Semi-leptonic form factors for $B_s \rightarrow K\ell\nu$ and $B_s \rightarrow D_s\ell\nu$

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introduction
Why $B_s$ meson decays?

- Alternative, tree-level determination of $|V_{cb}|$ and $|V_{ub}|$ from $B_s \to D_s \ell \nu$ and $B_s \to K \ell \nu$

  - Commonly used $B \to \pi \ell \nu$ and $B \to D^{(*)} \ell \nu$

  - Long standing $2-3\sigma$ discrepancy between exclusive ($B \to \pi \ell \nu$) and inclusive ($B \to X_u \ell \nu$)

  - $B \to \tau \nu$ has larger error

- Alternative, exclusive ($\Lambda_b \to p \ell \nu$) determination

  [Detmold, Lehner, Meinel, PRD92 (2015) 034503]

[http://ckmfitter.in2p3.fr]
Why $B_s$ meson decays?

- Alternative tests of lepton flavor violations
  - Determine e.g. $R_{D_s^(*)}$ from $B_s$ decays to compare with $R_{D^(*)}$ from $B$ decays

\[
R_{D^(*)}^{\tau/\mu} = \frac{BF(B \to D^{(*)}\tau\nu_\tau)}{BF(B \to D^{(*)}\mu\nu_\mu)}
\]

- Nonperturbative lattice calculation favor $B_s$ over $B$ decays (higher precision)
- Only the spectator quark differs: $R_{D_s^{(*)}}$ may be a good proxy for $R_{D^{(*)}}$
Why $B_s$ meson decays?

- Alternative tests of lepton flavor violations
  - Determine e.g. $R_{D(*)}$ from $B_s$ decays to compare with $R_{D(*)}$ from $B$ decays

$$R_{D(*)}^{\tau/\mu} = \frac{BF(B \to D^{(*)}\tau\nu_{\tau})}{BF(B \to D^{(*)}\mu\nu_{\mu})}$$

- HFLAV updated SM prediction, $R_{D(*)}$:
  - averaging [Bigi, Gambino PRD94(2016)094008]
  - [Bernlochner, Ligeti, Papucci, Robinson PRD95(2017)11500]

- Nonperturbative lattice calculation favor $B_s$ over $B$ decays (higher precision)
- Only the spectator quark differs: $R_{D_s(*)}$ may be a good proxy for $R_{D(*)}$
$|V_{ub}|$ from exclusive semileptonic $B_s \to K \ell \nu$ decay

\[
q^2 = M_{B_s}^2 + M_K^2 - 2M_{B_s}E_K
\]

Conventionally parametrized by (neglecting term $\propto m_\ell f_0^2$)

\[
\frac{d\Gamma(B_s \to K \ell \nu)}{dq^2} = \frac{G_F^2}{192\pi^3 M_{B_s}^3} \left[ (M_{B_s}^2 + M_K^2 - q^2)^2 - 4M_{B_s}^2 M_K^2 \right]^{3/2} \times |f_+(q^2)|^2 \times |V_{ub}|^2
\]
Nonperturbative input

- Parametrizes interactions due to the (nonperturbative) strong force
- Use operator product expansion (OPE) to identify short distance contributions
- Calculate the flavor changing currents as point-like operators using lattice QCD

⇒ Nonperturbative calculation: lattice QCD

→ Additional challenge $m_b = 4.18\text{GeV} \sim 1000 \times m_d$

$m_c = 1.28\text{GeV} \sim 270 \times m_d$
Set-up

- RBC-UKQCD’s 2+1 flavor domain-wall fermion and Iwasaki gauge action ensembles
  - Three lattice spacings $a \sim 0.11$ fm, 0.08 fm, 0.07 fm; one ensemble with physical pions
    

- Unitary and partially quenched domain-wall up/down quarks
  
  \[ \text{Kaplan PLB 288 (1992) 342], [Shamir NPB 406 (1993) 90] } \]

- Domain-wall strange quarks at/near the physical value

- Charm: Möbius domain-wall fermions optimized for heavy quarks
  - Simulate 3 or 2 charm-like masses then extrapolate/interpolate

- Effective relativistic heavy quark (RHQ) action for bottom quarks
  

  - Builds upon Fermilab approach
  
  \[ \text{El-Khadra et al. PRD 55 (1997) 3933] } \]

  - Allows to tune the three parameters ($m_0a$, $c_P$, $\zeta$) nonperturbatively
    
    \[ \text{PRD 86 (2012) 116003] } \]

  - Smooth continuum limit; heavy quark treated to all orders in $(m_ba)^n$
$B_s \rightarrow K\ell\nu$
**$B_s \rightarrow K \ell \nu$ form factors**

- Parametrize the hadronic matrix element for the flavor changing vector current $V^\mu$ in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$

\[
\langle K | V^\mu | B_s \rangle = f_+(q^2) \left( p_{B_s}^\mu + p_{K}^\mu - \frac{M_{B_s}^2 - M_K^2}{q^2} q^\mu \right) + f_0(q^2) \frac{M_{B_s}^2 - M_K^2}{q^2} q^\mu
\]

- Calculate 3-point function by
  - Inserting a quark source for a “light” propagator at $t_0$
  - Allow it to propagate to $t_{\text{sink}}$, turn it into a sequential source for a $b$ quark
  - Use another “light” quark propagating from $t_0$ and contract both at $t$
Determining $B_s \to K \ell \nu$ form factors $f_+$ and $f_0$ on the lattice

- Updating calculation [PRD 91 (2015) 074510] with new values for $a^{-1}$ and RHQ parameters

- New analysis directly fitting form factors and accounting for excited state contributions

- On the lattice we prefer using the $B_s$-meson rest frame and compute

\[
f_{\parallel}(E_K) = \frac{\langle K|V^0|B_s \rangle}{\sqrt{2M_{B_s}}} \quad \text{and} \quad f_{\perp}(E_K)p_K^i = \frac{\langle K|V^i|B_s \rangle}{\sqrt{2M_{B_s}}} \]

- Both are related by

\[
f_0(q^2) = \frac{\sqrt{2M_{B_s}}}{M_{B_s}^2 - M_K^2} \left[ (M_{B_s} - E_K)f_{\parallel}(E_K) + (E_K^2 - M_K^2)f_{\perp}(E_K) \right]
\]

\[
f_+(q^2) = \frac{1}{\sqrt{2M_{B_s}}} \left[ f_{\parallel}(E_K) + (M_{B_s} - E_K)f_{\perp}(E_K) \right]
\]
Chiral-continuum extrapolation using SU(2) hard-kaon $\chi$PT

$$f_\perp(M_K, E_K, a^2) = \frac{1}{E_K + \Delta} c_\perp^{(1)}$$
$$\times \left[ 1 + \frac{\delta f_\perp}{(4\pi f)^2} + c_\perp^{(2)} \frac{M_K^2}{\Lambda^2} + c_\perp^{(3)} \frac{E_K}{\Lambda} + c_\perp^{(4)} \frac{E_K^2}{\Lambda^2} + c_\perp^{(5)} \frac{a^2}{\Lambda^2 a_{32}^4} \right]$$

$$f_\parallel(M_K, E_K, a^2) = \frac{1}{E_K + \Delta} c_\parallel^{(1)}$$
$$\times \left[ 1 + \frac{\delta f_\parallel}{(4\pi f)^2} + c_\parallel^{(2)} \frac{M_K^2}{\Lambda^2} + c_\parallel^{(3)} \frac{E_K}{\Lambda} + c_\parallel^{(4)} \frac{E_K^2}{\Lambda^2} + c_\parallel^{(5)} \frac{a^2}{\Lambda^2 a_{32}^4} \right]$$

with $\delta f$ non-analytic logs of the kaon mass and hard-kaon limit is taken by $M_K/E_K \rightarrow 0$

$\bm{\text{Error budget not yet released for presentation}}$
Kinematical extrapolation (z-expansion)

- Map $q^2$ to $z$ with minimized magnitude in the semileptonic region: $|z| \leq 0.146$

$$z(q^2, t_0) = \frac{\sqrt{1-q^2/t_+} - \sqrt{1-t_0/t_+}}{\sqrt{1-q^2/t_+} + \sqrt{1-t_0/t_+}}$$

with

$$t_\pm = (M_B \pm M_\pi)^2$$

$$t_0 \equiv t_{\text{opt}} = (M_B + M_\pi)(\sqrt{M_B} - \sqrt{M_\pi})^2$$


[Bourrely, Caprini, Lellouch, PRD 79 (2009) 013008]

- Express $f_+$ as convergent power series
- $f_0$ is analytic, except for $B^*$ pole
- BCL with poles $M_+ = B^* = 5.33$ GeV and $M_0 = 5.63$ GeV
- Exploit kinematic constraint $f_+ = f_0$ at $q^2 = 0$

$\rightarrow$ Include HQ power counting to constrain size of $f_+$ coefficients
Kinematical extrapolation (z-expansion)

- Map $q^2$ to $z$ with minimized magnitude in the semileptonic region: $|z| \leq 0.146$

$$z(q^2, t_0) = \frac{\sqrt{1 - q^2/t_+} - \sqrt{1 - t_0/t_+}}{\sqrt{1 - q^2/t_+} + \sqrt{1 - t_0/t_+}} \quad \text{with} \quad t_+ = (M_B \pm M_\pi)^2$$

$$t_0 \equiv t_{opt} = (M_B + M_\pi)(\sqrt{M_B} - \sqrt{M_\pi})^2$$

[Bourrely, Caprini, Lellouch, PRD 79 (2009) 013008]

- Allows to compare shape of form factors
  - Obtained by other lattice calculations
  - Predicted by QCD sum rules and alike

- Combination with experiment leads to the overall normalization: $|V_{ub}|$
$B_s \rightarrow D_s \ell \nu$
$|V_{cb}|$ from exclusive semileptonic $B_s \rightarrow D_s \ell \nu$ decay

Conventionally parametrized by (neglecting term $\propto m_{\ell}^2 f_0^2$)

$$
\frac{d\Gamma(B_s \rightarrow D_s \ell \nu)}{dq^2} = \frac{G_F^2}{192\pi^3 M_{B_s}^3} \left[ \left( M_{B_s}^2 + M_{D_s}^2 - q^2 \right)^2 - 4 M_{B_s}^2 M_{D_s}^2 \right]^{3/2} \times |f_+(q^2)|^2 \times |V_{cb}|^2
$$

experiment known nonperturbative input CKM
Charm extra-/interpolation for $B_s \rightarrow D_s \ell \nu$

- Simulate charm quarks using MDWF
  - Similar action as for $u, d, s$ quarks
  - “Fully” relativistic setup simplifies renormalization
  - Established by calculating $f_{D(s)}$
    [Boyle et al. JHEP 1712 (2017) 008]

- Coarse ensembles
  - Extrapolate three charm-like masses

- Medium and fine ensembles
  - Interpolate between two charm-like masses

- Analysis of data at third, finer lattice spacing will help to better estimate uncertainty
Chiral-continuum extrapolation

- No light valence quarks, no need for $\chi$PT
- Account for dependence on
  - charm quark mass
  - lattice spacing
  - light sea-quark mass

$$f(q, a) = \frac{\alpha_0 + \alpha_1 M_{D_s} + \alpha_2 a^2 + \alpha_3 M_{\pi}^2}{1 + \alpha_4 q^2 / M_{B_s}^2}$$
z-expansion

- BCL with poles $M_+ = B_c^* = 6.33$ GeV and $M_0 = 6.42$ GeV
conclusion
Conclusion

- In the final stages to complete $B_s \rightarrow K \ell \nu$ and $B_s \rightarrow D_s \ell \nu$ form factor calculation
  - As usual, carefully estimating all systematic uncertainties is tedious
  - Can even require additional simulations

- Our lattice calculation also includes
  - $B \rightarrow \pi \ell \nu$, $B \rightarrow \pi \ell^+ \ell^-$
  - $B \rightarrow K^* \ell^+ \ell^-$
  - $B \rightarrow D(\ast) \ell \nu$
  - $B_s \rightarrow K^* \ell^+ \ell^-$
  - $B_s \rightarrow D_s^* \ell \nu$
  - $B_s \rightarrow \phi \ell^+ \ell^-$
  - ...
Resources and Acknowledgments

**USQCD**: Ds, Bc, and pi0 cluster (Fermilab), qcd12s cluster (Jlab)

**RBC** qcdcl (RIKEN) and cuth (Columbia U)

**UK**: ARCHER, Cirrus (EPCC) and DiRAC (UKQCD)
appendix
## 2+1 Flavor Domain-Wall Iwasaki ensembles

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