QED corrections to pion and kaon leptonic decay rates

James Richings

RBC/UKQCD

The 36th Annual International Symposium on Lattice Field Theory
Michigan State University
July 22nd - 28th 2018

25/07/2018
**RBC/UKQCD Collaboration**

<table>
<thead>
<tr>
<th>BNL and BNL/RBRC</th>
<th>University of Liverpool</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yasumichi Aoki (KEK)</td>
<td>Nicolas Garron</td>
</tr>
<tr>
<td>Mattia Bruno</td>
<td>MIT</td>
</tr>
<tr>
<td>Taku Izubuchi</td>
<td>David Murphy</td>
</tr>
<tr>
<td>Yong-Chull Jang</td>
<td>Peking University</td>
</tr>
<tr>
<td>Chulwoo Jung</td>
<td>Xu Feng</td>
</tr>
<tr>
<td>Christoph Lehner</td>
<td>University of Southampton</td>
</tr>
<tr>
<td>Meifeng Lin</td>
<td>Jonathan Flynn</td>
</tr>
<tr>
<td>Aaron Meyer</td>
<td>Vera Gülpers</td>
</tr>
<tr>
<td>Hiroshi Ohki</td>
<td>James Harrison</td>
</tr>
<tr>
<td>Shigemi Ohta (KEK)</td>
<td>Andreas Jüttner</td>
</tr>
<tr>
<td>Amarjit Soni</td>
<td>James Richings</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>UC Boulder</th>
<th>Stony Brook University</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oliver Witzel</td>
<td>Jun-Sik Yoo</td>
</tr>
<tr>
<td>Columbia University</td>
<td>Sergey Syritsyn (RBRC)</td>
</tr>
<tr>
<td>Ziyuan Bai</td>
<td>York University (Toronto)</td>
</tr>
<tr>
<td>Norman Christ</td>
<td>Renwick Hudspith</td>
</tr>
<tr>
<td>Duo Guo</td>
<td></td>
</tr>
<tr>
<td>Christopher Kelly</td>
<td></td>
</tr>
<tr>
<td>Bob Mawhinney</td>
<td></td>
</tr>
<tr>
<td>Masaaki Tomii</td>
<td></td>
</tr>
<tr>
<td>Jiqun Tu</td>
<td></td>
</tr>
<tr>
<td>Bigeng Wang</td>
<td></td>
</tr>
</tbody>
</table>

| Edinburgh University | |}
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Peter Boyle</td>
<td>Fionn Ó hÓgáin</td>
</tr>
<tr>
<td>Guido Cossu</td>
<td>Brian Pendleton</td>
</tr>
<tr>
<td>Luigi Del Debbio</td>
<td>Antonin Portelli</td>
</tr>
<tr>
<td>Tadeusz Janowski</td>
<td>Tobias Tsang</td>
</tr>
<tr>
<td>Richard Kenway</td>
<td>Azusa Yamaguchi</td>
</tr>
<tr>
<td>Julia Kettle</td>
<td></td>
</tr>
</tbody>
</table>

| KEK | |}
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Julien Frison</td>
<td></td>
</tr>
</tbody>
</table>
Introduction

- Work in progress: QED corrections to leptonic decay processes.
- Progress towards an All-to-All set up and creation of meson fields.
- How we plan to use domain wall fermion meson fields to calculate the leptonic decay correlators at the physical point.

Outlook
Dominant contributions to error budgets are from isospin breaking effects.
The leading order ($\alpha^0$) decay rate is:

$$\Gamma(\pi^+ \to l^+ \nu) = \frac{m_{\pi}}{8\pi} G_F^2 |f_\pi|^2 |V_{ud}|^2 m_l^2 \left(1 - \frac{m_l^2}{m_{\pi}^2}\right)^2$$

$$\langle 0| \bar{d} \gamma_\mu \gamma_5 u |\pi^+(p)\rangle = ip_\mu f_{\pi^+}$$

IR finite order $\alpha$ decay rate:

$$\Gamma_\alpha = \Gamma_0 + \Gamma_1$$


- $\Gamma_0$: Order alpha corrections without a final state photon.
- $\Gamma_1$: Order alpha corrections with a final state photon.
The order alpha correction can be calculated by using a perturbative approach:

\[
\langle O \rangle = \langle O \rangle_0 + \frac{e^2}{2} \frac{\partial^2}{\partial e^2} \langle O \rangle \bigg|_{e=0} + O(\alpha^2)
\]

If the operator is \( \alpha \) independent then the correction has the form:

\[
\langle O \rangle = \langle O \rangle_0 - \frac{e^2 q_f q_{f'}}{2} \langle OV_\mu^c(x)V_\nu^c(y) \rangle_0 \Delta_{\mu\nu}(x - y) - \frac{(eq_f)^2}{2} \langle OT_\mu(x) \rangle_0 \Delta_{\mu\mu}
\]

We use QEDL with photons in the Feynman gauge:

\[
\Delta_{\mu\nu}(x - y) = \delta_{\mu\nu} \frac{1}{N} \sum_{k,k\neq0} \frac{e^{ik.(x-y)}}{\hat{k}^2}
\]

Photon propagator generated by inserting stochastic photons:

\[
\Delta_{\mu\nu}(x - y) = \langle A_\mu(x)A_\nu(y) \rangle
\]
$\Gamma_0$ Connected Diagrams

The connected diagrams that contribute to the order $\alpha$ QED correction to leptonic decays:

- Lepton coupling diagram
- Exchange diagram
- Self-energy diagram
- Tadpole diagram
Implementation and Production Plan

Implementation:
- **Hadrons:**
  - Grid-powered Workflow Management System for lattice calculations
  - High modularity
  - Automatic Scheduling
- **GRID:** [www.github.com/paboyle/Grid](http://www.github.com/paboyle/Grid)

Plan:
- Solve for the correlators at physical point (physical pion mass).
- $48^3 \times 96$ Lattice, $a^{-1} = 1.73$ GeV  
  
- Use 2000 eigenvectors we have generated to deflate the solves.
Deflation

- Using deflation we are seeing a factor 10 decrease CG iterations required per solve on the unphysical $24^3$ ensembles.

- We see a $\approx 20$ speedup at physical quark masses with 2000 eigenvectors on the $48 \times 96$ lattice using z-Möbius DWF.

[Fionn Ó hÚgáin]
Lepton coupling diagram test results

- Test run: $M_\pi = 340$ MeV; $a^{-1} = 1.78$ GeV; $2 + 1$ flavour; Electro-quenched.
- Self energy, exchange & tadpole diagram calculated in previous project.


- First look at lepton coupling decay correlator:
All to All Propagator and the meson field

All to all propagator:

\[ D_{A2A}^{-1}(x, y) = \sum_{i=0}^{N_l+N_h} v_i(x) w_i^\dagger(y) = \sum_{l=0}^{N_l} v_l(x) w_l^\dagger(y) + \sum_{h=N_l}^{N_l+N_h} v_h(x) w_h^\dagger(y) \]
All to All Propagator and the meson field

- All to all propagator:

\[ D_{A2A}^{-1}(x, y) = \sum_{i=0}^{N_l+N_h} v_i(x) w_i^\dagger(y) = \sum_{l=0}^{N_l} v_l(x) w_l^\dagger(y) + \sum_{h=N_l}^{N_l+N_h} v_h(x) w_h^\dagger(y) \]

- Low modes (from eigenvectors):

\[ v_l(x) = \phi_l(x) \]
\[ w_l(x) = \phi_l(x)/\lambda_l \]

- High modes (from stochastic solves):

\[ v_h(x) = D^{-1}\eta_h(x) \]
\[ w_h(x) = \eta_h(x) \]
All to All Propagator and the meson field

- All to all propagator:
  \[ D_{A2A}^{-1}(x, y) = \sum_{i=0}^{N_l+N_h} v_i(x) w_i^\dagger(y) = \sum_{l=0}^{N_l} v_l(x) w_l^\dagger(y) + \sum_{h=N_l}^{N_l+N_h} v_h(x) w_h^\dagger(y) \]

- Low modes (from eigenvectors):
  \[ v_l(x) = \phi_l(x) \]
  \[ w_l(x) = \phi_l(x)/\lambda_l \]

- High modes (from stochastic solves):
  \[ v_h(x) = D^{-1} \eta_h(x) \]
  \[ w_h(x) = \eta_h(x) \]

- Two point function:
  \[ \gamma_5 \rightarrow \gamma_5 \]

- Meson Field:
  \[ \Pi_{ji}(t_x; \gamma_5) = \sum_{\vec{x}} w_j^\dagger(\vec{x}) \gamma_5 v_i(\vec{x}) \]

[J. Foley et al, CPC 172 (2005)0010-4655]
All to All Propagator and the meson field

- All to all propagator:
  \[ D_{A2A}^{-1}(x, y) = \sum_{i=0}^{N_l+N_h} v_i(x)w_i^\dagger(y) = \sum_{l=0}^{N_l} v_l(x)w_l^\dagger(y) + \sum_{h=N_l}^{N_l+N_h} v_h(x)w_h^\dagger(y) \]

- Low modes (from eigenvectors):
  \[ v_l(x) = \phi_l(x) \]
  \[ w_l(x) = \phi_l(x)/\lambda_l \]

- High modes (from stochastic solves):
  \[ v_h(x) = D^{-1}\eta_h(x) \]
  \[ w_h(x) = \eta_h(x) \]

- Two point function:
  \[ v_i(x) \quad w_i^\dagger(y) \quad \gamma_5 \quad \gamma_5 \]

- Meson Field:
  \[ \Pi_{ji}(t_x; \gamma_5) = \sum_{\vec{x}} w_j^\dagger(x)\gamma_5 v_i(x) \]

- 3,4,... pt functions can be made contracting the relevant meson fields with the correct gamma structure.

[J. Foley et al, CPC 172 (2005)0010-4655]
All to All conserved current meson field

Take the Wilson Fermion vector current:

\[ V_\mu^c(x) = \frac{1}{2} \left[ \bar{\psi}(x + \hat{\mu})(1 + \gamma_\mu)U_\mu(x)\psi(x) - \bar{\psi}(x)(1 - \gamma_\mu)U_\mu(x)\psi(x + \hat{\mu}) \right] \]
Take the Wilson Fermion vector current:

\[ V^c_{\mu}(x) = \frac{1}{2} [ \bar{\psi}(x + \hat{\mu})(1 + \gamma_\mu)U_\mu^\dagger(x)\psi(x) - \bar{\psi}(x)(1 - \gamma_\mu)U_\mu(x)\psi(x + \hat{\mu}) ] \]
Take the Wilson Fermion vector current:

\[ V^c_\mu(x) = \frac{1}{2} \left[ \bar{\psi}(x + \hat{\mu})(1 + \gamma_\mu) U^{\dagger}_\mu(x) \psi(x) - \bar{\psi}(x)(1 - \gamma_\mu) U_\mu(x) \psi(x + \hat{\mu}) \right] \]
Take the Wilson Fermion vector current:

\[ V_{\mu}^c(x) = \frac{1}{2} \left[ \bar{\psi}(x + \hat{\mu})(1 + \gamma_{\mu})U_{\mu}^\dagger(x)\psi(x) - \bar{\psi}(x)(1 - \gamma_{\mu})U_{\mu}(x)\psi(x + \hat{\mu}) \right] \]

Wilson fermion vector current meson field:

\[ \Pi_{ij}(t_x) = \sum_{\vec{x}, \mu} \frac{1}{2} \left[ w_{i}^\dagger(x + \hat{\mu})(1 + \gamma_{\mu})U_{\mu}^\dagger(x)v_{j}(x) - w_{i}^\dagger(x)(1 - \gamma_{\mu})U_{\mu}(x)v_{j}(x + \hat{\mu}) \right] \]

Generalisation to 5D DWF current (work in progress)
Leptonic coupling correlator using meson fields:

- $\Pi_{ij}(t; \gamma_5)$
- $\Pi_{ik}(t; V_{\mu}^c)$
- $\Pi_{jk}(t; \gamma_0 \gamma_5)$

Similarly, the other diagrams that contribute to the decay rate can be split into meson fields:
We can also construct the disconnected diagrams from the same meson fields:

No extra inversions are required to calculate the disconnected diagrams!
Current status & Outlook

Status:
- Testing implementation and analysis on $24^3 \times 64$ lattice.
- All to All set up into Hadrons in the process of testing.
- 2000 eigenvectors for physical mass ensemble generated and on disc.
- Optimizing $48^3 \times 96$ set up with an aim to start generating meson fields and form correlators offline.

Outlook:
- Calculate the isospin breaking correction to leptonic kaon/pion decays at the physical point. Using meson fields.
- Isospin breaking corrections to Semi-leptonic decay rates.
- $\Gamma_1$ leptonic decays with final state photons.
- Use stored meson fields to form correlators for other processes.