Short Range Operator Contributions to $0\nu\beta\beta$ decay from LQCD

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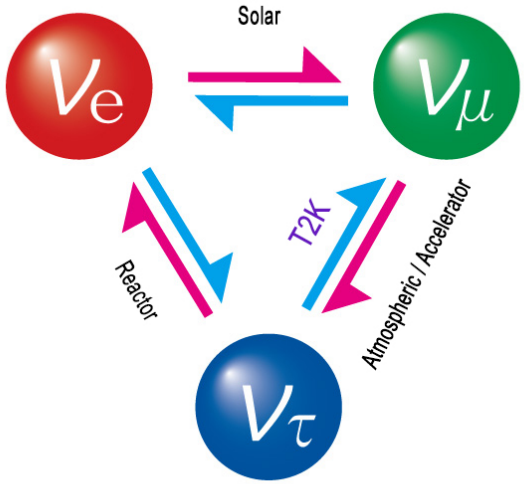
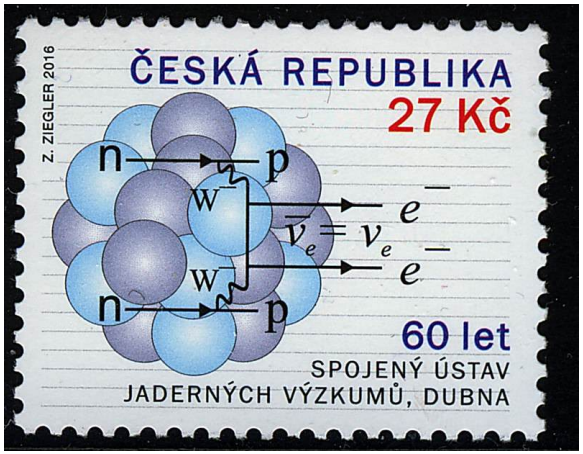
Motivation



Neutrino oscillation between three generations



Motivation

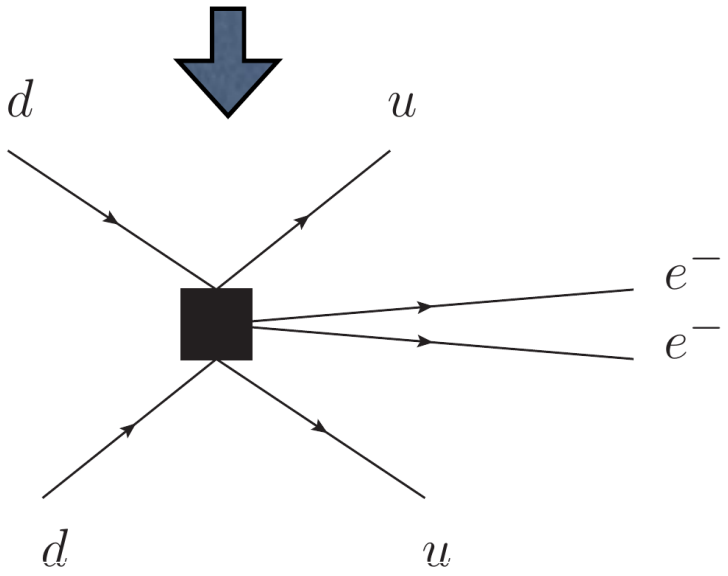
$0\nu\beta\beta$ Half life

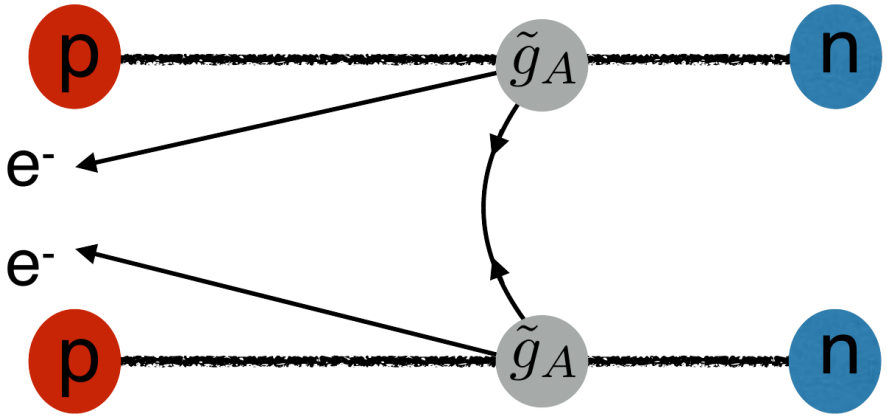
$$\frac{1}{T_{1/2}} = G(Q_{\beta\beta}, Z)|M|^2\eta_{\beta\beta}$$

$\eta_{\beta\beta}$

Light neutrino	Heavy neutrino
$\frac{1}{m_e}\sum U_{el}m_l$	$m_N \sum U_{eh}/m_h$

Experiments focused on $0^+ \to 0^+$

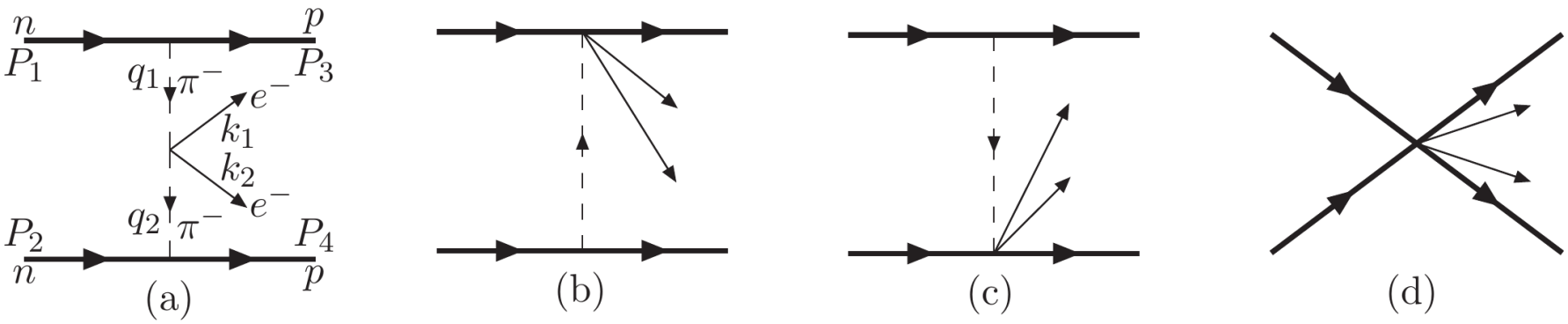




B. C. Tiburzi, M. L. Wagman, F. Winter, E. Chang, Z. Davoudi, W. Detmold, K. Orginos, M. J. Savage, and P. E. Shanahan (2017). In: *Phys. Rev.* D96.5, p. 054505. arXiv: 1702.02929 [hep-lat]

Contributing Diagrams

G. Prezeau, M. Ramsey-Musolf, and P. Vogel (2003). In: *Phys. Rev.* D68, p. 034016. arXiv: hep-ph/0303205 [hep-ph] used Effective Field Theory to Integrate out heavy modes and obtain the contributing operators are found to be:



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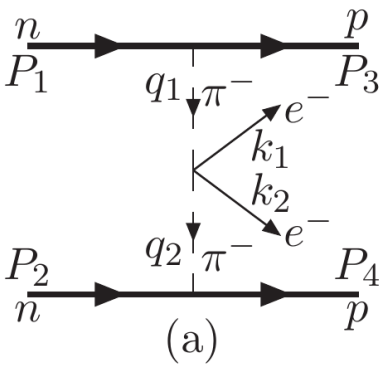
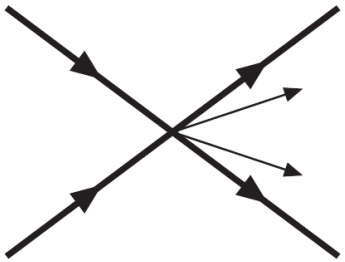


(a)


(d)

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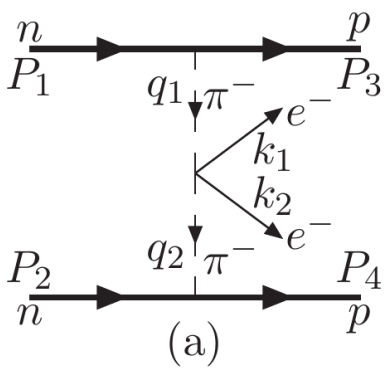

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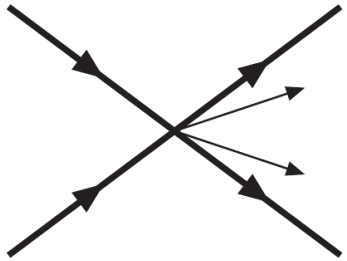
(d)

Decay Operators

$$\mathcal{O}_{1+}^{++} = (\bar{q}_L\tau^+\gamma^\mu q_L)(\bar{q}_R\tau^+\gamma^\mu q_R)$$

$$\mathcal{O}_{2\pm}^{++} = (\bar{q}_R\tau^+ q_L)\ (\bar{q}_R\tau^+ q_L) + (\bar{q}_L\tau^+ q_R)(\bar{q}_L\tau^+ q_R)$$

$$\mathcal{O}_{3\pm}^{++} = (\bar{q}_L\tau^+ q_L)(\bar{q}_L\tau^+ q_L) + (\bar{q}_R\tau^+ q_R)(\bar{q}_R\tau^+ q_R)$$

$$\mathcal{O}_{4\pm}^{++} = (\bar{q}_L\tau^+\gamma^\mu q_L \mp \bar{q}_R\tau^+\gamma^\mu q_R)(\bar{q}_L\tau^+ q_R - \bar{q}_R\tau^+ q_L)$$

$$\mathcal{O}_{5\pm}^{++} = (\bar{q}_L\tau^+\gamma^\mu q_L \pm \bar{q}_R\tau^+\gamma^\mu q_R)(\bar{q}_L\tau^+ q_R + \bar{q}_R\tau^+ q_L)$$

$\pi^- \to \pi^+$ Matrix Element

The operators contributing to $\pi^- \to \pi^+$ process are $\mathcal{O}^{++}_{1+}$, $\mathcal{O}^{++}_{2+}$ $\mathcal{O}^{++}_{3+}$ and $\mathcal{O}'^{++}_{1+}$, $\mathcal{O}'^{++}_{2+}$ (color mixed).

The corresponding 3-point correlation functions are computed as follows:

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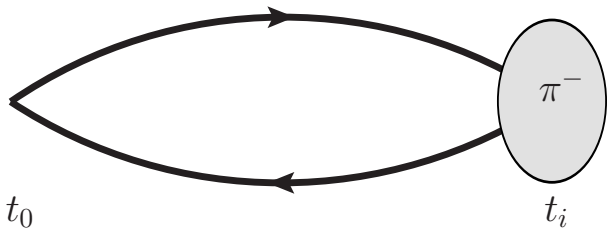

$$\bar{d}\gamma^5 u \rangle$$

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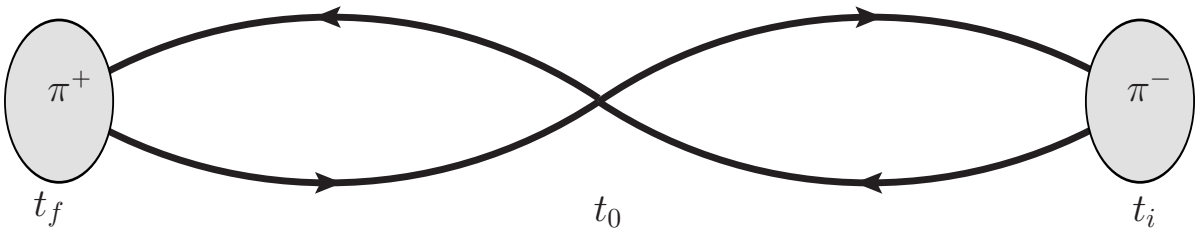

$$\langle\, \bar{d}\gamma^5 u \qquad \bar{d}\gamma^5 u \,\rangle$$

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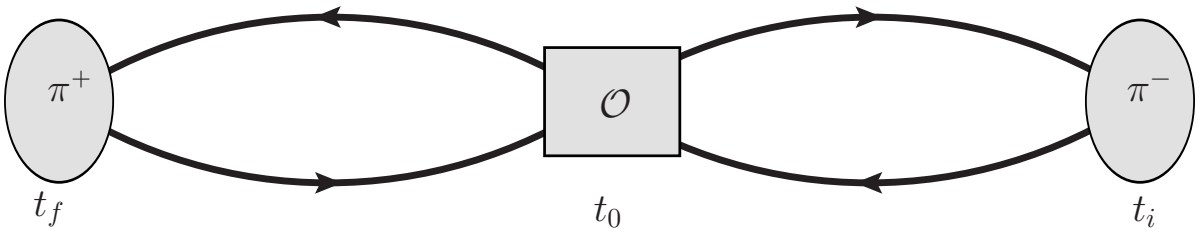
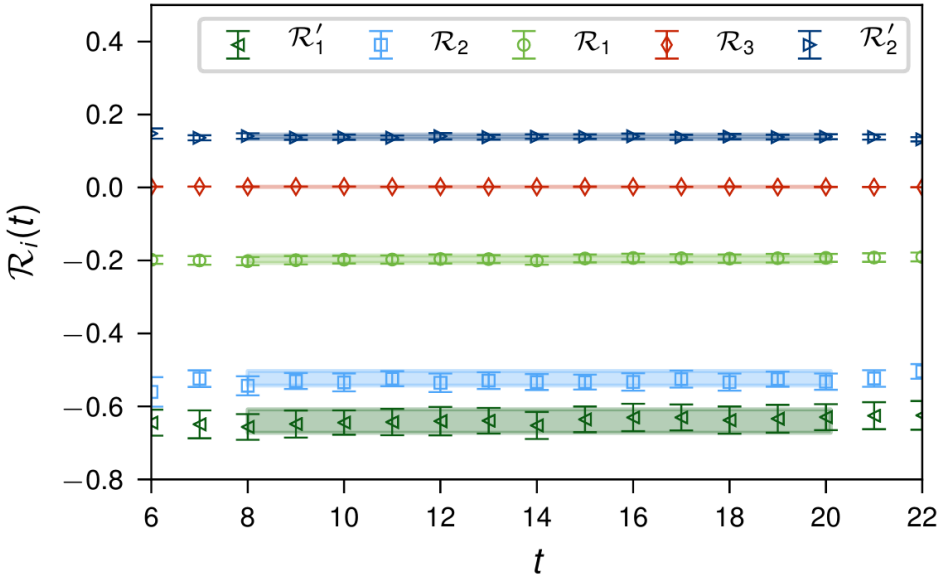
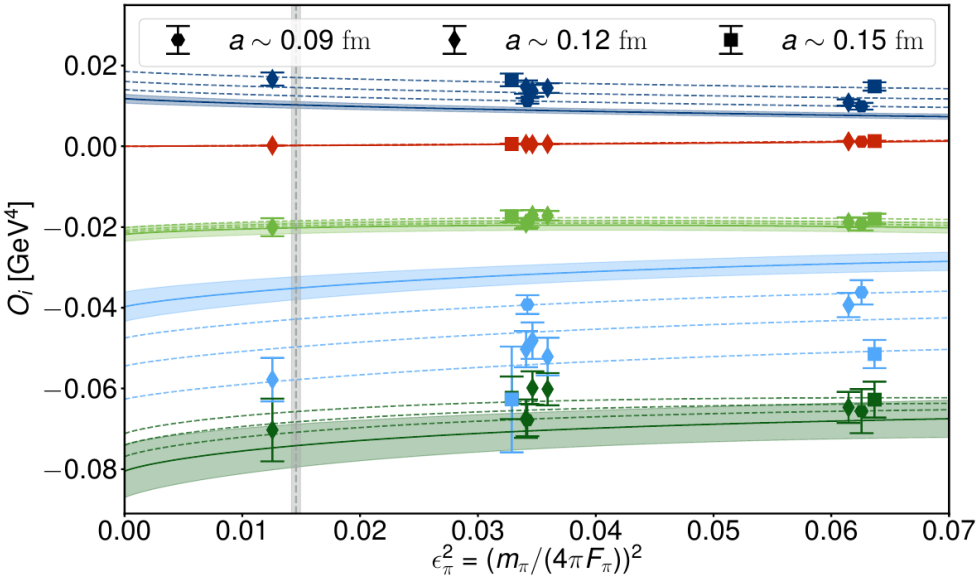


$$\langle\, \bar{d}\gamma^5 u \quad |\bar{u}\ \Gamma^1\ d\ \bar{u}\ \Gamma^2\ d| \quad \bar{d}\gamma^5 u\ \rangle$$

$\pi^- \to \pi+$

A. Nicholson et al. (2018). In: arXiv: 1805.02634 [nucl-th]





$$\mathcal{R}_i(t) = \frac{a^4 \langle \pi | \mathcal{O}_{i+}^{++} | \pi \rangle}{(a^2 Z_0^\pi)^3} + \mathcal{R}_{e.s.}(t)$$

$$O_1 = \frac{\beta_1 \Lambda_\chi^4}{(4\pi)^2}\left[1 + \frac{7}{3}\epsilon_\pi^2 \ln(\epsilon_\pi^2) + c_1 \epsilon_\pi^2\right]$$

$$O_2 = \frac{\beta_2 \Lambda_\chi^4}{(4\pi)^2}\left[1 + \frac{7}{3}\epsilon_\pi^2 \ln(\epsilon_\pi^2) + c_2 \epsilon_\pi^2\right]$$

$$\frac{O_3}{\epsilon_\pi^2} = \frac{\beta_3 \Lambda_\chi^4}{(4\pi)^2}\left[1 + \frac{4}{3}\epsilon_\pi^2 \ln(\epsilon_\pi^2) + c_3 \epsilon_\pi^2\right]$$

$\pi^- \to \pi^+$ Results

These matrix elements become inputs for nucleon potentials.
For example:

$$V_i^{nn\to pp}(|\mathbf{q}|) = -O_i \frac{g_A^2}{4F_\pi^2}\tau_1^+\tau_2^+ \frac{\sigma_1\cdot\mathbf{q}\sigma_2\cdot\mathbf{q}}{(|\mathbf{q}|^2+m_\pi^2)^2}$$

Non-perturbative Renormalization

A. Nicholson et al. (2018). In: arXiv: 1805.02634 [nucl-th]
C. C. Chang et al. (2018). In: *Nature* 558.7708, pp. 91–94. arXiv: 1805.12130 [hep-lat]

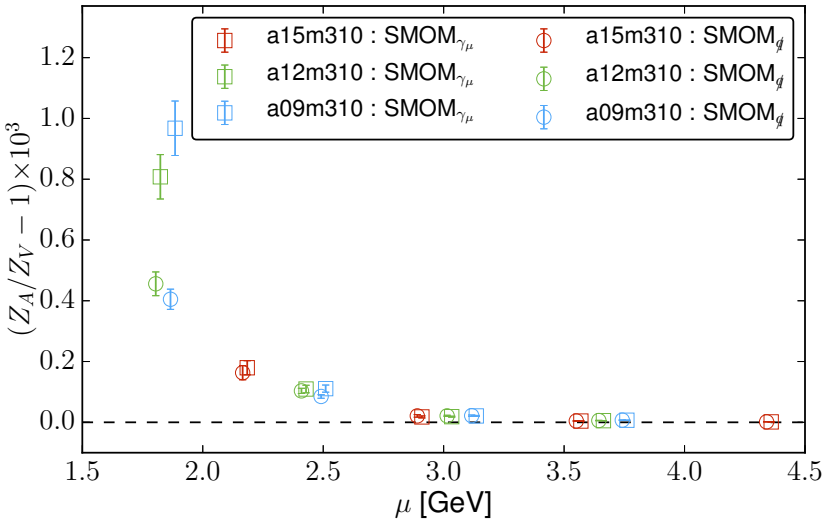


TABLE II. Resulting matrix elements extrapolated to the physical point, renormalized in RI/SMOM and $\overline{\text{MS}}$, both at $\mu = 3$ GeV.

O_i[GeV]^4	RI/SMOM $\mu = 3$ GeV	$\overline{\text{MS}}$ $\mu = 3$ GeV
O_1	$-1.96(14) \times 10^{-2}$	$-1.94(14) \times 10^{-2}$
O_1'	$-7.21(53) \times 10^{-2}$	$-7.81(57) \times 10^{-2}$
O_2	$-3.60(30) \times 10^{-2}$	$-3.69(31) \times 10^{-2}$
O_2'	$1.05(09) \times 10^{-2}$	$1.12(10) \times 10^{-2}$
O_3	$1.89(09) \times 10^{-4}$	$1.90(09) \times 10^{-4}$

Method RI-SMOM:[1]

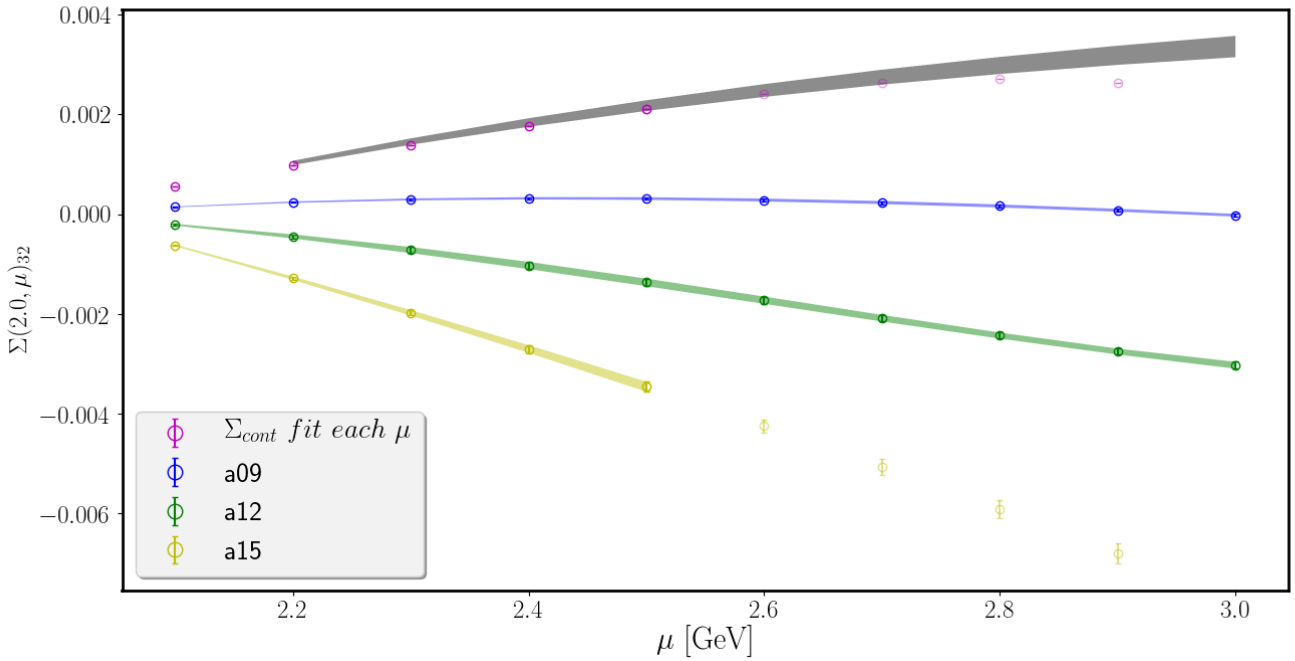
Three Lattice spacings:0.09,0.12,0.15fm

Projectors: γ and $\not q$ show agreement after $\overline{MS}$ conversion

Step scaling functions are used to handle reduced renormalization windows (0.15)

[1] C. Sturm, Y. Aoki, N. H. Christ, T. Izubuchi, C. T. C. Sachrajda, and A. Soni (2009). In: *Phys. Rev.* D80, p. 014501. arXiv: 0901.2599 [hep-ph]

Renormalization Constants Running



Renormalization Group $\Rightarrow$ cont. running $\Sigma(\mu_1, \mu_2) = Z(\mu_1)Z(\mu_2)^{-1}$

In the Lattice: $\Sigma(\mu_1, \mu_2, a) = \Sigma(\mu_1, \mu_2)_{cont} + \Delta a^2$

Fit assuming smooth μ dependence to obtain $\Sigma(\mu_1, \mu_2)_{cont}$

R. Arthur and P. A. Boyle (2011). In: *Phys. Rev.* D83, p. 114511. arXiv: 1006.0422 [hep-lat]

Four-quark Feynman-Hellman Method: $\pi^- \to \pi^+$

Analog of method implemented for baryons and bilinear currents [2]

$\partial_\lambda E_\lambda = \langle n| H_\lambda |n\rangle \qquad S_\lambda = \lambda \int d^4x \bar{\psi}\Gamma^1\psi\bar{\psi}\Gamma^2\psi$

$\partial_\lambda E_\lambda$

For a meson effective mass:

$$\left.\frac{\partial m_{eff}}{\partial_\lambda}\right|_{\lambda=0} = -\frac{\partial_\lambda C(t+\tau) + \partial_\lambda C(t-\tau) - 2cosh(m_{eff}\tau)\partial_\lambda C(t)}{2\tau C(t) sinh(m_{eff}\tau)}$$

For long enough t $\left.\frac{\partial m_{eff}}{\partial_\lambda}\right|_{\lambda=0} \approx \frac{\mathcal{J}_{00}}{2E_0^2}$

$\partial_\lambda C(t)$

Matrix element is pulled down with ∂_λ

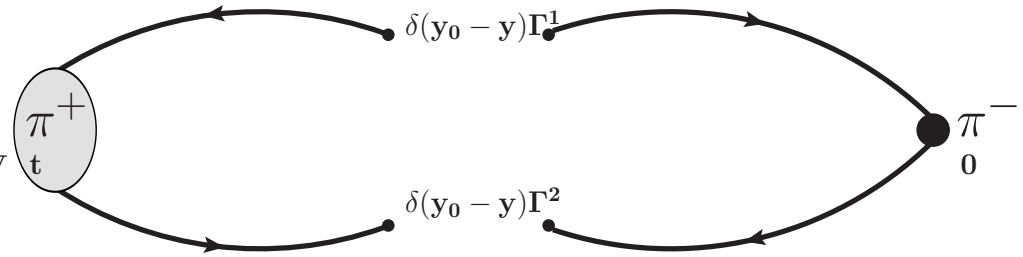
$$N(t) = \int d^4x \left\langle \Omega | T\mathcal{O}(t)\mathcal{J}(x)\mathcal{O}^\dagger(0)|\Omega \right\rangle$$

[2] C. Bouchard, C. C. Chang, T. Kurth, K. Orginos, and A. Walker-Loud (2017). In: *Phys. Rev.* D96.1, p. 014504. arXiv: 1612.06963 [hep-lat]

Lattice Implementation:

Brute force calculation on small Lattice:

$$\int \mathbf{d^4x} \left\langle \mathbf{\Omega}|\mathbf{T}\mathcal{O}(\mathbf{t})\mathcal{J}(\mathbf{x})\mathcal{O}^{\dagger}(\mathbf{0})|\mathbf{\Omega}\right\rangle = \sum_{\mathbf{y_0}\in\mathbf{V}}$$



Hubbard-Stratanovich Transformation:

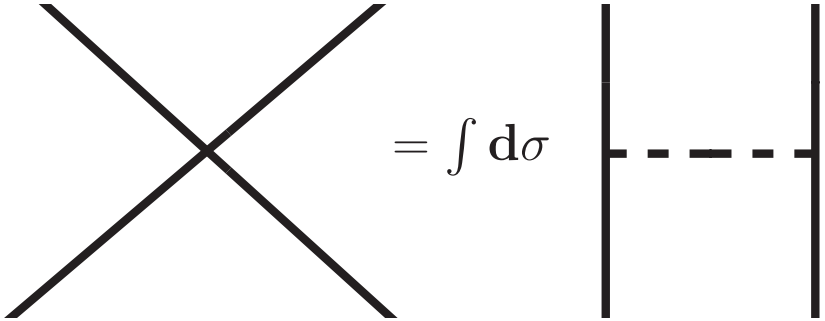
$$e^{-\lambda^2 \int d^4x(\overline{\psi}\Gamma\psi)^2} = \alpha \int_{-\infty}^{\infty} d\sigma e^{-\int d^4x\{\frac{\sigma^2}{4}+\lambda i\sigma(\overline{\psi}\Gamma\psi)\}}$$

D. J. Gross and A. Neveu (1974). In: *Phys. Rev.* D10, p. 3235

R. L. Stratonovich (1957). In: *Doklady Akad. Nauk S.S.S.R.* 115, p. 1097,J. Hubbard (1959). In: *Phys. Rev. Lett.* 3, pp. 77–80

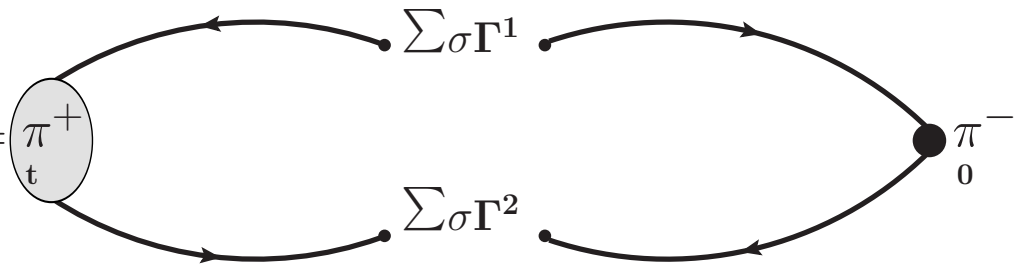
Lattice Implementation:

Four-quark is recovered after σ integration:



Numerical implementation:

$$\int \mathbf{d^4x} \left\langle \mathbf{\Omega} | \mathbf{T}\mathcal{O}(\mathbf{t})\mathcal{J}(\mathbf{x})\mathcal{O}^\dagger(\mathbf{0}) | \mathbf{\Omega} \right\rangle =$$



Conclusions and Future Work:

Reproduce $\pi^- \to \pi^+$ calculation with the new method

Implement calculation using the Hubbard-Stratanovich transformation

Apply method to $nn \to pp$ calculation

LBNL: Chia Cheng Chang, André Walker-Loud,

W&M and LLNL: David Brantley

BNL: Enrico Rinaldi

FZJ: Evan Berkowitz

JLab: Bálint Jóo

Liverpool Univ.: Nicolas Garron

LLNL: Pavlos Vranas,Arjun Gambhir

NERSC: Thorsten Kurth

UNC: Amy Nicholson

nVidia: Kate Clark

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