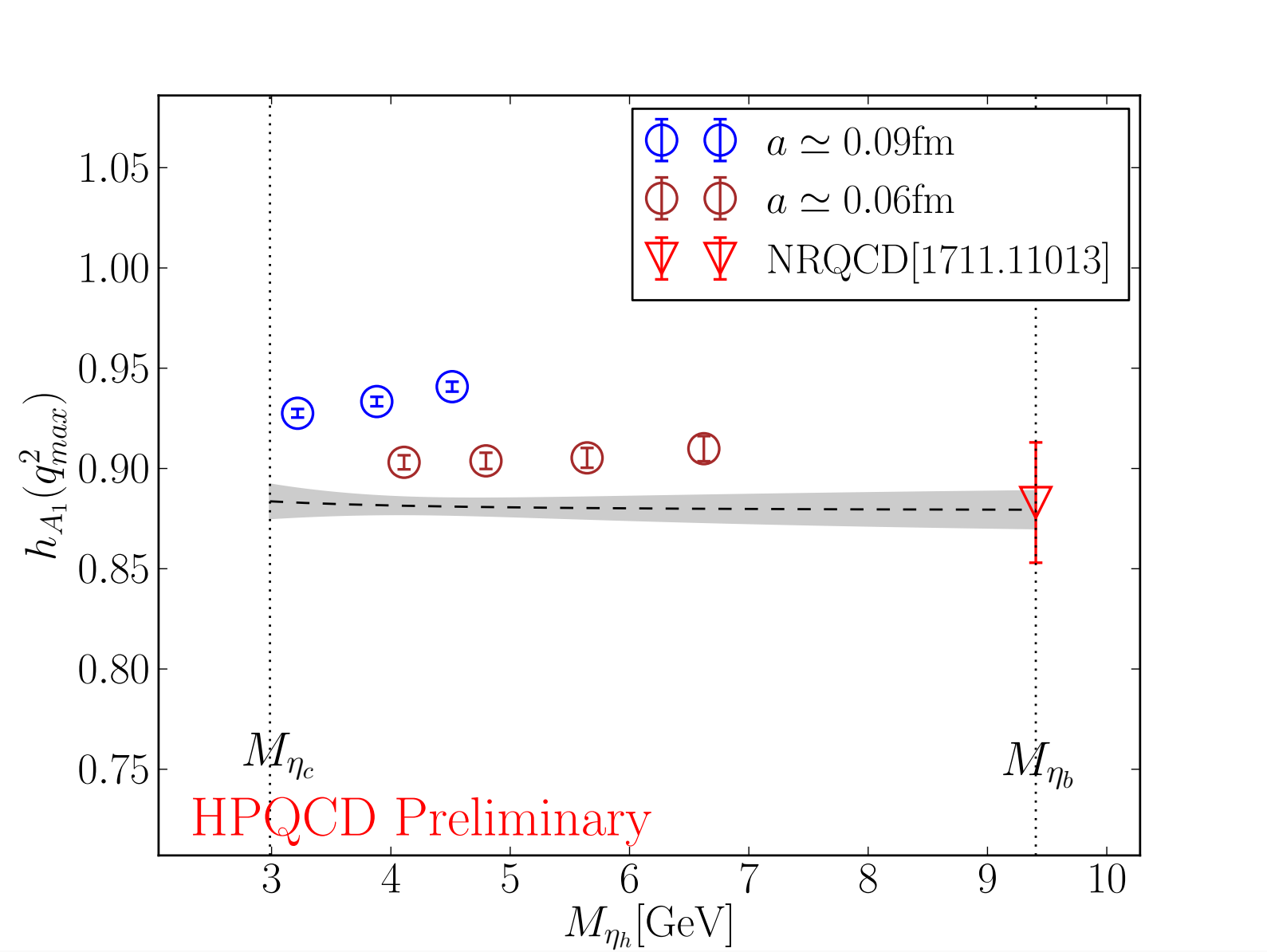
Continuum, Heavy Mass and Kinematic Extrapolation

$$\frac{f_{0,+}(q^2)}{f_{H_c}\sqrt{M_{H_c}}} = \frac{1 - q^2_{\max}/(M_{H_c^{0,*}})^2}{1 - q^2/(M_{H_c^{0,*}})^2} \left(1 + p \log\left(\frac{M_{\eta_h}}{M_{\eta_c}}\right)\right) \sum_{l=0}^{2} A_l z(q^2)^l$$

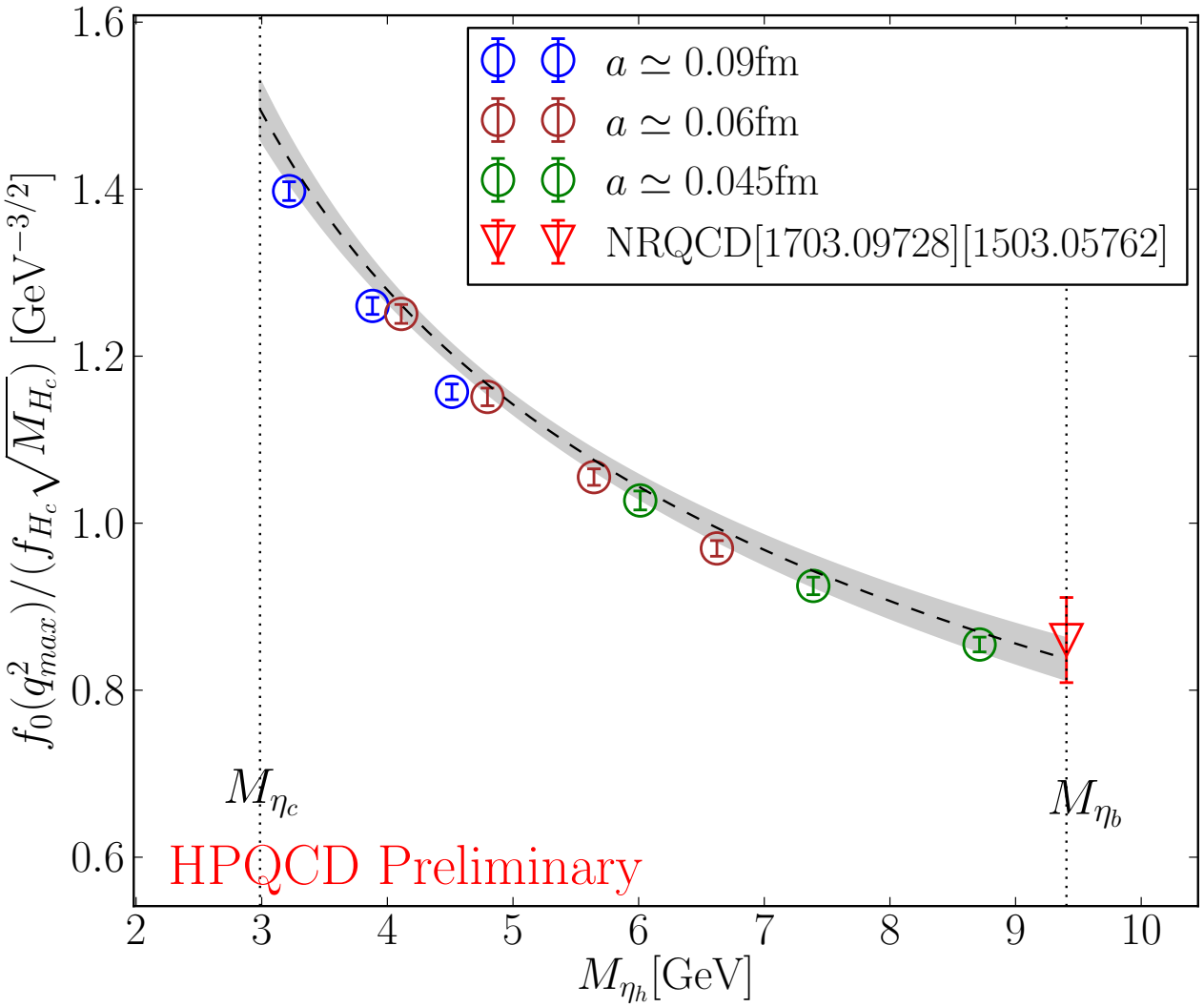
$$\begin{aligned} A_l = & \sum_{i,j,k=0}^{2,2,2} b_{ijkl} \left(\frac{\Lambda_{\mathrm{QCD}}}{M_{\eta_h}}\right)^i \left(\frac{am_h}{\pi}\right)^{2j} \left(\frac{am_c}{\pi}\right)^{2k} \\ & + \left(c_{0l} + c_{1l}\frac{\Lambda_{\mathrm{QCD}}}{M_{\eta_h}} + c_{2l}\left(\left(\frac{am_c}{\pi}\right)^2 + \left(\frac{am_h}{\pi}\right)^2\right)\right)(M_{\eta_c} - M_{\eta_c}^{\mathrm{phys}}) \\ & + \left(s_{0l} + s_{1l}\frac{\Lambda_{\mathrm{QCD}}}{M_{\eta_h}} + s_{2l}\left(\left(\frac{am_c}{\pi}\right)^2 + \left(\frac{am_h}{\pi}\right)^2\right)\right)(M_{\eta_s}^2 - M_{\eta_s}^{\mathrm{phys2}}) \end{aligned}$$

Results

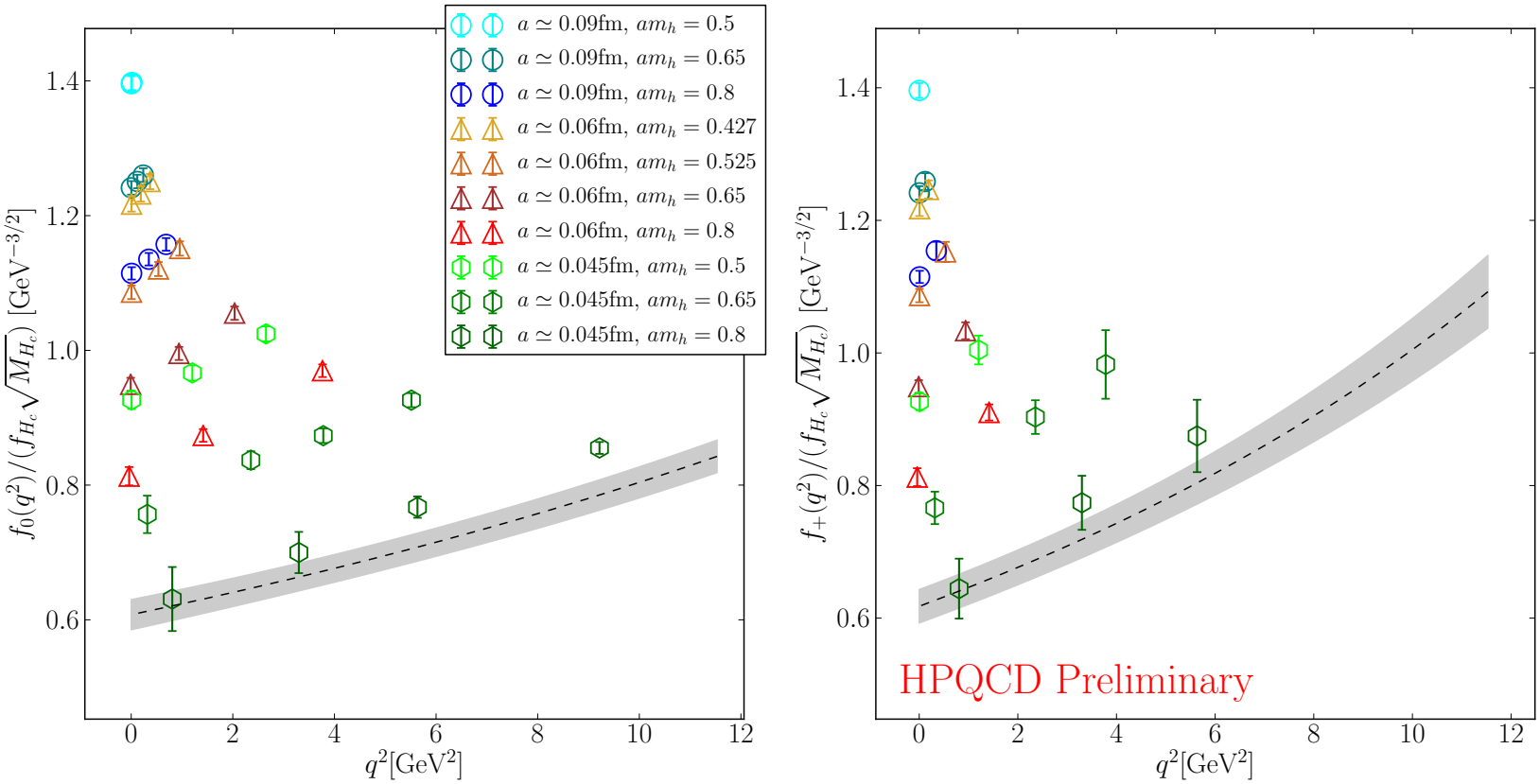
$B_s \to D_s^*$ @ zero recoil



$B_s \to D_s$ @ zero recoil

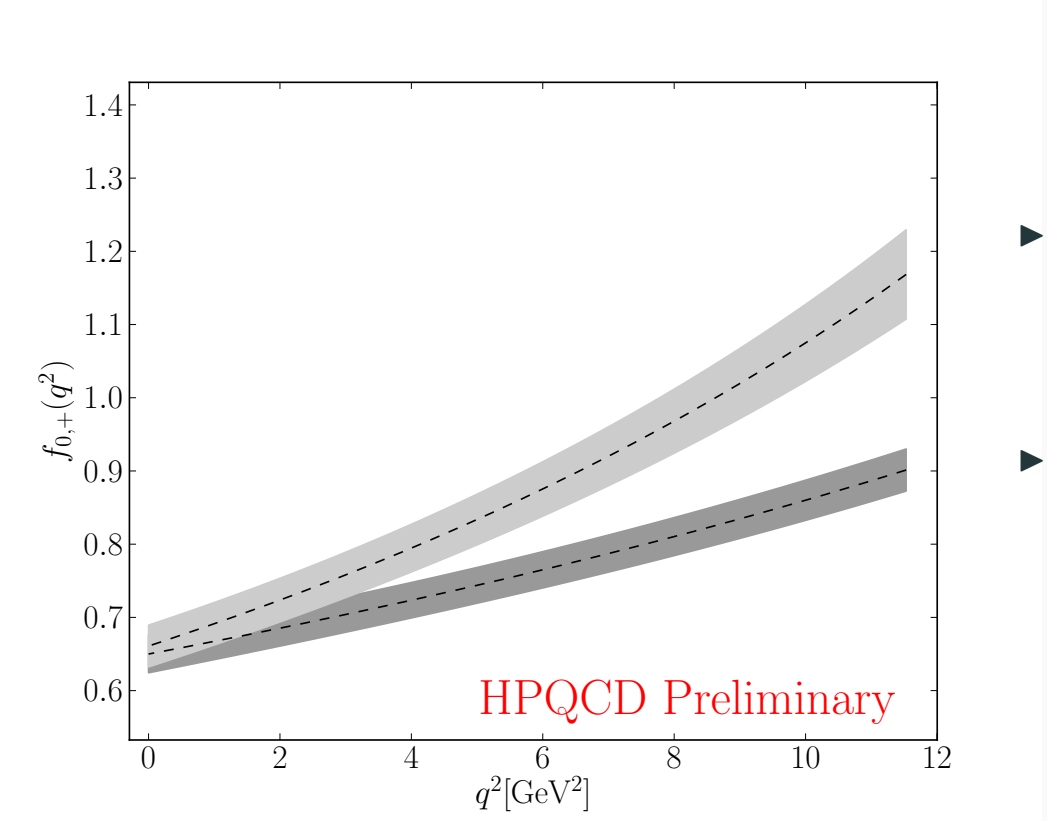


$B_s \to D_s \quad \forall q^2$



$B_s \to D_s \quad \forall q^2$

$f_{B_c}\sqrt{M_{B_c}}$ multiplied out by $f_{B_c} = 0.427(6)$GeV [McNeile et al. 1207.0994] and $M_{B_c} = 5.3667(1)$GeV from PDG.



- This work:
  $f_0(0) = f_+(0) = 0.650(26)$
  (missing chiral & finite vol. errors)
- Monahan et al. (NRQCD) 1703.09728 :
  $f_0(0) = f_+(0) = 0.656(31)$

Conclusions

- Have obtained $h_{A_1}(q^2_{\max})$ at zero recoil and $f_{0,+}$ at all q^2.
- Heavy-HISQ method in good agreement with other approaches e.g. NRQCD.
- In future:
  - $R(D_s^{(*)})$
  - Find HQET insights, e.g. low energy constants l_V,l_A,l_P
  - extend $B_s \to D_s^*$ to non-zero recoil?
  - $B \to D^{(*)}$?

thanks for listening!

Backup Slides

$z(q^2)$ definition

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

$$t_+ = (M_{H_s} + M_{D_s^{(*)}})^2 \quad , \quad t_0 = t_- = (M_{H_s} - M_{D_s^{(*)}})^2$$

- t_0 chosen so that $z(q^2_{max}) = 0$.
- $0 < z < 0.06$ for $B_s \to D_s$,
  $0 < z < 0.05$ for $B_s \to D_s^*$.

$M_{H_c}^{*,0}$ poles

Have M_{H_c} from simulation for each heavy mass, but how to get $M_{H_c^{*,0}}$ for z-expansion?

- $M_{H_c}^0$: $\Delta_0 = M_{H_c^0} - M_{H_c} = 0.429\text{GeV}$ [1207.5149], independant of heavy quark mass. So just use

$$M_{H_c^0} = M_{H_c} + \Delta_0$$

- $M_{H_c^*}$: $\Delta_* = M_{H_c^*} - M_{H_c} \to 0$ in infinite mass limit by heavy quark symmetry, so $\Delta_* \simeq x/M_{\eta_h}$. Fix x at physical point, $x = 0.508\text{GeV}^2$. Then

$$M_{H_c^*} = M_{H_c} + \frac{0.508\text{GeV}^2}{M_{\eta_h}}$$

Priors for continuum, heavy mass and kinematic extrapolation

- $f_{0,+}(q^2)/f_{H_c}\sqrt{M_{H_c}}$:

p	b_{ijkl}	c_i	s_i
0(1)	0(2)	0(0.1)	0(0.1)

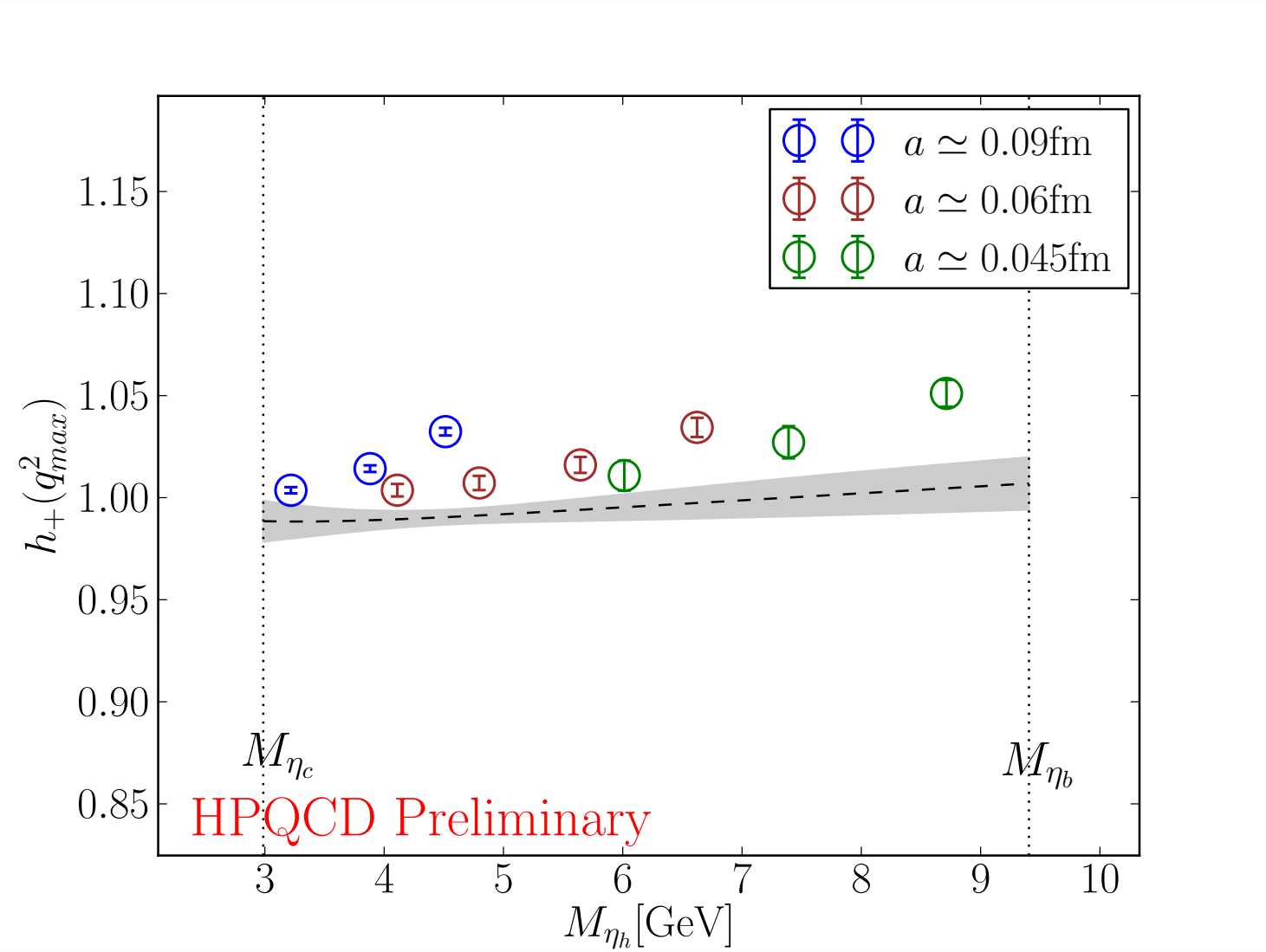
- $h_{A_1}(q^2_{\max})$:

p	b_{0jk}	b_{1jk}	b_{2jk}	c_i	s_i
0(1)	0.93(1.40)	0.19(29)	-0.25(50)	0(0.1)	0(0.1)

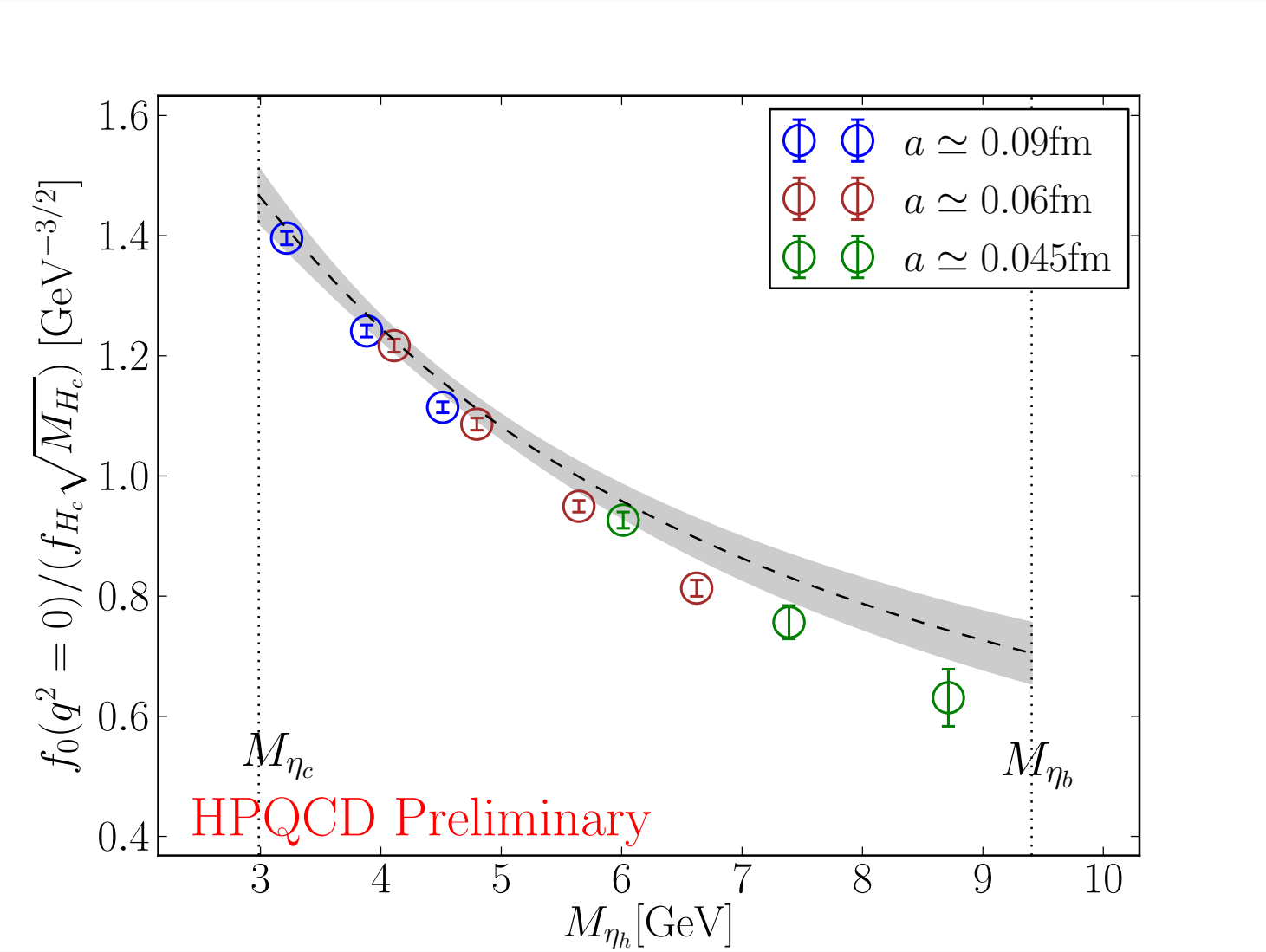
b_{ijk} priors inspired by HQET expression:

$$h_{A_1} = \eta_A \left[1 - \frac{l_V}{(2m_c)^2} + \frac{2l_A}{2m_c 2m_b} - \frac{l_P}{(2m_b)^2}\right]$$

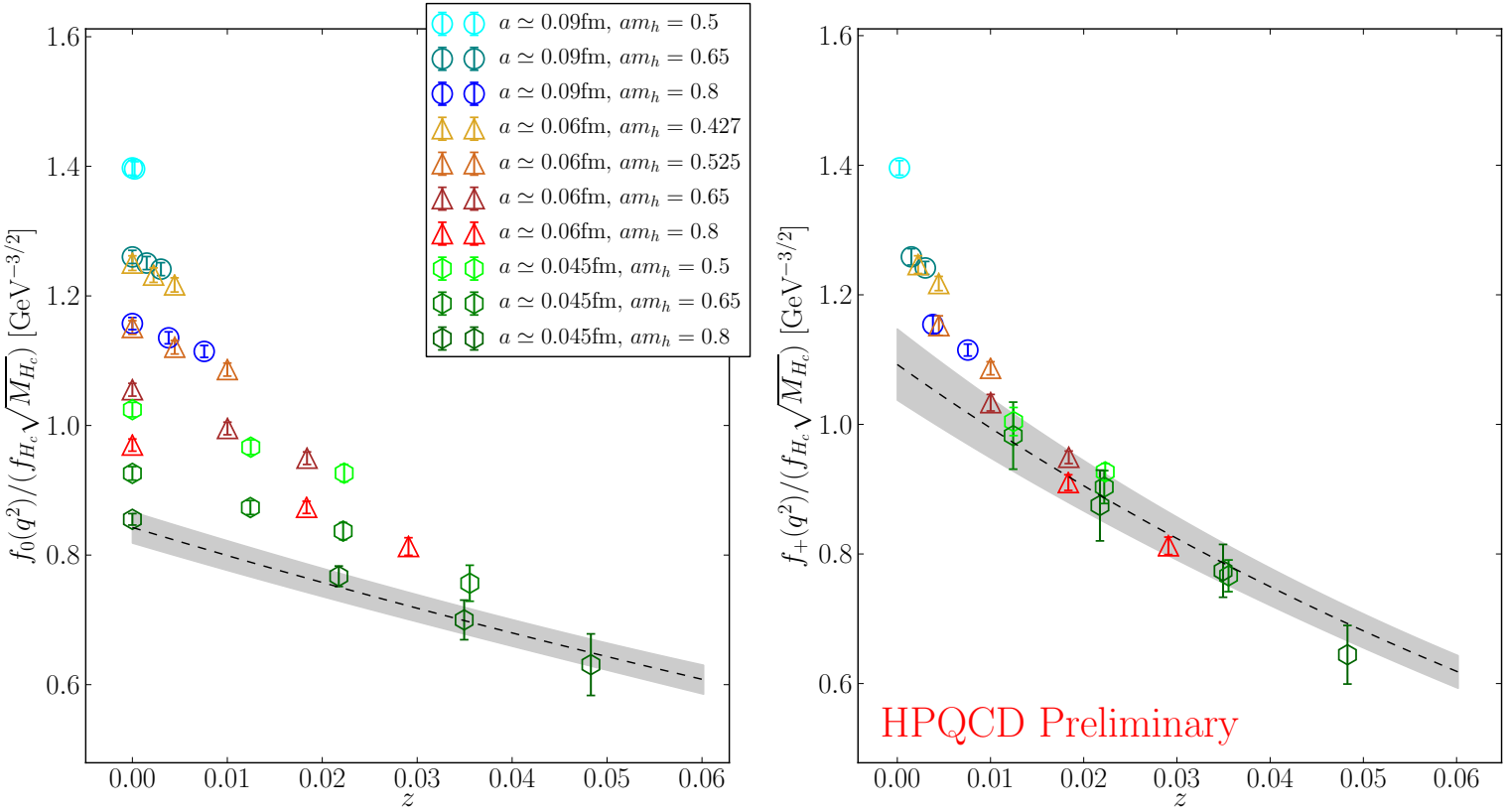
h_+ for $B_s \to D_s$


$a \simeq 0.09$fm
$a \simeq 0.06$fm
$a \simeq 0.045$fm
1.15
1.10
1.05
1.00
0.95
0.90
0.85
$h_+(q^2_{max})$
M_{η_c}
M_{η_b}
HPQCD Preliminary
3
4
5
6
7
8
9
10
M_{η_h}[GeV]

$f_0(q^2 = 0)$ for $B_s \to D_s$



$f_{0,+}(q^2)$ vs. z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$a \simeq 0.09$fm, $am_h = 0.8$
$a \simeq 0.06$fm, $am_h = 0.427$
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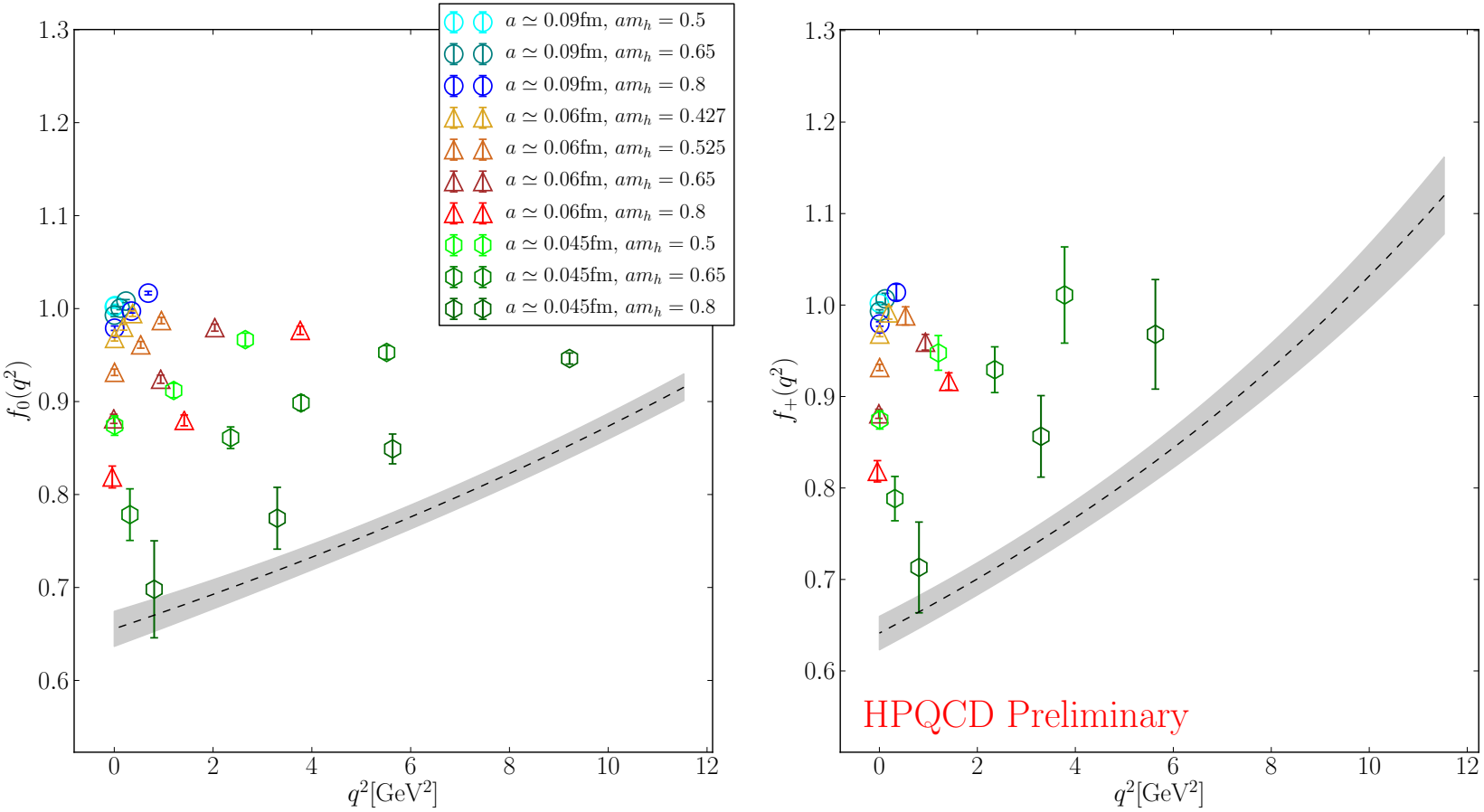
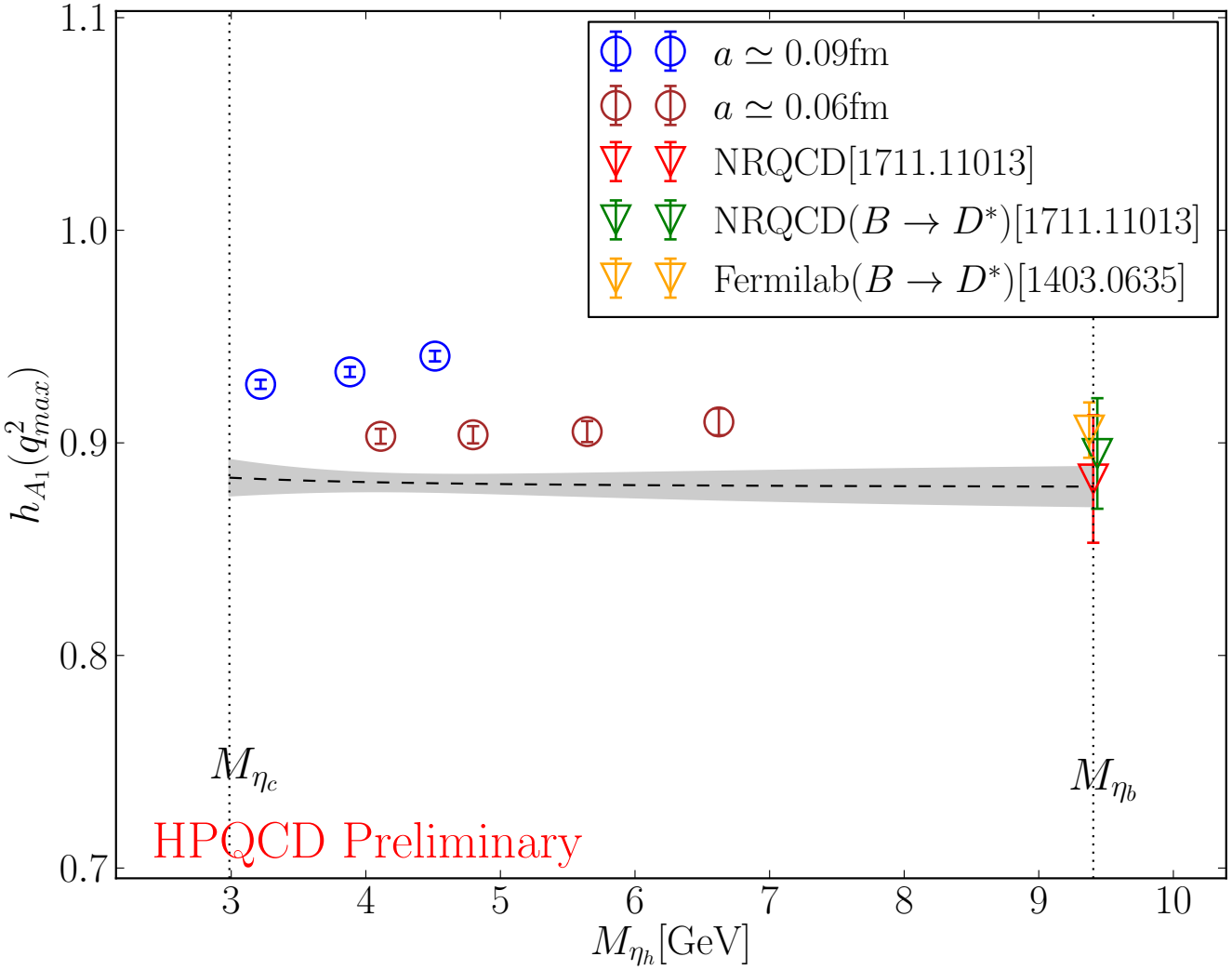
$a \simeq 0.06$fm, $am_h = 0.65$
$a \simeq 0.06$fm, $am_h = 0.8$
$a \simeq 0.045$fm, $am_h = 0.5$
$a \simeq 0.045$fm, $am_h = 0.65$
$a \simeq 0.045$fm, $am_h = 0.8$
$f_0(q^2)/(f_{H_c}\sqrt{M_{H_c}})$ [GeV$^{-3/2}$]
$f_+(q^2)/(f_{H_c}\sqrt{M_{H_c}})$ [GeV$^{-3/2}$]
z
HPQCD Preliminary

$B_s \to D_s \quad \forall q^2$, no ratio with f_{H_c}



$B_s \to D_s^*$ @ zero recoil



HISQ Details

$$\mathcal{F}_\mu = \prod_{\rho\neq\mu}\left(1+\frac{a^2\delta_\rho^{(2)}}{4}\right)$$

$$X_\mu(x) \equiv \mathcal{U}\mathcal{F}_\mu U_\mu(x)$$

$$W_\mu(x) \equiv \left(\mathcal{F}_\mu - \sum_{\rho\neq\mu}\frac{a^2(\delta_\rho)^2}{2}\right)\mathcal{U}\mathcal{F}_\mu U_\mu(x)$$

$$S_{\text{HISQ}} = \sum_x \bar{\psi}(x)\left(\gamma\cdot\left(\nabla_\mu(W) - \frac{a^2}{6}(1+\epsilon)\nabla_\mu^3(X)\right)+m\right)\psi(x)$$

ϵ tuned to satisfy

$$\lim_{\underline{p}\to 0}\frac{E^2(\underline{p})-m^2}{\underline{p}^2} = 1$$